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Robust control of delayed Cohen–Grossberg neural networks

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SUMMARY

In this paper, robust control of Cohen–Grossberg neural networks with time delays is considered based on Lyapunov functional method and matrix inequality technique. Several new controllers with time delays and without time delays are designed to ensure the global asymptotic stability of equilibrium point, respectively. Finally, simulation examples are constructed to justify the proposed theoretical analysis. Copyright © 2007 John Wiley & Sons, Ltd.

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KEY WORDS: global asymptotic stability; time delays; matrix inequality technique; Lyapunov functional method; Cohen–Grossberg neural network

1. INTRODUCTION

During recent decades, a lot of attention [1–27] has been devoted to the study of artificial neural networks due to the fact that neural networks can be applied in signal processing, image processing, pattern recognition and optimization problems. Some of these applications require the knowledge of dynamical behaviours of neural networks, such as the uniqueness and asymptotic stability of equilibrium point of a designed neural work. Therefore, the problem of stability analysis of neural networks has been an important topic for researchers. However many real neural networks are not stable, so some effective controllers should be designed to ensure the stability of neural networks.

Practically, systems with time delays are frequently encountered [1–36]. A time delay will occur in the activation between neurons in electronic implementation of neural networks, which will affect the dynamical behaviours of the neuron system. On the other hand, besides time-delayed features of such neural networks, there might also be some uncertainties such as perturbations and component variations, which might lead to very complex dynamical behaviours.

In deterministic neural networks, the neurons fire rates, the synaptic interconnection weights and the signal transmission delays, etc. are usually acquired and processed by means of statistical

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estimation. In addition, parameter fluctuation in neural network implementation on very large-scale integration chips is also unavoidable. Nevertheless, in practice, it is possible to explore the range of the above-mentioned vital data as well as the bounds of circuit parameters by engineering experience even from incomplete information, which paves the way for introducing the theory of interval matrices and interval dynamics to investigate the global asymptotic stability of interval neural networks. In the design of neural networks, it is important to ensure that system can be stable with respect to these uncertainties.

Recently, uncertain time-delayed dynamical systems have been intensively investigated. Some new controllers have already been designed based on linear matrix inequality (LMI) technique, adaptive control method, backstepping approach, and so on. Ge [18–20] has studied adaptive control of nonlinear systems with unknown time delays by using backstepping approach. Yu and Cao [26, 27, 32, 34] have considered the synchronization and stability control of delayed neural networks based on LMI technique and adaptive control method. Though LMI is an useful tool in the control field, some conditions must be satisfied in order to stabilize the system. In addition, adaptive and backstepping controllers are more difficult to be implemented than feedback controller, which is a memoryless controller and can be easily implemented. In this case, Lyapunov functional design has been shown to be an effective tool in controller design for uncertain nonlinear systems.

In recent years, there have been tremendous efforts in the control of certain class of nonlinear systems, but little work has been made on uncertain general neural networks with time delays. In this paper, a general delayed Cohen–Grossberg neural network with uncertainties of parameters and unknown time delays is considered. Feedback control has proven its great capability in compensating for linearly parameterized uncertainties. To obtain global asymptotic stability, some restrictions have to be made to system. Several controllers are designed to ensure the global asymptotic stability of neural network.

The rest of the paper is organized as follows: In Section 2, formulation and preliminaries for the main results are introduced. In Section 3, some designed controllers with time delays and without time delays are presented to ensure the global asymptotic stability of Cohen–Grossberg neural networks, respectively. In Section 4, simulation examples are constructed to show the effectiveness and feasibility of theoretical analysis in this paper. In Section 5, the conclusions are drawn.

2. MODEL FORMULATION AND PRELIMINARIES

In this paper, a general delayed Cohen–Grossberg neural network model is considered as follows:

$$\dot{x}(t) = -a(x(t))[b(x(t)) - (A+\Delta A)f(x(t)) - (B+\Delta B)f(x(t-\tau)) + I] \tag{1}$$

or

$$\dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n}(a_{ij}+\Delta a_{ij})f_j(x_j(t)) - \sum_{j=1}^{n}(b_{ij}+\Delta b_{ij})f_j(x_j(t-\tau_{ij})) + I_i\right], \quad i=1,2,\ldots,n \tag{2}$$

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where n denotes the number of units in a neural network, $x(t)=(x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$ is the state vector associated with the neurons, $I=(I_1, I_2, \ldots, I_n)^{\mathrm{T}} \in R^n$ is an external input vector, $f(x(t))=(f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}} \in R^n$ corresponds to the activation functions of neurons, $\tau=\tau_{ij}$ $(i, j=1, 2, \ldots, n)$ are the time delays, and the initial conditions of (1) are given by $x_i(t)=\phi_i(t) \in \mathscr{C}([-r, 0], R)$ with $r=\max_{1\leqslant i,j\leqslant n}\{\tau_{ij}\}$, where $\mathscr{C}([-r, 0], R)$ denotes the set of all continuous functions from $[-r, 0]$ to R. $a(x(t))=\mathrm{diag}(a_1(x_1(t)), a_2(x_2(t)), \ldots, a_n(x_n(t))) \in R^{n\times n}$ and $a_i(x_i(t))$ represents an amplification function. $b(x(t))=(b_1(x_1(t)), b_2(x_2(t)), \ldots, b_n(x_n(t)))^{\mathrm{T}} \in R^n$ and $b_i(x_i(t))$ is a behaved function. $A+\Delta A=(a_{ij}+\Delta a_{ij})_{n\times n}$ and $B+\Delta B=(b_{ij}+\Delta b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. A and B are known matrices, while ΔA and ΔB are unknown.

The matrices A and B describe the nominal motion of system (1), whereas the matrices ΔA and ΔB represent the uncertainties in the system parameters, respectively, which are possibly time varying or random. It is assumed that the right-hand side of system (1) is continuous so as to ensure the existence and uniqueness of the solution through every initial condition. The objective is to design a controller to let system (1) be stable.

First, some notations are introduced, which will be used later for convenience.

For any vector $v=(v_1, v_2, \ldots, v_n)$, $|v|=(|v_1|, |v_2|, \ldots, |v_n|)$, $\|v\|^2=v^{\mathrm{T}}v$. Similarly, for any matrix $W=(w_{ij})_{n\times n}$, $|W|=(|w_{ij}|)_{n\times n}$. Let $\|\bullet\|$ denote the Euclidean norm in the Euclidean space R^n. W is a symmetric matrix with $\lambda_{\max}(W)$ and $\lambda_{\min}(W)$ as its largest and smallest eigenvalue, respectively. Then its norm is defined by: $\|W\|=\sup\{\|Wx\| : \|x\|=1\}=\sqrt{\lambda_{\max}(W^{\mathrm{T}}W)}$.

To establish the main results for model (1), it is necessary to make the following assumptions:

(A1) Each function $a_i(\cdot)$ is positive, continuous and bounded.

(A2) Each function $f_i : R \to R$ are nondecreasing and globally Lipschitz with a constant $k_i>0$, i.e.

$$|f_i(u)-f_i(v)|\leqslant k_i|u-v| \quad \forall u, v \in R,\ i=1, 2, \ldots, n \tag{3}$$

(A3) The unknown matrices ΔA and ΔB are norm bounded

$$\|\Delta A\|\leqslant\sqrt{\rho_A}, \quad \|\Delta B\|\leqslant\sqrt{\rho_B}$$

where ρ_A and ρ_B are positive constants.

It is recalled that a state $x^* \in R^n$ is called an equilibrium point of (1) if it satisfies

$$-a(x^*)[b(x^*)-(A+\Delta A+B+\Delta B)f(x^*)+I]=0 \tag{4}$$

Notice that $a_i(x_i(t))$ is positive, (4) is equivalent to

$$b(x^*)-(A+\Delta A+B+\Delta B)f(x^*)+I=0 \tag{5}$$

Assume that for a given I model (1) has an equilibrium $x^*=(x_1^*, x_2^*, \ldots, x_n^*)$, which is known and can be considered as the control object. To simplify the proofs, the equilibrium point x^* of (1) can be shifted to the origin by using the following transformation:

$$y(t)=x(t)-x^*, \quad y(t-\tau)=x(t-\tau)-x^* \tag{6}$$

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Then, system (1) can be transformed into the following form:

$$\dot{y}(t) = -\overline{a}(y(t))[\overline{b}(y(t)) - (A + \Delta A)g(y(t)) - (B + \Delta B)g(y(t-\tau))] \tag{7}$$

or

$$\dot{y}_i(t) = -\overline{a}_i(y_i(t))\left[\overline{b}_i(y_i(t)) - \sum_{j=1}^{n}(a_{ij} + \Delta a_{ij})g_j(y_j(t)) - \sum_{j=1}^{n}(b_{ij} + \Delta b_{ij})g_j(y_j(t-\tau_{ij}))\right], \quad i = 1, 2, \ldots, n \tag{8}$$

where

$$\begin{aligned}
y(t) &= (y_1(t), y_2(t), \ldots, y_n(t))^{\mathrm{T}} \in R^n \\
\overline{a}(y(t)) &= \mathrm{diag}(\overline{a}_1(y_1(t)), \overline{a}_2(y_2(t)), \ldots, \overline{a}_n(y_n(t)))^{\mathrm{T}} \in R^{n\times n} \\
\overline{b}(y(t)) &= (\overline{b}_1(y_1(t)), \overline{b}_2(y_2(t)), \ldots, \overline{b}_n(y_n(t)))^{\mathrm{T}} \in R^n \\
g(y(t)) &= (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)))^{\mathrm{T}} \in R^n \\
\overline{a}_i(y_i(t)) &= a_i(y_i(t) + x_i^*) \\
\overline{b}_i(y_i(t)) &= b_i(y_i(t) + x_i^*) - b_i(x_i^*) \\
g_i(y_i(t)) &= f_i(y_i(t) + x_i^*) - f_i(x_i^*)
\end{aligned}$$

It is easy to see that

$$g_i(0) = 0 \quad \forall i = 1, 2, \ldots, n$$

Moreover, from (3), one knows that

$$|g_i(y_i)| \leqslant k_i|y_i| \quad \forall y_i \in R, \ i = 1, 2, \ldots, n \tag{9}$$

and

$$\|g(y(t))\|^2 \leqslant y^{\mathrm{T}}(t)Kg(y(t)) \leqslant y^{\mathrm{T}}(t)K^2y(t) \tag{10}$$

where $K = \mathrm{diag}(k_1, k_2, \ldots, k_n) > 0$.

In order to let the system be globally asymptotically stable, a controller u is added to system (7). Then the following system is obtained:

$$\dot{y}(t) = -\overline{a}(y(t))[\overline{b}(y(t)) - (A + \Delta A)g(y(t)) - (B + \Delta B)g(y(t-\tau))] + u(y(t)) \tag{11}$$

or

$$\dot{y}_i(t) = -\overline{a}_i(y_i(t))\left[\overline{b}_i(y_i(t)) - \sum_{j=1}^{n}(a_{ij} + \Delta a_{ij})g_j(y_j(t)) - \sum_{j=1}^{n}(b_{ij} + \Delta b_{ij})g_j(y_j(t-\tau_{ij}))\right] + u_i(y_i(t)) \tag{12}$$

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for $i=1,2,\dots,n$, where u is a function of state vector. Note that if the controller u is used to stabilize system (11), then the goal of ensuring the asymptotical stability of system (1) can be obtained by transformation (6). Thus, in this paper the asymptotical stability of system (11) is investigated instead.

To obtain the main results, the following two elementary lemmas are needed:

Lemma 1 (*Cao and Wang* [*1*])
For any vectors $x, y \in R^n$ and positive-definite matrix $G \in R^{n\times n}$, the following matrix inequality holds:

$$2x^{\mathrm{T}}y \leqslant x^{\mathrm{T}}Gx + y^{\mathrm{T}}G^{-1}y$$

Lemma 2 (*Cao and Wang* [*1*])
If $V, W \in R^{n\times n}$ are two matrices with property that $|V| = (|v_{ij}|_{n\times n}) \leqslant W = (w_{ij})_{n\times n}$, i.e. $|v_{ij}| \leqslant w_{ij}$, then $\|V\|_2 \leqslant \|W\|_2$.

Proof
For $\forall x$ satisfying $\|x\|=1$

$$\begin{aligned}
\|Vx\|^2 &= \sum_{i=1}^{n}\left(\sum_{j=1}^{n} v_{ij}x_j\right)^2 \\
&\leqslant \sum_{i=1}^{n}\left(\sum_{j=1}^{n} |v_{ij}||x_j|\right)^2 \\
&\leqslant \sum_{i=1}^{n}\left(\sum_{j=1}^{n} w_{ij}|x_j|\right)^2 \\
&= \|W(|x_1|, |x_2|, \dots, |x_n|)^{\mathrm{T}}\|^2 \\
&\leqslant \left(\|W\|_2 \sqrt{\sum_{j=1}^{n} |x_j|^2}\right)^2 \\
&= \|W\|_2^2
\end{aligned}$$

□

3. ROBUST CONTROL

In this section, some controllers with known and unknown time delays are designed to control system (11), respectively.

3.1. Known time delays

In this section, the designed controllers are dependent on the time delays τ.

Theorem 1
Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrix $P = \mathrm{diag}(p_1, p_2, \dots, p_n) > 0$, positive-definite matrix

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$H=(h_{ij})_{n\times n}>0$, and positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, such that

$$u(y)=\tfrac{1}{2}\overline{a}(y(t))P^{-1}[2P\overline{b}(y(t))-2PAg(y)-2PBg(y(t-\tau))-(\varepsilon_A+\varepsilon_B)P^2y \\ -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})Kg(y)-Hy] \tag{13}$$

Proof
Consider the following Lyapunov functional candidate:

$$V(y(t))=\sum_{i=1}^{n}p_i\int_0^{y_i(t)}\frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s+\gamma\int_{t-\tau}^{t}g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s \tag{14}$$

where γ is a positive constant.

Taking the derivative of $V(y)$ along the trajectories of (11), one obtains

$$\begin{aligned}\dot{V}(y(t))|_{(11)}&=\sum_{i=1}^{n}p_i\frac{2y_i(t)}{\overline{a}_i(y_i(t))}\dot{y}_i(t)+\gamma g^{\mathrm{T}}(y(t))g(y(t))-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\&=2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+(A+\Delta A)g(y(t))+(B+\Delta B)g(y(t-\tau))\\&\quad+\overline{a}(y(t))^{-1}u(y(t))]+\gamma g^{\mathrm{T}}(y(t))g(y(t))-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\&=2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\&\quad+2y^{\mathrm{T}}(t)P\Delta A\times g(y(t))+2y^{\mathrm{T}}(t)P\Delta Bg(y(t-\tau))+\gamma g^{\mathrm{T}}(y(t))g(y(t))\\&\quad-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned} \tag{15}$$

From Lemma 1 and Assumption (A3), one has

$$\begin{aligned}2y^{\mathrm{T}}(t)P\Delta Ag(y(t))&\leqslant\varepsilon_Ay^{\mathrm{T}}(t)P^2y(t)+\varepsilon_A^{-1}g^{\mathrm{T}}(y(t))\Delta A^{\mathrm{T}}\Delta Ag(y(t))\\&\leqslant\varepsilon_Ay^{\mathrm{T}}(t)P^2y(t)+\rho_A\varepsilon_A^{-1}g^{\mathrm{T}}(y(t))g(y(t))\end{aligned} \tag{16}$$

and

$$\begin{aligned}2y^{\mathrm{T}}(t)P\Delta Bg(y(t-\tau))&\leqslant\varepsilon_By^{\mathrm{T}}(t)P^2y(t)+\varepsilon_B^{-1}g^{\mathrm{T}}(y(t-\tau))\Delta B^{\mathrm{T}}\Delta Bg(y(t-\tau))\\&\leqslant\varepsilon_By^{\mathrm{T}}(t)P^2y(t)+\rho_B\varepsilon_B^{-1}g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned} \tag{17}$$

Substituting (16) and (17) into (15), one obtains

$$\begin{aligned}\dot{V}(y(t))|_{(11)}&\leqslant2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\&\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm{T}}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\gamma)g^{\mathrm{T}}(y(t))g(y(t))\\&\quad+(\rho_B\varepsilon_B^{-1}-\gamma)g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned} \tag{18}$$

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Choose $\gamma=\rho_B\varepsilon_B^{-1}$, and it follows that

$$\begin{aligned}\dot V(y(t))|_{(11)} &\leqslant 2y^{\mathrm T}(t)P[-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\ &\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm T}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm T}(y(t))g(y(t))\end{aligned} \tag{19}$$

From (10), one obtains

$$\begin{aligned}\dot V(y(t))|_{(11)} &\leqslant 2y^{\mathrm T}(t)P[-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\ &\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm T}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm T}(t)Kg(y(t))\\ &= y^{\mathrm T}(t)[-2P\overline{b}(y(t))+2PAg(y(t))+2PBg(y(t-\tau))+(\varepsilon_A+\varepsilon_B)P^2y(t)\\ &\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})Kg(y(t))+2P\overline{a}(y(t))^{-1}u(y(t))]\\ &= -y^{\mathrm T}(t)Hy(t)\end{aligned} \tag{20}$$

Therefore, from (20), one knows that under the designed controller (13), $\dot V(y(t))=0$ if and only if $y(t)=0$, otherwise $\dot V(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. This completes the proof. □

Corollary 1

Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrix $P=\mathrm{diag}(p_1,p_2,\ldots,p_n)>0$, positive-definite matrix $H=(h_{ij})_{n\times n}>0$, and positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, such that

$$\begin{aligned}u(y) &= \tfrac{1}{2}\overline{a}(y(t))P^{-1}[2P\overline{b}(y(t))-2PAg(y)-2PBg(y(t-\tau))-(\varepsilon_A+\varepsilon_B)P^2y\\ &\quad-(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})K^2y-Hy]\end{aligned} \tag{21}$$

Proof

Choosing the same Lyapunov functional as in Theorem 1

$$V(y(t))=\sum_{i=1}^{n}p_i\int_0^{y_i(t)}\frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s+\gamma\int_{t-\tau}^{t}g^{\mathrm T}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Substituting (10) into (19), one has

$$\begin{aligned}\dot V(y(t))|_{(11)} &\leqslant 2y^{\mathrm T}(t)P[-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\ &\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm T}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm T}(t)K^2y(t)\\ &= y^{\mathrm T}(t)[-2P\overline{b}(y(t))+2PAg(y(t))+2PBg(y(t-\tau))+(\varepsilon_A+\varepsilon_B)P^2y(t)\\ &\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})K^2y(t)+2P\overline{a}(y(t))^{-1}u(y(t))]\\ &= -y^{\mathrm T}(t)Hy(t)\end{aligned} \tag{22}$$

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Therefore, from (22), one knows that under the designed controller (21), $\dot{V}(y(t))=0$ if and only if $y(t)=0$, otherwise $\dot{V}(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. This completes the proof. □

Theorem 2
Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrices $P=\operatorname{diag}(p_1, p_2, \ldots, p_n)>0$ and $Q=\operatorname{diag}(q_1, q_2, \ldots, q_n)>0$, positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, such that

$$\begin{aligned} u(y) = \tfrac{1}{2}\overline{a}(y(t))[2\overline{b}(y(t)) - 2Ag(y) - 2Bg(y(t-\tau)) - (\varepsilon_A+\varepsilon_B)(Py+Qg(y)) \\ - (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1})P^{-1}Kg(y)] \end{aligned} \tag{23}$$

Proof
Consider the Lyapunov functional

$$(y(t)) = \sum_{i=1}^{n} p_i \int_0^{y_i(t)} \frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s + 2\sum_{i=1}^{n} q_i \int_0^{y_i(t)} \frac{g_i(s)}{\overline{a}_i(s)}\,\mathrm{d}s + \gamma\int_{t-\tau}^{t} g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s \tag{24}$$

where γ is positive and real.

Taking the derivative of $V(y)$ along the trajectories of (11), one obtains

$$\begin{aligned} \dot{V}(y(t))|_{(11)} &= \sum_{i=1}^{n} p_i \frac{2y_i(t)}{\overline{a}_i(y_i(t))}\dot{y}_i(t) + 2\sum_{i=1}^{n} q_i \frac{g_i(y_i(t))}{\overline{a}_i(y_i(t))}\dot{y}_i(t) + \gamma g^{\mathrm{T}}(y(t))g(y(t)) \\ &\quad - \gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau)) \\ &= 2y^{\mathrm{T}}(t)P[-\overline{b}(y(t)) + (A+\Delta A)g(y(t)) + (B+\Delta B)g(y(t-\tau)) \\ &\quad + \overline{a}(y(t))^{-1}u(y(t))] + 2g^{\mathrm{T}}(y(t))Q[-\overline{b}(y(t)) + (A+\Delta A)g(y(t)) \\ &\quad + (B+\Delta B)g(y(t-\tau)) + \overline{a}(y(t))^{-1}u(y(t))] + \gamma g^{\mathrm{T}}(y(t))g(y(t)) \\ &\quad - \gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau)) \\ &= 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t)) + Ag(y(t)) + Bg(y(t-\tau)) \\ &\quad + \overline{a}(y(t))^{-1}u(y(t))] + 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]\Delta Ag(y(t)) \\ &\quad + 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]\Delta Bg(y(t-\tau)) + \gamma g^{\mathrm{T}}(y(t))g(y(t)) \\ &\quad - \gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau)) \end{aligned} \tag{25}$$

From Lemma 1 and assumption (A3), one has

$$\begin{aligned} 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]\Delta Ag(y(t)) &\leqslant \varepsilon_A[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ &\quad + \varepsilon_A^{-1}g^{\mathrm{T}}(y(t))\Delta A^{\mathrm{T}}\Delta Ag(y(t)) \\ &\leqslant \varepsilon_A[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ &\quad + \rho_A\varepsilon_A^{-1}g^{\mathrm{T}}(y(t))g(y(t)) \end{aligned} \tag{26}$$

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and

$$\begin{aligned}2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]\Delta Bg(y(t-\tau)) &\leqslant \varepsilon_B[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}}\\&\quad+\varepsilon_B^{-1}g^{\mathrm{T}}(y(t-\tau))\Delta B^{\mathrm{T}}\Delta Bg(y(t-\tau))\\&\leqslant \varepsilon_B[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}}\\&\quad+\rho_B\varepsilon_B^{-1}g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned}\tag{27}$$

Substituting (26) and (27) into (25), one obtains

$$\begin{aligned}\dot{V}(y(t))|_{(11)} &\leqslant 2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\&\quad+(\varepsilon_A+\varepsilon_B)[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}}\\&\quad+(\rho_A\varepsilon_A^{-1}+\gamma)g^{\mathrm{T}}(y(t))g(y(t))+(\rho_B\varepsilon_B^{-1}-\gamma)g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned}\tag{28}$$

Choose $\gamma=\rho_B\varepsilon_B^{-1}$, it follows that

$$\begin{aligned}\dot{V}(y(t))|_{(11)} &\leqslant 2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\&\quad+(\varepsilon_A+\varepsilon_B)[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}}\\&\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm{T}}(y(t))g(y(t))\end{aligned}\tag{29}$$

From (10), one has

$$\begin{aligned}\dot{V}(y(t))|_{(11)} &\leqslant 2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t))+Ag(y(t))+Bg(y(t-\tau))\\&\quad+\overline{a}(y(t))^{-1}u(y(t))]+(\varepsilon_A+\varepsilon_B)[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}}\\&\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm{T}}(t)Kg(y(t))\\&=[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-2\overline{b}(y(t))+2Ag(y(t))+2Bg(y(t-\tau))\\&\quad+(\varepsilon_A+\varepsilon_B)(Py(t)+Qg(y(t)))+2\overline{a}(y(t))^{-1}u(y(t))]\\&\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm{T}}(t)Kg(y(t))\\&=-(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]P^{-1}Kg(y(t))\\&\quad+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm{T}}(t)Kg(y(t))\\&=-(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm{T}}(y(t))QP^{-1}Kg(y(t))\end{aligned}\tag{30}$$

Therefore, from (30), one knows that under the designed controller (23), $\dot{V}(y(t))=0$ if and only if $y(t)=g(y(t))=0$, otherwise $\dot{V}(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. This completes the proof. □

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Corollary 2
Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrices $P=\mathrm{diag}(p_1, p_2, \ldots, p_n)>0$ and $Q=\mathrm{diag}(q_1, q_2, \ldots, q_n)>0$, positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, such that

$$\begin{aligned} u(y) = {} & \tfrac{1}{2}\overline{a}(y(t))[2\overline{b}(y(t)) - 2Ag(y) - 2Bg(y(t-\tau)) - (\varepsilon_A+\varepsilon_B)(Py+Qg(y)) \\ & -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})Q^{-1}g(y)] \end{aligned} \tag{31}$$

Proof
Choose the same Lyapunov functional as in Theorem 2

$$V(y(t)) = \sum_{i=1}^{n} p_i \int_0^{y_i(t)} \frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s + 2\sum_{i=1}^{n} q_i \int_0^{y_i(t)} \frac{g_i(s)}{\overline{a}_i(s)}\,\mathrm{d}s + \gamma \int_{t-\tau}^{t} g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Equation (29) can be still obtained. Substituting (10) into (29), one has

$$\begin{aligned} \dot{V}(y(t))|_{(11)} \leqslant {} & 2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t)) + Ag(y(t))+Bg(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))] \\ & + (\varepsilon_A+\varepsilon_B)[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + (\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ = {} & [y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-2\overline{b}(y(t)) + 2Ag(y(t)) + 2Bg(y(t-\tau)) \\ & + (\varepsilon_A+\varepsilon_B)(Py(t)+Qg(y(t))) + 2\overline{a}(y(t))^{-1}u(y(t))] \\ & + (\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ = {} & -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]Q^{-1}g(y(t)) \\ & +(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ = {} & -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm{T}}(t)PQ^{-1}g(y(t)) \\ \leqslant {} & -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1})y^{\mathrm{T}}(t)PQ^{-1}K^{-1}y(t) \end{aligned} \tag{32}$$

Therefore, from (32), one can see that under the designed controller (31), $\dot{V}(y(t))=0$ if and only if $y(t)=0$, otherwise $\dot{V}(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. This completes the proof. □

3.2. Unknown time delays

In this subsection, some designed controllers are independent of the time delays τ.

Theorem 3
Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrix $P=\mathrm{diag}(p_1, p_2, \ldots, p_n)>0$, positive-definite matrix

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$H=(h_{ij})_{n\times n}>0$, and positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, $\varepsilon>0$, such that

$$u(y)=\tfrac{1}{2}\overline{a}(y(t))P^{-1}[2P\overline{b}(y(t))-2PAg(y)-(\varepsilon_A+\varepsilon_B)P^2y-\varepsilon PBB^{\mathrm{T}}Py - (\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})Kg(y)-Hy] \quad (33)$$

Proof

Consider the following Lyapunov functional candidate:

$$V(y(t))=\sum_{i=1}^{n}p_i\int_0^{y_i(t)}\frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s+\gamma\int_{t-\tau}^{t}g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Taking the derivative of $V(y)$ along the trajectories of (11), one obtains

$$\begin{aligned}\dot V(y(t))|_{(11)}&=\sum_{i=1}^{n}p_i\frac{2y_i(t)}{\overline{a}_i(y_i(t))}\dot y_i(t)+\gamma g^{\mathrm{T}}(y(t))g(y(t))-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\ &=2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+(A+\Delta A)g(y(t))+(B+\Delta B)g(y(t-\tau))\\ &\quad+\overline{a}(y(t))^{-1}u(y(t))]+\gamma g^{\mathrm{T}}(y(t))g(y(t))-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\ &=2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]+2y^{\mathrm{T}}(t)PBg(y(t-\tau))\\ &\quad+2y^{\mathrm{T}}(t)P\Delta Ag(y(t))+2y^{\mathrm{T}}(t)P\Delta Bg(y(t-\tau))+\gamma g^{\mathrm{T}}(y(t))g(y(t))\\ &\quad-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned}\quad (34)$$

From Lemma 1, one has

$$2y^{\mathrm{T}}(t)PBg(y(t-\tau))\leqslant\varepsilon y^{\mathrm{T}}(t)PBB^{\mathrm{T}}Py(t)+\varepsilon^{-1}g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau)) \quad (35)$$

Substituting (16), (17) and (35) into (34), one obtains

$$\begin{aligned}\dot V(y(t))|_{(11)}&\leqslant 2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]+\varepsilon y^{\mathrm{T}}(t)PBB^{\mathrm{T}}Py(t)\\ &\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm{T}}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\gamma)g^{\mathrm{T}}(y(t))g(y(t))\\ &\quad+(\rho_B\varepsilon_B^{-1}+\varepsilon^{-1}-\gamma)g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned}\quad (36)$$

Choose $\gamma=\rho_B\varepsilon_B^{-1}+\varepsilon^{-1}$, it follows that

$$\begin{aligned}\dot V(y(t))|_{(11)}&\leqslant 2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]+\varepsilon y^{\mathrm{T}}(t)PBB^{\mathrm{T}}Py(t)\\ &\quad+(\varepsilon_A+\varepsilon_B)y^{\mathrm{T}}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})g^{\mathrm{T}}(y(t))g(y(t))\end{aligned}\quad (37)$$

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From (10) and (33), one has

$$\begin{aligned}\dot{V}(y(t))|_{(11)} &\leqslant 2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]+\varepsilon y^{\mathrm{T}}(t)PBB^{\mathrm{T}}Py(t)\\ &\quad +(\varepsilon_A+\varepsilon_B)y^{\mathrm{T}}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})y^{\mathrm{T}}(t)Kg(y(t))\\ &= y^{\mathrm{T}}(t)[-2P\overline{b}(y(t))+2PAg(y(t))+2PBg(y(t-\tau))+(\varepsilon_A+\varepsilon_B)P^2y(t)\\ &\quad +\varepsilon PBB^{\mathrm{T}}Py(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})Kg(y(t))+2P\overline{a}(y(t))^{-1}u(y(t))]\\ &= -y^{\mathrm{T}}(t)Hy(t)\end{aligned}\tag{38}$$

Therefore, from (38), one knows that under the designed controller (33), $\dot{V}(y(t))=0$ if and only if $y(t)=0$, otherwise $\dot{V}(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. This completes the proof. □

Corollary 3

Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrix $P=\mathrm{diag}(p_1,p_2,\ldots,p_n)>0$, positive-definite matrix $H=(h_{ij})_{n\times n}>0$, and positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, $\varepsilon>0$, such that

$$\begin{aligned}u(y) &= \tfrac{1}{2}\overline{a}(y(t))P^{-1}[2P\overline{b}(y(t))-2PAg(y)-(\varepsilon_A+\varepsilon_B)P^2y-\varepsilon PBB^{\mathrm{T}}Py\\ &\quad -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})K^2y-Hy]\end{aligned}\tag{39}$$

Proof

Choosing the same Lyapunov functional as in Theorem 3

$$V(y(t))=\sum_{i=1}^{n}p_i\int_0^{y_i(t)}\frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s+\gamma\int_{t-\tau}^{t}g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Equation (37) can be still obtained. Substituting (10) into (37), one has

$$\begin{aligned}\dot{V}(y(t))|_{(11)} &\leqslant 2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]+\varepsilon y^{\mathrm{T}}(t)PBB^{\mathrm{T}}Py(t)\\ &\quad +(\varepsilon_A+\varepsilon_B)y^{\mathrm{T}}(t)P^2y(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})y^{\mathrm{T}}(t)K^2y(t)\\ &= y^{\mathrm{T}}(t)[-2P\overline{b}(y(t))+2PAg(y(t))+2PBg(y(t-\tau))+(\varepsilon_A+\varepsilon_B)P^2y(t)\\ &\quad +\varepsilon PBB^{\mathrm{T}}Py(t)+(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})K^2y(t)+2P\overline{a}(y(t))^{-1}u(y(t))]\\ &= -y^{\mathrm{T}}(t)Hy(t)\end{aligned}\tag{40}$$

Therefore, from (40), we know that under the designed controller (39), $\dot{V}(y(t))=0$ if and only if $y(t)=0$, otherwise $\dot{V}(y(t))\leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t))\to\infty$ as $\|y(t)\|\to\infty$. □

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Theorem 4

Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrices $P=\mathrm{diag}(p_1, p_2, \dots, p_n)>0$ and $Q=\mathrm{diag}(q_1, q_2, \dots, q_n)>0$, and positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, $\varepsilon>0$, such that

$$u(y)=\tfrac{1}{2}\overline{a}(y(t))[2\overline{b}(y(t))-2Ag(y)-(\varepsilon_A+\varepsilon_B+\varepsilon BB^{\mathrm{T}})(Py+Qg(y)) \\ -(\rho_A\varepsilon_A^{-1}+\rho_B\varepsilon_B^{-1}+\varepsilon^{-1})P^{-1}Kg(y)] \tag{41}$$

Proof

Consider the following Lyapunov functional candidate:

$$V(y(t))=\sum_{i=1}^{n}p_i\int_0^{y_i(t)}\frac{2s}{\overline{a}_i(s)}\,\mathrm{d}s+2\sum_{i=1}^{n}q_i\int_0^{y_i(t)}\frac{g_i(s)}{\overline{a}_i(s)}\,\mathrm{d}s+\gamma\int_{t-\tau}^{t}g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Taking the derivative of $V(y)$ along the trajectories of (11), one obtains

$$\begin{aligned}\dot V(y(t))|_{(11)}&=\sum_{i=1}^{n}p_i\frac{2y_i(t)}{\overline{a}_i(y_i(t))}\dot y_i(t)+2\sum_{i=1}^{n}q_i\frac{g_i(y_i(t))}{\overline{a}_i(y_i(t))}\dot y_i(t)+\gamma g^{\mathrm{T}}(y(t))g(y(t))\\ &\quad-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\ &=2y^{\mathrm{T}}(t)P[-\overline{b}(y(t))+(A+\Delta A)g(y(t))+(B+\Delta B)g(y(t-\tau))\\ &\quad+\overline{a}(y(t))^{-1}u(y(t))]+2g^{\mathrm{T}}(y(t))Q[-\overline{b}(y(t))+(A+\Delta A)g(y(t))\\ &\quad+(B+\Delta B)g(y(t-\tau))+\overline{a}(y(t))^{-1}u(y(t))]\\ &\quad+\gamma g^{\mathrm{T}}(y(t))g(y(t))-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\\ &=2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t))+Ag(y(t))+\overline{a}(y(t))^{-1}u(y(t))]\\ &\quad+2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]Bg(y(t-\tau))+2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]\Delta Ag(y(t))\\ &\quad+2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]\Delta Bg(y(t-\tau))+\gamma g^{\mathrm{T}}(y(t))g(y(t))\\ &\quad-\gamma g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned} \tag{42}$$

From Lemma 1, one has

$$2[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]Bg(y(t-\tau))\leqslant\varepsilon[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]BB^{\mathrm{T}}[y^{\mathrm{T}}(t)P+g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ +\varepsilon^{-1}g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau)) \tag{43}$$

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Substituting (26), (27) and (43) into (42), one obtains

$$\begin{aligned}\dot{V}(y(t))|_{(11)} \leqslant\; & 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t)) + Ag(y(t)) + \overline{a}(y(t))^{-1}u(y(t))] \\ & + (\varepsilon_A + \varepsilon_B)[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + \varepsilon[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]BB^{\mathrm{T}}[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + (\rho_A\varepsilon_A^{-1} + \gamma)g^{\mathrm{T}}(y(t))g(y(t)) \\ & +(\rho_B\varepsilon_B^{-1} + \varepsilon^{-1} - \gamma)g^{\mathrm{T}}(y(t-\tau))g(y(t-\tau))\end{aligned} \tag{44}$$

Choose $\gamma = \rho_B\varepsilon_B^{-1} + \varepsilon^{-1}$, it follows that

$$\begin{aligned}\dot{V}(y(t))|_{(11)} \leqslant\; & 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t)) + Ag(y(t) + \overline{a}(y(t))^{-1}u(y(t))] \\ & + (\varepsilon_A + \varepsilon_B)[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + \varepsilon[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]BB^{\mathrm{T}}[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})g^{\mathrm{T}}(y(t))g(y(t))\end{aligned} \tag{45}$$

From (10) and (45), one obtains

$$\begin{aligned}\dot{V}(y(t))|_{(11)} \leqslant\; & 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-\overline{b}(y(t)) + Ag(y(t) + \overline{a}(y(t))^{-1}u(y(t))] \\ & + (\varepsilon_A + \varepsilon_B)[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + \varepsilon[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]BB^{\mathrm{T}}[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ & + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})y^{\mathrm{T}}(t)Kg(y(t)) \\ =\; & [y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-2\overline{b}(y(t)) + 2Ag(y(t)) + (\varepsilon_A + \varepsilon_B + \varepsilon BB^{\mathrm{T}})(Py(t) \\ & + Qg(y(t))) + 2\overline{a}(y(t))^{-1}u(y(t))] + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})y^{\mathrm{T}}(t)Kg(y(t)) \\ =\; & -(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]P^{-1}Kg(y(t)) \\ & + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})y^{\mathrm{T}}(t)Kg(y(t)) \\ =\; & -(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})g^{\mathrm{T}}(y(t))QP^{-1}Kg(y(t))\end{aligned} \tag{46}$$

Therefore, from (46), one knows that under the designed controller (41), $\dot{V}(y(t)) = 0$ if and only if $y(t) = g(y(t)) = 0$, otherwise $\dot{V}(y(t)) \leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t)) \to \infty$ as $\|y(t)\| \to \infty$. □

Corollary 4

Under assumptions (A1)–(A3), the origin of model (11) is globally asymptotically stable if there are positive-definite diagonal matrices $P = \mathrm{diag}(p_1, p_2, \ldots, p_n) > 0$ and $Q = \mathrm{diag}(q_1, q_2, \ldots, q_n) > 0$,

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positive real values $\varepsilon_A>0$, $\varepsilon_B>0$, $\varepsilon>0$, such that

$$\begin{aligned} u(y) = \tfrac{1}{2}\bar{a}(y(t))[2\bar{b}(y(t)) - 2Ag(y) - (\varepsilon_A + \varepsilon_B + \varepsilon BB^{\mathrm{T}})(Py + Qg(y)) \\ - (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})Q^{-1}g(y)] \end{aligned} \tag{47}$$

Proof

Choosing the same Lyapunov functional as in Theorem 4

$$V(y(t)) = \sum_{i=1}^{n} p_i \int_0^{y_i(t)} \frac{2s}{\bar{a}_i(s)}\,\mathrm{d}s + 2\sum_{i=1}^{n} q_i \int_0^{y_i(t)} \frac{g_i(s)}{\bar{a}_i(s)}\,\mathrm{d}s + \gamma \int_{t-\tau}^{t} g^{\mathrm{T}}(y(s))g(y(s))\,\mathrm{d}s$$

where γ is positive and real.

Equation (45) can still be obtained. Substituting (47) into (45), one has

$$\begin{aligned} \dot{V}(y(t))|_{(11)} &\leqslant 2[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-\bar{b}(y(t)) + Ag(y(t)) + \bar{a}(y(t))^{-1}u(y(t))] \\ &\quad + (\varepsilon_A + \varepsilon_B)[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ &\quad + \varepsilon[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]BB^{\mathrm{T}}[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]^{\mathrm{T}} \\ &\quad + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ &= [y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q][-2\bar{b}(y(t)) + 2Ag(y(t)) + (\varepsilon_A + \varepsilon_B + \varepsilon BB^{\mathrm{T}})(Py(t) \\ &\quad + Qg(y(t))) + 2\bar{a}(y(t))^{-1}u(y(t))] + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ &= -(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})[y^{\mathrm{T}}(t)P + g^{\mathrm{T}}(y(t))Q]Q^{-1}g(y(t)) \\ &\quad + (\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})g^{\mathrm{T}}(y(t))g(y(t)) \\ &= -(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})y^{\mathrm{T}}(t)PQ^{-1}g(y(t)) \\ &\leqslant -(\rho_A\varepsilon_A^{-1} + \rho_B\varepsilon_B^{-1} + \varepsilon^{-1})y^{\mathrm{T}}(t)PQ^{-1}K^{-1}y(t) \end{aligned} \tag{48}$$

Therefore, from (48), one knows that under the designed controller (47), $\dot{V}(y(t)) = 0$ if and only if $y(t) = 0$, otherwise $\dot{V}(y(t)) \leqslant 0$. Moreover, on the other hand, $V(y)$ is radially unbounded since $V(y(t)) \to \infty$ as $\|y(t)\| \to \infty$. □

Remark

In this paper, some new controllers are designed to ensure the global asymptotic stability of the origin of system (11), and thus the equilibrium point of (1) is globally asymptotically stable by using transformation (6). The controllers are both designed with time delays and without time delays, since in some conditions the time delays are unknown. In the literature, there is little work about the robust control of delayed Cohen–Grossberg neural network. The controllers are designed based on matrix inequality technique and Lyapunov functional method.

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4. SIMULATION EXAMPLES

In this section, some simulation examples are constructed to show the effectiveness of the theoretical analysis in this paper.

Example 1
Consider the following delayed Cohen–Grossberg neural network:

$$\begin{pmatrix} \dfrac{\mathrm{d}y_1(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y_2(t)}{\mathrm{d}t} \end{pmatrix} = -\left\{ \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - (A+\Delta A)\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - (B+\Delta B)\begin{pmatrix} \tanh(y_1(t-1)) \\ \tanh(y_2(t-1)) \end{pmatrix} \right\} \tag{49}$$

where $b_i(y_i(t)) = y_i(t)$ for $i=1,2$. It is easy to see that assumptions (A1)–(A2) are satisfied. $a_i(y_i(t)) = 1$ $(i=1,2)$ are bounded, positive continuous functions, $f_i(y_i(t)) = \tanh(y_i(t))$ is globally Lipschitz and nondecreasing. Obviously, one has $K=E$, where E is the identity matrix.

$$A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix}, \quad B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}$$

Suppose each element of uncertain matrices ΔA and ΔB are random in $[-0.02, 0.02]$ every time, since the parameters of system (49) are perturbed by many factors at each time. From Lemma 2, one knows that

$$\|\Delta A\| \leqslant \left\| \begin{pmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{pmatrix} \right\| = 0.04$$

and

$$\|\Delta B\| \leqslant \left\| \begin{pmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{pmatrix} \right\| = 0.04$$

It is easy to see that $\rho_A = \rho_B = 0.04$.

According to Theorem 1, the designed controller is as follows:

$$u(y) = \frac{1}{2}\left\{ 2\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - 2\begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix}\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - 2\begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix} \right.$$
$$\left. \times \begin{pmatrix} \tanh(y_1(t-1)) \\ \tanh(y_2(t-1)) \end{pmatrix} - 0.02\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - 3\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \right\} \tag{50}$$

The trajectories of system (49) without controller and with controller (50) are shown in Figures 1 and 2, respectively. It is easy to see that without controller, it is chaotic in system (49), but it is stable by adding the above controller (50). The designed controller (50) is effective and can control the stability of system (49).

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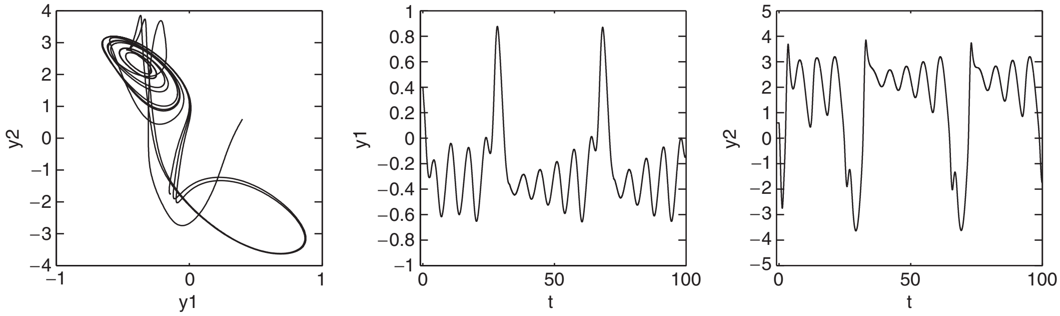

Figure 1. Trajectories of state variables $y_1(t)$ and $y_2(t)$ without control.

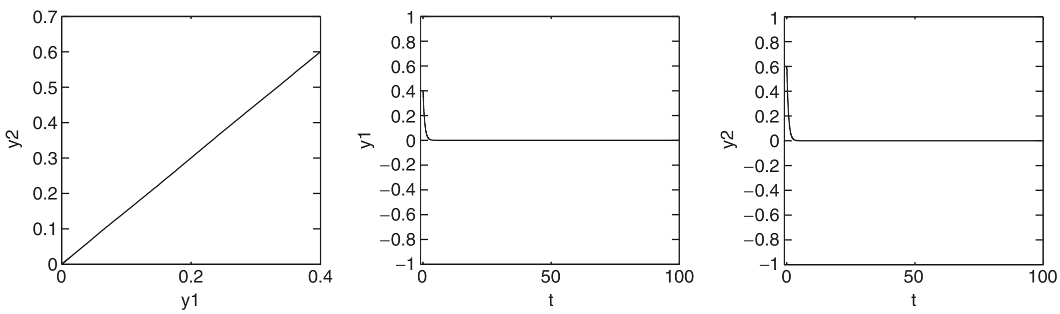

Figure 2. Trajectories of state variables $y_1(t)$ and $y_2(t)$ with control.

Example 2
Consider the following delayed Cohen–Grossberg neural network:

$$\begin{pmatrix} \dfrac{\mathrm{d}y_1(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y_2(t)}{\mathrm{d}t} \end{pmatrix} = -\begin{pmatrix} 2+\cos y_1(t) & 0 \\ 0 & 2+\sin y_2(t) \end{pmatrix} \left\{ \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - (A+\Delta A)\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} \right.$$
$$\left. -(B+\Delta B)\begin{pmatrix} \tanh(y_1(t-1)) \\ \tanh(y_2(t-1)) \end{pmatrix} \right\} \tag{51}$$

where $b_i(y_i(t)) = y_i(t)$ for $i = 1, 2$. It is easy to see that assumptions (A1)–(A2) are satisfied. $a_1(y_1(t)) = 2+\cos y_1(t)$ and $a_2(y_2(t)) = 2+\sin y_2(t)$ are bounded, positive continuous functions, $f_i(y_i(t)) = \tanh(y_i(t))$ is globally Lipschitz and nondecreasing. Obviously, one has $K = E$, where E is the identity matrix.

$$A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{pmatrix}, \quad B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -3.0 \end{pmatrix}$$

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Suppose each element of uncertainty matrices $\Delta A(t) = 0.05 \sin t$ and $\Delta B(t) = 0.05 \cos t$ are time varying functions. From Lemma 2, one knows that

$$\|\Delta A\| \leqslant \left\| \begin{pmatrix} 0.05 & 0.05 \\ 0.05 & 0.05 \end{pmatrix} \right\| = 0.1$$

and

$$\|\Delta B\| \leqslant \left\| \begin{pmatrix} 0.05 & 0.05 \\ 0.05 & 0.05 \end{pmatrix} \right\| = 0.1$$

It is easy to see that $\rho_A = \rho_B = 0.1$.

According to Theorem 3, the controller is designed as follows:

$$\begin{aligned} u(y) = \frac{1}{2} \begin{pmatrix} 2+\cos y_1(t) & 0 \\ 0 & 2+\sin y_2(t) \end{pmatrix} \Bigg\{ & 2\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - 2\begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{pmatrix} \begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} \\ & - \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -3.0 \end{pmatrix} \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -3.0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \\ & -1.2 \begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - 3\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \Bigg\} \end{aligned} \tag{52}$$

which is independent of time delay. The trajectories of system (51) without controller and with controller (52) are shown in Figures 3 and 4, respectively. It is easy to see that without controller, it is periodic in system (51), but it is stable by adding controller (52). The designed controller (52) is effective and can control the stability of system (51).

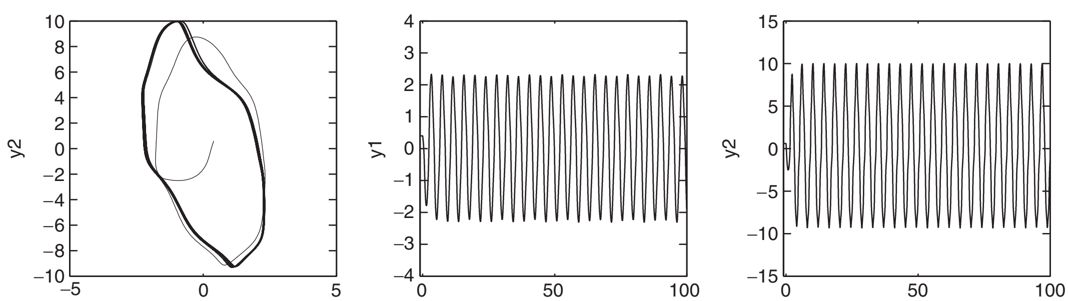

Figure 3. Trajectories of state variables $y_1(t)$ and $y_2(t)$ without control.

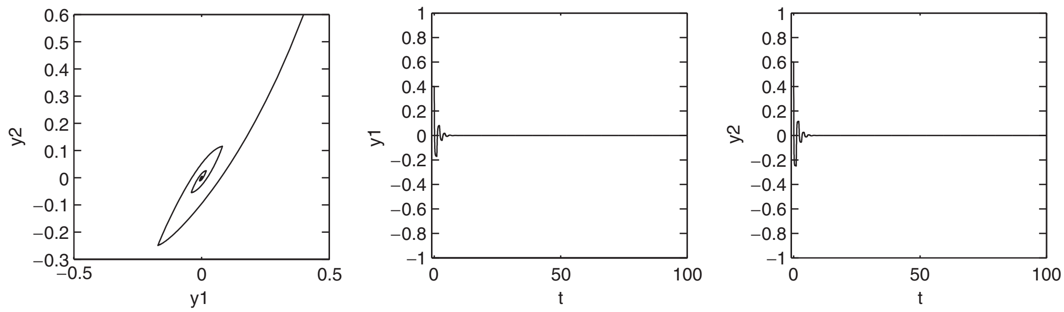

Figure 4. Trajectories of state variables $y_1(t)$ and $y_2(t)$ with control.

Example 3

Consider the following delayed Cohen–Grossberg neural network:

$$\begin{pmatrix} \dfrac{dy_1(t)}{dt} \\ \dfrac{dy_2(t)}{dt} \end{pmatrix} = -\left\{ \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - (A+\Delta A) \begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - (B+\Delta B) \begin{pmatrix} \tanh(y_1(t-1)) \\ \tanh(y_2(t-1)) \end{pmatrix} \right\} \quad (53)$$

where $b_i(y_i(t)) = y_i(t)$ for $i = 1, 2$. It is easy to see that assumptions (A1)–(A2) are satisfied. $a_i(y_i(t)) = 1$ $(i = 1, 2)$ are bounded, positive continuous functions, $f_i(y_i(t)) = \tanh(y_i(t))$ is globally Lipschitz and nondecreasing. Obviously, one has $K = E$, where E is the identity matrix.

$$A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{pmatrix}, \quad B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -8.5 \end{pmatrix}$$

Suppose each element of uncertainty matrices ΔA and ΔB are random in $[-0.2, 0.2]$ every time, since the parameters of system (61) are perturbed by many factors every time. From Lemma 2, one knows that

$$\|\Delta A\| \leqslant \left\| \begin{pmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{pmatrix} \right\| = 0.4$$

and

$$\|\Delta B\| \leqslant \left\| \begin{pmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{pmatrix} \right\| = 0.4$$

It is easy to see that $\rho_A = \rho_B = 0.4$.

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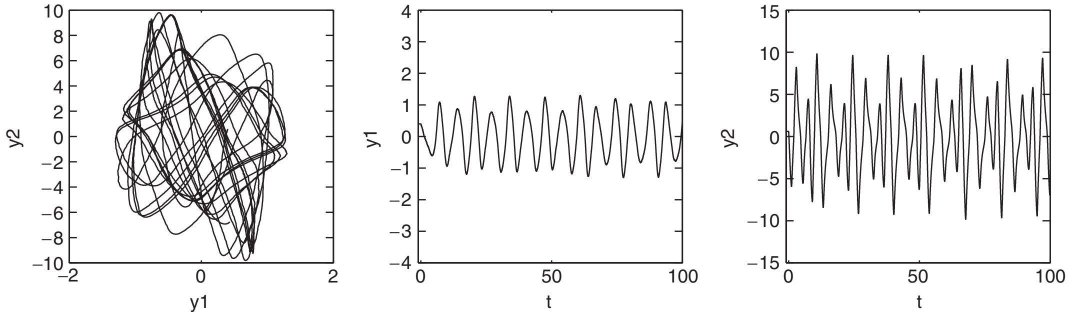

Figure 5. Trajectories of state variables $y_1(t)$ and $y_2(t)$ without control.

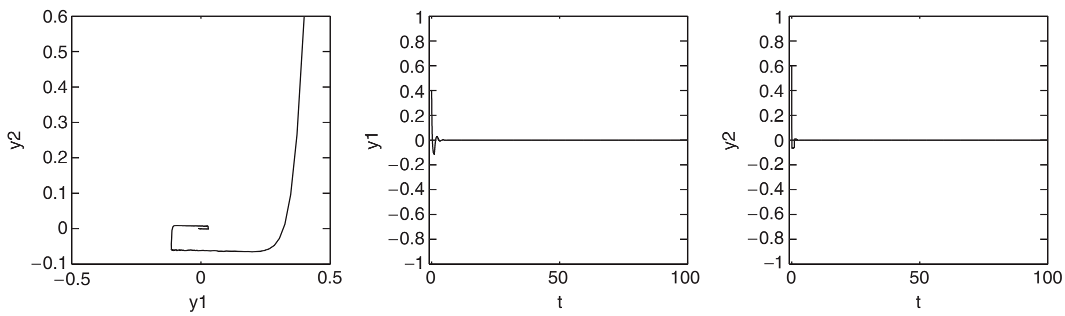

Figure 6. Trajectories of state variables $y_1(t)$ and $y_2(t)$ with control.

According to Theorem 4, the controller is as follows:

$$
\begin{aligned}
u(y) = \frac{1}{2}\Bigg\{ & 2\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - 2\begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 4.5 \end{pmatrix}\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} - \left(\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -8.5 \end{pmatrix}\right. \\
& \left. \times \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -8.5 \end{pmatrix}^{\mathrm{T}}\right)\left(\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix}\right) - 1.8\begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix}\Bigg\} \qquad (54)
\end{aligned}
$$

which is independent of time delay. The trajectories of system (53) without controller and with controller (54) are shown in Figures 5 and 6, respectively. It is easy to see that without controller, it is chaotic in system (53), but it is stable by adding controller (54). The designed controller (54) is effective and can control the stability of system (53). In this example, the perturbation is more larger than Example 1 and simulation result shows that the designed controller can also deal with large perturbation problems.

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5. CONCLUSIONS

In this paper, robust control of delayed Cohen–Grossberg neural networks is investigated. Several effective controllers have been designed to ensure the global asymptotic stability for the delayed neural networks. It is easy to apply these algorithms to the real control system. Finally, some simulation examples are given to justify the obtained results.

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