PERFORMANCE OF FLOW-CONTROLLED COMMUNICATIONS SYSTEMS UNDER BURSTY TRAFFIC*

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ABSTRACT

This study deals with a multi-access queueing/communication system where the arrival process is a doubly stochastic Poisson process with fluctuating arrival rates (bursty traffic). This model is used to evaluate the performance of systems in which the arrival rate is state-dependent and which include finite source, finite buffer and admission flow-controlled queueing and communications systems. Performance results are presented for the z-transform of the steady-state queue size distribution (as well as mean and variance) and for measures of throughput, message delay and blocking probability. Numerical results are presented to demonstrate the effect of traffic burstiness on the flow control mechanism.

1. INTRODUCTION

A multi-access communications channel (or server/processor) shared by a multitude of network stations is considered. Stations can be all locally connected to the communication system, in which case the sharing of the system is dictated by the underlying multiplexing policy. In turn, when stations are geographically distributed, a reservation multiple-access scheme (ref. 5) can be employed. In this case, requests-for-transmission generated by the active stations are queued at the central controller, which grants them transmission slots across the available service channel on a FCFS, or priority-induced, basis.

In many practical situations, the message arrival rate at the system (channel/processor) would be dependent on the system state. This is the case for flow-controlled communications systems whereby the rate of admitted messages is reduced as the system message queue size increases. This is also the case when the arrival message stream is modeled as a finite source process, under which a prescribed finite number of terminals can generate messages, each terminal being able to have, at any time, at most a single message within the system. In the latter case, the offered traffic rate is also state dependent, decreasing as the underlying system queue size increases.

In particular, it is important to employ admission flow control policies when the offered traffic stream is *bursty*, as is typically the case for numerous communication network systems. Many real-life queueing systems exhibit random fluctuation in their arrival rate and, typically, oscillate between periods of heavy and light traffic. This is clearly the case for many digital radio networks, 2-way CATV networks, cellular radio data termination systems, packet radio metropolitan area networks, satellite communication networks, local area networks operating under image/document/batch distribution services, PBX and switching systems, and others. In this paper we model such traffic burstiness by using the *Switched Poisson Arrival Process* (SPAP) model.

The arrival process here (SPAP) is a doubly stochastic Poisson process, where the intensity function $\{\lambda(t),t>0\}$ is itself a stochastic process. In particular, $\lambda(t)$ alternates between two levels, λ_0 and λ_1, corresponding to two arrival modes 0 and 1 respectively, spending a random amount of time during each visit to each of the two arrival modes. We assume that the time spent in arrival mode m $(m=0,1)$ is exponentially distributed with rate $c\alpha_m (m=0,1)$. We set $S=\{S(t),t\geq 0\}$ to be the *mode process* which alternates between two values, 0 or 1.

As observed in ref. 8 and ref. 11 where no admission control policy is employed, one may encounter long queueing delays when the system is subject to fluctuating (bursty) arrival rate processes. In this paper, we examine the effect of introducing admission control policies on such bursty systems. Also, applications to other queueing systems with state-dependent arrivals, such

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as finite source and infinite buffer systems, are studied.

We set λ_{im} to denote the admitted message arrival rate given that the queue size is i; i=0,1,2,..., and the arrival mode is m; m=0,1. Also, we assume that, for a prescribed level K, $\lambda_{im} = \lambda_{Km}$ for $i \geq K$ and m=0,1. For many practical applications, λ_{im}=0 ; $i \geq K$, to ensure acceptable delays for admitted messages. The service time provided by the service channel to each message is exponentially distributed with service rate parameter μ.

For queueing models with state dependent arrival rates but no allowance for fluctuations, see ref. 1 and ref. 2. On the other hand, in refs. 4, 6-11 extraneous mode changes are analyzed in the absence of admission control (i.e., state dependent arrivals).

For the above described model, we analyze the statistical characteristics of the queue size steady state behavior, by first obtaining the z-transform and then the expectation and the variance of the queue size steady state distribution. Using Little's formula, the average message delay can then be calculated. The steady state performance of this queueing system is examined as a function of the switching rate between the arrival modes (as reflected by the parameter c). The purpose of the constant c is to regulate how fast the system changes modes. Intuitively, for the case where the traffic intensity for one mode is greater than 1 (indicating very heavy traffic load during this period) and for the other it is smaller than 1, the smaller c is (i.e., longer average periods between mode transitions) the more nonhomogeneous the arrival process is and, therefore, the queue is more *bursty*. It is shown here that, in this case, the queue size increases rapidly for small c values. Also, blocking probabilities and throughput measures are calculated as functions of c. It is demonstrated here that loss probabilities are increased and throughput levels are decreased as c decreases.

Application of this model to admission flow control and finite-source systems is defined in the following.

Admission Flow Control

We define admission flow control mode dependent functions, $f_0(i)$ and $f_1(i)$, which are used to input-throttle arrivals in mode 0 and mode 1, respectively. The function $f_m(i)$ is defined as the probability of admission of a newly-arriving message, given that there are i messages in the system and that the arrival mode is m. In this case, the mode and state dependent arrival rates are given by

$$\lambda_{im} = \lambda_m f_m(i) \quad i=0,1,2,...; \quad m=0,1 \qquad (1.1)$$

The service time is again taken to be exponentially distributed with parameter μ. Note that a finite buffer system is also a special case of this model (i.e., for a buffer of size K, we have $f_m(i) = 0 \quad i > K$).

We follow with an example of a common admission control configuration. Consider a service resource (e.g., computer) shared by M users, whereby each user (station) is of different priority. The demand (message arrival rate) of the j-th user in mode m is Poisson distributed with parameter $\lambda_m(j)$. Assume that the j-th user can use the system only if the overall queue size is less than or equal to q_j. In this case we have:

$$\lambda_m = \sum_{j=1}^{M} \lambda_m(j) \qquad m=0,1$$

$$f_m(i) = \frac{\sum_{j=1}^{M} I(q_j \leq i)\, \lambda_m(j)}{\lambda_m} \quad m=0,1$$

$$\lambda_{im} = \lambda_m f_m(i) = \sum_{j-1}^{M} I(q_j \leq i)\, \lambda_m(j) \quad m=0,1 \qquad (1.2)$$

where $I(\cdot)$ is defined as the indicator function.

Finite Source

In this case, assuming a finite source of size K (so that each source can have at most a single active message at a time), we set

$$\lambda_{im} = \begin{cases} (K-i)\lambda_m & 0 \leq i < K \quad m=0,1 \\ 0 & i \geq K \quad m=0,1 \end{cases} \qquad (1.3)$$

The service time is exponentially distributed with parameter μ.

We first present a queueing analysis of this model for a general λ_{im} function. We then analyze extreme cases (leading to highly bursty or non-bursty traffic processes). Finally, we present numerical results to illustrate system performance for the applications described above.

2. QUEUEING ANALYSIS

In this section, we carry out steady state queue size analysis. First, we obtain the z-transform of the steady state queue size distribution, and then we derive expressions for the expectation and the variance. Finally, we present expressions for the throughput, the block-

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ing probabilities and the delay distribution. We set $X=\{X(t),t\geq 0\}$ as the queue size process, and $[X,S]=\{X(t),S(t),t\geq 0\}$ as a two-dimensional Markov process, where $X(t)$ represents the queue size (system size) and $S(t)$ the arrival mode at time t, $t\geq 0$.

We denote by π_{im} the steady state probabilities of having $X=i$ and $S=m$ (i.e., queue size i and arrival mode m) $i=0, 1, 2 \cdots$; $m=0, 1$.

The probabilities $\{\pi_{im}\}$ satisfy the following equilibrium equations:

$$0=-(c\alpha_m+\lambda_{0m})\pi_{0m}+c\alpha_{1-m}\pi_{0,1-m}+\mu\pi_{1m} \qquad m=0,1 \tag{2.1}$$

$$0=\lambda_{i-1,m}\pi_{i-1,m}-(\mu+c\alpha_m+\lambda_{im})\pi_{im}+c\alpha_{1-m}\pi_{i,1-m}+\mu\pi_{i+1,m} \qquad i=1,2,\ldots,K;\quad m=0,1; \tag{2.2}$$

$$0=\lambda_{Km}\pi_{i-1,m}-(\mu+c\alpha_m+\lambda_{Km})\pi_{im}+c\alpha_{1-m}\pi_{i,1-m}+\mu\pi_{i+1,m} \qquad i=K+1,K+2,..;\quad m=0,1 \tag{2.3}$$

Define π_i as the steady state probability of the queue size being equal to i; i.e., $\pi_i=\pi_{i0}+\pi_{i1}$, and

$$\Pi_m(z)\triangleq\sum_{i=0}^{\infty}z^i\pi_{im} \qquad m=0,1;\ |z|\leq 1,$$

$$\Pi(z)=\Pi_0(z)+\Pi_1(z)=\sum_{i=0}^{\infty}z^i\pi_i \qquad |z|\leq 1 \tag{2.4}$$

Multiplying the i-th equation of (2.1) through (2.3) by z^i, and summing up, we obtain, after some algebra:

$$\Pi_m(z)=N_m(z)D(z)^{-1} \qquad m=0,1 \tag{2.5}$$

where

$$D(z)=\lambda_{K0}\lambda_{K1}z^3-[\lambda_{K0}\lambda_{K1}+\mu(\lambda_{K0}+\lambda_{K1})+\lambda_{K0}c\alpha_1+\lambda_{K1}c\alpha_0]z^2+\mu(\lambda_{K0}+\lambda_{K1}+c\alpha_0+c\alpha_1+\mu)z-\mu^2 \tag{2.6}$$

$$N_m(z)=A_{1-m}(z)B_m(z)+zB_{1-m}(z)c\alpha_{1-m} \quad m=0,1 \tag{2.7}$$

$$A_m(z)=zc\alpha_m-(1-z)(\mu-z\lambda_{Km}) \quad m=0,1 \tag{2.8}$$

$$B_m(z)=\mu\pi_{0m}+z\sum_{i=0}^{K-1}(\lambda_{im}-\lambda_{Km})\pi_{im}z^i \tag{2.9}$$

Next we present a way to obtain 2K independent linear equations for the 2K parameters appearing in (2.9):

$$\pi_{0m}, \pi_{1m}, \ldots, \pi_{K-1,m}, \qquad m=0,1.$$

Consider the first $2(K-1)$ equations in (2.1) and (2.2), excluding the equations for $i=K-1$ and $i=K$. In addition, we use the equation $\Pi(1)=1$. Thus, altogether, we have $(2K-1)$ independent equations. The last equation will be derived by analyzing the denominator roots. We introduce the following proposition.

Proposition 2.1

Assume

$$\mu>\hat{\lambda}\triangleq\alpha_1(\alpha_0+\alpha_1)^{-1}\lambda_{K0}+\alpha_0(\alpha_0+\alpha_1)^{-1}\lambda_{K1} \tag{2.10}$$

Then, the polynomial $D(z)$ possesses a unique root, denoted by z^*, in the open interval (0,1). Moreover, z^* is the only root within the unit circle, $|z|<1$.

To prove this proposition, we show that $D(0)<0$, $D(1)>0$ and $D(\infty)>0$. Also under $\lambda_{K0}<\lambda_{K1}$, we have that $\mu\lambda_{K0}^{-1}>1$ and that $D(\mu\lambda_{K0}^{-1})<0$. Noticing that $D(z)$ is a 3rd degree polynomial, we conclude that the first root of $D(z)$ lies within (0,1), the second within $(1,\mu\lambda_{K0}^{-1})$ and the third is located within $(\mu\lambda_{K0}^{-1},\infty)$.

Since $\Pi(z)$ is analytic within $|z|<1$, then the root of the denominator must also be a root of the numerator. We denote by z^* the root of $D(z)$ within $|z|<1$; thus, the extra equation needed is: $N(z^*)=0$ where $N(z)\triangleq N_0(z)+N_1(z)$.

Having obtained $2K$ equations to solve for our $2K$ parameters, we are able to obtain the z-transform for the steady state queue size distribution. Then, taking derivatives and setting $z=1$, we obtain the expected value and the variance of the steady state queue size distribution. The expected value is given by $E[X]=\Pi'(1)$ and the variance by $Var[X]=\Pi''(1)+E[X]-E^2[X]$.

The throughput, denoted by TH, is given by

$$TH=\sum_{m=0}^{1}\left\{\left[\sum_{i=0}^{K-1}\lambda_{im}\pi_{im}\right]+\lambda_{Km}\left[\frac{\alpha_{1-m}}{\alpha_0+\alpha_1}-\sum_{i=0}^{K-1}\pi_{im}\right]\right\} \tag{2.11}$$

The blocking probability, denoted by P_B, is readily obtained for the flow control case, to be given by

$$P_B=1-\frac{TH}{\lambda^*} \tag{2.12}$$

where

$$\lambda^*\triangleq\frac{\alpha_0}{\alpha_0+\alpha_1}\lambda_1+\frac{\alpha_1}{\alpha_0+\alpha_1}\lambda_0$$

Note that $f_m(0)=1$; thus, $\lambda_{0m}=\lambda_m$ $m=0,1$, so that λ^* represents the offered traffic rate. Using Little's for-

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mula, the average delay, denoted by $E[D]$, is given by:

$$E[D] = TH^{-1}E[X]$$

3. EXTREME CASES

In this subsection we consider two extreme cases, $c \downarrow 0$ and $c \uparrow \infty$, in order to learn more about the effect of the mode parameter c on system performance. In particular, we study the limiting behavior of $\pi_{im}(c)$ (or $\pi_i(c)$)(i.e., π_{im} or π_i as a function of c, i=0,1,2,...; m=0,1) in the cases $c \downarrow 0$ and $c \uparrow \infty$. The analysis is performed for the single server case but can easily be adjusted to the multi-server case. Before we present the analysis of these extreme cases, we shall introduce some new notations. The traffic intensity for mode m (m=0,1) is denoted by ρ_m (m=0,1) and is defined as $\rho_m \stackrel{\Delta}{=} \lambda_{Km}\mu^{-1}$, m=0,1. The traffic intensity for the whole system is denoted by $\hat{\rho}$ and is defined as $\hat{\rho} = \hat{\lambda}\mu^{-1}$ where $\hat{\lambda}$ is given in (2.10). $\hat{\rho}$<1 ensures stability (i.e., existence of steady state distribution).

Also, define

$$\hat{\lambda}_i = \frac{\alpha_0}{\alpha_0+\alpha_1}\lambda_{i1} + \frac{\alpha_1}{\alpha_0+\alpha_1}\lambda_{i0}$$

Case 1: $c \uparrow \infty$

In this case, very rapid fluctuations occur between the two arrival modes and the time spent in each mode is very small. We show that the steady state behavior for this case is the same as the steady state behavior of a single-server queueing system with service rate μ and state-dependent arrival rate $\hat{\lambda}_i$. Define $\pi_j^* \stackrel{\Delta}{=} \lim_{c\uparrow\infty}\pi_j(c)$.

By dividing (2.1) through (2.3) by c, substituting $\pi_j = \pi_{j0} + \pi_{j1}$, and taking limit on both sides, noting that $\pi_{jm}(c) \le 1$ for all c and each j,m so that $\pi_{jm}(c)c^{-1}$ is approaching 0 as $c \uparrow \infty$, we obtain, after some algebra,

$$0 = \pi^*_0 \hat{\lambda}_0 - \pi^*_1 \mu$$

$$0 = \pi^*_{i-1}\hat{\lambda}_{i-1} - \pi^*_i(\hat{\lambda}_i+\mu) + \pi^*_{i+1}\mu \quad i=1,2,...,K$$

$$0 = \pi^*_{i-1}\hat{\lambda}_K - \pi^*_i(\hat{\lambda}_K+\mu) + \pi^*_{i+1}\mu \quad i=K+1,K+2,...$$

Also, by $\Pi(1) = 1$, we obtain that $\sum_{j=0}^{\infty}\pi^*_j = 1$. We have obtained that π^*_j j=0,1,... is a solution to the set of steady state equations for a single-server queueing system with state dependent arrival rate $\hat{\lambda}_i$ and constant service rate μ. The solution to these sets of equations is well-known. (See, for example, ref. 3, where the π^*_j's and the expectation are obtained.) The throughput is readily obtained by

$$TH = \sum_{i=0}^{\infty}\hat{\lambda}_i\,\pi^*_i$$

Case 2: $c \downarrow 0$

In this case, fluctuations between modes occur very seldom. We shall distinguish between two subcases.

Subcase 2A: ρ_0<1, ρ_1<1

Similarly to the previous case, we can prove the existence of $\lim_{c\downarrow 0}\pi_{im}(c)$, and we define $\pi^* \stackrel{\Delta}{=} \lim_{c\downarrow 0}\pi_{im}(c)$ (m=0,1; i=0,1,2,...). Notice that $\pi_{jm}(c)c$ is $O(c)$. Since $\pi_{jm}(c) \le 1$ for all c and each j,m, we obtain for this case, by (2.1) through (2.3), the following:

$$0 = -\pi^*_{0m}\lambda_{im} + \pi^*_{1m}\mu \qquad m=0,1$$

$$0 = \pi^*_{i-1,m}\lambda_{i-1,m} - \pi^*_{im}(\lambda_{im}+\mu) - \pi^*_{i+1,m}\mu$$
$$i=1,2,...,K \;;\; m=0,1$$

$$0 = \pi^*_{i-1,m}\lambda_{Km} - \pi^*_{im}(\lambda_{Km}+\mu) + \pi^*_{i+1,m}\mu$$
$$i=K+1,K+2,... \;;\; m=0,1$$

We have obtained that π^*_{im} is solved by two separate sets of equations; one set is for the case m=0 involving just the probabilities π^*_{i0}, i=0,1,2,..., and the parameters λ_{i0}; while the other set for the case m=1 involves just the probabilities π^*_{i1}, i=0,1,..., and arrival rates λ_{i1}. Each of these two separate sets is noted to be identical to the steady state set of equations describing a queueing system with state dependent arrivals and with constant service rate μ, for which the solution is known and is given in ref. 3. Note that, since ρ_0<1 and ρ_1<1, each of the two systems has a unique solution.

We conclude that if fluctuations between modes occur very seldom and the traffic intensity for each mode is less than one, the system settles into one of two distinct steady state conditions, one at a time, and each is of the state dependent arrival type. Denote by $E[X_m]$ m=0,1 the steady state queue size expectation for a state dependent arrival queueing system with parameters λ_{im}, m=0,1, respectively. The average queue size for this case is given by

$$E[X] = \alpha_1(\alpha_0+\alpha_1)^{-1}E[X_0] + \alpha_0(\alpha_0+\alpha_1)^{-1}E[X_1]$$

Subcase 2B: ρ_0<1, ρ_1>1, $\hat{\rho}$<1, $c \downarrow 0$

For this case, while the system is in mode 1, the queue-size continuously grows (ρ_1>1). When the system transitions to mode 0, the latter high queue size settles down. We show here that the steady state average queue size approaches infinity hyperbolically in c as

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$c \downarrow 0$.

Under the assumption that $\lambda_1 > \lambda_0$, by (2.10) and the assumptions of this subsection ($\rho_0 < 1$, $\rho_1 > 1$ and $\hat{\rho} < 1$) we have that $0 < \mu\lambda_1^{-1} < 1$. Substituting $z = \mu\lambda_1^{-1}$ into (2.6) we obtain that $D(\mu\lambda_1^{-1}) = O(c)$. Recall that $\Pi(z)$ is analytic and $0 < \Pi(z) \leq 1$, thus $N(\mu\lambda_1^{-1}) = O(c)$ Thus, $\pi_{i1}(c) = O(c)$ $i = 0, \cdots, K$. Then, $B_1(1) = O(c)$ and $B_0(1) = \mu - \hat{\lambda} + O(c) = \mu(1-\hat{\rho}) + O(c)$. We know that $E[X] = [D(1)N'(1) - N(1)D'(1)][D(1)]^{-2}$ and, therefore, after substituting the above results and some algebra, we obtain that $E[X] = c^{-1}[T + O(c)]$ where T is given by $T = (\mu - \lambda_1)(\lambda_0 - \hat{\lambda})(\alpha_0 + \alpha_1)^{-1}(\mu - \hat{\lambda})^{-1}$. Notice that, since $\mu < \lambda_1$, $\lambda_0 < \hat{\lambda}$ and $\mu > \hat{\lambda}$, we have that $T > 0$, and therefore, $E[X]$ approaches infinity as c approaches zero.

4. NUMERICAL RESULTS

In this section we present numerical examples in order to illustrate the effect of the mode parameter c on the performance of state-dependent arrival and admission flow control systems. We also compare the results with those cases in which admission control is not employed. The arrival rates in the admission control examples are set to be given as $\lambda_m = a\bar{\lambda}_m$; $m = 0,1$. We vary the parameter a to express system performance as a function of the offered traffic rate.

First we consider an admission control example with the following data:

$$\alpha_0 = 1 \quad \alpha_1 = 2 \quad \bar{\lambda}_0 = 3 \quad \bar{\lambda}_1 = 12 \quad \mu = 3$$

$$f_m(0) = 1 \quad f_m(1) = 2/3 \quad f_m(i) = 1/3 \quad i \geq 2; \quad m = 0,1$$

In this case, we obtain

$$\lambda_{00} = 3a \quad \lambda_{01} = 12a \quad \lambda_{10} = 2a \quad \lambda_{11} = 8a$$

$$\lambda_{i0} = a, \quad i \geq 2 \qquad \lambda_{i1} = 4a, \quad i \geq 2$$

In Figure 1 we present performance curves for the average queue size versus throughput behavior for different values of c and a. For this example, according to (2.10), the stability condition is $a < 1.5$. For each c level, as a approaches 1.5, we observe the throughput approaching μ (in this case, $\mu = 3$), and the queue size mean and variance approaching infinity, as shown in Table 1, in which we present results for $E[X]$, Var$[X]$ and the throughput levels for different c values and $a = 1.499$.

Table 1: Behavior of Queue Size Moments and Throughput Levels vs. c for $a = 1.4999$.

c	$E[X]$	Var$[X]$	TH
000.002	4.3×10^5	1.8×10^{11}	2.9991
000.010	7.6×10^4	5.8×10^9	2.9992
000.050	1.6×10^4	2.7×10^8	2.9993
000.500	3.0×10^3	9.0×10^6	2.9995
100.000	1.5×10^3	2.3×10^6	2.9999

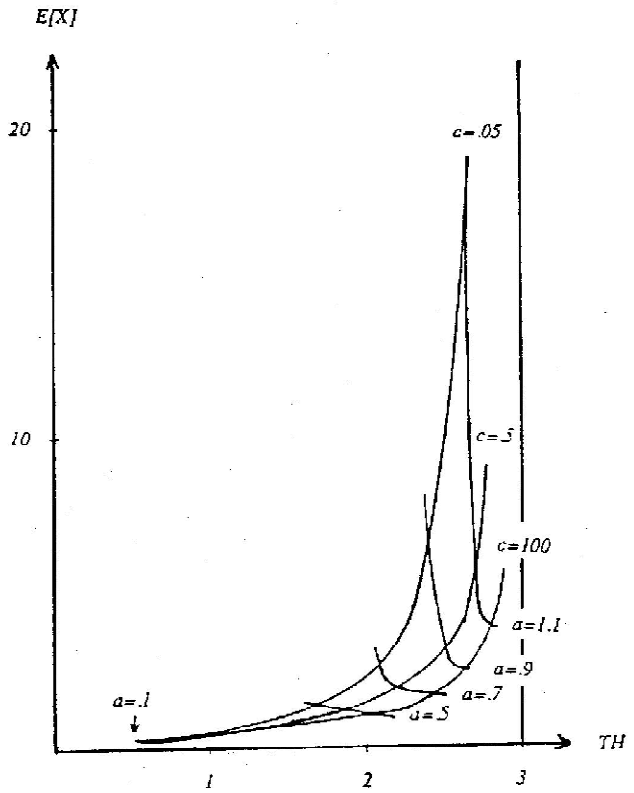

Figure 1: Performance curves for mean queue size vs. throughput for an admission flow control problem with the following data: $\alpha_0 = 1$, $\alpha_1 = 2$, $\bar{\lambda}_0 = 3$, $\bar{\lambda}_1 = 12$ and $\mu = 3$. $f_m(0) = 0$, $f_m(1) = 2/3$, $f_m(i) = 1/3$, $f_m(0) = 0$; $i \geq 2$, $m = 0,1$.

In this example (see Fig. 1) for the case where $\rho_0 < 1$ and $\rho_1 > 1$ (i.e., for $0.75 < a < 1.5$), we have observed that $E[X]$ monotonically decreases as c increases. Also, the throughput monotonically increases as c increases. We can, therefore, conclude that, with the more homogeneous arrival process (larger c values), we obtain better queue-size and throughput performance and lower blocking probabilities. (Note that, according to eq. 2.12 for a fixed a, the blocking probability is inversely linearly proportional to the throughput.)

To illustrate the system performance when admission control policy is not applied, consider the case in which $c = .5$ and $a = .4996667$ (with stability condition $a < .5$). We obtain $E[X] = 2996$ and $TH = 2.998$. By employing the above admission flow control policy, we obtain $E[X] = 5.4$ and $TH = 2.701$, thereby reducing the average queue size to a much lower level while having to somewhat increase the blocking probability level or reduce system throughput. Consider next a case of lower traffic intensity and set $c = .5$ and $a = .3$. In this case, the average queue size is reduced from $E[X] = 2.146$ to $E[X] = .6807$, and the throughput is reduced from 1.8 to 1.335.

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Next, we illustrate the effect of reducing c (i.e., lowering the rate of fluctuation between arrival modes). Consider a case where c is very small, $c = .001$ and $a = .3$. In this case, the above admission flow control policy reduces the queue size from $E[X] = 155.6$ to $E[X] = .3777$ and the throughput from 1.8 to 1.273.

We thus conclude that admission flow control proves most effective in extreme cases, where either c is very small (i.e., the arrival process is extremely nonhomogeneous) or a is very large (high level of average arrival rate).

To illustrate the performance of the system under the conditions of Subcase 2A (i.e., $\rho_0<1$, $\rho_1<1$), we next present an example of a finite source system. (Note that, since the queue size is bounded, we have, in this case, $\rho_0 = \rho_1 = 0$). We consider a system with the following data: $\alpha_0 = 1$, $\alpha_1 = 2$, $K = 3$, $\lambda_0 = 3$, $\lambda_1 = 12$ and $\mu = 2$. Representative results are given in Table 2.

Table 2: Average queue size and throughput for different c and a values for a finite source system with $\alpha_0 = 1$, $\alpha_1 = 2$, $K = 3$, $\lambda_0 = 3$, $\lambda_1 = 12$ and $\mu = 2$.

c	a	$E[X]$	TH
00.05	0.1	0.321	0.506
00.05	0.5	1.239	1.374
00.05	1.1	1.874	1.757
00.50	0.1	0.322	0.519
00.50	0.5	1.316	1.454
00.50	1.1	1.958	1.801
100.00	0.1	0.321	0.536
100.00	0.5	1.420	1.577
100.00	1.1	2.135	1.896

We notice here that the mean queue size and the throughput level increases as c increases (excluding the case of $a = .1$, where the queue size stays constant. As opposed to the case of Subcase 2B (i.e., $\rho_0<1$, $\rho_1>1$ and $\hat{\rho}<1$), in which case the queue size increases as c decreases, we may have a decrease in queue size as c decreases when the traffic intensity for both modes is less than 1.

5. SUMMARY AND CONCLUSIONS

We have carried out performance analysis of a queueing/communication system where the arrival rate fluctuates based on an extraneous process of modes and when an admission flow control procedure is employed. We have obtained expressions for the z-transform of the steady state queue size distribution from which the expectation and variance of the queue size, the system throughput and message blocking probabilities are obtained. Performance analysis of the queueing system under traffic activity limiting (extreme) conditions has been carried out. We demonstrate that the rate of fluctuations between the arrival modes, as reflected by the value assumed by the parameter c, can lead to higher queue size and blocking probability levels when flow control schema are employed as compared with the behavior of the corresponding flow-controlled system with nonfluctuating offered traffic processes. Our analysis permits the system designer to select the appropriate admission flow control function under designated traffic burstiness levels so that desired message delay and throughput/blocking probability values are obtained.

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