CHAPTER 4

Problems

P4.1 The time constant is given by \( \tau = RC \). Thus to attain a long time constant, we need large values for both \( R \) and \( C \).

P4.2 The leakage resistance must be greater than 11.39 M\( \Omega \).

P4.3

\[

v_c(t) = 100 - 100 \exp\left(-\frac{t}{10^{-3}}\right)

\]

P4.4*

\[

v_c(t) = 100 - 150 \exp\left(-\frac{t}{10^{-3}}\right)

\]
P4.5  
(a)  
\[ v_c(t) = \begin{cases} 
50 & \text{for } t < 0 \\
50 \exp(-50t) & \text{for } t > 0 
\end{cases} \]
\[ v_r(t) = \begin{cases} 
0 & \text{for } t < 0 \\
50 \exp(-50t) & \text{for } t > 0 
\end{cases} \]

(b)  
\[ p_r(t) = 2500 \exp(-100t) \mu W \]

(c)  
\[ W = 25 \mu J \]

(d) The initial energy stored in the capacitance is  
\[ W = 25 \mu J \]

P4.6  
\[ t_{\text{half}} = 0.6931 \tau \]

P4.7  
\[ v(t) = 10 - 10 \exp(-100t) \text{ for } t > 0 \]

P4.8*  
\[ R = 4.328 \, \text{M}\Omega \]

P4.9*  
\[ t_2 = 0.03466 \, \text{seconds} \]

P4.10  The current through the resistance is  
\[ i_r(t) = \frac{V}{R} \exp(-t/RC) \]

P4.11  In dc steady-state analysis, we replace capacitances with open circuits and inductances with short circuits.
P4.12
\[ i_4 = \frac{(100 \, V)}{(1 \, k\Omega)} = 100 \, mA \]
\[ i_3 = 0 \]
\[ i_2 = \frac{(100 \, V)}{(1 \, k\Omega)} = 100 \, mA \]
\[ i_4 = i_2 + i_3 + i_4 = 200 \, mA \]
\[ v_C = 100 \, V \]

P4.13* In steady state,
\[ i_1 = 0 \]
\[ i_3 = i_2 = 2 \, A \]

P4.14 Prior to \( t = 0 \), \( v_c = 25 \, V \).
A long time after \( t = 0 \), \( v_c = 15 \, V \).

P4.15* \[ v_c(t) = 10 - 10 \exp(-100t) \]
\[ t = 46.05 \, ms \]

P4.16 \( \tau = L / R \). Thus to attain a long time constant, we need a large value for \( L \) and a small value for \( R \).

P4.17 \[ i(t) = 0.5 - 0.5 \exp(-200t) \]
\[ v_L(t) = 100 \exp(-200t) \]
**P4.18**

\[ i_L(t) = \begin{cases} 
0 & \text{for } t < 0 \\
0.1 - 0.1\exp(-10^6 t) & \text{for } t > 0
\end{cases} \]

The voltage is:

\[ v(t) = \begin{cases} 
0 & \text{for } t < 0 \\
100\exp(-10^6 t) & \text{for } t > 0
\end{cases} \]

**P4.19**

The solution is similar to that for Problem P4.18.

\[ i_L(t) = 0.1 - 0.3\exp(-10^6 t) \text{ for } t > 0 \]

\[ v(t) = 300\exp(-10^6 t) \text{ for } t > 0 \]
P4.20

\[ i(t) = \begin{cases} 1 & t < 0 \text{ (switch open)} \\ 4 - 3\exp(-12.5t) & t \geq 0 \text{ (switch closed)} \end{cases} \]

P4.21

(a) \[ i(t) = I_i \exp(-Rt/L) \text{ for } t \geq 0 \]

(b) \[ p_R(t) = R i^2(t) = R(I_i)^2 \exp(-2Rt/L) \text{ for } t \geq 0 \]

(c) \[ W = \frac{1}{2} L(I_i)^2 \]

which is precisely the expression for the energy stored in the inductance at \( t = 0 \).

P4.22*

\[ i(t) = \begin{cases} 0 & t < 0 \\ 1 - \exp(-20t) & t \geq 0 \end{cases} \]

P4.23

In steady state with the switch closed, the current is \( i(t) = 2 \) A for \( t < 0 \).

The resistance of a voltmeter is very high -- ideally infinite. Thus, there is no path for the current in the inductance when the switch opens, and \( di/dt \) is very large in magnitude at \( t = 0 \). Consequently, the voltage induced in the inductance is very large in magnitude and an arc occurs across the switch. With an ideal meter and switch, the voltage would be infinite. The voltmeter could be damaged in this circuit.
\[ P4.24^* \quad R \leq 399.6 \mu \Omega. \]

\[ P4.25 \quad v_c(t) = -RC + t + RC \exp(-t/RC) \]

\[ P4.26 \]

(a) \( i(0^+) = 1 \) mA

(b) Applying KVL, we have
\[
-v_1(t) + Ri(t) + v_2(t) = 0
\]
\[
\frac{1}{C_1} \int_0^t i(t) dt - 100 + \frac{1}{C_2} \int_0^t i(t) dt + 0 = 0
\]
Taking a derivative with respect to time and rearranging, we obtain
\[
\frac{di(t)}{dt} + \frac{1}{R} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) i(t) = 0 \quad (1)
\]

(c) The time constant is \( \tau = 50 \) ms.

(d) \( i(t) = \exp(-20t) \) mA.

(e) The final value of \( v_2(t) \) is
\[
v_2(\infty) = 50 \text{ V}
\]

\[ P4.27^* \quad \text{Applying KVL, we obtain the differential equation:}
\]
\[
L \frac{di(t)}{dt} + Ri(t) = 5 \exp(-t) \text{ for } t > 0 \quad (1)
\]
The current is given by
\[
i(t) = -\exp(-t) + \exp(-Rt/L) \text{ for } t \geq 0.
\]
P4.28 Applying KVL to the circuit, we have
\[ L \frac{di(t)}{dt} + Ri(t) = v(t) \]
\[ 2 \frac{di(t)}{dt} + 10i(t) = 10t \]
The solution is
\[ i(t) = 0.2 \exp(-5t) - 0.2 + t \]

P4.29 Using KVL, we obtain the differential equation
\[ L \frac{di(t)}{dt} + Ri(t) = v(t) \]
\[ \frac{di(t)}{dt} + 300i(t) = 10 \sin(300t) \]
The solution for the current in amperes is
\[ i(t) = \frac{1}{60} \exp(-300t) - \frac{1}{60} \cos(300t) + \frac{1}{60} \sin(300t) \]

P4.30 First we write the differential equation for the system and put it in the form
\[ \frac{d^2x(t)}{dt^2} + 2 \alpha \frac{dx(t)}{dt} + \omega_0^2(t) = f(t) \]
Then compute the damping ratio \( \zeta = \alpha / \omega_0 \).

If we have \( \zeta < 1 \), the system is underdamped and the complementary solution is of the form
\[ x_c(t) = K_1 \exp(-\alpha t) \cos(\omega_n t) + K_2 \exp(-\alpha t) \sin(\omega_n t) \]
in which \( \omega_n = \sqrt{\omega_0^2 - \alpha^2} \).

If we have \( \zeta = 1 \), the system is critically damped and the complementary solution is of the form
\[ x_c(t) = K_1 \exp(s_1 t) + K_2 t \exp(s_1 t) \]
in which \( s_1 \) is the root of the characteristic equation \( s^2 + 2\alpha s + \omega_0^2 = 0 \).

If we have \( \zeta > 1 \), the system is overdamped and the complementary solution is of the form
\[ x_c(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t) \]
in which \( s_1 \) and \( s_2 \) are the roots of the characteristic equation \( s^2 + 2\alpha s + \omega_0^2 = 0 \).
P4.31 We look at a circuit diagram and combine all of the inductors that are in series or parallel. Then we combine all of the capacitances that are in series or parallel. Next we count the number of energy storage elements (inductances and capacitances) in the reduced circuit. If there is only one energy-storage element, we have a first-order circuit. If there are two, we have a second-order circuit and so forth.

P4.32 The unit step function is defined by
\[
 u(t) = \begin{cases} 
 0 & \text{for } t < 0 \\
 1 & \text{for } t \geq 0 
\end{cases}
\]
The unit step function is illustrated in Figure 4.27 in the book.

P4.33 The sketch should resemble the response shown in Figure 4.29 for \( \zeta = 0.1 \). Second-order circuits that are severely underdamped (\( \zeta \ll 1 \)) have step responses that display considerable overshoot and ringing.

P4.34 One way to determine the particular solution is to assume that it is a constant \( \mathcal{X}_p(t) = K \), substitute into the differential equation, and solve for \( K \).

A second method is to replace the inductors by short circuits the capacitances by open circuits and solve for the steady-state dc response.

P4.35* Applying KVL to the circuit, we obtain
\[
 L \frac{di(t)}{dt} + Ri(t) + v_c(t) = v_s = 50
\]
Finally, the solution is
\[
 v_c(t) = 50 - 53.87 \exp(s_1 t) + 3.867 \exp(s_2 t) 
\]

P4.36* \( v_c(t) = 50 - 50 \exp(s_1 t) - (50 \times 10^4) t \exp(s_1 t) \)

P4.37* \( v_c(t) = 50 - 50 \exp(-\alpha t) \cos(\omega_n t) - (28.86) \exp(-\alpha t) \sin(\omega_n t) \)
(a) Damping coefficient is \[ a = \frac{1}{2RC} = 20 \times 10^6 \]

Undamped resonant frequency \[ \omega_0 = \frac{1}{LC} = 10 \times 10^6 \]

Damping ratio \[ \zeta = \frac{a}{\omega_0} = 2 \]

(b) \( \nu'(0+) = 1/C = 10^9 \)

(c) The particular solution for \( \nu(t) \) is: \( \nu_p(t) = 0 \)

(d) \( \nu(t) = 28.87 \exp(s_1 t) - 28.87 \exp(s_2 t) \)

(a) Damping coefficient is \[ a = \frac{1}{2RC} = 10 \times 10^6 \]

Undamped resonant frequency: \[ \omega_0 = \frac{1}{LC} = 10 \times 10^6 \]

Damping ratio: \[ \zeta = \frac{a}{\omega_0} = 1 \]

(b) \( \nu'(0+) = 1/C = 10^9 \)

(c) The particular solution for \( \nu(t) \) is: \( \nu_p(t) = 0 \)

(d) \( \nu_c(t) = 10^9 t \exp(-10^7 t) \)
P4.40 (a) Damping coefficient is
\[ a = \frac{1}{2RC} = 10^6 \]
Undamped resonant frequency:
\[ \omega_0 = \frac{1}{LC} = 10 \times 10^6 \]
Damping ratio:
\[ \zeta = \frac{\alpha}{\omega_0} = 0.1 \]
(b) \( \nu'(0+) = \frac{1}{C} = 10^9 \)
(c) The particular solution for \( \nu(t) \) is:
\[ \nu_p(t) = 0 \]
(d) \( \nu(t) = 100.5 \exp(-at) \sin(\omega_0 t) \)