Multiple-Stage Switch/Switching Network/Link Systems
Learning Objectives

• Why multiple-stage switch/switching network/link system is needed for a large network?
• What is the definition/function of three different kinds of switches (concentrator, route switch/router and expander)?
• What is the big picture of telephone network with concentrator, route switch and expander?
• What is the connection requirement for concentrator, route switch and expander respectively (i.e. under different modes/requirements)?
• How to design a link system and compute its cost (i.e. number of crosspoints)?
• Why three-stage switching network is better than two-stage switching network to be used as a router or an expander?
• How to derive and compute the blocking probabilities of a multiple-stage switch under different modes/requirements?
• Why the expansion is needed? How to use it and compute its performance?
Outline

• Two Switching Techniques
• Crossbar (One-Stage) Switch
• Multiple-Stage Switch/ Switching Network/ Link System
• Types of Switches
• Switch Design
• Grades of Service
• Expansion
Two Switching Techniques

• Space-Division Switching
  – Used in old days
  – Carry analog voice signal

• Time-Division Switching
  – Used nowadays
  – Carry digital voice signal
Space-Division Switching

• Signal paths are physically separated from one another (i.e. divided in space)

• Each connection requires the establishment of a physical path through the switch connecting two endpoints

• The basic unit is a metallic crosspoint (or semiconductor gate) which is enabled or disabled the connection by a control unit
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Crossbar Space-division Switch
Features

- 10 inputs and 10 outputs in this example switch
- Each station is attached to the switch via one input (line) and one output (line)
- A connection is made possible by enabling the appropriate crosspoint
- Advantage: no internal blocking
Internal Blocking

• The blocking is due to lack of resources (e.g. links) inside the switch rather than congestion of the external trunks
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Limitations of Crossbar Switch

• Number of crosspoints increases rapidly as square of number of stations
• Loss of crosspoint prevents connection
• Inefficient utilization of crosspoints (e.g. even when all the stations are active, only a small percentage of the crosspoints are engaged)
Solution

• Multiple-stage switch:
  – reduce the number of crosspoints required
  – increase the number of paths between two endpoints
    => increase reliability

• However, it introduces more control effort and internal blocking (i.e. unable to connect two stations because all possible paths are already in use)
Three Stage Switch

FIRST STAGE

SECOND STAGE

THIRD STAGE

5 × 2 switch

2 × 2 switch

2 × 5 switch

5

6

7

8

9

10

1

2

3

4

5

6

7

8

9

10
Multiple-stage switch

Multiple-stage switch may have any no. of stages and any possible patterns of links between stages

Terminology

Switch module, inlet, outlet, incoming trunk, outgoing trunk, link
Public Switched Telephone Network (PSTN)
Four Major Components of PSTN:

- **Subscribers**: the devices (e.g. telephones) which are attached to the PSTN
- **Local/Subscriber loop**: the connection between the subscriber and the PSTN
- **Exchanges**: the multiple-stage switches in the network, e.g. local and core switches
- **Trunks**: the connections between exchanges
One two-stage switching network (Fig 5.11)

• There is (only) one link from each secondary switch (module) to each primary switch (module)
• The no. of links is set to the no. of incoming/outgoing trunks (Why?)
• In practice, switch units of fixed sizes are often used, e.g. 8X8 (Bell) or 5X5 (British Telecom)
• For large networks, it becomes more economic to use networks with more than two stages
Two-stage switching network

Figure 5.11 Two-stage switching network.
One three-stage switching network (Fig. 5.13)

- There is one link from each primary switch (module) to each secondary switch (module) and one link from each secondary switch to each tertiary switch.
- The no. of primary-secondary links ($A$ links) and secondary-tertiary links ($B$ links) are each set to the no. of incoming/outgoing trunks.
Three-stage switching networks

Figure 5.13  Fully interconnected three-stage switching network.
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Types of (Multiple-Stage) Switches

The switches can serve as

• **Concentrator** (Mode 3): connection may be made to any free outgoing trunk

• **Route Switch/Router** (Mode 2): connection is required to a particular outgoing route, but any free outgoing trunk on that route may be used

• **Expander** (Mode 1): connection is required to one particular free outgoing trunk
A big picture of telephone network with concentrator, router and expander
Two-stage switch (Fig. 3.15)

Figure 3.15 Two-stage link network (using switches of size $10 \times 10$ to interconnect 100 incoming and 100 outgoing trunks).
Two-stage switch (Cont’ed)

**Expander**: a connection can only be made from a given incoming trunk to a selected outgoing trunk if the link is available (not being used for another connection) between that primary switch and that secondary switch.
Two-stage switch (Cont’ed)

Route Switch:

• The router servers ten outgoing routes with ten trunks on each route, then trunk no. 1 of each route is connected to secondary switch no. 1, trunk no. 2 is connected to switch no. 2, and so on.

• An incoming trunk can obtain a connection to the selected outgoing route via any of the links outgoing from its primary switch

• The call is lost only if all the paths associated to the selected outgoing route are blocked
Two-stage switch (Cont’ed)

Concentrator:

• An incoming trunk can use any outgoing trunk via any free link from its primary switch.

• If there are as many links as outgoing trunks, a connection can always be made if there is a free outgoing trunk (i.e. no internal blocking)
Three-stage switch

Expander

• A connection can be made through a path from a given inlet on a primary switch to a selected outlet on a tertiary switch via any secondary switch, unless the path associated that secondary switch is busy.

• The call is blocked if all such paths are busy/blocked simultaneously.

• Remark: The probability of being unable to set up a connection because of blocking is thus much less than for a two-stage switching network. Therefore, the three-stage switching network is suitable for operation as an expander.
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Designing a two-stage switch

Figure 5.11 Two-stage switching network.
Designing a two-stage switch (cont’d)

No. of primary switches (SWs) = no. of secondary SWs = no. of outlets per primary SW => \( g = \frac{N}{n} \)

No. of crosspoints (CPs) per primary SW = no. of CPs per secondary SW = \( gn = N \)

Total no. of CPs in the network = (no. of SWs) \( \times \) (CPs per SW), i.e. \( C_2 = 2gN = 2N^2/n \)
Designing a two-stage switch (cont’d)

Since there is one link from each primary SW to each secondary switch, the number of links is equal to no. of primary SWs \( \times \) no. of secondary SWs, i.e.

No. of links = \( g^2 = (N/n)^2 \)
Designing a two-stage switch (cont’d)

Let no. of links = no. of incoming/outgoing trunks $\Rightarrow g^2 = N$

$\Rightarrow n = N^{1/2}$

Then, total no. of CPs is

$C_2 = 2N^{3/2}$
Example 5.3

Design a two-stage switching network for connecting 200 incoming trunks to 200 outgoing trunk.

\[ n = (200)^{1/2} = 14.14. \] However, \( n \) must be a factor of 200, so the nearest practicable values are \( n = 10 \) and \( n = 20 \).
Example 5.3 (cont’d)

(a) $n = 10$: 20 primary SWs of size $10 \times 10$
10 secondary SWs of size $20 \times 20$

$g_1 = 20 \quad g_2 = 10$

No. of crosspoints = $20 \times 10 \times 10 + 10 \times 20 \times 20 = 6,000$

Asymmetry!!
Example 5.3 (cont’d)

(b) \( n = 20 \): 10 primary SWs of size \( 20 \times 20 \)
20 secondary SWs of size \( 10 \times 10 \)

\[ g_1 = 10 \quad g_2 = 20 \]

No. of crosspoints = \( 10 \times 20 \times 20 + 20 \times 10 \times 10 = 6,000 \)
Example 5.3 (cont’d)

(c) \( n = 10 \): 20 primary SWs of size \( 10 \times 20 \)
20 secondary SWs of size \( 20 \times 10 \)

\[
\begin{align*}
&10 \times 20 & 20 \times 10 \\
&10 \times 20 & 20 \times 10
\end{align*}
\]

\( g_1 = 20 \quad g_2 = 20 \)

No. of crosspoints = \( 20 \times 10 \times 20 + 20 \times 20 \times 10 = 8,000!! \)

\( \Rightarrow \) Trade off between symmetry and no. of crosspoints

Symmetry!!

400 links!!
Designing a three-stage switch

Figure 5.13 Fully interconnected three-stage switching network.
Designing a three-stage switch (cont’d)

No. of primary SWs \((g_1)\) = no. of tertiary SWs \((g_3)\) = \(N/n\)

The secondary SWs have \(N/n\) inlets and outlets

No. of secondary SWs is \(g_2 = N/(N/n) = n\) = no. of outlets per primary SW = no. of inlet per tertiary SW.
Designing a three-stage switch (cont’d)

No. of CPs in primary stage = \( n^2(N/n) = nN \)
No. of CPs in secondary stage = \( n(N/n)^2 = N^2/n \)
No. of CPs in tertiary stage = \( n^2(N/n) = nN \)
Total no. of CP is \( C_3 = N(2n+N/n) \)
Differentiating it with respect to \( n \) and equating to zero, we have \( n = (N/2)^{1/2} \)
Designing a three-stage switch (cont’d)

And then

\[ C_3 = 2^{3/2} N^{3/2} = 2^{1/2} \]

\[ C_2 = 2^{3/2} N^{-1/2} C_1 \]
Example 5.4

Design a three-stage network for connecting 100 incoming trunks to 100 outgoing trunks:

\[ n = (100/2)^{1/2} = 7.07 \Rightarrow n = 5 \text{ or } n = 10 \]

**Solution 1: \( n = 5 \)**

- 20 primary SWs of size 5×5
- 5 secondary SWs of size 20 ×20
- 20 tertiary SWs of size 5×5
Example 5.4 (cont’d)

Solution 2: \( n = 10 \)

All primary, secondary and tertiary switches, each of size \( 10 \times 10 \).

Both networks contain 3,000 crosspoints. However, the second design has more secondary switches. It therefore provides a greater number of paths between an incoming and an outgoing trunk and will exhibit less blocking.
Four-stage switch design

We have

\[ C_4 = 4 \, N^{4/3} = 2^{1/2} \, N^{-1/6} \quad C_3 = 2 \, N^{-1/6} \, C_2 \]
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Grades of service of link systems

Independence Assumption

- Trunks and links being busy constitute independent random events
- If two random events are independent, the probability of both happening at the same time is given by the product of their separate probabilities of occurrence at that time.
Examples for Independence Assumption

If two links are to be connected in tandem, and the probability of one being busy is $a$ and of the other being busy is $b$, then the probabilities of each being free are $1-a$ and $1-b$, respectively, so the probability of both being free is $(1-a)(1-b)$. Therefore, the probability of the path being blocked is $1 - (1-a)(1-b)$. 
The unit of traffic - **Erlang**

- On a group of trunks, the average number of calls in progress depends on both the no. of calls which arrive and their duration (or call holding time).
- The traffic in **Erlangs** is equal to the mean no. of calls arriving during a period equal to the mean call holding time.
Example for Erlang

Assume

• Call arrival rate = 30 calls per minute
• Call duration = 3 minutes

Then,

• Traffic load = Call arrival rate \times \text{Call duration} = 90 \text{ Erlangs}
Occupancy, carried traffic and offered traffic

- **Occupancy** for a link/trunk at each stage is the system carried traffic divided by the no. of links/trunks at that stage.
- If the loss is small (as it should be), the carried traffic can be well approximated by the offered traffic.
Two-stage switching network

![Diagram of a two-stage switching network](image)

**Figure 5.11** Two-stage switching network.
Two-stage switching network (1)

• Let occupancy of links = \( a \)
  occupancy of outgoing trunks = \( b \)

• For an **expander**: the blocking
  \( B_1 = a \)

• For a **route switch**: the blocking
  \( B_2 = [1 - (1 - a)(1 - b)]^{g_2} \)
  where \( g_2 \) is the no. of secondary switches
Two-stage switching network (2)

For an **Concentrator**: the blocking can be approximated by **Erlang Loss Formula** with parameter $\rho = bN$ and $K = N$:

$$P_B = \frac{\rho^K}{K!} \frac{\rho^K}{\sum_{k=0}^{K} \frac{\rho^k}{k!}}$$
Figure 3.15 Two-stage link network (using switches of size $10 \times 10$ to interconnect 100 incoming and 100 outgoing trunks).
Example 5.6

1. Find the grade of service when a total of 30 E is offered to the two-stage switching network of Figure 3.15 and the traffic is evenly distributed over the 10 outgoing routes.

The link and trunk occupancies are \( a = b = 30/100 = 0.3 \ E \).

\[
B_2 = [1 - (1 - 0.3)(1 - 0.3)]^{10} = 0.51^{10} = 0.0012
\]
Example 5.6 (cont’d)

2. Find the traffic capacity of this network if the grade of service is not to exceed 0.01.

\[ B \leq 0.01 = [1 - (1 - a)^2]^{10} \]

\[ 1 - (1 - a)^2 \leq 0.01^{0.1} = 0.631 \]

\[ a \leq 0.39 \quad \text{and} \quad A \leq 39 \quad \text{E} \]
Three-stage switching networks

Figure 5.13  Fully interconnected three-stage switching network.
Three-stage switching networks

- Occupancy of $A$ links = $a$
  Occupancy of $B$ links = $b$
  Occupancy of outgoing trunks = $c$.

- For an expander, probability that all $g_2$ independent paths are simultaneously blocked is

$$B_1 = [1 - (1 - a)(1 - b)]^{g_2}$$

where $g_2$ is the no. of secondary switches.
Three-stage switching networks (cont’d)

For a route switch (i.e. a connection to any free trunk in a particular route):

Probability of blocking for a particular trunk

\[ B_1 = 1 - (1 - B_1)(1 - c) \]

Probability of simultaneous blocking for all \( g_3 \) independent paths is

\[ B_2 = \left[ B_1 + c(1 - B_1) \right]^{g_3} \]

where \( g_3 \) is the no. of tertiary switches.
Example 5.7

1. Compare the grades of service provided by the two networks of Example 5.4 when each operates as an expander and is offered 30 E of traffic:

   \[ a = b = \frac{30}{100} = 0.3 \text{ E} \]

   For network (a): \( g_2 = 5 \)
   \[ B_1 = \left[ 1 - (1 - 0.3)(1 - 0.3) \right]^5 = 0.51^5 = 0.035 \]

   For network (b): \( g_2 = 10 \)
   \[ B_1 = 0.51^{10} = 0.0012 \]
Example 5.7 (cont’d)

2. What is the traffic capacity of each network if the required grade of service is 0.01?

For network (a): \[ 1 - (1 - a)^2 \]^{5} = 0.01 \Rightarrow a = 0.224

Total traffic capacity = 100 x 0.224 = 22.4 E

For network (b): \[ 1 - (1 - a)^2 \]^{10} = 0.01 \Rightarrow a = 0.393

Total traffic capacity = 100 \times 0.393 = 39.3 E
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Use of expansion

• The blocking loss of a network can be reduced by increasing the connectivity by providing more links between switching stages than there are outgoing trunks. This is known as expansion.

• Expansion is not used in local exchanges. Indeed, because customers’ line are lightly loaded.

• Tandem exchanges do not use expansion either because an adequately low grade of service is obtained without it.
Use of expansion (cont’d)

• **Long-distance circuits**, particularly international ones, are expensive; however, trunks/links within an exchange (and their associated switches) are relatively cheap.

• It is undesirable for expensive circuits to be idle (and losing revenue) because of blocking within an exchange. Expansion is therefore used in trunk-transit and international exchanges to ensure that loss due to blocking is much less than loss due to congestion of outgoing routes.
Example 5.8

A fully interconnected three-stage network has 100 incoming trunks, 100 $A$ links, 100 $B$ links and 100 outgoing trunks. At each stage, it uses ten switches of size 10 x 10. As shown in Example 5.7, the grade of service as an expander is 0.01 when the link occupancy is 0.39 E.

For the same total offered traffic, what grade of service is obtained if the numbers of links and secondary switches are increased by (1) 20%, (2) 50%?
Example 5.8 (Cont’d)

1. The link occupancy of the modified network is
   \( a = b = 0.39 \times \frac{100}{120} = 0.325 \text{ E} \)
   From equation (5.26), the grade of service is:
   \( B_1 = [1 - (1 - 0.325)^2]^{12} = 0.544^{12} = 6.7 \times 10^{-4} \)

2. The link occupancy of the modified network is
   \( a = b = 0.39 \times \frac{100}{150} = 0.26 \text{ E} \)
   From equation (5.26), the grade of service is:
   \( B_1 = [1 - (1 - 0.26)^2]^{15} = 0.452^{15} = 6.7 \times 10^{-6} \)