Question 1

Consider the procedure described in VoIP’s notes for estimating average delay $d_i$. Suppose that $u = 0.1$. Let $r_1 - t_1$ be the most recent sample delay, let $r_2 - t_2$ be the next most recent sample delay, and so on.

a) For a given audio application suppose five packets have arrived at the receiver with sample delays $r_5 - t_5$, $r_4 - t_4$, $r_3 - t_3$, and $r_2 - t_2$, and $r_1 - t_1$. Express the estimate of delay $d$ in terms of the five samples. [7 marks]

b) Generalize your formula for $n$ sample delays. [7 marks]

c) For the formula in Part b, let $n$ approach infinity and give the resulting formula. Comment on why this averaging procedure is called an exponential moving average. [6 marks]

a) Denote $d^{(n)}$ for the estimate after the $n$th sample.

\[
d^{(1)} = r_5 - t_5
\]
\[
d^{(2)} = u(r_4 - t_4) + (1-u)(r_5 - t_5)
\]
\[
d^{(3)} = u(r_3 - t_2) + (1-u)[u(r_3 - t_3) + (1-u)(r_4 - t_4)]
\]
\[= u(r_3 - t_3) + (1-u)u(r_4 - t_4) + (1-u)^2(r_5 - t_5)
\]
\[
d^{(4)} = u(r_2 - t_2) + (1-u)d^{(3)}
\]
\[= u(r_2 - t_2) + (1-u)u(r_3 - t_3) + (1-u)^2u(r_4 - t_4) + (1-u)^3(r_5 - t_5)
\]
\[
d^{(5)} = u(r_1 - t_1) + (1-u)d^{(4)}
\]
\[= u(r_1 - t_1) + (1-u)u(r_2 - t_2) + (1-u)^2u(r_3 - t_3) + (1-u)^3u(r_4 - t_4) + (1-u)^4(r_5 - t_5)
\]

b) 
\[
d^{(n)} = u \sum_{j=1}^{n-1} (1-u)^{j-1}(r_j - t_j) + (1-u)^{n-1}(r_n - t_n)
\]

c) 
\[
d^{(\infty)} = \frac{u}{1-u} \sum_{j=1}^{\infty} (1-u)^{j-1}(r_j - t_j)
\]
\[= \frac{1}{9} \sum_{j=1}^{\infty} 9^{j}(r_j - t_j)
\]

The weight given to past samples decays exponentially.
Question 2

Consider the figure below. A sender begins sending packetized audio periodically at $t = 1$. The first packet arrives at the receiver at $t = 8$.

a) What are the delays (from sender to receiver, ignoring any playout delays) of packets 2 through 8? Note that each vertical and horizontal line segment in the figure has a length of 1, 2, or 3 time units. [4 marks]

b) If audio playout begins as soon as the first packet arrives at the receiver at $t = 8$, which of the first eight packets sent will not arrive in time for playout? [4 marks]

c) If audio playout begins at $t = 9$, which of the first eight packets sent will not arrive in time for playout? [4 marks]

d) If audio playout begins at $t = 10$, which of the first eight packets sent will not arrive in time for playout? [4 marks]

e) What is the minimum playout delay at the receiver that results in all of the first eight packets arriving in time for their playout? [4 marks]

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a) The delay of packet 2 is 7 slots. The delay of packet 3 is 9 slots. The delay of packet 4 is 8 slots. The delay of packet 5 is 10 slots. The delay of packet 6 is 9 slots. The delay of packet 7 is 8 slots. The delay of packet 8 is 8 slots.

b) Packets 3, 4, 5, 6, 7, and 8 will not be received in time for their playout if playout begins at $t = 8$.

c) Packets 3, 5 and 6 will not be received in time for their playout if playout begins at $t = 9$.

d) Packet 5 will not be received in time for their playout if playout time begins at $t = 10$.

e) The minimum playout delay at the receiver that results in all of the first eight packets arriving in time for their playout is 11.
**Question 3**

(a) Design a fully interconnected three-stage network with 1,250 incoming and 1,250 outgoing trunks with minimum number of crosspoints. How many crosspoints does the network contain? Note that \( n = (N/2)^{0.5} \).

\[ N = 1,250, \quad n = 25 \]
\[ g_2 = n = 25 \]
\[ C_3 = 2Ng_2 + g_2 (N/n)^2 = 125,000 \]

(b) If the occupancy of the trunks is 0.6 \( E \) and connections are to be made to particular outgoing trunks, estimate the grade of service. State any assumptions you make.

\[ a = b = 0.6 \ E \]
\[ B_1 = [1-(1-a)^2]^{g_2} = [1-(1-0.6)^2]^{25} = 0.013 \]

Assumptions: 1) the traffic load is evenly distributed among the links
2) all links, trunks and paths behave independently

(c) Redesign the above network by increasing the number of the secondary switches to provide a grade of service better than 1 in 500. How many crosspoints does this network require? Draw the resulting design.

\[ B_1 = [1-(1-15/g_2)^2]^{g_2} \]
For \( g_2 = 26, \quad B_1 = 0.0059 \)
For \( g_2 = 27, \quad B_1 = 0.0026 \)
For \( g_2 = 28, \quad B_1 = 0.0011 \) which is less than 0.002
\[ C_3 = 2Ng_2 + g_2 (N/n)^2 = 140,000 \]

(d) If connection is required to a particular outgoing route, but any free trunk on that route may be used, what are the answers for parts (b) and (c), respectively?

\[ g_3 = N/n = 50 \] and \( c = 0.6 \ E \)
\[ B_2 = [B_1 + c(1-B_1)]^{g_3} = 1.24 \times 10^{-11} \]

The grade of service is already better than 1 in 500 so there is no need to increase the number of the secondary switches. The number of crosspoints is unchanged, i.e. 125,000.
Alternatively, we can increase the number of the secondary switches as in (b):

\[ B_1 = 0.0011 \text{ and } c = 0.6 \ E \]
\[ B_2 = [B_1 + c(1-B_1)]^3 = 8.38 \times 10^{-12} \]

**Question 4**

(a) A T–S–T network has twelve incoming highways and twelve outgoing highways, each carrying 32 PCM channels. The average occupancy of the incoming channels is 0.3 E.

(i) Draw an equivalent space-division network.
(ii) Estimate the blocking probability as an expander.
(iii) Estimate the grade of service when an incoming call must be connected to a selected outgoing highway but may use any free channel on it.

(i) \( m = 12, \ n = 32, \ b = c = 0.3 \)

(ii) \( B_1 = [1 - (1 - b)^2]^n = [1 - (1 - 0.3)^2]^{32} = 4.39 \times 10^{-10} \)

(iii) \( B_2 = 1 - (1 - B_1)(1 - b^n) \approx B_1 \approx 4.39 \times 10^{-10} \) since \( b^n = 0.3^{32} = 1.85 \times 10^{-17} \approx 0 \)

(b) A T–S–T and S–T–S switch both have 32 incoming and outgoing highways, each having 12 PCM channels. The S–T–S network has 32 time-switch links, so both networks give the same blocking probability. Compare the numbers of crosspoints and bytes of storage required for these two networks with the assumption that each memory location in the connection/speech store requires one byte memory.

For a time switch:
No. of crosspoints = 0
Storage required = 2 \times \text{no. of PCM channels (for both speech store and connection store)}

For a space switch:
No. of crosspoints = \text{no. of incoming highways} \times \text{no. of outgoing highways}
Storage required = \text{no. of PCM channels (only for connection store since there is no speech store in a space switch)}
(i) For T–S–T switch,
Crosspoints required = $32 \times 32 = 1024$
Storage required = $2 \times 12 \times 32 + 12 \times 32 + 2 \times 12 \times 32 = 1920$ bytes

(ii) For S–T–S switch,
Crosspoints required = $2 \times 32 \times 32 = 2048$
Storage required = $12 \times 32 + 2 \times 12 \times 32 + 12 \times 32 = 1536$ bytes

(c) A two-stage digital switching network is to make connections between $m$ incoming PCM highways and $m$ outgoing PCM highways, each having $n$ channels. There are two cases: 1) Each call from an incoming PCM highway is to be connected to a particular channel in a selected outgoing PCM highway, and 2) Each call from an incoming PCM highway is to be connected to a selected outgoing PCM highway but may use any free channel on it. For T-S and S-T switching networks, determine the grade of service in the above two cases when the network has 25 incoming and 25 outgoing 20-channel PCM systems, each channel having an average occupancy of 0.3 E. [9 marks]

For T-S switch, $a = b = 0.3$
$m$ primary switches of size $n \times n$
$n$ secondary switches of size $m \times m$

Case 1:
$B_1 = a = 0.3$

Case 2
$B_2 = [1 - (1 - a)(1 - b)]^n = [1 - (1 - 0.3)^2]^{20} = 1.4 \times 10^{-6}$

For S-T switch, $a = b = 0.3$
$n$ primary switches of size $m \times m$
$m$ secondary switches of size $n \times n$

Case 1:
$B_1 = a = 0.3$

Case 2
$B_2 = 1 - (1 - a)(1 - b^n) \sim a \sim 0.3$ since $b^n = 0.3^{20} = 3.5 \times 10^{-11} \sim 0$