I. INTRODUCTION

In radar and sonar data processing, a radiated signal often arrives at a receiver through more than one propagation path. By making use of the passive time delay measurements between the multipath arrivals, useful source localization information can be obtained [1, 2]. Other applications of multipath time delay estimation include wireless communication systems [3], seismology [4], and biomedical engineering [5]. Since multipath delays are not known a priori and may be nonstationary due to relative source/receiver motion, adaptive techniques are required for delay estimation and tracking.

In this work, a new adaptive multipath canceller (AMC) is proposed that provides direct measurements of multipath gain and delay on a sample-by-sample basis. It is assumed that the transmitted signal arrives at the sensor from two paths in the same plane with the receiver and source. The received signal of the multipath time delay estimation model is given by

$$r(k) = s(k) + \alpha s(k - \Delta) + n(k)$$

where the unknown source signal $s(k)$ and the corrupting noise $n(k)$ are zero-mean Gaussian processes and mutually uncorrelated with each other. They are also assumed to be ideal low-pass processes [6] with a bandwidth of 0.5 and unity sampling period. The multipath transmission is characterized by a gain factor $\alpha$ and an interpath delay $\Delta$. The multipath delay is larger than zero and the gain is supposed to lie between 0 and 1 which is true in many applications such as sonar and underwater systems where the transmitters are mostly omni-directional. Since the source location is also related to the multipath gain [7], it is desired to estimate both $\Delta$ and $\alpha$ from the received signal $r(k)$.

Basically, the AMC is a computationally efficient adaptive recursive filter that eliminates the multipath component of the received signal. In Section II, the structure and algorithm of the AMC are derived for integral multipath delays. Global convergence is proved and performance analysis is given. The AMC is modified in Section III such that it can be used to estimate any real-valued interpath delay. Section IV discusses the effects of corrupting noise while Section V shows that the performance of the AMC can approach the Cramér-Rao lower bound (CRLB) at high signal-to-noise ratio (SNR). Numerical examples are presented in Section VI to corroborate the analytical derivations and to evaluate the system performance. Finally, conclusions are drawn in Section VII.

II. AN ADAPTIVE MULTIPATH CANCELLER

To simplify the analysis, we first assume a noise free condition, and that the multipath delay is an
integral multiple of the sampling interval. Taking Z transform of (1) yields

$$R(z) = (1 + \alpha z^{-\Delta})S(z).$$

The underlying principle of the AMC is to cancel out the multipath component of \( r(k) \) by passing it through an adaptive infinite-duration impulse response (IIR) filter whose transfer function is given by

$$A(z) = \frac{1}{1 + \sum_{i=1}^{M} a_i z^{-i}}$$

where \( M \) is chosen to be the maximum allowable delay. The system block diagram is shown in Fig. 1 where \( r'(k) \) represents the filtered output of \( r(k) \). If the mean square value of \( r'(k) \) is minimized, it is expected that \( A(z) \) approaches \( 1/(1 + \alpha z^{-\Delta}) \) and \( r'(k) \to s(k) \). In our studies, the filter coefficients of \( A(z) \) are adapted according to the simplified recursive least mean square (LMS) algorithm proposed by Feintuch [8],

$$a_i(k + 1) = a_i(k) + 2\mu a_i r'(k) r'(k - i),$$

$$i = 1, 2, \ldots, M$$

where

$$r'(k) = r(k) - \sum_{i=1}^{M} a_i(k) r'(k - i)$$

and \( \mu_a \) is a positive scalar that controls the convergence rate and the system stability.

The performance surface of \( A(z) \) is obtained by taking the mean square value of \( r'(k) \),

$$E\{r'^2(k)\} = \frac{\sigma_s^2}{2\pi j} \oint |A(z)(1 + \alpha z^{-\Delta})|^2 \frac{dz}{z}$$

where \( \sigma_s^2 \) represents the power of \( s(k) \). Considering the following inequality

$$\frac{1}{2\pi j} \oint |R'(z) - S(z)|^2 \frac{dz}{z} \geq 0$$

where \( R'(z) \) stands for the Z transform of \( r'(k) \), and using [9], we obtain

$$E\{r'^2(k)\} \geq \sigma_s^2.$$  

Using (6), the equality of (8) holds if and only if \( |A(z)(1 + \alpha z^{-\Delta})|^2 = 1 \) which implies that the global minimum of \( E\{r'^2(k)\} \) occurs when \( A(z) = 1/ (1 + \alpha z^{-\Delta}) \). It has also been investigated [9] that \( E\{r'^2(k)\} \) is unimodal. Consequently, \( r'(k) \) tends to \( s(k) \) in steady state.

Since \( A(z) \) will approach \( 1/(1 + \alpha z^{-\Delta}) \), its peak weight can be used to estimate the multipath gain. Moreover, after \( \Delta \) has been determined from the peak of \( \{a_i(k)\} \), only the largest coefficient \( a_{\Delta}(k) \) needs to be updated while all other filter weights are set to zero. The adaptive algorithm is then simplified to

$$a_{\Delta}(k + 1) = a_{\Delta}(k) + 2\mu a_{\Delta} r'(k) r'(k - \Delta)$$

where

$$r'(k) = r(k) - a_{\Delta}(k) r'(k - \Delta).$$

The variance of the system parameters and the computational complexity of the algorithm are both reduced considerably. As a matter of fact, the AMC algorithm only requires 2 additions and 4 multiplications at each iteration.

Assume that \( s(k) \) is uncorrelated with \( a_{\Delta}(k) \) [10]. When \( \mu_a \) is chosen within the range \((0, 1/\sigma_s^2)\) and as \( r'(k) \) converges to \( s(k) \), taking the expected value of (9) yields the learning behavior of the gain estimate as follows,

$$E\{a_{\Delta}(k)\} = \alpha + (a_{\Delta}(0) - \alpha)(1 - 2\mu_a \sigma_s^2)^k$$

where \( a_{\Delta}(k) \) is an unbiased estimate of \( \alpha \) as \( k \to \infty \), and \( a_{\Delta}(0) \) is its initial value. The mean square error of the gain estimate, denoted by \( \epsilon_{a}(k) \), can be derived as

$$\epsilon_{a}(k) \Delta E\{(a_{\Delta}(k) - \alpha)^2\}$$

$$\approx \mu_a \sigma_s^2 + ((a_{\Delta}(0) - \alpha)^2 - \mu_a \sigma_s^2)(1 - 4\mu_a \sigma_s^2)^k$$

provided that

$$0 < \mu_a < \frac{1}{2\sigma_s^2}.$$  

Equation (13) is a more stringent bound for \( \mu_a \) since the mean square weight error is ensured to be finite [11]. The variance of the gain estimate, \( \text{var}(\alpha) \), which equals the steady state value of \( \epsilon_{a}(k) \), is thus given by

$$\text{var}(\alpha) \approx \mu_a \sigma_s^2.$$  

It is seen that the value of \( \text{var}(\alpha) \) increases with the step size \( \mu_a \) and the signal power \( \sigma_s^2 \).

III. EXTENSION TO REAL-VALUED DELAY

When \( \Delta \) is not restricted to be an integral multiple of the sampling interval, \( r(k) \) can be expressed as [12]

$$r(k) = s(k) + \alpha \sum_{i=-\infty}^{\infty} \text{sinc}(i - \Delta)s(k - i)$$

$$E\{r'^2(k)\} \geq \sigma_s^2.$$
where sinc(ν) ≜ \sin(πν)/(πν). That means the delayed version of a bandlimited signal can be represented by convolving a sinc function and the signal itself. Apparently, the transfer function of the multipath canceller for any real multipath delay, \( A^0(z) \), is of the form

\[
A^0(z) = \frac{1}{1 + \alpha \sum_{i=-M}^{M} \text{sinc}(i - \Delta) z^{-i}}. \tag{16}
\]

Here, \( M \) should be chosen larger than \( \Delta \) to reduce truncation error [13]. However, it has been shown (see Appendix I) that due to the noncausality of the filter structure, \( A^0(z) \) is an unstable system and thus its practical realization is prohibited. To alleviate the problem, we again apply \( A(z) \) to cancel out the replica but with \( a_i(k) \) replaced by \( \hat{\alpha}(k)\text{sinc}(i - \hat{\Delta}(k)) \), for \( 1 \leq i \leq M \), where \( \hat{\alpha}(k) \) and \( \hat{\Delta}(k) \) are the estimates of \( \alpha \) and \( \Delta \), respectively. Upon convergence, \( \hat{\alpha}(k) \rightarrow \alpha \) and \( \hat{\Delta}(k) \rightarrow \Delta \) are expected and in this case direct multipath parameter measurements can be obtained.

The updating rule for the multipath gain is similar to (9),

\[
\hat{\alpha}(k + 1) = \hat{\alpha}(k) + 2\mu_{a} r'(k) \sum_{i=1}^{M} r'(k - i) \text{sinc}(i - \hat{\Delta}(k)) \tag{17}
\]

where

\[
r'(k) = r(k) - \hat{\alpha}(k) \sum_{i=1}^{M} \text{sinc}(i - \hat{\Delta}(k)) r'(k - i). \tag{18}
\]

We use [8] and the explicit delay estimation algorithm [14] to derive the iterative equation for the multipath delay estimate,

\[
\hat{\Delta}(k + 1) = \hat{\Delta}(k) - 2\mu_{\Delta} r''(k) \sum_{i=1}^{M} f(i - \hat{\Delta}(k)) r'(k - i) \tag{19}
\]

where \( \mu_{\Delta} \) is the convergence step size for \( \hat{\Delta}(k) \) and \( f(\nu) \equiv \Delta (\cos(\pi \nu) - \text{sinc}(\nu))/\nu \). This error gradient estimate is obtained by differentiating \( r'^{2}(k) \) with respect to \( \hat{\Delta}(k) \). It is worthy to note that (17) and (19) are the generalized algorithm for the AMC and they can be applied for both integral and nonintegral multipath delays. Equation (9) is only a special case of (17) when \( \Delta \) is an integer. To significantly reduce the computational load of the AMC algorithm, look-up tables are constructed [14]. As a result, \((3M + 1)\) additions and \((3M + 7)\) multiplications are needed at each iteration.

For highly resolvable multipath [15], that is, \( \Delta > 1 \), the learning characteristics of \( \hat{\alpha}(k) \) and \( \hat{\Delta}(k) \) can be approximated by

\[
E\{\hat{\alpha}(k)\} \approx \alpha + (\hat{\alpha}(0) - \alpha)(1 - 2\mu_{a}\sigma_{a}^{2})^{k}, \tag{20}
\]

and

\[
E\{\hat{\Delta}(k)\} \approx \Delta + (\hat{\Delta}(0) - \Delta)(1 - \frac{3}{2}\mu_{\Delta}\alpha\sigma_{\Delta}^{2})^{k} \tag{21}
\]

provided that \( 0 < \mu_{a} < 1/\sigma_{a}^{2} \) and \( 0 < \mu_{\Delta} < 3/(\alpha\sigma_{\Delta}^{2}). \)

At the beginning of the adaptation, we use (4) for a few hundred iterations to determine the value of \( \Delta(0) \), which is given by the value of \( i \) at which the maximum of \( a_i(k) \) occurs, in order to avoid the delay estimate from being locked up at any local minimum. Whereas the initial estimate of the multipath gain, \( \hat{\alpha}(0) \), can be arbitrarily chosen between 0 and 1.

The variance of \( \hat{\alpha}(k) \) is given by (14) while the variance of \( \hat{\Delta}(k) \), namely, \( \text{var}(\Delta) \), can be shown to be

\[
\text{var}(\Delta) \approx \frac{\mu_{\Delta}\sigma_{\Delta}^{2}}{\alpha}. \tag{22}
\]

IV. EFFECTS OF ADDITIVE NOISE

In this section, the performance of the AMC is investigated when corrupting noise is present in \( r(k) \). For ease of analysis, only the case for integral multipath delays is considered and \( A(z) \) is adapted according to (9) assuming that \( \Delta \) has been determined. Using (1) and (10), the performance surface of \( A(z) \), \( E\{r'^{2}(k)\} \), can be found to be

\[
E\{r'^{2}(k)\} = \frac{(1 + \alpha^{2} - 2\alpha\Delta)\sigma_{a}^{2} + \sigma_{n}^{2}}{1 - \sigma_{n}^{2}} \tag{23}
\]

where \( \sigma_{n}^{2} \) denotes the noise power. Differentiating (23) with respect to \( \Delta \) and then setting the result to zero yields the mean gain estimate \( \hat{a}_{\Delta} \) which has a value of

\[
\hat{a}_{\Delta} = \frac{(1 + \alpha^{2})\sigma_{a}^{2} + \sigma_{n}^{2} - \sqrt{(1 + \alpha^{2})^{2}\sigma_{a}^{4} + \sigma_{n}^{4} + 2(1 + \alpha^{2})\sigma_{a}^{2}\sigma_{n}^{2}}}{2\alpha\sigma_{a}^{2}}. \tag{24}
\]

When \( \sigma_{n}^{2} = 0 \), \( \hat{a}_{\Delta} \) equals \( \alpha \) and similar result is obtained in Section II. However, it can be seen from (24) that \( \hat{a}_{\Delta} < \alpha \) under noisy environment. In addition, the difference between \( \hat{a}_{\Delta} \) and \( \alpha \) increases with noise power.

In the presence of noise, the steady state value of \( r'(k) \) can be approximated by

\[
r'(k) \approx s(k) + (\alpha - \hat{a}_{\Delta})s(k - \Delta) + n(k). \tag{25}
\]

The replica is not eliminated or reduced through the AMC when the SNR is small. It is because from (24), \( \hat{a}_{\Delta} \rightarrow 0 \) as \( \sigma_{n}^{2} \rightarrow \infty \). When noise is present, \( a_{\Delta}(k) \) will no longer be an unbiased estimate of \( \alpha \) upon convergence. If we use \( a_{\Delta}(k) \) as the estimate of \( \alpha \), the steady state mean square error of \( a_{\Delta}(k) \), namely, \( \epsilon_{\alpha}(\infty) \), is equal to the sum of \( \text{var}(\alpha) \) and the bias square,

\[
\epsilon_{\alpha}(\infty) \approx \mu_{a}\sigma_{a}^{2} + (\alpha - \hat{a}_{\Delta})^{2}. \tag{26}
\]
V. PERFORMANCE COMPARISON WITH THE CRLB

From (18), an optimal realization of the AMC is to search the minimum of the following cost function

\[ C(a, b) = \sum_{k=1}^{N} \left( r(k) - a \sum_{i=1}^{M} \text{sinc}(i - b) r'(k - i) \right)^2 \]  

(27)

where \( N \) is the observation time. Denote the multipath gain and delay estimate by \( \hat{\alpha} \) and \( \hat{\Delta} \) respectively, where \((\hat{\alpha}, \hat{\Delta})\) represents the global minimum of \( C(a, b) \). When \( a \) and \( b \) are at the neighborhood of \( \alpha \) and \( \Delta \), respectively, the delay variance, \( \text{var}(\hat{\Delta}) \), is given by [15]

\[
\text{var}(\hat{\Delta}) = \frac{E \left\{ \left( \frac{\partial C(a, b)}{\partial b} \right)^2 \right\}^2}{E \left\{ \frac{\partial^2 C(a, b)}{\partial b^2} \right\}^2_{b=\hat{\Delta}}} \]  

(28)

Assume that \( \Delta \gg 1 \), \( \alpha \ll 1 \) and the SNR is high. Then, (28) can be simplified to

\[
\text{var}(\hat{\Delta}) = \frac{3}{N \pi^2 \alpha^2} \]  

(29)

which is identical to the CRLB for multipath time delay estimation [16].

VI. NUMERICAL EXAMPLES

Extensive computer simulations have been done to corroborate the theoretical derivations and to evaluate the performance of the AMC for multipath time delay estimation. The source signal \( s(k) \) and the corrupting noise \( n(k) \) were white Gaussian random variables and they were produced by a pseudorandom noise generator. The power of \( s(k) \) was fixed to unity and different SNRs, where the SNR was defined as \( \sigma_s^2 / \sigma_n^2 \), were obtained by proper scaling of the random noise sequence. The delayed signal \( s(k-\Delta) \) was produced by passing \( s(k) \) through a finite-duration impulse response (FIR) filter whose transfer function was \( \sum_{i=-\infty}^{\infty} \text{sinc}(i-\Delta)z^{-i} \). We fixed \( \Delta \in (0, 15) \) and \( M \) was chosen to be 15 to cover all possible delays. The initial values of \( \{a_i(k)\} \) were all assigned to zero.

\( A(z) \) was updated according to (4) at the beginning of the adaptation for 400 iterations to determine an estimate of \( \Delta \) from the peak of \( \{a_i(k)\} \). The AMC was then adjusted using (9) for integral interpath delays or (17) and (19) for real-valued multipath delays. When applying (17), the initial value of \( \hat{\alpha}(k) \) was arbitrarily selected to be 0.5. The step sizes \( \mu_a \) and \( \mu_\Delta \) had a value of 0.004. All simulation results provided were averages of 200 independent runs.

The learning trajectory of the multipath gain estimate for an integral delay under a noise-free condition is shown in Fig. 2. In this test, \( \alpha = 0.8 \) and \( \Delta = 5.0 \). After the first 400 iterations, \( \Delta \) was accurately obtained, and \( a_\delta(k) \) was then adapted with all other filter weights being set to zero. It can be seen that \( a_\delta(k) \to 0.78 \) at approximately the 700th iteration. There is a bias in the gain estimate because the received signal was not totally uncorrelated with \( a_\delta(k) \), however, the bias will become negligible if the step size is small enough. Furthermore, the convergence time was slightly longer than the theoretical value because the assumption that \( r'(k) \approx s(k) \) when deriving (11) is not true at the beginning of the adaptation.

Upon convergence, the measured variance of \( a_\delta(k) \) was close to the theoretical value of 0.004. When \( A(z) \) was adjusted according to (4) instead, the convergence behavior of the gain estimate was found to be similar to the trajectory in Fig. 2. However, the measured gain variance had a value of 0.016 which was much larger than that of the constrained adaptation.

The above experiment was repeated for \( \Delta = 5.4 \) and the results were shown in Fig. 3. In this case, \( A(z) \) was adjusted according to (17) and (19). It can be observed that \( \hat{\alpha}(k) \to 0.81 \) and \( \hat{\Delta}(k) \to 5.41 \) at approximately the 10000th iteration. The convergence rate of \( \hat{\Delta}(k) \) was close to the predicted value whereas that of the gain estimate was slightly slower than the analytical calculation. This is due to the fact that the multipath delay estimate is assumed to have approached to its optimal value when deriving (20).
Upon convergence, the measured variance of \( \hat{\alpha}(k) \) and \( \hat{\Delta}(k) \) were 0.0039 and 0.0042, respectively, which agreed well with their theoretical values of 0.004 and 0.005.

Figs. 4 and 5 demonstrate the ability of the AMC to estimate nonstationary system parameters. The actual multipath gain and delay were given step offsets after each 2000 iterations. It can be seen that the AMC tracked all these step changes in less than a thousand iterations for both noise-free condition and \( \text{SNR} = 10\text{dB} \). In Fig. 5, we see that the trajectory of \( \hat{\Delta}(k) \) was almost unaffected by the corrupting noise and the delay estimates were accurate. Moreover, the learning rate of \( \hat{\Delta}(k) \) decreased as the multipath gain decreased, and it can be explained easily by (21). Fig. 4 shows that the performance of gain estimation under noise-free condition was better than those under noisy environment, which has confirmed the analysis in Section IV. At lower SNRs, the accuracy of the gain estimate is expected to get worse since it will shift away from the desired value as the noise power increases.

The mean square delay errors for different \( \alpha \) and \( \Delta \) in a noise-free condition are shown in Fig. 6. In this test, \( \Delta \) was varied from 1.0 to 14.75 and two values of \( \alpha \), namely, 0.2 and 0.8, were tried. It is noted that the AMC provided a smaller mean square error for the larger gain and this has been indicated in (22). Due to the inexact inverse modeling, the mean square delay error had a relatively large value when the multipath delay was close to 1 and 15. Nevertheless, when \( \Delta \in (5, 14) \), the delay variances were comparable to the predicted values of 0.02 and 0.005, although there are some peaks with decreasing heights in the plot for \( \alpha = 0.8 \).

Due to stability reasons, the AMC cannot be extended to cases of more than two propagation paths. However, we have found that this method can still get an accurate delay estimate of the dominant multipath, as long as the interpath delays are highly resolvable. The following multipath signals had been generated for demonstration purpose: 1) \( r(k) = s(k) + 0.8s(k - 5.4) + 0.5s(k - 8.7) \), and 2) \( r(k) = s(k) + 0.8s(k - 5.4) + 0.5s(k - 8.7) + 0.2s(k - 12.5) \).

The delay estimates of the AMC were measured to be 5.45 and 5.46, respectively, which were close to the optimum value of 5.4. Whereas the gain estimates were inaccurate and had values of 0.53 and 0.49, respectively.

VII. CONCLUSIONS

A simple adaptive system for estimating and tracking the interpath delay of a random signal in a multipath environment has been proposed. The AMC uses an adaptive IIR filter to cancel out the multipath component in the received signal. Using an LMS-style algorithm, the estimate of the multipath parameters are adjusted explicitly and simultaneously. The performance of the AMC is analyzed rigorously for both integral and real-valued delay. When the delay is an integral multiple of the sampling period, the AMC provides exact multipath cancellation. The AMC can also estimate real-valued delay accurately if the multipath is highly resolvable. Computer simulation results are provided to validate the theoretical analysis and to demonstrate its capability of tracking time-varying parameters.

APPENDIX I

Here we show that the optimal multipath canceller \( A^0(z) \) as given by (16) is an unstable system by applying Rouché's Theorem [17]. We define two
functions, \( u(z) \) and \( v(z) \), which are analytic on the contour \( |z| \leq 1 \), as follows,

\[
u(z) = \alpha \sum_{i=-M}^{M} \text{sinc}(i - \Delta)z^{M-i}
\]

such that \((u(z) + v(z))z^{-M}\) equals the denominator of \( A^0(z) \). We then evaluate the magnitudes of \( u(z) \) and \( v(z) \) when \(|z| = 1\),

\[
|u(z)| = |z|^M = 1
\]

and

\[
|v(z)| = |\alpha| \cdot \left| \sum_{i=-M}^{M} \text{sinc}(i - \Delta)z^{i} \right| |z|^M
\]

\[
\leq |\alpha| \cdot \left| \sum_{i=-M}^{M} \text{sinc}(i - \Delta) \right| |z|^M
\]

If \( M \) is chosen such that \( M = \text{trun}(|\Delta|) \), where \( \text{trun}[x] \) equals the integral part of \( x \), it is an even number, it can be shown that \( \sum_{i=-M}^{M} \text{sinc}(i - \Delta) \) is always less than one, implying \(|v(z)| < \alpha| |v(z)| \) is also less than one when \( \alpha \) is not very close to unity and if \( M \) is chosen sufficiently large since \( \sum_{i=-\infty}^{\infty} \text{sinc}(i - \Delta) = 1 \). In general, we can say \(|u(z)| > |v(z)|\) for \(|z| = 1\). Using Rouche’s Theorem, since \( u(z) \) has \( M \) zeros interior to the unit circle, so does \( u(z) + v(z) \). This means that there are also \( M \) zeros, which are outside or on the unit circle, in \( u(z) + v(z) \). Consequently, \( A^0(z) \) only has \( M \) poles lying inside the unit circle and thus its instability is proved.

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