A NOVEL CONSTRAINED ALGORITHM FOR DELAY ESTIMATION IN THE PRESENCE OF MULTIPATH TRANSMISSIONS


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ABSTRACT

A delayed version of a bandlimited signal can be represented by convolving a sinc function and the signal itself. By using this property, a novel constrained LMS algorithm that estimates the difference in arrival times of signal at two spatially separated receivers with multipath propagations is developed. The new model consists of two adaptive FIR filters to compensate the multipath effect in each channel and a third adaptive FIR filter to act as a delay estimator. With constraints, all filter coefficients are expressed in terms of the derived time delay as well as the multipath parameters. The error surface is multimodal and proper initialization is required to give good estimation of the system variables. Derivation of the proposed algorithm is given and simulations are included to demonstrate the effectiveness and validity of the algorithm.

I. INTRODUCTION

Time delay estimation (TDE) has applications in many areas such as radar, seismology and sonar[1]. The presence of multipath propagations, however, has made delay estimation very difficult, if not impossible. Physically, these multipaths might come from bottom bounces or reflections from the ocean surface in sonar and reflections from buildings or mountains in radar transmission. Ching et al[2] have proposed a solution to tackle the problem if there is only a single multipath in either one of the two sensors. The solution is obtained from the property that a delay can be modelled by an adaptive FIR filter. This is extended here to design an adaptive algorithm to estimate the time delay in the presence of more than one multipath. We assume, again, a two-path model and that each sensor output consists of a signal plus a delayed and attenuated version of itself.

The received signals can be represented by

\[ x(k) = s(k) + \alpha_s s(k - \Delta_s) \]  \hspace{1cm} (1a)

\[ y(k) = s(k - D) + \alpha_s s(k - D - \Delta_s) \]  \hspace{1cm} (1b)

where \( s(k) \) is a white Gaussian process with variance \( \sigma_s^2 \) and \( D \) is the delay to be estimated. The multipaths are characterized by \( \alpha_s \) and \( \alpha_{s} \), the attenuation constants, and \( \Delta_s \) and \( \Delta_s \), the multipath delays, respectively. Notice that the attenuation constants must lie between 0 and 1 and the multipath delays should be greater than 0.

From the convolution theorem, these signals can also be expressed as

\[ x(k) = s(k) + \alpha_s \sum_{i=-\infty}^{\infty} \text{sinc}(i - \Delta_s) s(k - i) \]  \hspace{1cm} (2a)

\[ y(k) = s(k - D) + \alpha_s \sum_{j=-\infty}^{\infty} \text{sinc}(j - \Delta_s) s(k - D - j) \]  \hspace{1cm} (2b)

where the infinity sign in the summation can be replaced by an integer \( P \) which is chosen large enough to minimize truncation error[3], and \( \text{sinc}(v) = \sin(\pi v) / \pi v \).

Recently, Ching et al[4] has proposed an adaptive algorithm that can extract the time delay in the above situation. It consists of two IIR filters and one FIR filter which is shown in Figure 1. The two IIR adaptive filters \( A(z) \) and \( B(z) \) are used to cancel out the multipath effects and \( W(z) \) simply serves as an ordinary delay estimator. Accurate prediction of all multipath variables is required in order to have the best result. This system works perfectly well if \( \Delta_s \) and \( \Delta_s \) are both integral multiples of the sampling period \( T \). However, if \( \Delta_s \) and \( \Delta_s \) are any arbitrary values, the optimum solutions of \( A(z) \) and \( B(z) \) will be given by

\[ A^*(z) = 1 + \sum_{i=\Delta_s}^{\infty} \alpha_i \text{sinc}(i - \Delta_s) z^{-i} \]  \hspace{1cm} (3a)

\[ B^*(z) = 1 + \sum_{j=\Delta_s}^{\infty} \alpha_j \text{sinc}(j - \Delta_s) z^{-j} \]  \hspace{1cm} (3b)

Practical realization of these IIR filters is not feasible, therefore, the multipaths are not completely eliminated and thus the delay estimation performance is notably degraded.

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In this paper, we present a novel algorithm to deal with this inaccurate multipath time delay modeling. Section II describes the proposed system and adaptive algorithm and Section III will provide simulation results for performance evaluation.

II. THE NEW TDE MODEL

The proposed configuration of the time delay estimator with multipath transmissions is depicted in Figure 2. It has a similar structure as the one shown in Figure 1 except that here $\hat{A}(z)$ and $\hat{B}(z)$ are both FIR filters instead and also a constrained delay estimator filter is used. The transfer function of $\hat{A}(z)$, $\hat{B}(z)$ and $\hat{W}(z)$ are of the form

$$\hat{A}(z) = 1 + \hat{\Delta}_1 \sum_{i=1}^{p} \text{sinc}(i - \hat{\Delta}_1)z^{-i}$$  \hspace{1cm} (4a)

$$\hat{B}(z) = 1 + \hat{\Delta}_2 \sum_{j=0}^{p} \text{sinc}(j - \hat{\Delta}_2)z^{-j}$$  \hspace{1cm} (4b)

$$\hat{W}(z) = \hat{\Delta}_3 \sum_{n=0}^{\infty} \text{sinc}(n - \hat{\Delta}_3)z^{-n}$$  \hspace{1cm} (4c)

respectively. The filter coefficients of $\hat{W}(z)$ are restricted to samples of a sinc function which was first proposed by Ching et al[5]. Ideally, $\hat{\Delta}_1$, $\hat{\Delta}_2$, $\hat{\Delta}_3$ and $\hat{\Delta}_4$ should converge to $\Delta_1$, $\Delta_2$, $\Delta_3$ and $\Delta_4$ such that their immediate filter outputs, $x'(k)$ and $y'(k)$, will become

$$x'(k) = s(k) + \alpha_1 s(k - \Delta_1) + \alpha_2 s(k - \Delta_2)$$

$$+ \alpha_3 s(k - \Delta_3 - \Delta_4)$$  \hspace{1cm} (5a)

$$y'(k) = s(k) + \alpha_1 s(k - \Delta_1 - \Delta_2) + \alpha_2 s(k - \Delta_4)$$

$$+ \alpha_3 s(k - \Delta_3 - \Delta_4)$$  \hspace{1cm} (5b)

In this case, the actual difference in arrival times of the original signals, $D$, can easily be obtained by $\hat{W}(z)$ since $x'(k)$ differs from $y'(k)$ by exactly the same delay.

The coefficients of the three adaptive filters are adjusted by using the LMS method to minimize the sum of the squares of the error sequences:

$$e(k) = y'(k) - \sum_{n=-p}^{p} \text{sinc}(n - \hat{D}(k))x'(k - n)$$  \hspace{1cm} (6a)

where

$$x'(k) = x(k) + \hat{\alpha}_1(k) \sum_{i=1}^{p} \text{sinc}(i - \hat{\Delta}_1(k))x(k - i)$$  \hspace{1cm} (6b)

$$y'(k) = y(k) + \hat{\alpha}_2(k) \sum_{j=0}^{p} \text{sinc}(j - \hat{\Delta}_2(k))y(k - j)$$  \hspace{1cm} (6c)

The variables $\hat{D}(k)$, $\hat{\Delta}_1(k)$, $\hat{\Delta}_2(k)$, $\hat{\alpha}_1(k)$ and $\hat{\alpha}_2(k)$ are estimates of the time delay, multipath delays and attenuation constants respectively.

Obviously, the error surface, $E[e^2(k)]$, is a function of $\hat{D}(k)$, $\hat{\Delta}_1(k)$, $\hat{\Delta}_2(k)$, $\hat{\alpha}_1(k)$ and $\hat{\alpha}_2(k)$, rather than a function of the filter coefficients. As a result, the error gradient estimate used by the LMS method will be quite different and it is obtained by differentiating the instantaneous error surface with respect to the estimated time delay and multipath parameters. For instance, the instantaneous gradient with respect to $\hat{D}(k)$ can be shown to be

$$\frac{\partial e^2(k)}{\partial \hat{D}(k)} = 2e(k) \sum_{n=-p}^{p} x'(k - n)f(n - \hat{D}(k))$$  \hspace{1cm} (7a)

where

$$f(x) = \cos(\pi x) - \text{sinc}(x)$$  \hspace{1cm} (7b)

The error gradients with respect to the multipath parameters can be computed in a similar way. Subsequently, the constrained LMS algorithm is summarized as follow

$$\hat{D}(k+1) = \hat{D}(k) - 2\mu_D e(k) \sum_{n=-p}^{p} x'(k - n)f(n - \hat{D}(k))$$  \hspace{1cm} (8a)

$$\hat{\Delta}_1(k+1) = \hat{\Delta}_1(k) + 2\mu_1 e(k) \hat{\alpha}_1(k) \sum_{i=1}^{p} \text{sinc}(i - \hat{\Delta}_1(k))$$  \hspace{1cm} (8b)

$$\hat{\Delta}_2(k+1) = \hat{\Delta}_2(k) + 2\mu_2 e(k) \hat{\alpha}_2(k) \sum_{j=0}^{p} \text{sinc}(j - \hat{\Delta}_2(k))$$  \hspace{1cm} (8c)

$$\hat{\Delta}_3(k+1) = \hat{\Delta}_3(k) + 2\mu_3 e(k) \sum_{n=-p}^{p} \text{sinc}(n - \hat{\Delta}_3(k))$$  \hspace{1cm} (8d)

where $\mu_D$, $\mu_1$, $\mu_2$ and $\mu_3$ are convergence step sizes.

Notice that the operation $\cos(\pi x)$ is equivalent to $(-1)^{t} \cos(\pi r)$ where $q$ and $r$ are the integral and decimal parts of $x$ respectively. By transforming to this form, only three cos operations are needed in (8). In order to further reduce the computational load, lookup tables of sinc and cos functions are constructed. The former is a matrix $M$ of dimension $K \times 2r + 1$ with element

$$m_{ij} = \text{sinc}(0.5i(K - 1) - j)$$  \hspace{1cm} (9)

for $0 \leq i \leq K - 1$ and $-r \leq j \leq r$ while the latter is a vector $N$ of length $L$ such that

$$n_j = \cos(\pi j / L)$$  \hspace{1cm} (10)

for $0 \leq j \leq L - 1$. At each iteration, the previous estimate of $\hat{D}$, $\hat{\Delta}_1$ and $\hat{\Delta}_2$ is compared with the lookup tables in turn to find the best fit column and element in $m_j$ and $n_i$ respectively which are used to calculate the required parameters.
Moreover, careful inspection reveals that the error surface is in fact multimodal and the desired solution might not be achieved in all cases. Proper initialization is needed and we have used the adaptive structure in Figure 1 to obtain initial values for the system parameters. This has reduced the occurrence of lock-up at the local minima and it also improves the convergence speed significantly. In our studies, we first freely adapt the old model for a few hundred iterations. It is noted that for \( \Delta_i > 1 \) and \( \Delta_j > 1 \), \( a_i \) and \( b_j \) will approach \( \alpha_i \text{sinc}(i - \Delta_i) \) and \( \alpha_j \text{sinc}(j - \Delta_j) \) respectively while \( w_n \) is close to \( \text{sinc}(n - D) \). Hence, rough estimate of \( \hat{D} \), \( \hat{\Delta_1} \) and \( \hat{\Delta_2} \) could be extracted from \( \{w_n\} \), \( \{a_i\} \) and \( \{b_j\} \) by interpolation. If no dominant peak is detected in \( \{a_i\} \) or \( \{b_j\} \), the corresponding multipath delay is set to \( 0.25T \), where \( T \) is the sampling period. An initial value of 0.5 is arbitrary assigned to both \( \hat{\Delta_1} \) and \( \hat{\Delta_2} \) and it does not seem to create any obvious problem in the course of adaptation.

III. SIMULATION RESULTS

Extensive simulation tests have been conducted to evaluate the performance of the new constrained adaptive time delay estimation algorithm in the presence of multipath propagations. We begin by updating all the filter parameters of the old model in the first 400 iterations. These parameters are then interpolated by using a sinc function and transferred to the new system for further constrained adaptation. \( \sigma_i^2 \) is set to unity and the size of the sinc table and cos table are \( 513 \times 31 \) and \( 1024 \) respectively. This has provided a resolution of approximately 0.0001T. We shall examine the trajectories of the time delay estimate and the multipath parameters which are shown in Figure 3. These plots are averages of 10 independent runs. They show the convergence characteristics of the estimated variables with input given by (1), where \( D = 1.7T \), \( \alpha_1 = 0.8 \), \( \alpha_2 = 0.6 \), \( \Delta_1 = 1.4T \) and \( \Delta_2 = 2.6T \), for noise free situation and when a 10dB novel level was added. The dotted line shown in Figure 3a illustrates the incometence of the old model to provide an accurate estimate of the time delay. Similar results are also obtained with the presence of additive noise. Again from Figure 3a, the system takes approximately 2500 iterations for the time delay estimate to come to the steady state. Moreover, it can be seen that all multipath parameters converge to the optimum solution after approximately 4000 iterations. It is worth to note that apart from the step sizes, the convergence characteristic is also affected by the absolute values of the time delay and the multipath parameters which essentially determines the error surface. Although global convergence is not necessarily guaranteed, however, we have tried many experiments under different conditions and so far the performance of the proposed time delay estimation is very promising which confirms the validity of the methodology.

References


![Figure 1 A multipath time delay estimator proposed by Ching et al.[4]](image)
Figure 2 Schematic block diagram of the proposed time delay estimator with multipath transmissions

Figure 3a Estimate of time delay

Figure 3b Estimate of multipath delays

Figure 3c Estimate of multipath attenuation constants