Adaptive Multipath Equalization Time Delay Estimation with Bias-Removal

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ABSTRACT
The multipath equalization time delay estimator (METDE) provides an adaptive approach for estimating the difference in arrival times of a signal received at spatially separated sensors as well as the multipath channel characteristics. However, the parameter estimates of the METDE are biased in the presence of noise. In this paper, the METDE algorithm is improved for unbiased parameter estimation via minimizing a modified cost function. Computer simulations are included to evaluate the multipath time delay estimation performance of the proposed algorithm in noisy environments.

I. Introduction
Time delay estimation between signals received at two spatially separated sensors has many important applications such as synchronization in communication systems [1], speech enhancement and source localization in radio systems [2]. The presence of unknown multipath propagation in practical situations has often made the task of delay estimation very difficult. If the multipaths are ignored, the delay estimation accuracy based on direct-path-only propagation will be degraded. On the other hand, when the multipath structure is utilized, fading effects can be alleviated and significant performance improvement can be achieved [3].

In the presence of multipaths, the received outputs of two separated sensors can be represented as

\[x(k) = s(k) + \sum_{i=1}^{M_1} a_{1i} s(k - \Delta_{1i}) + n_1(k)\]  \hspace{1cm} (1)

\[y(k) = s(k - D) + \sum_{j=1}^{M_2} a_{2j} s(k - \Delta_{2j}) + n_2(k)\]  \hspace{1cm} (2)

where \(s(k)\) is the unknown source signal while \(n_1(k)\) and \(n_2(k)\) are the uncorrelated zero-mean noises with variance \(\sigma_n^2\), which are independent of \(s(k)\). Without loss of generality, it is assumed that the sampling period is unity second and \(s(k)\) is bandlimited between 0 Hz and 0.5 Hz. The parameter \(D\) denotes the time delay between the two sensor outputs. The multipath transmissions are characterized by the gain factors, \(a_{1i}\) and \(a_{2j}\), as well as the interpath delays, \(\Delta_{1i}\) and \(\Delta_{2j}\), for \(M_1 \geq i \geq 1\) and \(M_2 \geq j \geq 1\), such that \(1 > a_{11} > a_{12} > \cdots > a_{1M_1} > 0, 1 > a_{21} > a_{22} > \cdots > a_{2M_2} > 0, 0 < \Delta_{11} < \Delta_{12} < \cdots < \Delta_{1M_1}\) and \(D < \Delta_{21} < \Delta_{22} < \cdots < \Delta_{2M_2}\). The integers \(M_1\) and \(M_2\) are the numbers of multipaths contained in \(x(k)\) and \(y(k)\), respectively, and we consider that they are known a priori. The goal is to estimate \(D, a_{1i}, a_{2j}, \Delta_{1i}\) and \(\Delta_{2j}\) from \(x(k)\) and \(y(k)\).

When the time difference of arrival and the multipath parameters are nonstationary due to either relative source/receiver motion or time-varying characteristics of the transmission medium, adaptive estimation is necessary to track them over time. Based on the property that a time-shifted version of a bandlimited signal can be modeled by passing the signal through an FIR filter whose coefficients are samples of a sinc function [4], an adaptive multipath equalization time delay estimator (METDE) was developed in [5] to tackle the problem. However, the estimated delay and multipath parameters provided by the METDE are biased even at high signal-to-noise ratio (SNR) condition, particularly for the multipath gain estimates. In this paper, the METDE algorithm is improved to acquire unbiased estimation for all system parameters.

The structure of the METDE is first reviewed in Section II. By examining the mean square error function of the METDE, a modified cost function whose global minimum gives the exact values of the time delay and multipath parameters, is developed. A bias-free (BF)-METDE algorithm is then proposed which applies the least mean square (LMS) [6] method to minimize the cost function and all parameter estimates are updated explicitly on a sample-by-sample basis. Simulation results are presented in Section III to demonstrate the unbiasedness of the proposed multipath time delay estimation method.

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II. Bias-Free Multipath Time Delay Estimation

Let \( X(z) \), \( Y(z) \), \( S(z) \), \( N_1(z) \) and \( N_2(z) \) be the z-transform of \( x(k) \), \( y(k) \), \( S(k) \), \( n_1(k) \) and \( n_2(k) \), respectively. Using the interpolation formula [4], \( X(z) \) and \( Y(z) \) are equal to

\[
X(z) = S(z) \left[ 1 + \sum_{i=1}^{M_1} \alpha_{1i} \sum_{m=-\infty}^{\infty} \text{sinc}(m - \Delta_{1i}) z^{-m} \right] + N_1(z) \tag{3}
\]

\[
Y(z) = S(z) \left[ \sum_{m=-\infty}^{\infty} \text{sinc}(m - D) z^{-m} + \sum_{j=1}^{M_2} \alpha_{2j} \sum_{m=-\infty}^{\infty} \text{sinc}(m - \Delta_{2j}) z^{-m} \right] + N_2(z) \tag{4}
\]

where \( \text{sinc}(v) \triangleq \sin(\pi v)/(\pi v) \). Multiplying the squared bracket term of (4) to (3) and the squared bracket of (3) to (4), it can be easily shown that the two resultant transfer functions are identical in the absence of noise. The METDE employs this idea of equalization and its system block diagram is depicted in Figure 1. The error function of the METDE, \( \epsilon(k) \), has the form [5]

\[
\epsilon(k) = y(k) + \sum_{m=-P}^{P} y(k-m) \sum_{i=1}^{M_1} \alpha_{1i} \text{sinc}(m - \Delta_{1i}) - \sum_{m=-P}^{P} x(k-m) \left( \text{sinc}(m - D) + \sum_{j=1}^{M_2} \alpha_{2j} \text{sinc}(m - \Delta_{2j}) \right) \tag{5}
\]

where \( D \), \( \Delta_{1i} \), \( \Delta_{2j} \) and \( \Delta_{2j} \), for \( M_1 \geq i \geq 1 \) and \( M_2 \geq j \geq 1 \), denote the estimate of \( D \), \( \Delta_{1i} \), \( \Delta_{2j} \) and \( \Delta_{2j} \), respectively. The parameter \( P \) is chosen much larger than \( \max\{\Delta_{1M_1}, \Delta_{2M_2}\} \) in order to accurately model the ideal time shift function [4].

Since the signal and noises are independent, squaring and taking expectation of (5) gives the mean square value of \( \epsilon(k) \):

\[
E\{\epsilon^2(k)\} = E \left\{ \left( \hat{y}(k) + \sum_{i=1}^{M_1} \alpha_{1i} \hat{y}(k - \Delta_{1i}) - \hat{x}(k - D) - \sum_{j=1}^{M_2} \alpha_{2j} \hat{x}(k - \Delta_{2j}) \right)^2 \right\} + \sigma_n^2 \gamma \tag{6}
\]

where

\[
\gamma = \sum_{i=0}^{M_1} \alpha_{1i}^2 + 2 \sum_{i=0}^{M_1} \sum_{j=0}^{M_2} \alpha_{1i} \alpha_{1j} \text{sinc}(\Delta_{1i} - \Delta_{1j}) + \sum_{j=0}^{M_2} \alpha_{2j}^2 + 2 \sum_{j=0}^{M_2} \alpha_{2j} \alpha_{2j} \text{sinc}(\Delta_{2j} - \Delta_{2j}) \tag{7}
\]

The signals \( \hat{x}(k) \) and \( \hat{y}(k) \) represent the noise free versions of \( x(k) \) and \( y(k) \), respectively, while \( \alpha_{10} = \alpha_{20} = 1, \Delta_{10} = 0 \) and \( \Delta_{20} = D \).

In [5], the METDE algorithm was devised for adaptive multipath time delay estimation by minimizing \( E\{\epsilon^2(k)\} \) with respect to \( D, \alpha_{1i}, \Delta_{1i}, \alpha_{2j} \), and \( \Delta_{2j} \). In a noise-free condition, all parameter estimates provided by this method will be unbiased. However, the METDE cannot give accurate estimates even at high SNR, particularly for the multipath gain parameters. It is because the noise component of \( E\{\epsilon^2(k)\} \) is also a function of the system variables, as seen from (6). To remove the effect of noise, we propose a modified mean square error function, \( E\{\zeta^2(k)\} \), which is expressed as

\[
E\{\zeta^2(k)\} = \frac{E\{\epsilon^2(k)\}}{\gamma} \tag{8}
\]

Since the noise component of \( E\{\zeta^2(k)\} \) is independent of the delay and multipath parameters, it is easy to verify that the new performance surface has the global minimum at \( D = D, \alpha_{1i} = \alpha_{1i}, \Delta_{1j} = \Delta_{1j}, \alpha_{2j} = \alpha_{2j} \) and \( \Delta_{2j} = \Delta_{2j} \), although \( E\{\zeta^2(k)\} \) is multimodal. In our study, two steps are involved for unbiased multipath time delay estimation. The LMS-based multipath
cancellation time delay estimator (MCTDE) algorithm [7] is used at the beginning of the adaptation to initialize the system parameters such that these coarse estimates correspond to a point on the error surface which is sufficiently close to the global minimum. After initialization, we employ the LMS method again to minimize \(E\{c^2(k)\}\) with respect to the system variables so that an unbiased parameter estimation can be achieved. The instantaneous value of \(E\{c^2(k)\}\), \(c^2(k)\), is given by \(c^2(k) = \epsilon(k)/\gamma(k)\) where \(\epsilon(k)\) and \(\gamma(k)\) are computed from (5) and (7) using the delay and multipath parameter estimates at time \(k\), viz., \(D(k), \alpha_{11}(k), \Delta_{11}(k), \alpha_{22}(k)\) and \(\Delta_{22}(k)\). The updating equations for the BF-METDE are summarized as follows,

\[
D(k + 1) = D(k) - \mu_D \epsilon(k) \left( x'(k - D(k)) - \frac{\epsilon(k)}{\gamma(k)} \right) - \frac{\epsilon(k)}{\gamma(k)} \sum_{i=1}^{M_2} \alpha_{22}(k)f(D(k) - \Delta_{22}(k))
\]  

(9)

\[
\alpha_{11}(k + 1) = \alpha_{11}(k) - \mu_\alpha \epsilon(k) \left( g(k - \Delta_{11}(k)) - \frac{\epsilon(k)}{\gamma(k)} \right) - \frac{\epsilon(k)}{\gamma(k)} \sum_{i=0}^{M_1} \alpha_{11}(k)\text{sinc}(\Delta_{11}(k) - \Delta_{11}(k))
\]  

(10)

\[
\Delta_{11}(k + 1) = \Delta_{11}(k) + \mu_\Delta \epsilon(k) \left( g'(k - \Delta_{11}(k)) + \frac{\epsilon(k)}{\gamma(k)} \sum_{i=0}^{M_1} \alpha_{11}(k)\text{sinc}(\Delta_{11}(k) - \Delta_{11}(k)) \right)
\]  

(11)

\[
\alpha_{22}(k + 1) = \alpha_{22}(k) + \mu_\alpha \epsilon(k) \left( x(k - \Delta_{22}(k)) + \frac{\epsilon(k)}{\gamma(k)} \sum_{i=0}^{M_2} \alpha_{22}(k)\text{sinc}(\Delta_{22}(k) - \Delta_{22}(k)) \right)
\]  

(12)

\[
\Delta_{22}(k + 1) = \Delta_{22}(k) - \mu_\Delta \epsilon(k) \left( x'(k - \Delta_{22}(k)) - \frac{\epsilon(k)}{\gamma(k)} \sum_{i=0}^{M_2} \alpha_{22}(k)\text{sinc}(\Delta_{22}(k) - \Delta_{22}(k)) \right)
\]  

(13)

where \(\epsilon(k) = \sum_{m=-P}^{P} \text{sinc}(m - v)u(k - m)\) and \(\epsilon'(k) = \sum_{m=-P}^{P} f(m - v)u(k - m)\). The positive scalars \(\mu_D, \mu_\alpha\) and \(\mu_\Delta\) are the step sizes of delay, multipath gain, and multipath delay estimates, respectively, and \(f(v) = (\cos(\pi v) - \text{sinc}(v))/v\). The quantities \(\alpha_{11}(k)\) and \(\alpha_{22}(k)\) are equal to one while \(\Delta_{11}(k) = 0\) and \(\Delta_{22}(k) = D(k)\). Note that if \(\gamma(k) \to \infty\), (9)-(13) will become the METDE algorithm.

III. Simulation Results

Computer simulations were conducted to evaluate the performance of the proposed algorithm for time delay estimation in the presence of multipath transmissions. The signal \(s(k)\) as well as the noises \(n_1(k)\) and \(n_2(k)\) were independent zero-mean white Gaussian processes. We assumed that \(D \in (-1, 1)\) s and the values of the multipath delays were less than 5 s. In order to allow for acceptable delay modeling error, \(P\) was chosen to be 15.

In our experiments, we freely adapted the MCTDE at the beginning for 600 iterations to determine the initial estimates of \(D, \Delta_{11}\) and \(\Delta_{22}\). While the values of \(\alpha_{11}(0)\) and \(\alpha_{22}(0)\) were arbitrarily selected to be 0.5. The step size \(\mu_\alpha\) had a value of 0.006 and \(\mu_D = \mu_\Delta = 0.0003\) were used. All results provided were averages of 100 independent runs.

Figures 2 to 6 show the learning behaviors for the parameter estimates of the METDE and BF-METDE when \(M_1 = 1, M_2 = 1\) and at an SNR of 10 dB. The actual values of the system parameters were given as follows, \(D = 0.5\) s, \(\alpha_{11} = 0.8, \Delta_{11} = 1.6\) s, \(\alpha_{22} = 0.7\) and \(\Delta_{22} = 3.3\) s. It can be seen that for the modified algorithm, \(D(k) \to 0.5\) s in about 1200 iterations while the multipath variables \(\alpha_{11}(k), \Delta_{11}(k), \alpha_{22}(k), \Delta_{22}(k)\) converged to their desired values at approximately the 1000th, 1400th, 1800th and 1500th iteration, respectively. On the other hand, the METDE provided similar learning speeds but its parameter estimates were biased in the presence of noise. At steady state, the METDE system variables were \(D(k) = 0.534\) s, \(\alpha_{11} = 0.706, \Delta_{11} = 1.561\) s, \(\alpha_{22} = 0.588\) and \(\Delta_{22} = 3.397\) s.

References


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Figure 2 Estimate of $D$ in METDE & BF-METDE
Figure 4 Estimate of $\Delta_{11}$ in METDE & BF-METDE
Figure 3 Estimate of $\alpha_{11}$ in METDE & BF-METDE
Figure 5 Estimate of $\alpha_{21}$ in METDE & BF-METDE
Figure 6 Estimate of $\Delta_{21}$ in METDE & BF-METDE

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