DETECTION OF NARROWBAND RANDOM SIGNALS VIA SPECTRUM MATCHING

H. C. So†, W. K. Ma‡ and Y. T. Chan*  
† Dept. of Electronic Engg., City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong  
‡ Dept. of Electronic Engg., The Chinese University of Hong Kong, Shatin, N.T., Hong Kong  
*Dept. of Electrical and Computer Engg., Royal Military College of Canada, Kingston, Ontario, Canada, K7K 5L0  
e-mail: hceso@ee.cityu.edu.hk, wkma@ee.cuhk.edu.hk, chan-yy@banyan.rmc.ca

ABSTRACT
Detection of narrowband signals is of interest in many areas such as radar and communications. Bartlett method is the conventional method to tackle this problem but it solely uses the peak information in the periodogram even when the signal structure is partially known. Using a priori knowledge of the signal power spectral density (PSD), a spectrum matching approach which effectively utilizes the available signal spectral shape is developed for narrowband signal detection in this paper. Two spectrum matching detector structures, which are practically implemented by correlogram and periodogram respectively, are examined. Theoretical calculation of their false alarm rates is derived and confirmed by simulations. It is also demonstrated that the proposed detectors outperform the standard periodogram and Bartlett method under constant false alarm rate (CFAR) condition for different models of narrowband signals.

1 INTRODUCTION
Ascertaining the presence of a narrowband random signal from a finite number of noisy discrete-time measurements has applications in many fields [1-3] such as radar, sonar, communications and spectroscopy. Due to electrical receiver noises, atmospheric disturbances, spurious reflections from the ground and other objects, etc., it is usually not possible to determine with absolute certainty whether or not the target is present.

Considering the narrowband noise process as a sinusoid that exhibits phase instability, Bartlett method, which is a variant of the periodogram, is commonly used as the detector because such a processor is close to optimal in several cases of interest [2-3]. The detection procedure involves segmenting the received signal, computing the periodogram for each segment at the centre frequency of the narrowband signal, averaging the periodograms to form a test statistic and comparing the test statistic with a preassigned threshold value. If the test statistic value exceeds the threshold, we decide that the signal is present. Otherwise, the signal is assumed to be absent. In applications where the signal structure is partially known, the averaged periodogram will not be an optimum detector because it only uses the peak of the signal spectrum as the test statistic and does not employ other accessible characteristics of the signal.

Assuming the signal PSD is available, a spectrum matching detector (SMD) which can effectively utilize both the signal shape and bandwidth information is proposed for narrowband signal detection. The idea is to cross correlate the squared magnitude spectrum of the received signal with the given PSD and the resultant correlation function is then employed as the test statistic. The SMD can be realized exactly using the correlogram but the computation is expensive. In order to reduce computational complexity, we have also suggested an approximate version, called approximate SMD (ASMD), which is based on the periodogram and can be implemented efficiently by discrete Fourier transform.

The organization of this paper is as follows. In Section 2, the problem of narrowband signal detection is formulated and the spectrum matching approach is developed. Based on this technique, two detector structures are proposed. By approximating the test statistics of the detectors as chi-square random variables, the false alarm rates are derived in Section 3. In Section 4, simulation results for performance evaluation are presented and conclusions are then drawn.

2 SPECTRUM MATCHING DETECTION
The detection problem of a narrowband signal in the presence of noise is formulated as follows. Given the received sequence \( x(n) \), a decision has to be made between the hypotheses:

\[
H_0 : \quad x(n) = v(n), \quad n = 0, 1, ..., N - 1 \\
H_1 : \quad x(n) = s(n) + v(n)
\]

(1)

where \( v(n) \) is a zero-mean white Gaussian noise with variance \( \sigma_v^2 \) while \( s(n) \) is the narrowband random signal to be detected and is given by

\[
s(n) = a(n) \sin(2\pi f_c n + \phi)
\]

(2)

where \( a(n) \), \( f_c \) and \( \phi \) represent the signal envelope, centre frequency and phase respectively. It is assumed that \( a(n) \) changes slowly with respect to the carrier period \( 1/f_c \) and is uncorrelated with \( \phi \), which is uniformly distributed over the interval \([0, 2\pi] \).

When the PSD of \( s(n) \), \( P_s(f) \), is known, the PSD of \( a(n) \), \( P_a(f) \), as well as the \( f_c \) should also be available.
Using the idea of matched filtering [2], we propose the spectrum matching detector (SMD) which transforms the received signal into the power spectrum domain to match with the $P_s(f)$. For simplicity, the spectrum matching is performed in baseband and the system block diagram is depicted in Figure 1. The periodogram of $x(n)$ is defined as $I_x(f) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-j2\pi fn/N}$ while $\hat{P}_s(f)$ is the normalized and modified version of $P_s(f)$ for compensating the finite length effect of $x(n)$. It is given by

$$\hat{P}_s(f) = \sum_{m=-\infty}^{\infty} \hat{r}_s(m)e^{-j2\pi fm}$$

(3)

where $\hat{r}_s(m) = \frac{r_s(m)}{\sigma_s^2}$ with $r_s(m)$ and $\sigma_s^2$ being denoted as the autocorrelation sequence (ACS) and power of $s(n)$, respectively. The test statistic $\Lambda$ of the SMD is the cross correlation function of $I_x(f) - F_c$ and $\hat{P}_s(f)$ and it is computed as

$$\Lambda = \int_{-0.5}^{0.5} \hat{P}_s(f)I_x(f - F_c)df$$

(4)

Hypothesis $H_1$ is chosen if $\Lambda$ is greater than the threshold value $V_T$ and $H_0$ is decided otherwise.

![Fig. 1: System Block Diagram of the SMD](image)

Following [5], we can rewrite (4) as

$$\Lambda = \sum_{m=-(N-1)}^{N-1} \hat{r}_x(m)\hat{r}_s(m)\exp(-2\pi f_cm)$$

(5)

so that the integration operation in computing $\Lambda$ is avoided. For implementation, we will use (5) instead of (4). The ACS of $x(n)$ is given by

$$\hat{r}_x(m) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)x(n + |m|).$$

(6)

It is worthy to note that (5) can be interpreted as the windowed correlogram or the Blackman-Tukey spectral estimator (BTSE) at $F_c$, with lag window given by $\hat{r}_s(m)$ [4].

On the other hand, the integral of (4) can be approximated by summing the sampled values of $\Lambda$, viz.,

$$\hat{\Lambda} = \frac{1}{N} \sum_{k=0}^{N-1} \hat{P}_s(k)I_x(k - F_c)$$

(7)

where we denote $\hat{P}_s(k) = \hat{P}_s(f)|_{f=k/N}$ and $I_x(k) = I_x(f)|_{f=k/N}$ for brevity. Notice that $k_0$ is not necessary to be an integer, and that $\hat{P}_s(-k) = \hat{P}_s(N - k)$ as it is a circular sequence. Since $s(n)$ is narrowband, the signal power should be concentrated only in a few values of $\hat{P}_s(k)$; say over the interval $[-B + 1, B - 1]$. As a result, the test statistic can be further simplified as follows,

$$\hat{\Lambda} = \frac{1}{NU} \sum_{k=-B}^{B} \hat{P}_s(k)I_x(k - k_0)$$

(8)

where $\hat{U} = \sum_{k=-B}^{B} \hat{P}_s(k)/N$ is a normalizing factor so that $E[\hat{\Lambda}|H_0] = \sigma_s^2$. This spectrum matching detection method with $\hat{\Lambda}$ as the test statistic is called the ASMD. The advantage of the ASMD is that it only involves a weighted sum of a relatively small region of the periodogram, thus providing an efficient computation of the test statistic. However, it should be noted that the selection of $B$ is always a tradeoff between detection performance and computation simplicity. Nevertheless, if $B$ is chosen in such a way that the interval $[-B + 1, B - 1]$ covers most of the power of $s(n)$ in spectral domain, the ASMD is capable of providing a satisfactory performance.

3 APPROXIMATE FALSE ALARM RATES

In applications where the CFAR criterion is chosen to adjust the decision threshold, it is necessary to analyze the false alarm rates ($P_{FA}$) with respect to the threshold. For SMD, calculating its $P_{FA}$ is as difficult as finding the exact distribution of the BTSE. Very often, in the field of classical spectral analysis, the chi-squared approximation is employed to evaluate the distribution of the BTSE [5]. Borrowing the result of [5] directly, we can obtain an approximate solution for $P_{FA}$ of the SMD:

$$P(\Lambda > V_T|H_0) \approx 1 - \gamma(cV_T/\sigma_s^2, c)$$

(9)

$$c = N\sqrt{\sum_{m=-(N-1)}^{N-1} \hat{r}_s^2(m)}$$

(10)

where $\gamma(x, y) = \int_0^y u^{x-1}e^{-u}du/\Gamma(y)$ denotes the Pearson's incomplete Gamma function.

Direct evaluation of $P_{FA}$ of ASMD is also a difficult problem. Here we utilize chi-squared approximation again to evaluate $P_{FA}$. The chi-squared approximation
method requires the first two moments to provide the approximation [5, p.87]. Assuming that $k_1 - k_0$ does not equal 0 or $N/2$ for any $k \in [-B + 1, B - 1]$, the moments of the ASMD can be derived as

$$E[\hat{\lambda}/H_0] = \sigma_v^2,$$  \hspace{1cm} (11)

$$\text{var}[\hat{\lambda}/H_0] = \sigma_v^4 \sum_{k=-(B-1)}^{B-1} \hat{P}_s^2(k),$$  \hspace{1cm} (12)

Applying the chi-squared approximation, the false alarm rate of ASMD is given by

$$P(\hat{\lambda} > V_T|H_0) \approx 1 - \gamma \left( dV_T/\sigma_v^2, d \right),$$  \hspace{1cm} (13)

$$d = \left( \sum_{k=-(B-1)}^{B-1} \hat{P}_s(k) \right)^2 / \left( \sum_{k=-(B-1)}^{B-1} \hat{P}_s^2(k) \right).$$  \hspace{1cm} (14)

4 SIMULATIONS & CONCLUSIONS

In the simulations, the signal envelope was modeled as a Gaussian process with ACS

$$\hat{r}_s(m) = \sin(2\pi \Delta f m)/2\pi \Delta f m,$$  \hspace{1cm} (15)

which corresponds to a rectangular-shape PSD:

$$P_s(f) = \begin{cases} \sigma_v^2/2\Delta f, & |f| < \Delta f \\ 0, & \text{otherwise} \end{cases}.$$  \hspace{1cm} (16)

The first experiment compared the receiver operating characteristics (ROCs) of the SMD and ASMD. The parameters for simulation are: signal-to-noise ratio (SNR) $= \sigma_v^2/2\sigma^2 = -9$dB, $\sigma_v^2 = 1$, $\Delta f = 3.5/N$, $f_c = 0.25$, and $N = 256$. The method suggested in [6] was utilized to perform the simulation and the number of ensembles was 10000. The results are shown in Fig. 2. It is clearly observed that the SMD gives the best performance, and that the ASMD gradually approaches the SMD as $B$ increases. Fig. 3 shows both theoretical and experimental values of $P_{FA}$ of the proposed detectors with respect to $V_T$. It can be seen that the theoretical false alarm rates in (9) and (13) agree with the simulated values.

In the second experiment, we aimed at testing the accuracy of equations (9) and (13) when dealing with small values of $P_{FA}$. The experiment was performed by fixing the value of $P_{FA}$, computing the corresponding threshold $V_T$ via equations (9) and (13), and running a Monte-Carlo simulation of 100000 trials to estimate $P_{FA}$. The results are tabulated in Table 1. It is, however, observed that the numerical accuracy reduces when $P_{FA}$ of the SMD is very small. Though relatively insignificant, similar problem also arises in the ASMD for $B = 4$. For the rest, the precision is acceptable.

In the third experiment, we compared the performance of the conventional and ASMD detectors. By fixing the probability of false alarm to 0.01, 10000 independent runs were carried out to evaluate the probability of detection ($P_D$) with respect to SNR. The parameters for the simulation were identical to those of the first experiment. The results are illustrated in Fig. 4. Note that the symbol $K$ in the figure stands for the number of segments in the Bartlett method. It can be seen that the best and the worst detectors are ASMD and periodogram without segmentation, respectively. On the other hand, the performance of the Bartlett procedure for various values of $K$ lies midway between the standard periodogram and ASMD. We also repeated the experiment with the ACS model altered as

$$\hat{r}_s(m) = 0.992^{m}.$$  \hspace{1cm} (17)

Fig. 5 shows the simulation results, and it can be seen that the outcome is similar to the previous one.

To conclude, the detection of narrowband signals based on a priori knowledge of PSD is investigated. Two detectors, SMD and ASMD, are proposed for this purpose. The ASMD is an approximate version of SMD for the sake of reducing computation. However, with an appropriate choice of $B$, the ASMD can approach the SMD in performance. The theoretical false alarm rates of the proposed detectors are also investigated. Since exact solutions are too complicated to obtain, the chi-squared approximation method is utilized to evaluate the false alarm rates. Simulation results show that the theoretical false alarm rates can reasonably approximate the simulated ones, though their precision tends to reduce when dealing with tails of the distribution. Accurate computation of the false alarm rates over distribution tails seems to be an interesting and challenging work in the future.

Finally, for a constant false alarm rate, it has been experimentally shown that the ASMD can surpass the conventional detectors in a higher probability of detection.

References


Table 1: The simulated $P_{FA}$ for the proposed detectors.

<table>
<thead>
<tr>
<th>True $P_{FA}$</th>
<th>Simulated $P_{FA}$</th>
<th>SMD</th>
<th>ASMD $B = 2$</th>
<th>ASMD $B = 3$</th>
<th>ASMD $B = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.09324</td>
<td>0.00973</td>
<td>0.10047</td>
<td>0.10040</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.00901</td>
<td>0.00911</td>
<td>0.00961</td>
<td>0.00990</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.00078</td>
<td>0.00101</td>
<td>0.00099</td>
<td>0.00091</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2: ROCs of SMD, and ASMDs for various values of $B$

Fig. 3a: $P_{FA}$ as a function of $V_T$ for SMD

Fig. 3b: $P_{FA}$ as a function of $V_T$ for ASMDs

Fig. 4: $P_D$ of various detectors as a function of SNR for $r_a(m) = \sin(2\pi f_m)/2\pi f_m$

Fig. 5: $P_D$ of various detectors as a function of SNR for $r_a(m) = 0.999^{2m}$