Unbiased FIR System Identification in the Presence of Input and Output Interference

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Abstract - In the presence of input interference, the Wiener solution for impulse response estimation is biased. In this paper, it is proved that bias removal can be achieved by proper scaling the optimal filter coefficients and a modified least mean squares algorithm is then developed for accurate system identification in noise. Simulation results are included to compare the impulse response estimation performances of the proposed method and two total least squares based adaptive algorithms under different interference conditions.

I. INTRODUCTION

Estimation of the impulse response of an unknown system from its measured input and output has found applications in many fields such as control and signal processing [1]-[2]. If measurement noise exists only in the output, the least mean squares (LMS) algorithm [3], which is based on the minimum mean squared error (MMSE) criterion, can provide accurate estimates of the system parameters. However, when both input and output contain additive noise, which is the more realistic case, the LMS solution is biased. The total least squares (TLS) approach [4]-[5] seems to be an appropriate choice for this problem because it considers both stimulation error and response error. Typical examples of TLS-based computationally efficient algorithms for adaptive impulse response estimation in noise include the constrained anti-Hebbian (CAH) method [6] and the total least mean squares (TLMS) algorithm [7]. The first algorithm is a local gradient descent algorithm that uses the linearized gradient vector of the instantaneous estimate of an TLS energy function while the second one intrinsically finds the TLS solution on a sample-by-sample basis via minimizing the Raleigh quotient. In this paper, estimation of a finite impulse response (FIR) system is investigated. By examining the MMSE estimates for system identification in the presence of input and output interference, a modified LMS algorithm is proposed to provide unbiased impulse response estimates.

The paper is organized as follows. In Section II, the FIR system identification problem is formulated and the MMSE solution is derived. It is proved that unbiased impulse response estimates can be obtained by weighting the Wiener filter weights with a correction factor and the LMS algorithm is then modified for its adaptive realization. Simulation results are presented in Section III to demonstrate the unbiasedness of the proposed algorithm and to compare its impulse response estimation performance with the CAH and TLMS algorithms under different input and output noise environments. Finally, conclusions and some possible extensions are given in Section IV.

II. The MODIFIED LMS ALGORITHM

Consider the system identification configuration in Figure 1. It is assumed that the unknown system \( H(z) \) is of causal FIR and has the form \( H(z) = \sum_{i=0}^{N-1} a_i z^{-i} \) where \( N \) is known a priori. The signals \( s(k) \) and \( d(k) \) represent the noise-free input and output of \( H(z) \), respectively, while \( n_i(k) \) and \( n_o(k) \) are the input and output measurement noise. Given the corrupted input and output,

\[
x(k) = s(k) + n_i(k)
\]

\[
r(k) = d(k) + n_o(k)
\]

Figure 1: System block diagram for system identification
the aim is to find $H(z)$ using the filter $W(z) = \sum_{i=0}^{N-1} w_i z^{-i}$. For ease of analysis, we assume that $\sigma^2$, $\sigma^2_n$, and $\sigma^2_{n_k}$ are independent white processes with variances $\sigma^2$, $\sigma^2_n$, and $\sigma^2_{n_k}$, respectively. For time-invariant filter coefficients $w_i$, the error function $e(k)$ is given by

$$e(k) = r(k) - \sum_{i=0}^{N-1} w_i x(k-i)$$  \hspace{1cm} (2)

The mean squared value of $e(k)$, $E\{e^2(k)\}$, can be shown to be

$$E\{e^2(k)\} = \sigma^2 + \sigma^2_n + \sigma^2_{n_k} \sum_{i=0}^{N-1} w_i^2$$  \hspace{1cm} (3)

Differentiating $E\{e^2(k)\}$ with respect to each $w_i$ and then equating the resultant expressions to zero, we obtain the Wiener filter weights as follows,

$$w_i^* = \alpha h_i, \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (4)

where

$$\alpha = \frac{\sigma^2}{\sigma^2 + \sigma^2_n}.$$  \hspace{1cm} (5)

It is clear that $0 < \alpha < 1$ and the Wiener solution equals the unknown impulse response only if $\sigma^2_n = 0$. In the presence of input interference, $w_i^*$ are biased estimates of $h_i$, and the bias increases with the power of $n_i(k)$. Notice that the output noise $n_i(k)$ has no effect on the biasedness of the system parameter estimates. However, if $\alpha$ is known, the desired filter coefficients can be acquired by dividing each $w_i^*$ by it. In our study, it is found that $\alpha$ can be calculated as

$$\alpha = \frac{\sum_{i=0}^{N-1} w_i^2 \cdot E\{x^2(k)\}}{E\{r(k) r(k-\tau)\}}$$  \hspace{1cm} (5)

where $\tau \in [1, N - 1]$ represents a time-shifted parameter. Notice that the terms in (5), namely, $\sum_{i=0}^{N-1} w_i^2$, $E\{x^2(k)\}$, and $E\{r(k) r(k-\tau)\}$, have values of $\sigma^2 + \sigma^2_n$, $\sigma^2_n$, $\sigma^2_n$, and $\sigma^2 \sum_{i=0}^{N-1} x_{i+r}^2$, $\sigma^2_n h_i + \sigma^2_n \delta(\tau)$, respectively, where $\delta(\tau)$ is the unit impulse function. If $n_i(k) = 0$, we can even choose $\tau = 0$. In practice, $\tau$ should be selected to maximize the magnitude of $\sum_{i=0}^{N-1} x_{i+r}^2$ in order to minimize the estimation error.

Based on (4) and (5), we propose the following modified LMS algorithm to estimate $H(z)$ adaptively:

$$w_i(k+1) = w_i(k) + \mu e(k) x(k-i), \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (6)

$$\hat{R}_{xx}(k) = \beta z^2(k) + (1 - \beta) \hat{R}_{xx}(k-1)$$  \hspace{1cm} (7)

$$\hat{R}_{rr}(k) = \beta r(k) r(k-\tau) + (1 - \beta) \hat{R}_{rr}(k-1)$$  \hspace{1cm} (8)

$$w_i(k) = w_i(k) \cdot \frac{\hat{R}_{rr}(k)}{\sum_{i=0}^{N-1} w_i^2(k) x_i(k) \cdot \hat{R}_{xx}(k)} \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (9)

The error signal $e(k)$ is computed from (2) where $w_i$ are now replaced by $w_i(k)$. Equation (6) is the standard LMS algorithm and the mean value of each $w_i(k)$ will converge to $w_i^*$ in steady state. The parameters $\hat{R}_{xx}$ and $\hat{R}_{rr}$ represent the estimates of $E\{x^2(k)\}$ and $E\{r(k) r(k-\tau)\}$, respectively, with $0 < \beta < 1$. The modified filter parameters $w_i(k)$ are given by (9) where the instantaneous values of $w_i(k)$ are used to estimate $\sum_{i=0}^{N-1} w_i^2(k) x_i(k)$ and $\hat{R}_{xx}(k)$. As $k$ tends to infinity, it is expected that $w_i(k) \to h_i, i = 0, 1, ..., N - 1$, in the mean sense. For each iteration, the algorithm requires $(3N + 10)$ multiplications and $(2N + 2)$ additions.

Recall the CAH updating equation [6]:

$$w_i(k+1) = w_i(k) + \mu e(k) x(k-i) + r(k) w_i(k), \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (10)

and the TLMS algorithm [7]:

$$w_i(k+1) = w_i(k) + \mu(w_i(k) - p(k) x(k-i) e(k)), \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (11)

$$w_N(k+1) = w_N(k) + \mu(w_N(k) - p(k) r(k) e(k))$$  \hspace{1cm} (12)

$$w_i(k) = \frac{w_i(k)}{w_N(k)}, \quad i = 0, 1, ..., N - 1$$  \hspace{1cm} (13)

where

$$e(k) = \sum_{i=0}^{N-1} w_i(k) x(k-i) + w_N(k) r(k)$$

and

$$p(k) = \sum_{i=0}^{N} w_i^2(k)$$

We see that at each sampling interval, the former requires $(3N + 1)$ multiplications and $3N$ additions while $(5N + 4)$ multiplications and $(4N + 2)$ additions are involved in the latter. Therefore, the computational complexity of the proposed method is comparable to the CAH and TLMS algorithms.
III. SIMULATION RESULTS

Computer simulations had been carried out to evaluate the impulse response estimation performance of the modified LMS algorithm in noise. Comparisons were also made with the CAH and TLMS methods. The noise-free input $s(k)$ and the measurement noises, $n_t(k)$ and $n_o(k)$, were independent white Gaussian random variables. The power of $s(k)$ was fixed to be unity and different input and output signal-to-noise ratios, which were denoted by $\text{SNR}_t = \sigma_s^2/\sigma_n^2$, and $\text{SNR}_o = \sigma_s^2/\sigma_o^2$, where $\sigma_s^2$ was the power of $d(k)$, were obtained by proper scaling of the respective noise sequences. The unknown impulse responses were $[h_0 \ h_1 \ h_2 \ h_3 \ h_4] = [-0.1941 \ -0.5822 \ 0.5175 \ -0.4528 \ 0.385]$, so that $\sigma_s^2 = \sigma_n^2$. All adaptive filter coefficients were set to be zero initially except that the initial weight for $r(k)$ in the TLMS method was $-1$. The step sizes of the three algorithms were chosen to make the convergence times to reach their steady state mean squared coefficient errors (MSCEs) almost identical, where

$$\text{MSCE} = \sum_{i=0}^{N-1} E\{(h_i - v_i(k))^2\}$$

(14)

The parameter $\tau$ was selected to be $1$ so that the magnitude of $\sum_{i=0}^{N-1} h_{i+\tau}$ was maximized. All simulation results provided were averages of 400 independent runs.

Figure 2 compares the learning trajectories of the MSCEs of the three algorithms at $\text{SNR}_t = \text{SNR}_o = 10$dB. All methods converged at approximately the 2500 iteration. It can be observed that the CAH method gave the smallest steady state MSCE of $-30.5$dB while the performances of the modified LMS and TLMS algorithms were similar and their steady state MSCEs were $-21.5$dB and $-20.9$dB. The above test was repeated for $\text{SNR}_o = 0$dB and the results are shown in Figure 3. In this case, the proposed method, with steady state MSCE of $-18.2$dB, outperformed significantly the TLS-based algorithms and there was about 13dB improvement in estimation accuracy. Figures 4 and 5 show the results when $\text{SNR}_t = 0$dB and $\text{SNR}_o = 10$dB, and $\text{SNR}_t = \text{SNR}_o = 0$dB, respectively. In Figure 4, the steady state MSCEs of the modified LMS, CAH and TLMS methods were $-18.6$dB, $-9.0$dB and $-8.9$dB, respectively, while they had values of $-16.0$dB, $-19.4$dB and $-19.0$dB in Figure 5. It is seen that the CAH and TLMS algorithms performed similarly and they provided more accurate impulse response estimates than the proposed method at $\text{SNR}_t = \text{SNR}_o = 0$dB but they were inferior to it when $\text{SNR}_t = 0$dB and $\text{SNR}_o = 10$dB.

Table 1 tabulates the values of their mean impulse response estimates for different input and output interference conditions. We observe that the proposed algorithm gave unbiased estimates in all noise conditions, while the CAH and TLMS estimates were unbiased only when $\text{SNR}_t = \text{SNR}_o$ but they were severely biased when the input and output noise powers were different.

IV. CONCLUSIONS & FUTURE WORK

The LMS algorithm has been modified for accurate FIR system identification in the presence of input interference. The idea is to weight the LMS coefficients by a correction factor whose value depends on the SNR of the input signal. It is shown that the proposed method always provides unbiased impulse response estimates for different interference conditions and is superior to the CAH and TLMS algorithms when the input and output noise powers are different.

A drawback of the proposed algorithm is that the variances of its filter weights are larger than those of the TLS-based methods. As a result, its estimation performance is inferior to the CAH and TLMS algorithms when they provide unbiased FIR estimates. Further investigation will be focused on reducing the filter coefficient variances in the modified LMS algorithm given in (6)-(9). There is also a need to extend the algorithm for nonwhite source signals, if not impossible.

References


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Figure 2: Mean squared coefficient errors at SNR_t=10dB and SNR_o=10dB

Figure 4: Mean squared coefficient errors at SNR_t=0dB and SNR_o=10dB

Figure 3: Mean squared coefficient errors at SNR_t=10dB and SNR_o=0dB

Figure 5: Mean squared coefficient errors at SNR_t=0dB and SNR_o=0dB

<table>
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<tr>
<th>Filter Coefficient</th>
<th>h_0</th>
<th>h_1</th>
<th>h_2</th>
<th>h_3</th>
<th>h_4</th>
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<td>-0.5822</td>
<td>0.5175</td>
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<td><strong>Proposed Method</strong></td>
<td>SNR_t = 10dB, SNR_o = 10dB</td>
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<td>0.5178</td>
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<td><strong>TLMS Algorithm</strong></td>
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<td>-0.5834</td>
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Table 1: Mean impulse response estimates