DIGITAL FILTER REALISATION BY PASSIVE NETWORK SIMULATION

Li Ping and J I Sewell

Recently it has been shown that the LU decomposition drawn from matrix theory can be employed to derive digital circuit topologies simulating passive LCR ladders [1–2]. These, so called LUD type, circuits have some advantageous properties for parallel processing while retaining the low-sensitivity of the prototypes.

In this paper, the basic principle of LUD design is outlined and a frequency transformation method is introduced. This will solve the instability problem encountered in high-pass and band-stop LUD design. The same technique is applied to band-pass filters with reduced hardware cost, when compared with a direct approach. The regularity of the structures allows easy programmable application.

The LU Decomposition Simulation Method

A passive RLC network, Fig.1 can always be described by the nodal equation

\[
(\begin{bmatrix} s C + s^{-1} \Gamma & G \end{bmatrix} \begin{bmatrix} \Gamma + G \end{bmatrix}) \begin{bmatrix} \mathbf{v} \end{bmatrix} = \begin{bmatrix} J \end{bmatrix}
\]

(1)

where C, \( \Gamma \) and G represent the contribution of capacitors, inductors and conductors respectively. Perform the bilinear transformation on (1),

\[
\begin{bmatrix} \frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right) & \frac{T}{2} \left( \frac{G}{1 - z^{-1}} \right) \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} = \begin{bmatrix} J \end{bmatrix}
\]

(2)

Eq. (2) is equivalent to

\[
\begin{bmatrix} \frac{2}{T} \left( \frac{C}{2} \Gamma + G \right) + \frac{z^{-1}}{(1-z^{-1})^2} & \frac{2T\Gamma}{2} + \frac{z^{-1}}{1-z^{-1}} \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} = \begin{bmatrix} \frac{1+z^{-1}}{1-z^{-1}} J \end{bmatrix}
\]

(3)

For simplicity rewrite (3) as

\[
\begin{bmatrix} \mathbf{A} + \Psi \Phi \mathbf{B} + \Psi \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} = \Psi \Theta \begin{bmatrix} J \end{bmatrix}
\]

(4)

with

\[
\begin{align*}
A &= \frac{2}{T} \left( \frac{C}{2} \Gamma + G \right) \\
B &= 2T \Gamma \\
D &= 2G \\
\Psi &= \frac{z^{-1}}{(1-z^{-1})} \\
\Phi &= \frac{1}{(1-z^{-1})} \\
\Theta &= 1 + z
\end{align*}
\]

(5a - 5f)

Notice that \( \mathbf{A} \) is symmetric when (1) is derived from a passive network hence it can be decomposed into symmetric LU form

\[
\mathbf{A} = \mathbf{LU} = \mathbf{U}^T \mathbf{D_a} \mathbf{U}
\]

(6)

The authors are with the Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow G12 8QQ.
where \( D \) is a diagonal matrix used to scale all diagonal entries of \( U \) to 1. Applying (6) to the eq. system (4) permits a realisation in recursive form

\[
    \begin{align*}
        U^T X &= - (\Phi B + D) V + \Theta J \\
        U V &= \Psi D_a^{-1} X 
    \end{align*}
\]

(7a) \hspace{1cm} (7b)

Now the LUD digital implementation Fig. 2 can be obtained from eqn. (7). Where \( B \) is realised by decomposition in terms of network topology

\[
    B = 2T \Gamma = 2T A_L D_L A_L^T 
\]

(8)

Several alternative structures can also be derived. For a detailed discussion, see [1]

**Frequency Transformation Method**

The methods introduced in the last section can be applied directly to low-pass and band-pass but not to high-pass and band-stop design. The difficulty is that for these latter cases the transfer function is not of zero value at \( z = -1 \) (corresponding to \( s = \infty \) in continuous domain). However the input function of (4), \( (1+z)^{-1} \), must be infinite at \( z = -1 \) for cancellation, which inevitably results in an unstable system. For the high-pass case this difficulty can be overcome by using a frequency transformation of \( z^{-1} \) to \( z^{-1} \) to obtain the desired system from a low-pass reference. For the bandstop case the following technique is introduced.

It is well known that in the continuous domain a symmetric band-stop function can be derived from a normalised low-pass one by transformation [3], Fig. 3b

\[
    s \rightarrow \frac{1}{a} \left( \frac{s}{\omega_m} + \frac{\omega_m}{s} \right)^{-1} \hspace{1cm} (9a)
\]

with

\[
    a = \frac{\omega_m}{\omega_m^T}, \hspace{1cm} \omega_m = \frac{\omega_m}{\omega_m^T} \hspace{1cm} (9b)
\]

Substitute (9) into (1) and perform the bilinear transformation,

\[
    \left[ a^{-1} \left( \frac{2}{\omega_m^T} - \frac{1}{1+z^{-1}} \right) \right] C + a \frac{2}{\omega_m^T} \left( \frac{1}{1+z^{-1}} \right) \Gamma + G \right] V = J \hspace{1cm} (10)
\]

After some manipulation (10) can be rewritten as

\[
    (A + \Psi_{bs} \Phi_{bs} \Gamma + \Theta_{bs} D) V = \Psi_{bs} \Theta_{bs} J \hspace{1cm} (11)
\]

with

\[
    \begin{align*}
        A &= \alpha^{-1} C + \alpha \Gamma + G \\
        B &= 4\alpha \Gamma \\
        D &= 2 \Gamma \\
        \Psi_{bs} &= (z^{-2} - \beta z^{-2})/(1-z^{-2}) \\
        \Phi_{bs} &= (1 - \beta z^{-1})/(1-z^{-2}) \\
        \Theta_{bs} &= (1-2\beta z^{-2})/(z^{-2} - \beta z^{-1}) \\
        \alpha &= \frac{2}{\omega_m^T} + \frac{\omega_m^T}{2}
    \end{align*}
\]

(12a) \hspace{1cm} (12b) \hspace{1cm} (12c) \hspace{1cm} (12d) \hspace{1cm} (12e) \hspace{1cm} (12f) \hspace{1cm} (12g)
\[
\beta = \frac{2/(\omega_m T) - (\omega_m T)/2}{2/(\omega_m T) + (\omega_m T)/2} \tag{12h}
\]

System (11) has the same form as (4) and can be realised by the same scheme, Fig.2, with \( \Psi, \Phi \) and \( \Theta \) replaced by \( \Psi_{bp}, \Phi_{bp} \) and \( \Theta_{bp} \), Table 1. A set of parameters is given in Table 2 as an example of the transformed simulation of the ladder of Fig.1. Notice that in this example, the system of (11) has been scaled by 0.5 to set the termination entries in D to 1, saving two multipliers. The stopband range is from 3000 Hz to 4000 Hz and the sampling frequency is 32000 Hz. The frequency response is shown in Fig.4. It can be verified that the zeros of \( \Theta_{bp} \) now lie exactly in the middle of stopband and the instability problem mentioned above is thus avoided.

Similarly a symmetric band-pass function can be derived from a normalised low-pass one by transformation, Fig. 3c.

\[
s \rightarrow a \left( \frac{a}{\omega_m} + \frac{\omega_m}{s} \right) \tag{13a}
\]

with
\[
a = \frac{\omega_m}{\omega_m^2 - \omega}, \quad \omega_m = \sqrt{\omega^2 - \omega} \tag{13b}
\]

Substitute (13) into (1) and perform the bilinear transformation,
\[
\left\{ a(\frac{2}{\omega_m T} + \omega_m T) (1 + \frac{1}{z}) \right\} C + a^{-1}(\frac{2}{\omega_m T} (1 + \frac{1}{z}) \frac{1}{\omega_m T} (1 + \frac{1}{z} - 1) \Gamma + G \right\} V = J \tag{14}
\]

Eq. (10) can be again rewritten as
\[
(A + \Psi_{bp}^\Phi_{bp} B + \Psi_{bp} D) V = \Psi_{bp} \Theta_{bp} J \tag{15}
\]

with
\[
A = \alpha C + \alpha^{-1} \Gamma + G \tag{16a}
\]
\[
B = 4\alpha^{-1} \Gamma \tag{16b}
\]
\[
D = 2 \Gamma \tag{16c}
\]
\[
\Psi_{bp} = (\beta z^{-1} - z^{-2})/(1 - 2\beta z^{-1} + z^{-2}) \tag{16d}
\]
\[
\Phi_{bp} = (1 - \beta z^{-1})/(1 - 2\beta z^{-1} + z^{-2}) \tag{16e}
\]
\[
\Theta_{bp} = (1 - z^{-2})/(\beta z^{-1} - z^{-2}) \tag{16f}
\]
\[
\alpha = a \left( \frac{a}{\omega_m T} + \frac{\omega_m}{J} \right) \tag{16g}
\]
\[
\beta = \frac{2/(\omega_m T) - (\omega_m T)/2}{2/(\omega_m T) + (\omega_m T)/2} \tag{16h}
\]

Again eq. (15) can be realised by the same scheme Fig. 2 with \( \Psi, \Phi \) and \( \Theta \) replaced by \( \Psi_{bp}, \Phi_{bp} \) and \( \Theta_{bp} \). It can even be verified that in this approach the number of multiplications and additions is much smaller than required in the direct simulation of a band-pass prototype ladder.

**Discussion and Conclusion**

It is shown that different types of LUD digital circuits can be derived in a unified procedure from a lowpass ladder prototype.

It is interesting to note some common properties shared by the block operators given in Table 1: (i) \( \Phi_* = \Psi_* = 1 \), (ii) the poles of \( \Psi_* \) and \( \Phi_* \) are on the unit circle and at the middle of passband, (iii) the zeroes of \( \Theta_* \) are on the unit circle and at the middle of stopband; where subscript "*" applies to all four types. This may be worth further investigation.
If it is possible to adjust the product of $\omega_m$ and $T$, $\beta$ may be scaled to a special number to facilitate easy multiplication. For example, if $\beta$ is set to $0.75 = 2^{-2} + 2^{-3}$, then it requires only two shifts and one addition to multiply a signal by $\beta$. As $\beta$ is repeatedly used, considerable saving of hardware cost and operation time can be gained. The frequency transformation technique can also be applied to active RC and switched capacitor circuit design.

REFERENCES

1. Li Ping and J.I.Sewell, "Filter realisation by passive network simulation", to be published, Proc. IEE Part G.

Fig. 1 A fifth order low-pass elliptic passive prototype.
Fig. 2  Standard LUD digital realisation of the elliptic filter.
Fig. 3
(a) A normalised low-pass reference.
(b) A band-stop specification.
(c) A band-pass specification.

<table>
<thead>
<tr>
<th>Reference Ladder (normalised values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 = 1 ) ( g_L = 1 )</td>
</tr>
<tr>
<td>( c_1 = 1.05298 ) ( L_2 = 1.24796 ) ( C_2 = 0.11789 )</td>
</tr>
<tr>
<td>( c_3 = 1.69738 ) ( L_4 = 1.02002 ) ( C_4 = 0.33388 )</td>
</tr>
<tr>
<td>( C_5 = 0.88240 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Digital simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 = 0.21618 ) ( a_2 = 0.17976 ) ( a_3 = 0.94811 )</td>
</tr>
<tr>
<td>( b_1 = 16.2715 ) ( b_2 = 19.908 )</td>
</tr>
<tr>
<td>( c_1 = 0.88070 ) ( c_2 = 0.89761 )</td>
</tr>
<tr>
<td>( d_1 = 1 ) ( d_2 = 1 ) ( \beta = 0.77675 )</td>
</tr>
</tbody>
</table>

Table 2. Parameters for the Band-stop Filter
<table>
<thead>
<tr>
<th>Type</th>
<th>Operator</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard and lowpass</td>
<td>( \frac{z^{-1}}{1-z^{-1}} )</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{1-z^{-1}} )</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>Highpass</td>
<td>( \frac{-z^{-1}}{1+z^{-1}} )</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{1+z^{-1}} )</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>Bandstop</td>
<td>( \frac{z^{-2}-\beta z^{-1}}{1-z^{-2}} )</td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( \frac{1-\beta z^{-1}}{1-z^{-2}} )</td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>Bandpass</td>
<td>( \frac{\beta z^{-1}-z^{-3}}{1-2\beta z^{-1}+z^{-2}} )</td>
<td><img src="image7" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( \frac{1-\beta z^{-1}}{1-2\beta z^{-1}+z^{-2}} )</td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Table 1: Various Types of Frequency Operators
Fig. 4 A 5th order elliptic band-stop response.