Canonical design of integrated ladder filters

R.K. Henderson, BSc, PhD
Li Ping, MS, PhD
Prof. J.L. Sewell, BSc, PhD, CEng, FIEE

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Abstract: Necessary conditions are given for the existence of a one-opamp-per-pole (canonical) realisation of a transfer function by an active ladder filter designed by operational simulation methods. The parity of the transfer function is shown to be critical. Some new design methods are applied to guarantee canonical low-sensitivity realisation of hitherto problematic filter types. These ensure a uniform progression of circuit structures with filter order. Hybrid matrix methods are shown to be useful to minimise the use of opamps in the simulation of arbitrary prototype ladder topologies.

1 Introduction

Low-sensitivity integrated active RC or switched-capacitor (SC) filter structures based on passive ladder simulation have been widely used for more than three decades [1, 2]. Among many alternatives, the most popular are the leapfrog and coupled-biquad circuits by virtue of their stray insensitivity [3, 4]. However, the advantages of these circuits have always been compromised by their relatively complicated design procedures, requiring different design techniques to meet different filter specifications. For this reason, matrix methods have been introduced to regularise the design procedures, ensuring that the steps involved do not change significantly according to the form of prototype and system order [5, 6]. Some inconsistencies still remain, and it is the purpose of this paper to address these. An efficient integrated filter design will normally minimise the use of opamps, since these consume power and are sources of noise. Realisations with less than one opamp per pole are normally sensitive to component deviations and, in particular for SC circuits, to top and bottom plate stray capacitance. Therefore, it is generally accepted that one-opamp-per-pole realisations are canonical for low-sensitivity ladder circuits. However, this is not always achievable. For example, a 'pure' even-order lowpass elliptic function cannot be realised by a prototype passive ladder, unless the function is modified at the cost of poorer attenuation in the stopband [7]. Another problem exists for bandstop or highpass functions where, although a prototype can be synthesised, its straightfor-ward active simulation would be unstable [2]. Hence it is of both theoretical and practical interest to develop a procedure for canonical ladder design for general forms of transfer function.

A necessary condition is derived to assess whether a given transfer function possesses a canonical ladder realisation. The outcome is shown to be dependent on the parity of the numerator. This condition is also believed to be sufficient, although a rigorous proof has not been obtained. For transfer functions with the wrong parity, a novel method is introduced to realise canonical ladder circuits by changing the parity of the numerator and compensating for the distortion in the simulation procedure. Thus a wide family of hitherto problematic transfer functions can be realised by active circuits. A regular progression of circuits with increasing order is now possible, in the same way as for biquadratic cascade realisations. The simulation of arbitrary forms of ladder prototype which do not belong to the minimum node type is considered. Hybrid matrix methods are shown to be useful to ensure canonical realisations in these cases.

2 Standard ladder simulation methods

A passive ladder can be described by the nodal equation

\[ YV = J \]  

where \[ Y(s) = sC + s^{-1} \Gamma + G \]  

and where \( C, \Gamma \) and \( G \) are matrices formed from the contributions of capacitors, inductors and resistors respectively, \( \Gamma \) is the vector of node voltages and \( J \) is the input vector. A design procedure has been developed to realise eqn. 1 by active RC and SC circuits [8]. This is done by creating a set of intermediate variables and decomposing the system eqn. 1 into two interrelated systems. For example, in active RC circuits this decomposition can be performed in the following ways.

2.1 Left matrix decomposition

Factorise the matrix \( C \) into

\[ C = C_1C_2 \]  

The following pair of equations is equivalent to eqn. 1:

\[ C_1W = (-s^{-1} \Gamma - G)V - (-J) \]  \hspace{1cm} (4a)

\[ C_2V = s^{-1}W \]  \hspace{1cm} (4b)

where \( W \) is the vector of intermediate variables.

2.2 Right matrix decomposition

\( \Gamma \) can also be factorised as

\[ \Gamma = \Gamma_1\Gamma_2 \]  \hspace{1cm} (5)

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Prof. Sewell is with the Department of Electronics and Electrical Engineering, University of Glasgow, G12 8QQ, United Kingdom
R.K. Henderson is with CSEM, Case Postale 41, CH-2007 Neuchatel, Switzerland
Li Ping is with the Department of Electrical and Electronics Engineering, University of Melbourne, Parkville, Victoria 3052, Australia

The following pair of equations is equivalent to eqn. 1:
\[
CV = -s^{-1}[C_1 W + G_v + (-J)] \\
IW = s^{-1} Y 
\]
(6a) (6b)

The details of the design procedure can be found in [5, 6, 8, 9], covering the well known leapfrog and coupled-biquad methods together with some new ones, notably those obtained by adopting LU decompositions [10]. As system eqn. 1 with size \( n \) can at most realise a 2nth-order or a \((2n - 1)\)th-order transfer function for an even-order or an odd-order system respectively, then:

**Definition 1:** System eqn. 1 with size \( n \) is said to represent a canonical ladder prototype if it realises a \( 2n \)-th-order or a \((2n - 1)\)th-order transfer function.

In [10] it has been shown that system eqn. 1 can always be simulated by an active RC or SC ladder with \( 2n \) opamps, alternatively, if the transfer function is of order \( 2n - 1 \) a simulation with \( 2n - 1 \) opamps is possible. Therefore the problem of finding a canonical standard ladder simulation becomes that of finding a canonical ladder prototype.

3 Canonical ladder prototypes

In the following, conditions will be derived for a ladder prototype eqn. 1 to be canonical. First some properties of the prototype eqn. 1 are stated as follows:

**Remark 1:** The nodal description of a doubly-terminated ladder synthesised from the minimum node two-port sections in Fig. 1 has the following properties:

(a) \( C, G \) and \( G \) are all tridiagonal matrices. So \( Y \) is also tridiagonal.
(b) \( J \) has only one nonzero element, i.e. \( J = [J_1, 0, \ldots, 0]^T \).
(c) \( G \) has only two nonzero elements \( g_{11} = g_{nn} \) and \( g_{nn} = g_L \), so that in general
\[
y_{i+1, i} = s c_{i+1, i} - s^{-1} y_{i+1, i} \\
(7)
\]
(d) The output is the nodal voltage \( v_y \).

The constraints for a transfer function to be realisable by a canonical doubly-terminated ladder are as follows:

**Theorem 1:**

(a) The numerator of the transfer function \( v_y/J_1 \) of a canonical even-order doubly-terminated ladder is an odd polynomial.
(b) The numerator of the transfer function \( v_y/J_1 \) of an odd-order doubly-terminated ladder is an odd polynomial if \( |C| \) is nonsingular or an even polynomial if \( |F| \) is nonsingular.

**Proof:** From remark 1 the nodal admittance matrix of a doubly-terminated ladder is tridiagonal. By Cramer's rule [12] it can be found for the output \( v_y \) that
\[
\frac{v_y}{J_1} = \pm \frac{\Delta_{1n}}{\Delta(s)} \\
(8)
\]
where \( \Delta(s) \) is the determinant of \( Y \) and \( \Delta_{1n} \) is the appropriate cofactor. It can be shown that
\[
\frac{v_y}{J_1} = \prod_{i=1}^{n-1} \left[ s c_{i+1, i} - s^{-1} y_{i+1, i} \right] \\
\Delta(s) \\
(9)
\]
where \( F = \{y_{ik}\} \) and \( C = \{c_{ik}\} \). Let \( v_y/J_1 \) be expressed in the form of a rational function
\[
\frac{v_y}{J_1} = \frac{N(s)}{D(s)} \\
(10)
\]
\( D(s) \) and \( N(s) \) are the denominator and numerator polynomials respectively, and they contain only non-negative powers of \( s \). Consider first the case of an even canonical realisation. Notice that by using the Laplace expansion [12] repeatedly the determinant of \( \Delta(s) \) can be expanded as follows, where \( n \) is the size of the coefficient matrices:
\[
\Delta(s) = |C|s^n + a_{n-1}s^{n-1} + \ldots + a_0 + |F|s^{-n} \\
+ \ldots + a_0 + \ldots + a_{-(n-1)}s^{-(n-1)} + |F|s^{-n} \\
(11)
\]
A ladder realising a \( 2n \)-th-order function must have a nonsingular \( F \), and so the denominator can be rearranged to be a \( 2n \)-th-order polynomial as
\[
D(s) = s^n \Delta(s) \\
(12a)
\]
and the numerator becomes
\[
N(s) = s^n \prod_{i=1}^{n-1} \left[ s c_{i+1, i} - s^{-1} y_{i+1, i} \right] \\
(12b)
\]
Here \( c_{i+1, i} \) and \( y_{i+1, i} \) (for all \( i \)) cannot both be zero, otherwise the transfer function would be zero. Suppose \( c_{i+1, i} \) are nonzero for all \( i \); then it can be seen that \( N(s) \) is a \((2n - 1)\)th-order polynomial with only odd terms. If
any \( c_{i+1,j} \) is zero then \( N(s) \) will reduce to a \((2n-3)\)th-order polynomial (since in this case \( \gamma_{i+1,j} \) must be nonzero) and \( N(s) \) will stay odd. It is easy to deduce that \( N(s) \) will remain odd for cases of more zero \( c_{i+1,j} \). The same reasoning can be applied to the cases that some \( \gamma_{i+1,j} \) are zero.

Now consider the case of odd-order design, where either \( C \) or \( \Gamma \) must be singular to make \( D(s) \) in eqn. 12a odd. If \( \Gamma \) is nonsingular, exactly the same reasoning as for the even case can be used to show that \( N(s) \) must be odd. If \( C \) is nonsingular then

\[
D(s) = s^{n} \Lambda(s) \tag{13a}
\]

\[
N(s) = s^{n} \prod_{i=1}^{n-1} \left[ \beta_{i+1,j} - s^{i} \gamma_{i+1,j} \right] \tag{13b}
\]

and it is easily shown that \( N(s) \) must be an even polynomial.

Theorem 1 establishes some necessary conditions for a transfer function to have a canonical realisation. It appears that these conditions are also sufficient for realizability provided that the transfer function is stable.

It is seen from theorem 1 that the constraint on the parity of the numerator is related to the singularity of the matrices \( C \) and \( \Gamma \). The singularities, however, cannot be arbitrarily chosen.

**Theorem 2:** A doubly-terminated ladder has a nonzero response at \( \omega = \infty \) if \( C \) is singular and has a nonzero response at \( \omega = 0 \) only if \( \Gamma \) is singular.

**Proof:** Let \( s = \omega \). From eqn. 12 it can be seen that when \( \omega \to \infty \),

\[
\Delta(s) = |C| s^n + a_{n-1} s^{n-1} \tag{14}
\]

and from eqn. 9 the numerator is at most to the power of \( s^{-1} \). Therefore if \( |C| \) is not zero then eqn. 9 must be zero. Similar reasoning can be used at \( \omega \to 0 \).

It is mandatory that lowpass transfer functions have nonzero values at \( \omega = 0 \) and highpass and bandstop functions at \( \omega = \infty \). This indicates that the singularity of the matrices is predetermined by the filtering types, and therefore the parity of the numerators of odd-order cases is also constrained.

Since the singularities of \( C \) and \( \Gamma \) mean that their rank can at most be \( n - 1 \), according to eqn. 14 a list of the upper bounds for various filtering types by a ladder with order \( n \) is obtained in Table 1.

<table>
<thead>
<tr>
<th>Classes</th>
<th>Constraint</th>
<th>Upper bound of system order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowpass</td>
<td>( H(0) \neq 0 ) ( I ) singular</td>
<td>( 2n - 1 )</td>
</tr>
<tr>
<td>Bandpass</td>
<td>( H(0) \neq 0 ) ( C ) singular</td>
<td>( 2n )</td>
</tr>
<tr>
<td>Highpass</td>
<td>( H(\infty) \neq 0 ) ( C ) singular</td>
<td>( 2n - 1 )</td>
</tr>
<tr>
<td>Bandstop</td>
<td>( H(0) \neq 0 ) ( H(\infty) \neq 0 ) both ( C ) and ( \Gamma ) singular</td>
<td>( 2n - 2 )</td>
</tr>
</tbody>
</table>

It is easily seen that canonical designs can be achieved only by bandpass, odd-order lowpass and odd-order highpass. For other cases, constraints given in theorem 2 make a canonical realisation impossible. In the succeeding sections it will be seen that a noncanonical ladder prototype will lead to an integrated circuit simulation of excessive size, unless some complicated procedure is adopted. It will also be shown that the wrong parity of numerator can be easily corrected, and a simple technique is introduced to eliminate the error caused by this modification. This results in a unified procedure, with very regular structures, to realise a wide family of transfer functions. There is a uniform progression in the form of the circuit structures regardless of the type of specification or order.

**4 Canonical ladder simulation by modified prototype**

The numerators of the transfer functions considered in this paper are restricted to be polynomials with purely even or odd terms. This means their zeros are restricted to lie on or have quadrantal symmetry about the imaginary axis. Such a constraint is valid for most filter design problems. With this constraint and from theorem 1 the problem concerned in this paper has been greatly simplified: whether a given transfer function can be realised by a canonical standard ladder depends solely on the numerator parity.

The most common example of a transfer function with the wrong parity is an even-order 'pure' elliptic function with an even-order numerator. It is well known that this kind of function cannot be realised by a doubly-terminated ladder [13]. Traditionally the solution to this problem is to modify the numerator by moving a pair of finite zeros to infinity [7], which incurs the penalty of a loss of stopband attenuation (Fig. 2).

![Fig. 2 Modified and 'pure' eighth-order bandpass elliptic functions](image)

A new scheme is introduced here to realise the transfer function exactly without any sacrifice of filtering quality. Let \( H(s) \) be a transfer function with all its zeros on the imaginary axis or at infinity. If the constraints of theorem 1 are not met, or more precisely if \( N(s) \) has the wrong parity, some simple manipulation of the transfer
function can be made to cope with the problem. Consider three possibilities:
(a) The numerator of \( H \) is a constant.
(b) The numerator of \( H \) has a single root at \( \omega_0 = 0 \).
(c) The numerator of \( H \) has a pair of imaginary roots at \( \pm \omega_0 (\omega_0 \text{ can be zero}) \).

To change the parity, manipulate as follows:
(a) Let \( H'(s) = H(s) \).
(b) Let \( H'(s) = H(s) \) or \( H'(s) = H(s)/s \).
(c) Let \( H'(s) = H(s)/s^2 + \omega_0^2 \).

Then the parity of \( H' \) is opposite to that of \( H \), and \( H'(s) \) can now be realised by a canonical prototype ladder described by the nodal equation

\[
(sC + s^{-1}T + G)V = J
\]

(16)

### 4.1 Canonical ladder simulation by active RC circuits

A system realising the original transfer function \( H(s) \) can be obtained by multiplying the input vector \( J \) by the inverse of the modification function. For case (e) we have

\[
(sC + s^{-1}T + G)V = (s + \omega_0 s^{-1})J
\]

(17)

This system can now be expressed in realisable form by matrix methods [5, 6].

#### 4.1.1 Left matrix decomposition form:
Let \( C = C_1 C_2 \).
Then the system can be written as

\[
C_1 W = -s^{-1}[T V + G V + \omega_0 T (J)]
\]

(18a) \[
C_2 V = s^{-1} W - C_1^{-1} (J)
\]

(18b)

#### 4.1.2 Right matrix decomposition form:
Let \( \Gamma = \Gamma_1 \Gamma_2 \).
Then the system can be written as

\[
C V = -s^{-1}[\Gamma_1 V + G V + \omega_0 \Gamma_1^{-1} (J)]
\]

(19a) \[
W = s^{-1}[\Gamma_2 V + \omega_0 \Gamma_1^{-1} (J)]
\]

(19b)

Active RC networks can be obtained directly from these two equations. It is found that the most efficient method in either case is to use UL factorisation, which minimises the required number of input branches (only two).

The prototype in Fig. 3 is simulated by the two canonical eighth-order left-ULD and right-ULD active RC circuits shown in Fig. 4. These canonical designs differ from standard ones in the position of the input stage branches.

![Fig. 4. Canonical eighth-order active RC ladder filters](image)

#### 4.2 Canonical discrete ladder simulations

The design of a discrete ladder simulation starts from the modified prototype system eqn. 17, which after bilinear transformation becomes

\[
\begin{align*}
2 \frac{1-z^{-1}}{T} \frac{1}{1+z^{-1}} (C + T \frac{1+z^{-1}}{1+z^{-1}} \Gamma + G) V \\
= 2 \frac{1-z^{-1}}{T} \frac{1}{1+z^{-1}} J + T \frac{1+z^{-1}}{1+z^{-1}} \omega_0^2 J
\end{align*}
\]

(20)

The system can be rearranged as

\[
\begin{align*}
\left( \frac{1}{\Psi} A + \Phi B + D \right) V = \left[ \omega_0^2 + 1 \frac{1}{\Psi} \Phi \omega_0^2 \right] J
\end{align*}
\]

(21a)

\[
A = 2T C + T \frac{1}{2} \Gamma + G \quad B = 2T \Gamma
\]

(21b)

\[
D = 2G \quad \Psi = z^{-1}/(1 - z^{-1})
\]

\[
\Phi = 1/(1 - z^{-1})
\]

(21c)

or

\[
\begin{align*}
\left( \frac{1}{\Psi} A + \Phi B + D \right) V = \left[ \omega_0^2 + 1 \frac{1}{\Psi} \Phi \omega_0^2 \right] J
\end{align*}
\]

(21a)

\[
A = 2T C + T \frac{1}{2} \Gamma + G \quad B = 2T \Gamma
\]

(21b)

\[
D = 2G \quad \Psi = z^{-1}/(1 - z^{-1})
\]

\[
\Phi = 1/(1 - z^{-1})
\]

(21c)

The above equations can be linearised respectively as follows.

#### 4.2.1 Left matrix Decomposition

\[
A V = -(\Phi B + G)V + \Phi 4\omega_0^2 (J)
\]

(22a)

\[
V = \Psi W - A^{-1}(J)
\]

(22b)

#### 4.2.2 Right matrix decomposition

\[
A V = -\Phi(B V + G V) (J)
\]

(23a)

\[
W = \Psi(B V + 4\omega_0^2 B^2 V) (J)
\]

(23b)

The prototype in Fig. 3 is simulated by the two canonical eighth-order left-ULD and right-ULD SC circuits shown in Fig. 5. The sensitivity behaviour of the new structures must be examined, as they are no longer strictly ladder simulations and would seem to depart from Orchard's low-sensitivity criterion [1]. From the following examples, and many others studied by computer simulation, the sensitivity for the new structures has been confirmed to be much better than their biquad counterparts, and very close to traditional ladder simulations.
5 Circuit design examples

The circuits of Fig. 5 are generated by the PANDA filter compiler [14] to realise the 'pure' eighth-order elliptic bandpass function of Fig. 2. The design data are given in Table 2. A passband sensitivity comparison is shown in Fig. 6 with a corresponding cascade biquad design. A left-ULD realisation proves preferable in all respects, having lower sensitivity and total capacitance.

Table 2: Design data for eighth-order bandpass filter realisations

<table>
<thead>
<tr>
<th>Comparator</th>
<th>ULD</th>
<th>Biquad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total capacitance</td>
<td>188.03</td>
<td>193.91</td>
</tr>
<tr>
<td>Capacitance spread</td>
<td>57.38</td>
<td>41.54</td>
</tr>
<tr>
<td>Average capacitor</td>
<td>5.88</td>
<td>6.48</td>
</tr>
<tr>
<td>Number of capacitors</td>
<td>32</td>
<td>30</td>
</tr>
<tr>
<td>Number of switches</td>
<td>34</td>
<td>36</td>
</tr>
<tr>
<td>Number of opamps</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Capacitor values of left-ULD SC ladder filter in Fig. 5a

- $C_1 = 18.20$  $C_2 = 87.38$  $C_3 = 2.28$  $C_4 = 7.69$  $C_5 = 3.82$
- $C_6 = 1.05$  $C_7 = 1.00$  $C_8 = 4.80$  $C_9 = 6.25$  $C_{10} = 14.06$
- $C_{11} = 3.12$  $C_{12} = 10.72$  $C_{13} = 1.00$  $C_{14} = 1.00$  $C_{15} = 1.36$
- $C_{16} = 5.36$  $C_{17} = 3.96$  $C_{18} = 12.66$  $C_{19} = 1.00$  $C_{20} = 2.87$
- $C_{21} = 1.00$  $C_{22} = 2.01$  $C_{23} = 1.26$  $C_{24} = 3.21$  $C_{25} = 2.57$
- $C_{26} = 11.05$  $C_{27} = 1.00$  $C_{28} = 2.16$  $C_{29} = 2.06$  $C_{30} = 1.00$
- $C_{31} = 1.00$

Clock frequency = 200 kHz; Passband ripple < 1 dB; Stopband attenuation > 50 dB; Lower passband edge = 9 kHz; Upper passband edge = 11 kHz

The passive ladder of Fig. 3 can also be used to realise the partitioned transfer function of an eighth-order lowpass function in Fig. 7. The circuits of Fig. 5 are once again employed. The design data for a right-ULD SC realisation are given in Table 3. A passband sensitivity comparison is shown in Fig. 8. The left-ULD has a serious peak towards low frequency, a known problem for lowpass design, and would not normally be considered. The right-ULD design has very low sensitivity, well below the cascade biquad realisation. In this case, the ladder has a larger total capacitance.

Clock frequency = 200 kHz; Passband edge = 10 kHz; Passband ripple < 0.1 dB; Stopband attenuation > 70 dB.
6 Hybrid matrix approaches

The above techniques, which guarantee the existence of a canonical ladder simulation, require restrictions to be made on the structure of the passive prototype. Given a prototype which does not have this structure, how can a canonical simulation be obtained? The restrictions were made in order to provide an efficient matrix nodal description. For arbitrary prototypes, more general hybrid matrix descriptions can be adopted to minimise the size of the matrix systems and their resulting simulated circuits. The drawback of the hybrid method is that there is no unified rule. The exact design method depends on the individual prototype structures and the selection of the internal variables, which can be seen from the following examples.

An even-order lowpass circuit shown in Fig. 9a has \( n + 1 \) nodes but the filter order is \( 2n \). The rank of \( \Gamma \) is \( n \) as there are \( n \) inductors in the circuit, making the total number of opamps required \( 2n + 1 \); so even a leapfrog design cannot directly provide a canonical circuit.

However, if a single mesh current \( i_n \) is selected as a variable to replace \( v_{n-1} \), the last two row equations will have the following form:

\[
\begin{pmatrix}
\cdots & c_{m-2} & c_{m-1} + c_{m-2} & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
L_m \\
\end{pmatrix}
+ \begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 1 & 0 & \cdots & 0 \\
1 & \frac{1}{g_{L_n}} & \cdots & \cdots & \cdots \\
\end{pmatrix}
\begin{pmatrix}
-v_{n-1} \\
v_n \\
0 \\
i_n \\
\end{pmatrix}
= J \quad (24)
\]

The output is now \( i_n \), which differs from \( v_{n+1} \) by only a constant \( g_{L_n} \). The rank of matrix \( \Gamma \) will be \( n - 1 \) since the contribution of the \( m \)th inductor is now moved to the first matrix. If a right-LUD decomposition method \( \Gamma = LU \) is used, two zero rows will appear in matrix \( U \). According to eqn. 6b the intermediate variable \( W = s^{-1}UV \) contains two variables which can be deleted. This means that the total number of variables is \( 2n \) and only \( n \) opamps are necessary, providing a canonical solution.

The same technique can be applied to the left decomposition designs for a \( 2n \)-order bandpass prototype (Fig. 9b) derived from an \( n \)-th-order lowpass reference with \( n \) even. There are \( n + 1 \) nodes in the ladder, so a nodal description is not efficient. If a single mesh current \( i_n \) is selected as a variable to replace \( v_{n-1} \) and \( v_{n+1} \), the last two row equations can be arranged into the following form, providing a canonical solution:

\[
\begin{pmatrix}
\cdots & c_{m-2} & c_{m-1} + c_{m-2} & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
L_m \\
\end{pmatrix}
+ \begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 1 & 0 & \cdots & 0 \\
1 & \frac{1}{g_{L_n}} & \cdots & \cdots & \cdots \\
\end{pmatrix}
\begin{pmatrix}
-v_{n-2} \\
v_{n-1} \\
0 \\
i_n \end{pmatrix}
= J \quad (25)
\]

Fig. 9 Canonical simulation scheme

- a Even-order lowpass prototype
- b Even-order bandpass prototype

7 Conclusion

An objection to the use of active ladder simulation filters is the difficulty of guaranteeing a canonic filter circuit. By standard methods, a passive ladder realisation may not exist or the simulation of the ladder would yield non-canonic circuits. This paper has presented a set of conditions on the transfer function and prototype structure whereby a canonic simulation is obtainable. For the exceptions, a new design method and new canonic ladder structures are proposed. These circuits are applicable to all active technologies and are less insensitive in switched-capacitor implementation. Despite departing from the strict conditions for low-sensitivity passive ladder simulation, the circuits are shown to have very good sensitivity properties. Although competitive with biquad cascade realisations for certain types of transfer function, there is some cost in component spread for others. The design is formulated in a very regular manner in terms of matrix equations, making it highly suited to computer implementation. The problem of obtaining a canonic simulation of an arbitrary ladder structure is demonstrated to be one of properly choosing the system variables to ensure a sparse matrix description. Some example techniques are illustrated for symmetric bandpass elliptic ladder filters.

8 Acknowledgment

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