Iterative detection of chip interleaved CDMA systems in multipath channels

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This study is concerned with iterative chip-by-chip mutuser detection for chip-interleaved code-division multiple-access (CDMA) systems in multipath channels. Different algorithms based on log-likelihood ratio combining (LLRC), maximal ratio combining (MRC) and joint Gaussian combining (JGC) are investigated and compared. It is shown that the LLRC approach provides a good trade-off between performance and complexity compared with the other two alternatives, achieving high throughputs with a complexity of $O(L)$, where $L$ is the tap number of the channel.

**Introduction:** Multiple access interference (MAI) and intersymbol interference (ISI) are two of the main factors limiting performance in cellular mobile systems. While turbo mutuser detection (MUD) [1] and turbo equalisation [2] are two promising techniques for mitigating MAI and ISI, respectively [1–4], the complexity of turbo MUD/equalisation is still a major concern for practical applications.

Chip-interleaving techniques have recently been introduced to address MAI and ISI in direct-sequence code-division multiple-access (DS-CDMA or simply CDMA) system types [3–5]. In this Letter, we provide a comparative study of several low-cost, sub-optimal detection techniques for chip-interleaved CDMA systems. The optimal algorithm requires an exponential complexity and therefore is not considered. We show that the log-likelihood ratio combining (LLRC) [4] method provides a good trade-off between performance and complexity. Its complexity is independent of the number of users $K$ and linear with the channel tap number $L$. This is considerably lower than other alternatives, such as the maximal ratio combining (MRC) [3] and joint Gaussian combining (JGC) methods [5], that achieve similar performance when the system loading is not excessively high.

**Transmitter structure:** We consider an uplink coded CDMA system with $K$ users in a multipath channel. The same transmitter structure as in [5] is employed. The input data sequence of user-$k$ is encoded by a lower-rate code $C$ (including both forward error control coding and spreading), and then permuted by a chip-level interleaver $\pi_k$. The resulting chip sequences are transmitted over a multiple access multipath channel. The interleavers $\{\pi_k\}$ are different for each user and act to disperse chips related to a common bit. This makes adjacent chips approximately uncorrelated, which is crucial for mitigating ISI, as analysed in [3, 5]. In this approach, interleaving in the transmitter provides the only means to distinguish the signals from different users and so the scheme is coded interleaver division multiple access (IDMA) [4].

We assume quasi-static multipath channels with $L$ tap-coefficients.

The signal (sampled at the chip rate) can be represented by

$$ r(j) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} h_{k,l} x_l(j - l) + n(j), \quad j = 1, 2, \ldots $$

where $x_l(j)$ is the chip transmitted by user-$k$ ($x_l(j) = 0$ for $j = 2L, \ldots, 0$), $h_{k,l} l = 0, 1, \ldots, L-1$) the fading coefficients related to user-$k$, and $n(j)$ a sample of an additive white Gaussian noise (AWGN) process with zero-mean and variance $\sigma^2 = N_0/2$ per dimension. We assume that the channel coefficients $\{h_{k,l}\}$ are known a priori at the receiver side.

**Receiver structure:** The overall detection process follows the well-known turbo MUD principle [1–5]. The receiver consists of an elementary signal estimator (ESE) and $K$ single-user a posteriori probability (APP) decoders (DECs), working in an iterative manner [5]. Fig. 1 shows part of the receiver structure related to user-$k$. We follow the notation in [5], denote $H_k = \{h_{k,l}, \forall k, l\}$, and denote the output of the DEC by $e_{ESE}(x_k(j))$, which is initialised to zero. The output of the ESE is calculated as [5]

$$ e_{ESE}(x_k(j)) = \log \left( \frac{p(r|\{x_k(j)\} = +1, H_k)}{p(r|\{x_k(j)\} = -1, H_k)} \right) $$

(2)

The DEC then use this output to compute the extrinsic LLRs about $(x_k(j), \forall k, j)$ based on $C$. In this Letter we focus on the calculation of $(r(j))$ for further details of the overall turbo MUD process the reader is referred to [1, 4, 5].

**ESE function for single-path channel:** Before discussing the multipath channel case, we summarise the chip-by-chip detection scheme for a single-path fading channel. In this case, the received signal in (1) reduces to

$$ r(j) = \sum_{k=1}^{K} h_{k,j} x_l(j) + n(j), \quad j = 1, 2, \ldots $$

where the fading coefficients are simply denoted by $\{h_k\}$. We consider BPSK signalling, i.e. $x_l(j) \in \{+1, -1\}$, and real fading coefficients. The ESE function for this case is as follows.

$$ E(r(x_k(j)) = \tanh(\frac{e_{ESE}(x_k(j))}{2}), \quad \forall k, j $$

(4a)

$$ \text{Var}(x_k(j)) = 1 - (E(r(x_k(j))^2)), \quad \forall k, j $$

(4b)

$$ E(r(j)) = \sum_{k=1}^{K} h_{k,j} E(x_l(j)), \quad \forall j $$

(5a)

$$ \text{Var}(r(j)) = \sum_{k=1}^{K} |h_{k,j}|^2 \text{Var}(x_l(j)) + \sigma^2 \quad \forall j $$

(5b)

$$ e_{ESE}(x_k(j)) = 2h_k \frac{r(j) - E(r(j)) + h_k E(x_l(j))}{\text{Var}(r(j)) - |h_k|^2 \text{Var}(x_l(j))} \quad \forall k, j $$

(6)

It can be shown that the normalised computational cost per chip per user per iteration in (4)–(6) is only six additions, six multiplications and a tanh function, and is independent of the number of users $K$ [4, 5]. The basic detection principle remains the same for QPSK signalling and complex channels [5], but the details are omitted due to space limitations.

**ESE function for multipath channel:** Now consider multipath fading channels. In the presence of ISI, a transmitted chip $x_l(j)$ is observed on $L$ successive samples $(r(j), r(j+1), \ldots, r(j+L-1))$ in the received signal. Again, we consider BPSK signalling and real fading coefficients. There are several alternative treatments in this case.

**A. LLR combining (LLRC):** With LLRC, the output of the ESE is calculated as

$$ e_{ESE}(x_k(j)) = \sum_{l=1}^{L-1} e_{ESE}(x_l(j)) \quad \forall k, j $$

(7)

where

$$ e_{ESE}(x_k(j)) = 2h_{k,j} \frac{r(j+l) - E(r(j+l)) + h_{k,j} E(x_l(j))}{\text{Var}(r(j+l)) - |h_{k,j}|^2 \text{Var}(x_l(j))} $$

(8)

In (8), $e_{ESE}(x_k(j))$ is the LLR value for $x_k(j)$ obtained by applying (6) to $r(j+l)$. The operation in (7) simply sums the estimates for $x_k(j)$ based on the consecutive received samples $(r(j), r(j+1), \ldots, r(j+L-1))$.

The normalised complexity (per chip per user per iteration) for each $e_{ESE}(x_k(j))$ is only several additions and multiplications (see (4)–(6)). Thus the normalised complexity of the LLRC algorithm given by (7) and (8) is $O(L)$.

**B. Maximal ratio combing (MRC):** With MRC [3], the received signal is passed through an MRC filter matched to the $L$ tap-coefficients for a particular user. For user-$k$, the output of the MRC filter is

$$ z_k(j) = \sum_{l=1}^{L} h_{k,l} r(j+l), \quad \forall k, j $$

(9)

Chip detection (similar to that given in (6)) is then applied to $z_k(j)$ to generate $e_{ESE}(x_k(j))$. Since $K$ different L-tap matched filters are used for $K$ users (see (9)), the calculations of the mean and variance of $z_k(j)$ are different for each user. These computations cannot be shared by different users as in (5). Consequently the normalised complexity is
We then rewrite (1) in vector form as:

\[ r = \sum_{k=1}^{K} h_k(j)x_k(j) + n = h_k(j)x_k(j) + \xi_k(j) \] (10)

where \( r \in [r(1), r(2), \ldots, r(j + L - 1)]^T \), \( n \in [n(1), n(2), \ldots, n(j + L - 1)]^T \) and \( \xi_k(j) \) as a Gaussian random vector with mean \( E(\xi_k(j)) \) and covariance \( Cov(\xi_k(j)) \) [5]. The ESE output is then calculated as:

\[ e_{\text{ESE}}(x_k(j)) = 2h_k(j)^T(Cov(\xi_k(j)))^{-1}(r - E(\xi_k(j))) \] (11)

Equation (11) can be computed with complexity \( O(L^2) \), see [5] for details.

Generally speaking, the LLRC, MRC and JGC techniques achieve similar performance provided the system loading is not too high, although when the system loading is very high the JGC algorithm exhibits better performance than the LLRC and MRC algorithms.

Conclusions: We have examined low-cost MUD receiver algorithms in multipath channels. For very heavily loaded situations, the MRC and JGC algorithms demonstrate better performance than the LLRC algorithm but at the cost of increased complexity. The LLRC algorithm appears to offer a good compromise between cost and performance, since its complexity is only \( O(L) \) per user.

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Numerical results: We now compare the LLRC, MRC and JGC receivers using simulations. Each user employs a convolutional code with generator polynomials \((23, 35)\) followed by a length-8 repetition code. The coding rate of each user is \( R_c = 1/2 \times 1/8 = 1/16 \). The information block size is 128. The signs of the repetition code are reversed alternatively, \([-1 + 1 + 1 - 1 + 1 - 1 - 1]\), to increase randomness. All of the interleavers are generated randomly and independently. Quasi-static fading channels are used.

Fig. 2 shows the performance of the three receivers in multipath fading channels with tap number \( L = 4 \). The LLRC receiver achieves similar performance to the MRC and JGC receivers when the number of users is less than \( K = 40 \). When \( K \) exceeds 48, the advantages of the MRC and JGC algorithms become significant with JGC performing better than MRC. However, when \( K > 56 \), the performance of all of the methods degrades considerably. Overall, the LLRC algorithm appears to offer a good compromise between performance and complexity.

References