Interleave Division Multiple Access

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Outline

- Introduction
  - IDMA
  - Chip-by-chip multiuser detection
    - Analysis and optimization
  - IDM space-time coding and IDM coded modulation
- Conclusions
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Background

- Low-rate coded systems: Viterbi and Verdu
- TCMA (2002): Brannstrom, Aulin and Rasmussen
- Graph-code based multiple access (2001): McEliece
- Chip-interleaved CDMA (2002): Mahavadevappa and Proakis
- CDMA power control (2003/2004): Verdu, Shaimai, Caire and Muller
Some Requirements for Future Wireless Systems

- low receiver cost
- de-centralized (i.e., asynchronous) control,
- simple treatment of ISI,
- cross-cell interference mitigation,
- diversity against fading,
- power efficiency (long battery life),

- multi-media services (e.g., mixed voice and IP),
- high user number,
- high throughput and high spectral efficiency,
CDMA Spectrum Efficiency (per Dimension)

![Graph showing CDMA Spectrum Efficiency (per Dimension)]

- **Spectral efficiency (bits/chip)**
- **Eb/N0 (dB)**
- **Optimal**
- **Matched Filter**

The graph illustrates the relationship between spectral efficiency (bits/chip) and Eb/N0 (dB) for CDMA systems. The optimal and matched filter lines are plotted, showing how spectral efficiency improves with increasing Eb/N0.
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Interleave Division Multiple Access (IDMA)

Key: The interleavers $\pi_1, \ldots, \pi_K$ must be user-specific.
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Interleave Division Multiple Access (IDMA)

Key: The interleavers \( \pi_1, \ldots, \pi_K \) must be user-specific.
Chip-by-Chip Multiuser Detection

\[ r(j) \rightarrow \text{Chip-by-Chip Processing} \]

\[ \pi_1 \rightarrow \pi_1^{-1} \rightarrow \text{APP DEC-1} \]

\[ \pi_k \rightarrow \pi_k^{-1} \rightarrow \text{APP DEC-k} \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]
Chip-by-Chip Detection

Step 1. Chip-level path model: 
\[ r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j) \]

Step 2. Gaussian approximation: 
\[ r(j) = h_k x_k(j) + \zeta_k(j) \]

Step 3. Estimation: 
\[ e(x_k(j)) = \log \frac{\Pr(x_k(j) = +1)}{\Pr(x_k(j) = -1)} \]
\[ = \log \frac{\exp\left(-\frac{(r(j) - \text{E}(\zeta_k(j)) - h_k)^2}{2\text{Var}(\zeta_k(j))}\right)}{\exp\left(-\frac{(r(j) - \text{E}(\zeta_k(j)) + h_k)^2}{2\text{Var}(\zeta_k(j))}\right)} \]
\[ = \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - \text{E}(\zeta_k(j))) \]
The Single-Path Chip-by-Chip Detection Algorithm

Step 1. \[ E(r(j)) = \sum_{k=1}^{K} h_k E(x_k(j)) \quad \text{Var}(r(j)) = \sum_{k=1}^{K} |h_k|^2 \text{Var}(x_k(j)) \]

Step 2. \[ E(\zeta_k(j)) = E(r(j)) - h_k E(x_k(j)) \quad \text{Var}(\zeta_k(j)) = \text{Var}(r(j)) - |h_k|^2 \text{Var}(x_k(j)) \]

Step 3. \[ e(x_k(j)) = \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - E(\zeta_k(j))) \]

Notes:
(1) This is an extremely simplified version of Wang-Poor Algorithm.
(2) No matrix operations.
Chip-by-Chip Multiuser Detection Again

\[ r(j) \rightarrow \text{Chip-by-Chip Processing} \]

\[ e(x_k(j)) \rightarrow \pi_k^{-1} \]

\[ \pi_k \rightarrow E(x_k(j)) \]

\[ \text{APP DEC-k} \]
Complexity

- 6 additions and 6 multiplications per chip per iteration per user.

- Complexity (per user) is independent of user number $K$.

Comparison: To achieve good performance, the cost for MMSE CDMA multi-user detection is $O(K^2)$ due to matrix operations.
IDMA with Repetition Coding

User-1 → repeater → $\pi_1$

User-k → repeater → $\pi_k$

User-K → repeater → $\pi_K$

+ → AWGN
Un-coded IDMA
(with rate-1/8 repetition coding)
Rate 1/8 Convolutional-Repeat Coded IDMA

![Graph showing BER vs. Average Eb/N0 (dB) for different numbers of users and CDMA with 6 users and matched filter. The graph includes IDMA for 8, 16, 32, and 64 users, and CDMA with 6 users and matched filter. The y-axis represents BER, ranging from 1E-05 to 1E+00, and the x-axis represents Average Eb/N0 (dB), ranging from 0 to 22.](image-url)
Multiuser Detection in Multipath Channels

Step 1. Chip-level path model

\[ r(j) = \sum_{l=0}^{L-1} \sum_{k=1}^{K} h_{k,l} x_k (j-l) + n(j) \]

Step 2. Gaussian approximation

\[ r(j) = h_{k,l} x_k (j-l) + \zeta_{k,l}(j) \]

Step 3. Estimation:

\[ e(x_k(j-l))_l = \frac{2 \cdot h_{k,l}}{\text{Var}(\zeta_{k,l}(j))} \cdot (r(j) - \text{E}(\zeta_{k,l}(j))) \]

Step 4. Rake combining:

\[ e(x_k(j)) = \sum_{l=0}^{L-1} e(x_k(j))_l \]

Note: Still no matrix operations here.
Rake Detector in Multipath Channels
(rate 1/2 convolutional & length-8 repetition, 32 users)
Multipath Performance
(rate-1/2 convolutional & length-8 repetition)

$L$= the number of taps. $M$ = the number of receive antennas. $K$=the number of users
Chip-by-Chip Joint Channel Estimation and Multi-User Detection
Performance with Joint Channel Estimation and Multi-user Detection

$E_b$ includes the pilot overhead.
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Chip-by-Chip Multiuser Detection Again

\[ r(j) \rightarrow \text{Chip-by-Chip Processing} \]

\[ e(x_k(j)) \rightarrow \pi_k^{-1} \rightarrow \text{APP DEC-}k \]

\[ \pi_k \rightarrow E(x_k(j)) \rightarrow \text{APP DEC-}k \]
SNR Evolution in the Chip-by-Chip Algorithm

\[ SNR_{k\_new} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 f(SNR_{k\_old}) + \sigma^2} \]
Number of Iterations Required by IDMA
(24 users, 1/2 convolutional + 1/8 repetition coding)
Power Allocation for Non-ideal Coding

Optimization:

Find \( \{h_k\} \) to maximize \( \{SNR_k\} \) after certain iterations.

\[
SNR_{k_{\text{new}}} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 f\left(SNR_{k_{\text{old}}}\right) + \sigma^2}
\]

Constraint: \( \sum |h_k|^2 = \text{fixed} \)
Power Allocation for Different Users

\[ |h_k|^2 \]
Un-coded IDMA
(with rate-1/8 repetition coding)
Rate 1/8 Convolutional-Repeat Coded IDMA

![Graph showing BER vs. Average Eb/N0 for IDMA with different numbers of users and CDMA with matched filter.](image)

- **IDMA**: 8 users, 16 users, 32 users, 64 users
- **CDMA**: 6 users, matched filter

**Axes:**
- **Y-axis**: BER
- **X-axis**: Average Eb/N0 (dB)

**Note:** Capacities are indicated by markers on the graph.
Impact of FEC Coding on IDMA

![Graph showing the impact of different FEC coding schemes on IDMA capacity]

- Turbo Hadamard
- Turbo
- Super-orthonoga-rate-1/32
- Convolutional
- Uncoded

capacity
Spectral Efficiency

1/8 repeating with 64 users, spectral efficiency = 8 bits/chip.

Equivalent to single-user 256-QAM.

Comparison: IS-95 CDMA efficiency?
with ideal coding
with ideal coding

Achieving overall capacity

\[ C = \log(1 + \frac{E_1 + E_2}{\sigma^2}) = \log(1 + \frac{E_1}{\sigma^2}) + \log(1 + \frac{E_2}{\sigma^2 + E_1}) \]

Onion-peeling capacity

Single-user capacity
with ideal coding

\[
\log(1 + \frac{E_1 + E_2 + E_3}{\sigma^2}) \\
= \log(1 + \frac{E_3}{\sigma^2 + E_1 + E_2}) + \log(1 + \frac{E_2}{\sigma^2 + E_1}) + \log(1 + \frac{E_1}{\sigma^2})
\]

We can achieve multi-user capacity provided that an ideal code is used for every user.
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Application 1:
IDM Space-Time Coding
IDM Space-Time Coding

The interleavers $\pi_1, \ldots, \pi_N$ are randomly chosen.

![Diagram showing data flow through interleavers]
Multi Layer IDM Space-Time Coding

\[ \Sigma \]

1-antenna

N-antenna

1-layer

K-layer

layer-1

layer- K

\[ d_1 \]

\[ c_1 \]

\[ \pi_1^{(1)} \]

\[ x_1^{(1)} \]

\[ \sqrt{p_1} \]

\[ \sum \]

\[ d_K \]

\[ c_K \]

\[ \pi_K^{(1)} \]

\[ x_K^{(1)} \]

\[ \sqrt{p_K} \]
Performance of IDM Space-Time Codes
(overall rate $R = 2$ bits/symbol)
Performance of IDM Space-Time Codes
(overall rate $R = 4$ bits/symbol)
For performance analysis of space-time codes, we have to consider all possible fading coefficients $\{h_n\}$.

This is usually very difficult, involving multi-dimensional integration over the distribution of $\{h_n\}$. 
Performance Bounds of IDM Space-Time Codes

**Theorem 1:** Worst performance at: \( h_1 = h_2 = \ldots = h_N \)

**Theorem 2:** Best performance at: \( h_1 = 1, \)

\[ h_2 = \ldots = h_N = 0 \]
Performance Bounds
(overall rate $R = 4$ bits/symbol)
Performance in Multi-Path Channels
(R = 2 bits/symbol, 2×2 system)
The Capacity Achieving Property

An IDM-ST code can achieve capacity if $C$ is low-rate and achieves capacity in AWGN.
Multi Layer IDM Space-Time Coding

\[
C \xrightarrow{d_1} c_1 \xrightarrow{\pi_1^{(1)}} x_1^{(1)} \sqrt{p_1} \xrightarrow{\Sigma} \text{antenna-1}
\]

\[
\vdots
\]

\[
C \xrightarrow{d_K} c_K \xrightarrow{\pi_K^{(1)}} x_K^{(1)} \sqrt{p_K} \xrightarrow{\Sigma} \text{antenna-N}
\]
Summary: Properties of IDM ST Codes

- Conceptually simple.
- Potentially capacity achieving.
- Low decoding complexity.
- Multi-path resolution.
Application 2:
IDM Coded Modulation
IDM Coded Modulation

- Sigma mapping: Duan Rimoldi and Urbanke.
- Multi-level codes: Imai and Hirakawa
IDM Coded Modulation

\[ S/P \]

layer-1 \[ \rightarrow C \rightarrow \pi_1 \]
layer-\( k \) \[ \rightarrow C \rightarrow \pi_k \]
layer-\( K \) \[ \rightarrow C \rightarrow \pi_K \]

[Diagram showing the process of IDM Coded Modulation with layers and coding blocks.]
Advantages of IDM Coded Modulation

- Simplicity
- Flexibility
- High performance
- Low-decoding cost
- Easy treatments for ISI
Rate-1/8-Repeating IDMA

BER

8 users

64 users

single-user

Average Eb/N₀ (dB)
Performance of IDM Coded Modulation (per real dimension)

![Graph showing the performance of IDM Coded Modulation. The graph plots the rate against the signal-to-noise ratio (Eb/N0) in dB. The line represents the Shannon capacity of the AWGN channels, and the markers represent simulation results at BER ≈ 10^{-5}.]
Conclusions Again

What makes IDMA work?
Randomness.
A Comparison between Un-coded IDMA and CDMA

![Graph showing BER vs. Eb/N0 for IDMA and CDMA for different values of K.]

- **IDMA**
- **CDMA**

For $K = 1, 16, 32$, the graph illustrates the performance of both systems under varying $E_b/N_0$ (dB) conditions, with IDMA generally outperforming CDMA at lower $E_b/N_0$ values.
For Details

http://www.ee.cityu.edu.hk/~liping/research/
Chip-by-Chip Detection

Step 1. Chip-level path model: \[ r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j) \]

Step 2. Gaussian approximation: \[ r(j) = h_k x_k(j) + \zeta_k(j) \]

Step 3. Estimation:

\[ e(x_k(j)) = \frac{2h_k}{\text{Var}(\zeta_k(j))} \cdot (r(j) - \text{E}(\zeta_k(j))) \]

For a chip, not much can be done. It must be simple.
Analysis of the Chip-by-Chip Algorithm

\[ e(x_k(j)) = \frac{2h_k}{\text{Var}(\xi_k(j))} \cdot \left( r(j) - \mathbb{E}(\xi_k(j)) \right) \]

\[ = \frac{2h_k}{\text{Var}(\xi_k(j))} \cdot \left( h_k x_k(j) + \xi_k(j) - \mathbb{E}(\xi_k(j)) \right) \]

**Signal and Noise**

\[ SNR_k = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 \text{Var}(x_{k'}(j)) + \sigma^2} \]