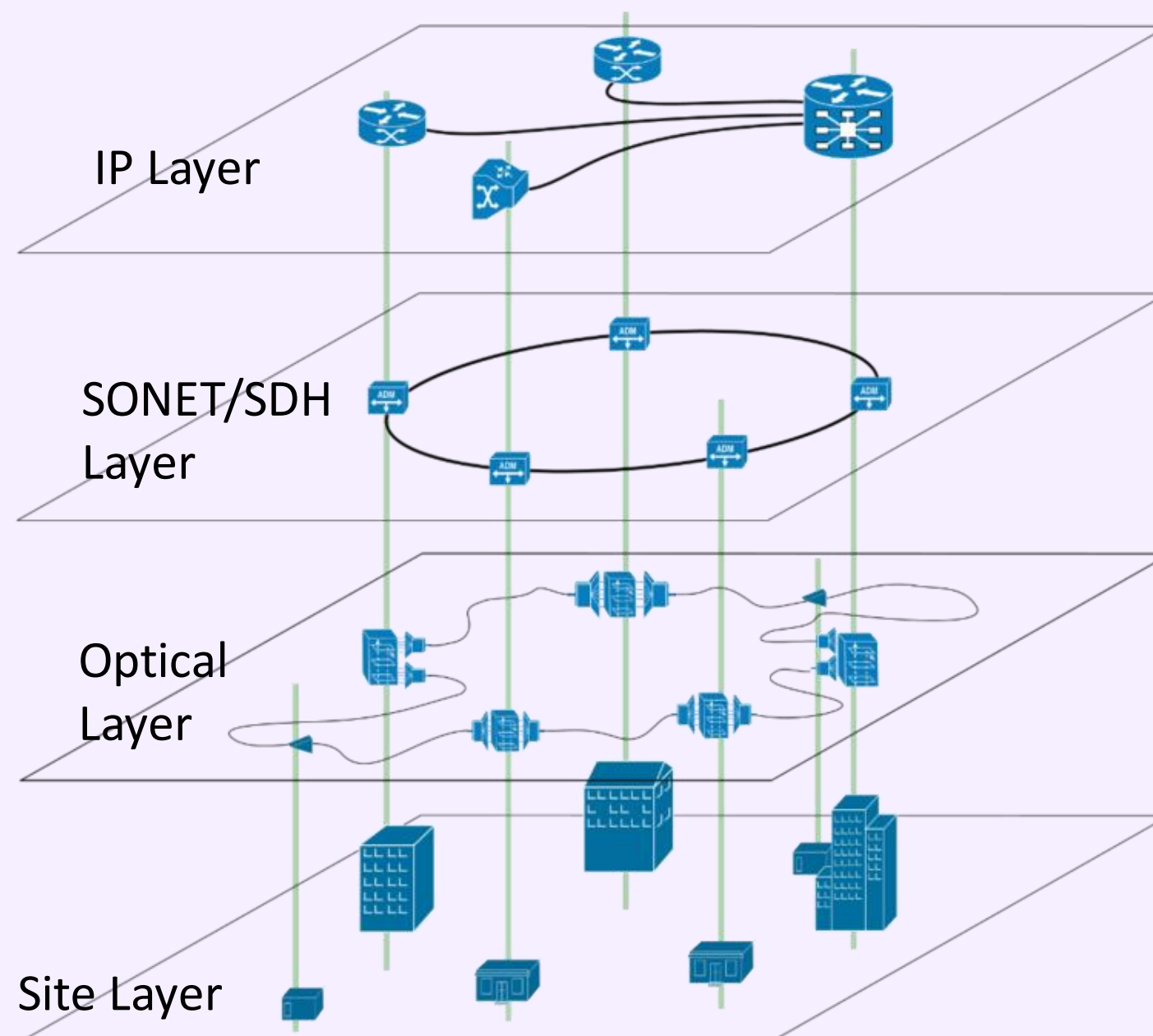


# Integer Linear Programming Modeling for Multi-layered Network Optimization Problems

## Background & Objectives

- Deployment of multiple technologies brings challenges to the design and operation of networks.
- Partitioning networks into layers can help simplify the network design and provide flexibility to upgrade the networks.
- The complexity resulting from this layering design requires an effective optimization model to support cost-effective resource provisioning.



## Methodology

- Two Integer Linear Programming (ILP) formulations are provided for multi-layered network optimization problems:

1. Link-Path ILP Formulation (LPIF)
2. Node-Link ILP Formulation (NLIF)

### Link-Path ILP Formulation

Minimize:

$$\sum_l \sum_e \xi_l \cdot M_{e,l} + \sum_e T \cdot P_{e,1} \cdot M_{e,1}$$

Demand Constraint:

$$\sum_{p^l} F_{e^{l+1}p^l}^l = Y_{e^{l+1}}^{l+1}$$

Capacity Constraint:

$$\sum_{e^{l+1}} \sum_{p^l} \delta_{e^l e^{l+1} p^l}^l \cdot F_{e^{l+1} p^l}^l \leq Y_{e^l}^l$$

$$Y_{e^l}^l \leq M_{e,l} \cdot C_l$$

### Node-Link ILP Formulation

Minimize:

$$\sum_l \sum_{m,n} \xi_l \cdot M_{mn,l} + \sum_{m,n} T \cdot P_{mn,1} \cdot M_{mn,1}$$

Flow Conservation Constraints:

$$\sum_n F_{in,l}^{ij} = \sum_m F_{mj,l}^{ij} = Y_{ij,l+1}^{ij}$$

$$\sum_n F_{nt,l}^{ij} = \sum_m F_{tm,l}^{ij}$$

Capacity Constraints:

$$\sum_{i,j} F_{mn,l}^{ij} = Y_{mn,l}$$

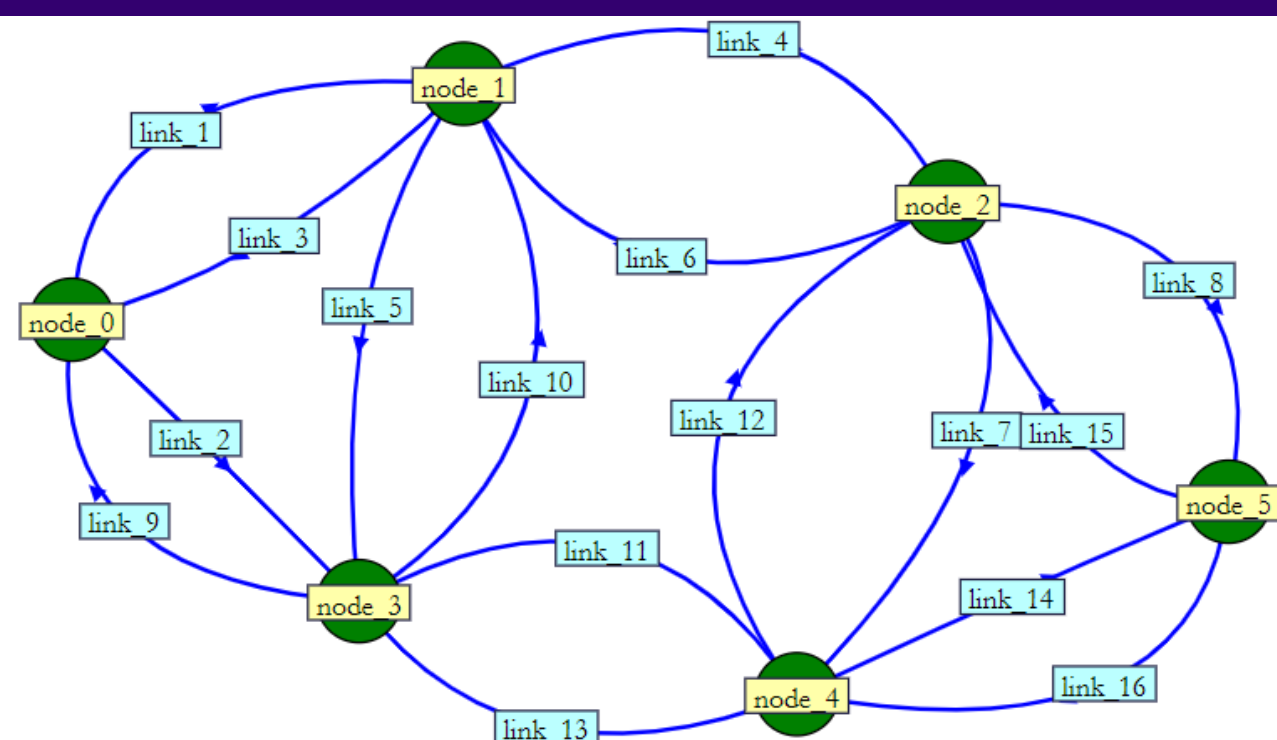
$$Y_{mn,l} \leq M_{mn,l} \cdot C_l$$

Other Constraints:

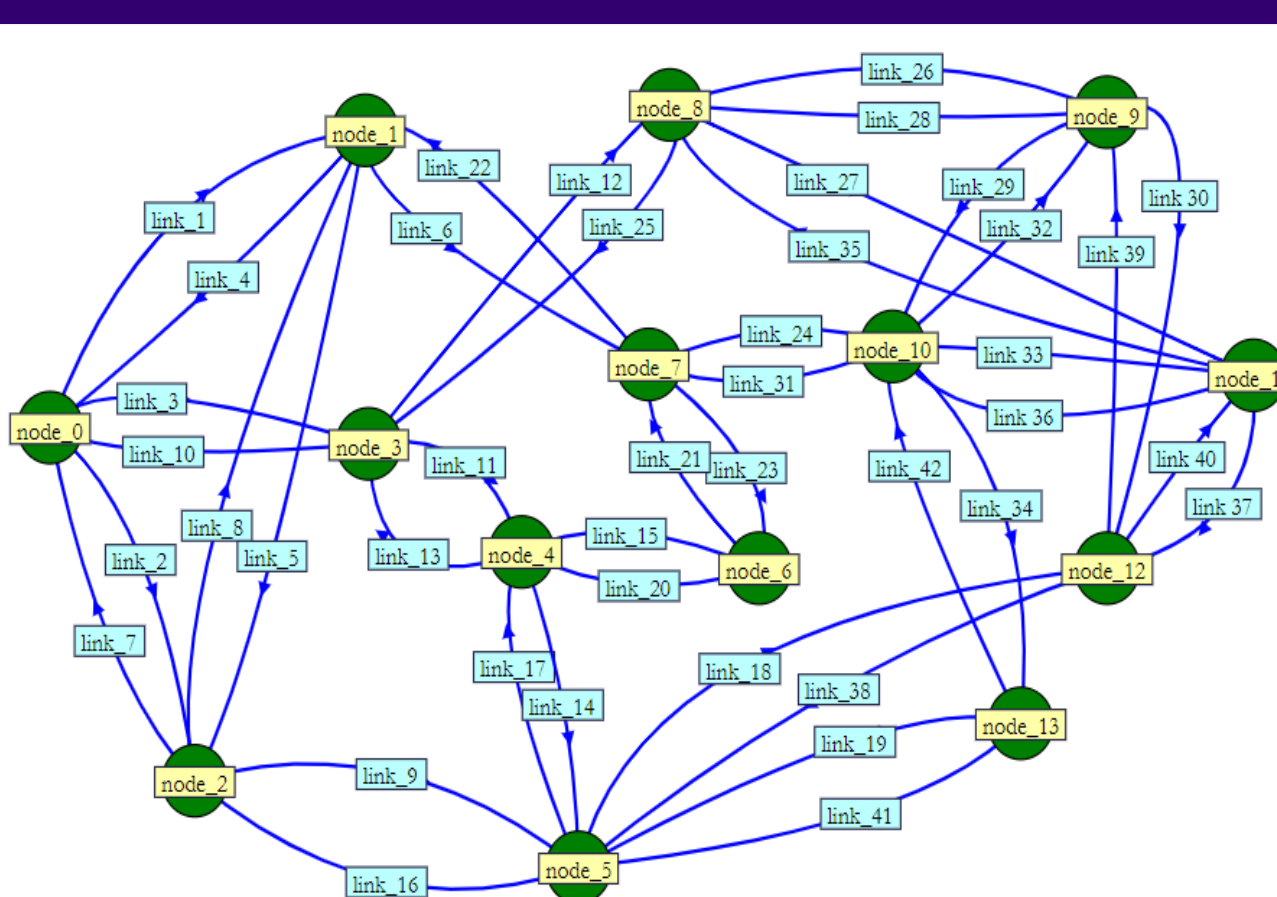
$$Y_{mn,l} \leq U \cdot P_{mn,l}$$

- Two testing networks:

### Six-node Network



### NSFNET



- Two multiplexing techniques:

### Deterministic Multiplexing

Resource allocation is based on the **sum** of **maximum** bandwidth required:

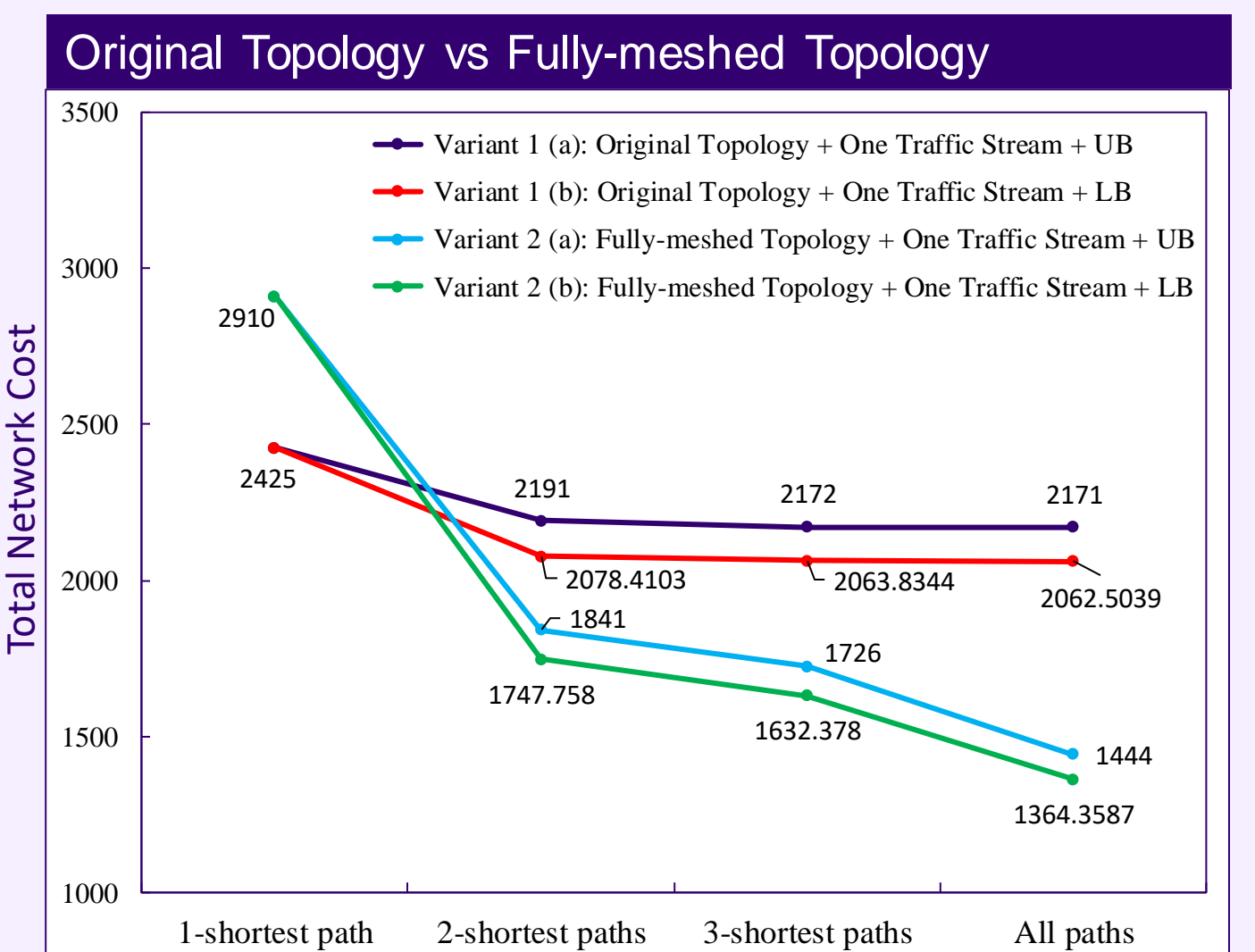
$$B = \sum_{i=1}^n \mu_i + 3 \sum_{i=1}^n \sigma_i$$

### Statistical Multiplexing

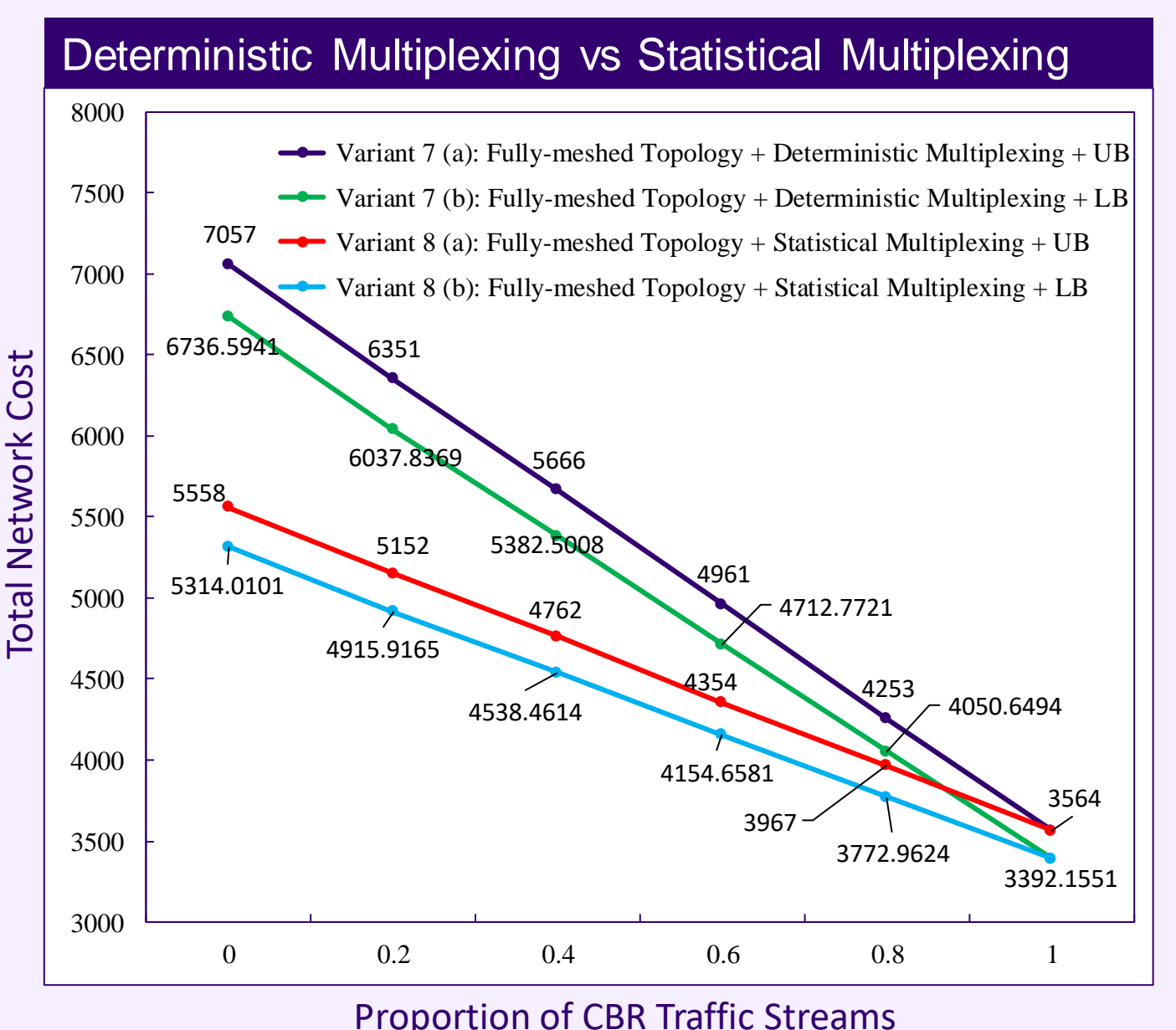
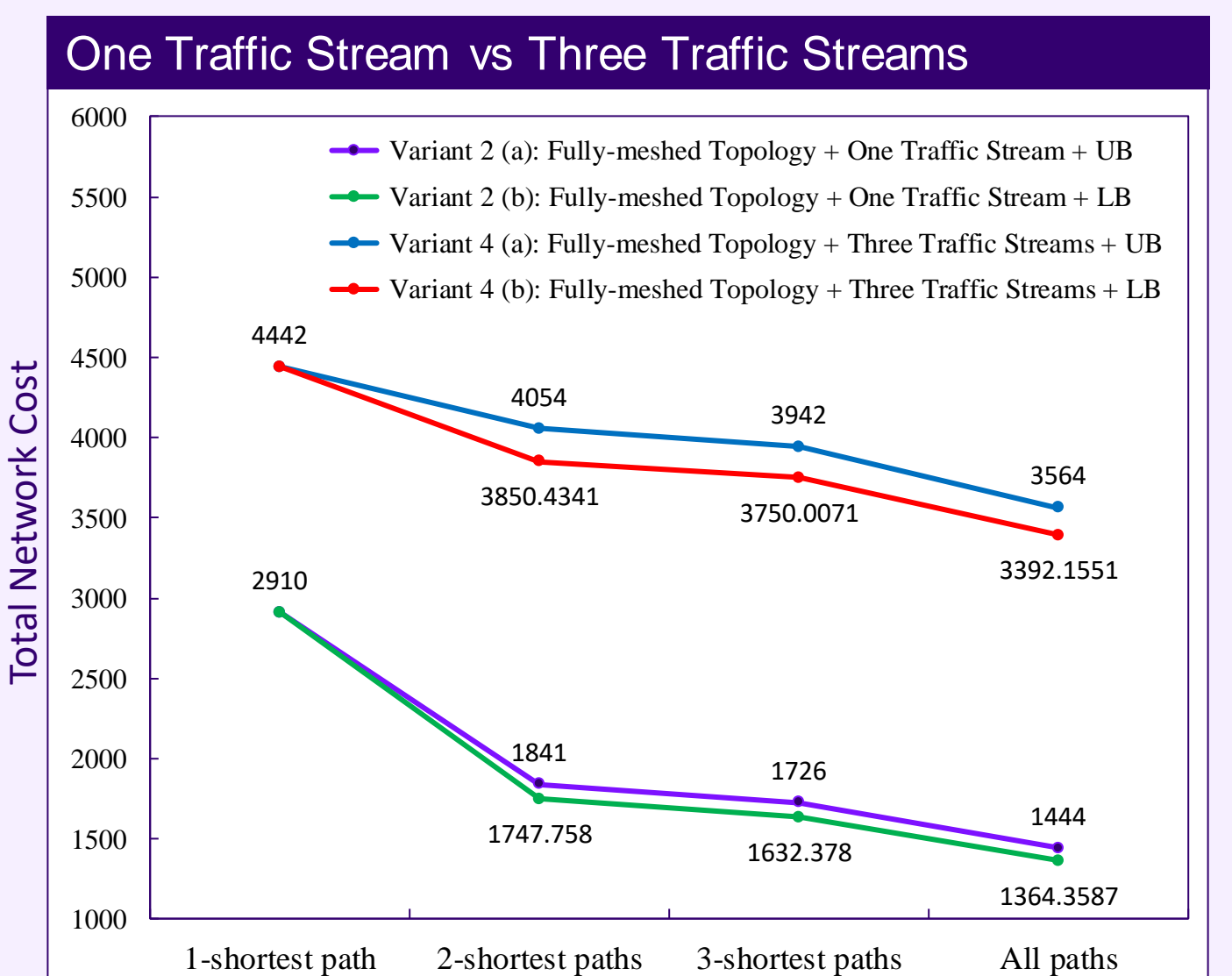
Resource allocation is **lower** than the sum of maximum bandwidth required:

$$B = \sum_{i=1}^n \mu_i + 3 \sqrt{\sum_{i=1}^n \sigma_i^2}$$

## Results



	1-shortest path	2-shortest paths	3-shortest paths	All paths
Original	0.158 secs	3.099 secs	6.855 secs	18.77 secs
Fully-meshed	0.717 secs	12.731 secs	~ 25 hours	~ 72 hours



- Number of routing choices  $\uparrow$
- Total Network Cost  $\downarrow$ , Optimization Time  $\uparrow$
- Network size  $\uparrow$ , ILP Efficiency  $\downarrow$
- ILP is not scalable, but we can set an appropriate gap tolerance to get tight upper and lower bounds on the optimal solution, serving as benchmarks for other heuristic algorithms.