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Sparse Portfolio Design

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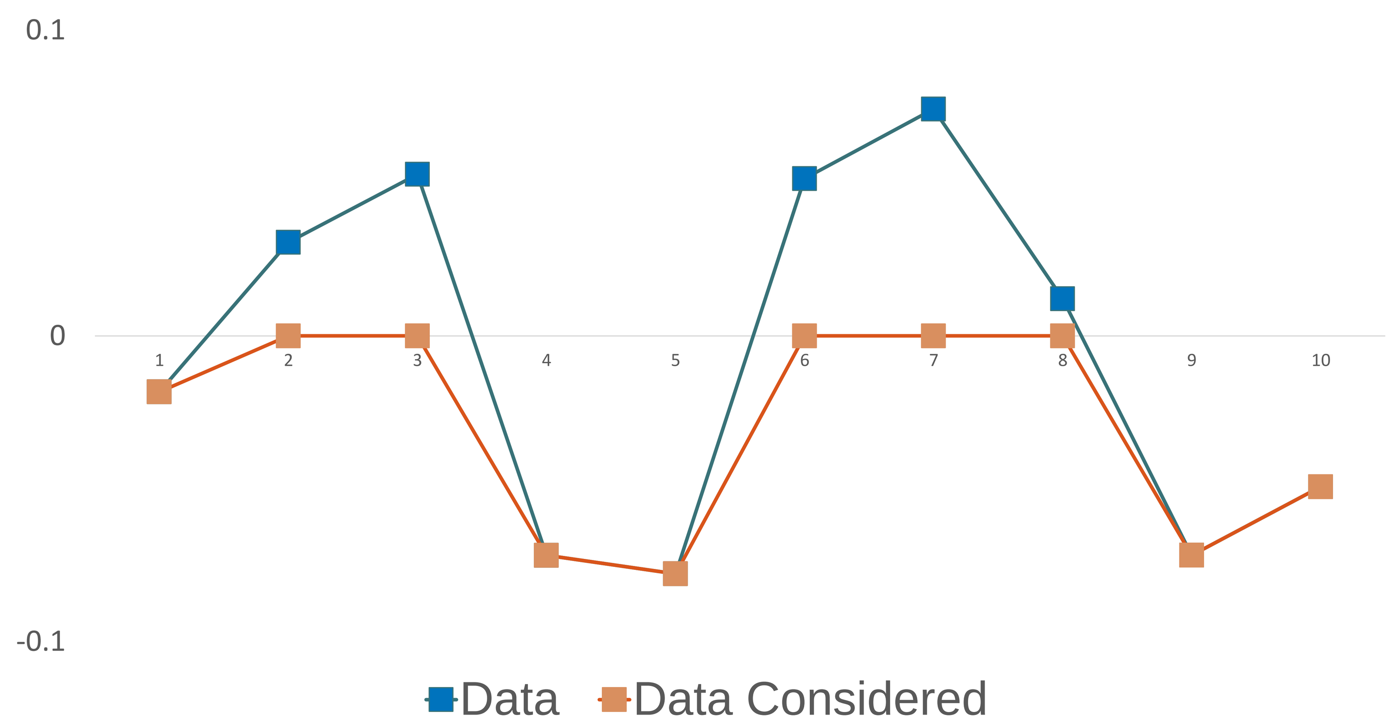
Background

- Portfolio Optimization: Optimizing the allocation of funds $\boldsymbol{w}$ in a portfolio to maximize return and minimize risk
- Mean-Variance Model: $min_{\boldsymbol{w}} \underbrace{\boldsymbol{w}^\top \boldsymbol{K} \boldsymbol{w}}_{\text{Risk}} - \beta \underbrace{\boldsymbol{u}^\top \boldsymbol{w}}_{\text{Return}} \;\; s.t. \underbrace{\boldsymbol{w}^\top \mathbf{1} = 1}_{\text{Budget}}$
- $\boldsymbol{K}$ is the covariance matrix and $\boldsymbol{u}$ is the average daily return of the assets, and β is risk tolerance
- MV model results in dense portfolio which is more expensive and complex
- $\boldsymbol{K}$ considers both overperformance and underperformance risky

How LPM considers data



Objective

Create a optimization which:

- Considers underperformance more risky
- Produces a long-only sparse portfolio
- Enforces reallocation limits

Methodology

Downside deviation diagonal matrix $\boldsymbol{D}$:

- Each value on the diagonal is $\sqrt{LPM_2(0)}$ of the returns of an asset
- Penalizes stocks with large or frequent underperformance

Modified $\widehat{\boldsymbol{K}} = \boldsymbol{K}_{\text{denoised}} + \gamma \boldsymbol{D} \times \frac{\text{mean}(\text{diag}(\boldsymbol{K}_{\text{denoised}}))}{\text{mean}(\text{diag}(\boldsymbol{D}))}$

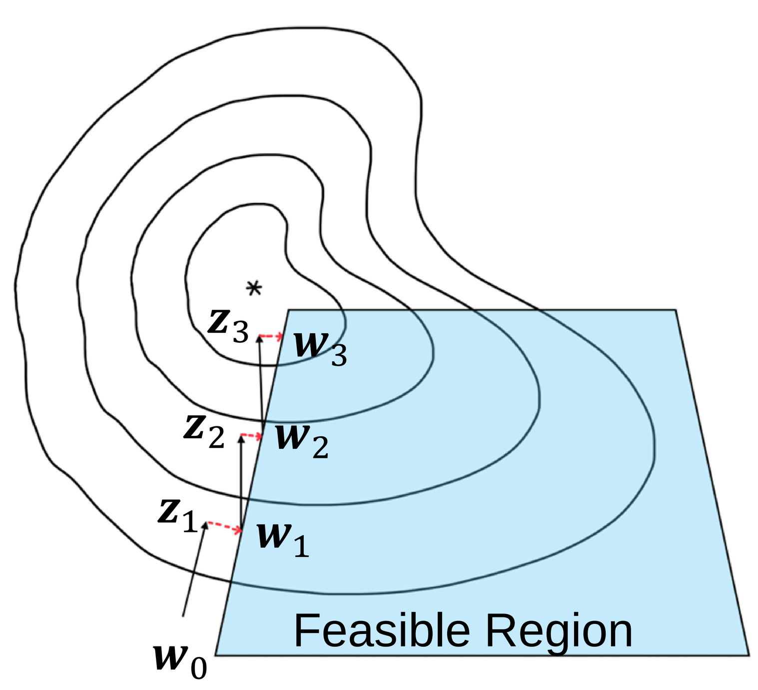
- γ to control weighting of $\boldsymbol{D}$

Formulation: $\min_{w} f(w) = \underbrace{w^\top \widehat{K} w}_{\text{Risk}} + \beta \underbrace{u^\top w}_{\text{Return}} + \alpha \underbrace{(w^\top 1 - 1)^2}_{\text{Budget}}$

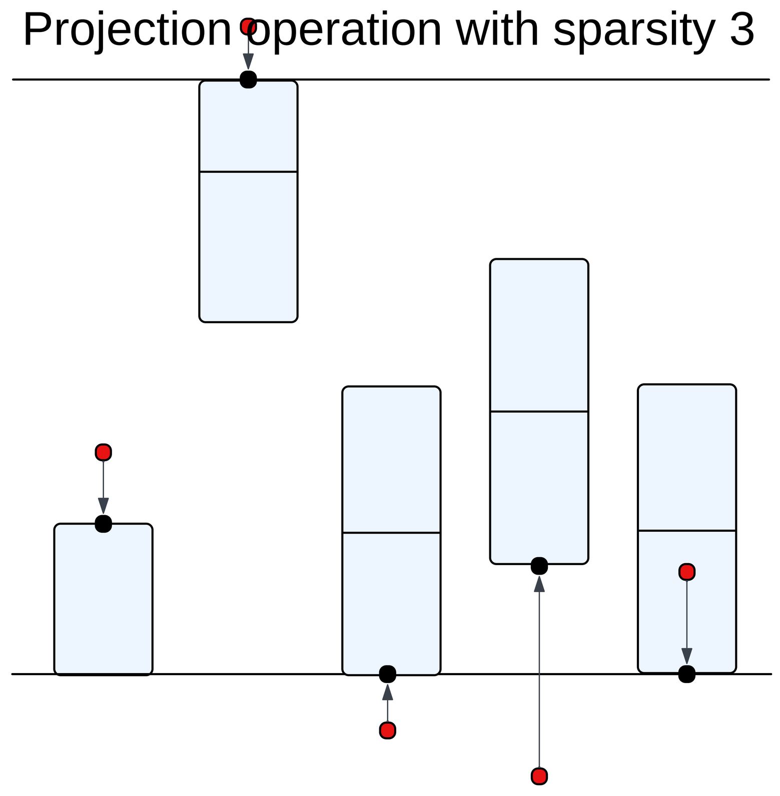
Projection for sparsity, weight and reallocation constraints

- First project values with non-zero lower bounds
- Project remaining until desired sparsity
- Done at each step of gradient descent

Projected Gradient Descent



Projection operation with sparsity 3

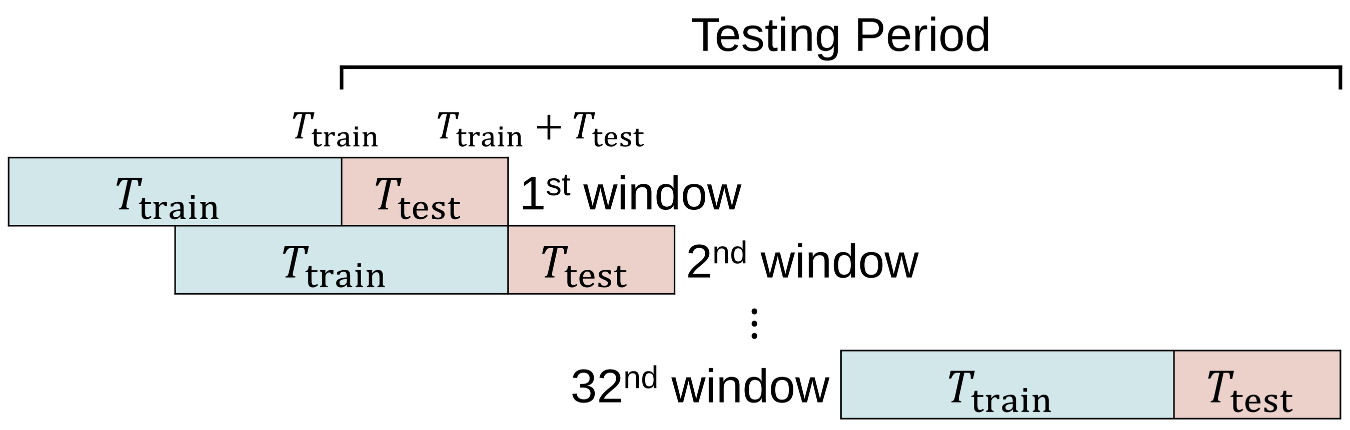


- Intermediate weight $\boldsymbol{z}$
- Updated weight $\boldsymbol{w}$

Results

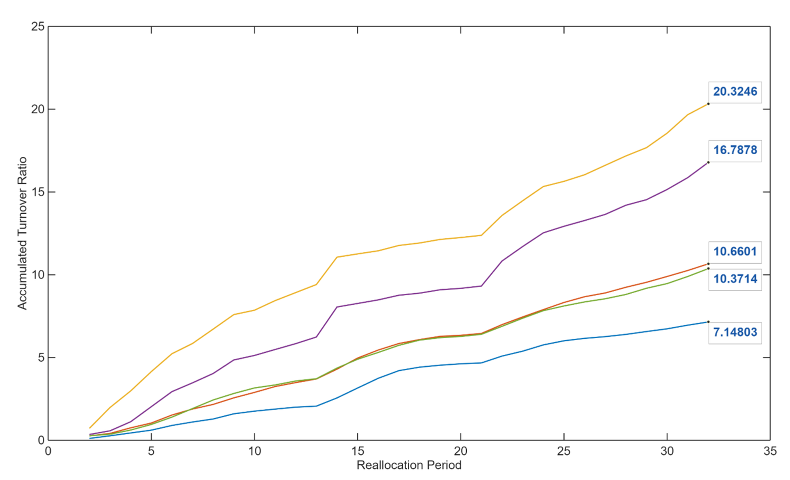
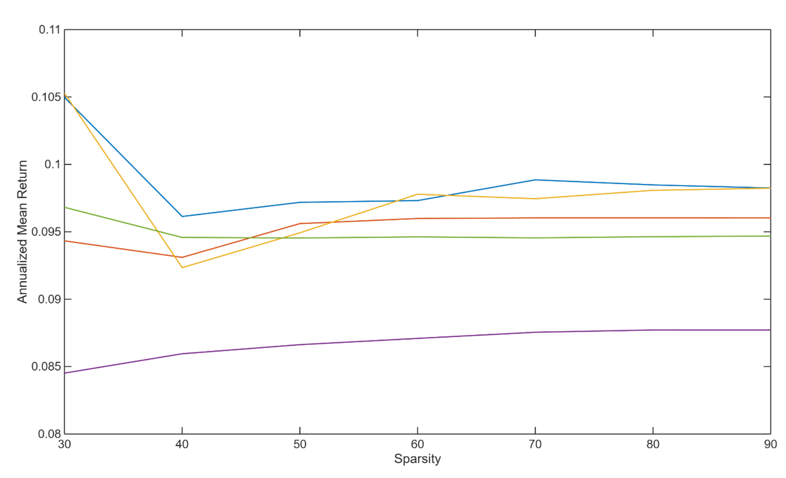
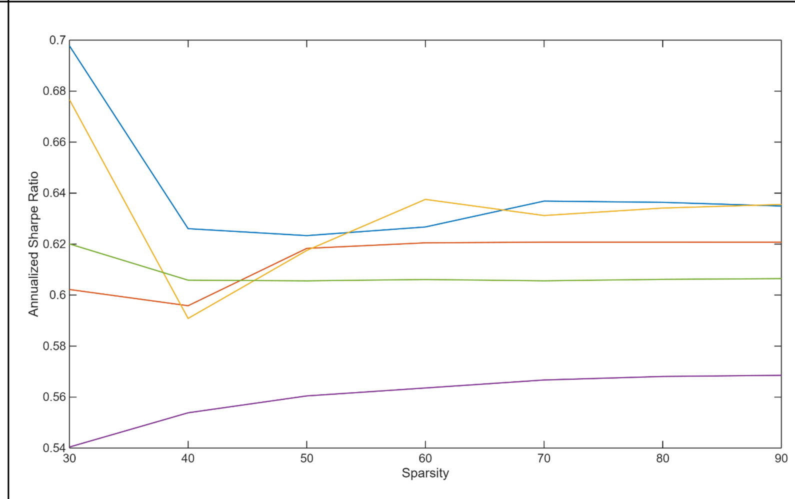
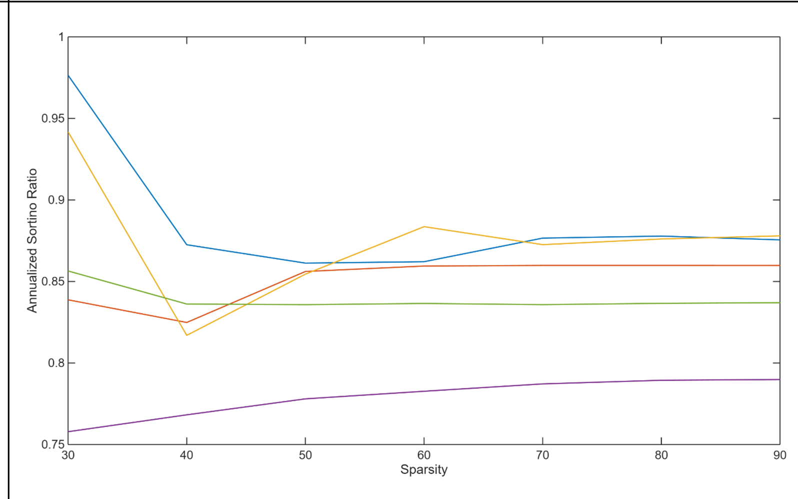
- Tested using sliding window model

Sliding window model



Index	Assets	T_{train}	T_{test}	Total days	Start Date	End Date
S&P 500	472	504	63	2520	3/2/2015	3/2/2025
Russell 1000	807					

	Algorithm	Bounded Reallocation	Note
(blue)	PGDBDR (Mine)	Yes	
(orange)	PGDB		Mean-Variance
(yellow)	PGDDR (Mine)	Yes	
(purple)	PGD		Mean-Variance
(green)	Fixed Target	Yes	Minimizes returns below target

Index	Turnover	Mean Return	Sharpe Ratio	Sortino Ratio
S&P 500				
Russell 1000