Hybrid Beamforming
for
5G Millimeter-Wave Systems

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Slides available at:

https://yuxianghao.github.io/slides/ICCC19.pdf

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Outline

- **Background and Motivation**
- **Preliminaries of Hybrid Beamforming**
- **Hybrid Beamforming Design**
  - **Improve Spectral Efficiency: Approaching the Fully Digital**
  - **Boost Computational Efficiency: Convex Relaxation**
  - **Fight for Hardware Efficiency: How Many Phase Shifters Are Needed?**
- **Conclusions**
- **Potential Research Directions**

Background and Motivation

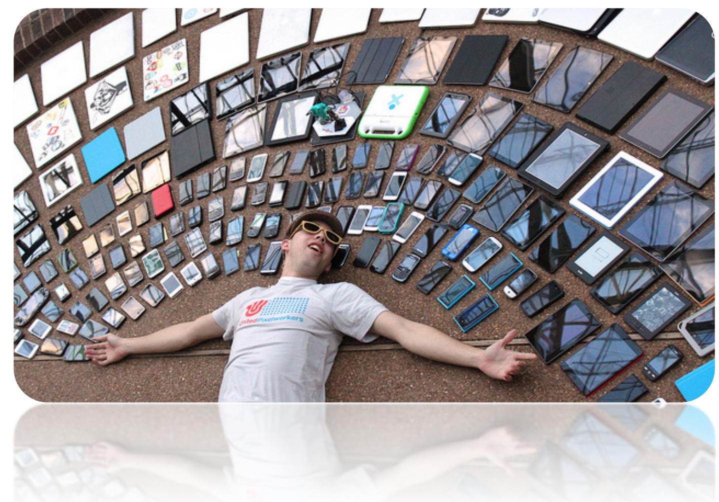
❖ Era of mobile data deluge

7 x

Data growth by 2021

8.0 Billion

Mobile devices/connections in 2016



60 %

Video traffic in 2016

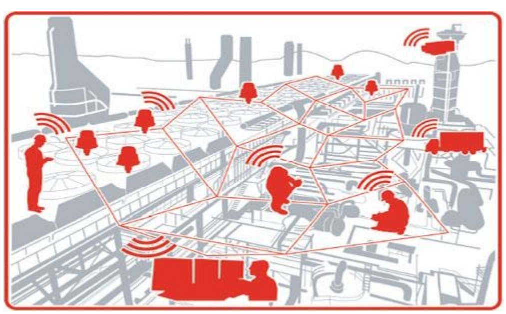
Cisco VNI, March 2017

Background and Motivation

- Requirements of 5G systems

High data rate

Massive connections



Uniform coverage

Green communications

Security & privacy

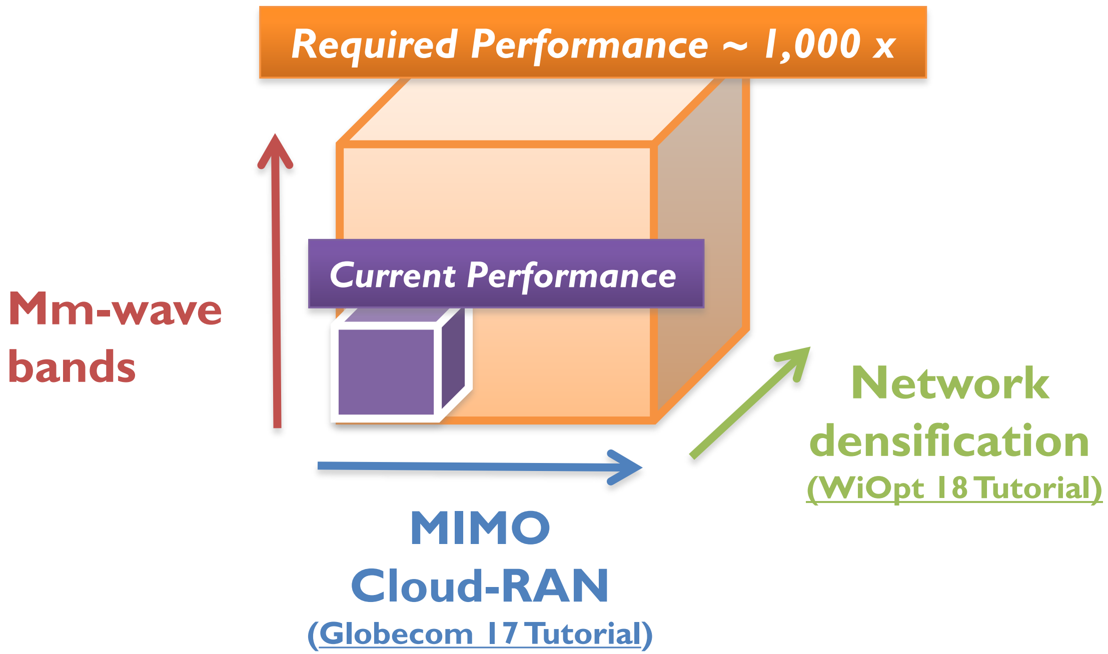
Background and Motivation

- The 1000x Capacity Challenge for 5G

Background and Motivation

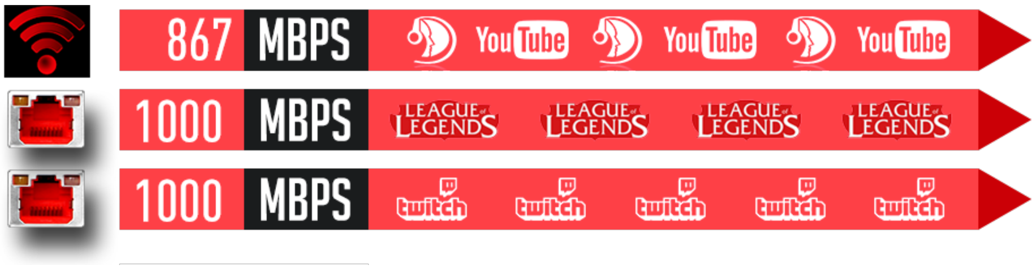
❖ The 1000x Capacity Challenge for 5G

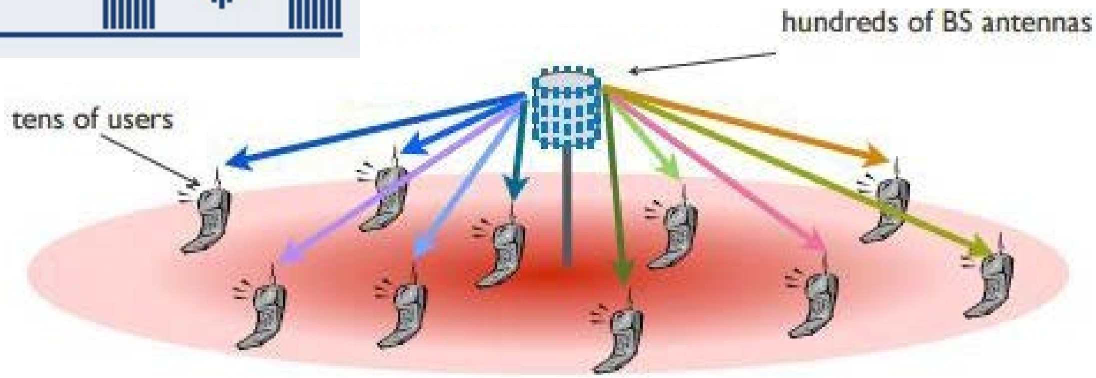
Capacity	=	Bandwidth (Hz)	x	Spectral Efficiency (bps/Hz)	x	# Links
1000	=	10	x	5	x	20



Background and Motivation

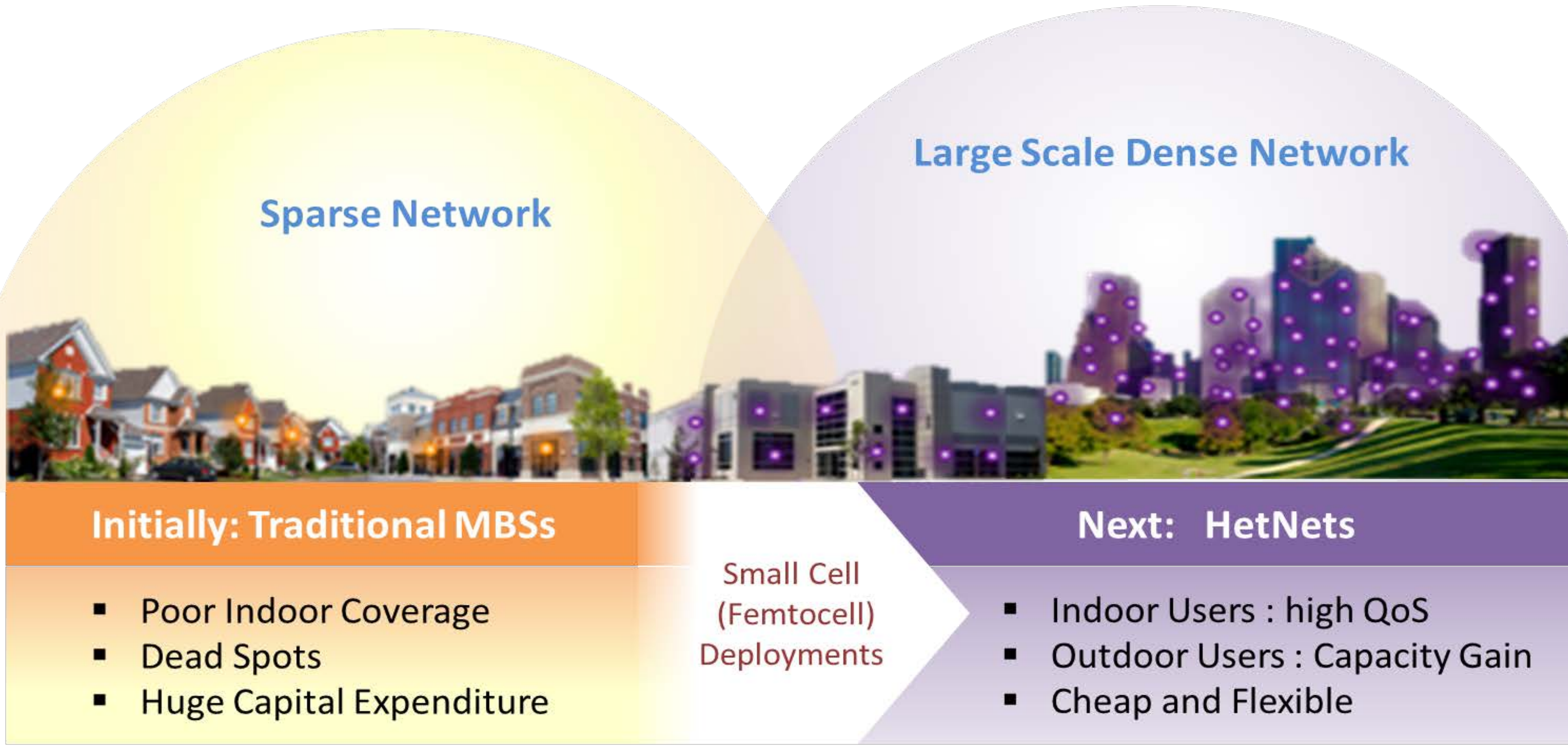
❖ Higher spectral efficiency





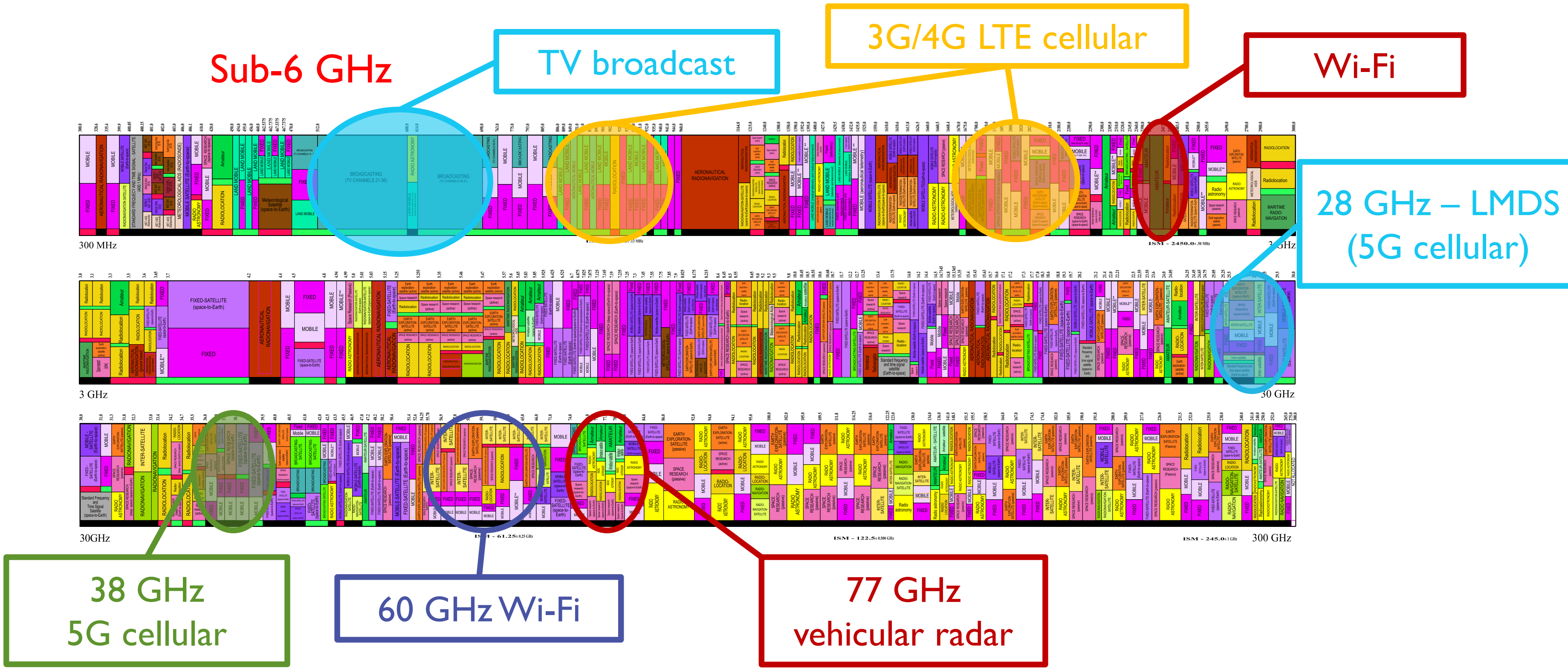
Background and Motivation

❖ Ultra dense networks



Background and Motivation

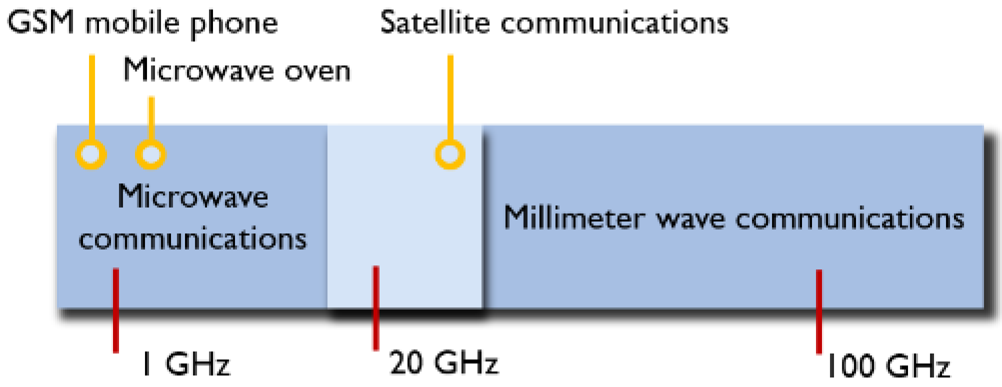
- Spectrum crunch: A fundamental bottleneck



[U.S. Frequency Allocation Chart as of October 2011]

Background and Motivation

❖ New Spectrum: Beyond sub-6 GHz



5G = Millimeter wave

At least to someone

Background and Motivation

- Latest activities at mm-wave bands

Standardization (IEEE 802.11 ad)

Hardware products

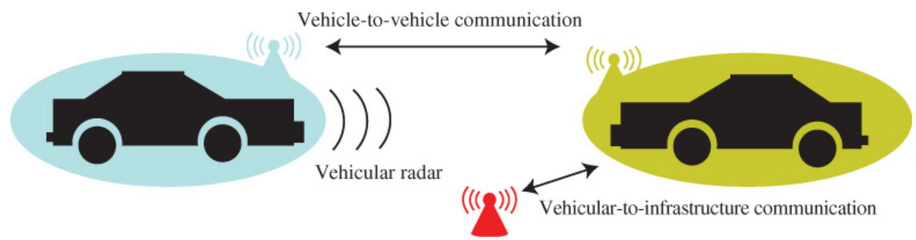
Channel models

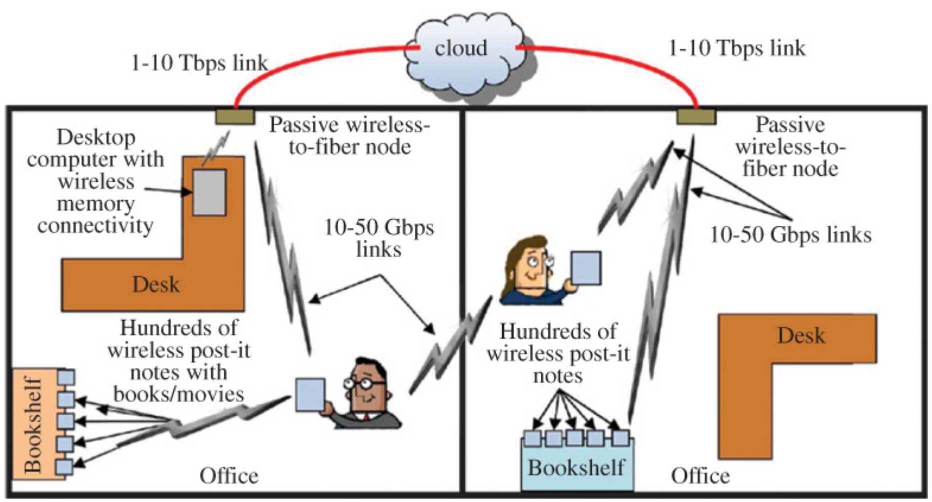
Small cell networks

mm-Wave trial

Background and Motivation

❖ **Emerging mm-wave applications** [T. S. Rappaport *et al.*, 2014]





Background and Motivation

Sub-6 GHz signals

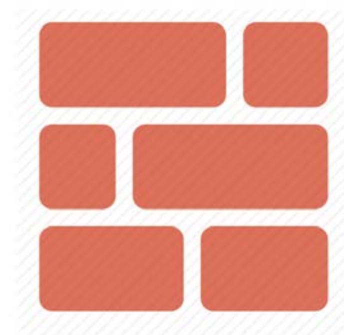
Receive power: $P_r = \frac{P_t}{4\pi d^2} \frac{\lambda^2}{4\pi}$ ▼

Noise power: $N_0 = kT_e B$ ▲

SNR ▼

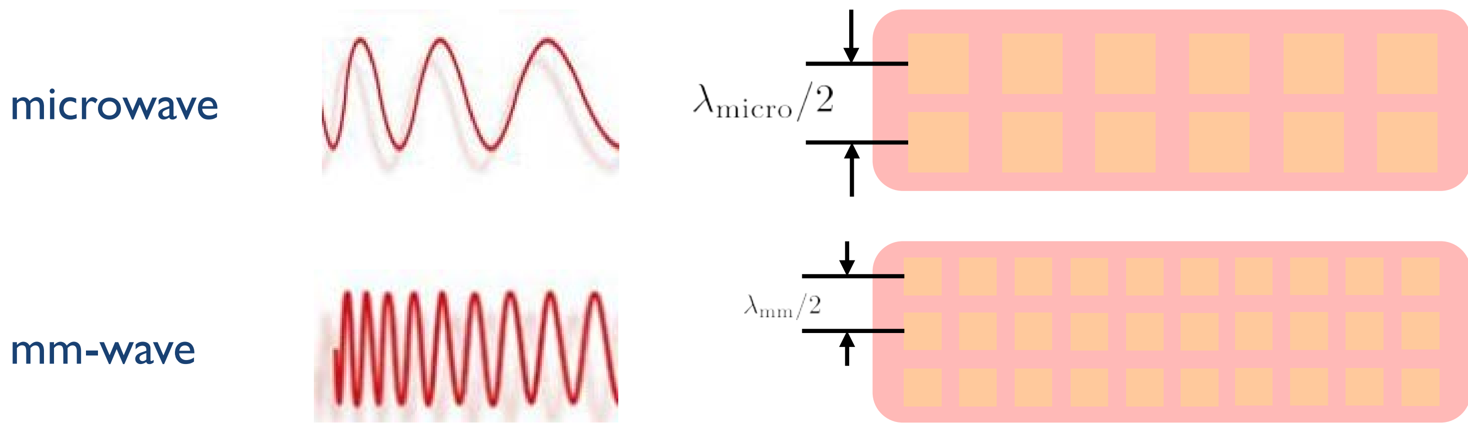
Huge path loss

Mm-wave signals



Sensitivity to blockages

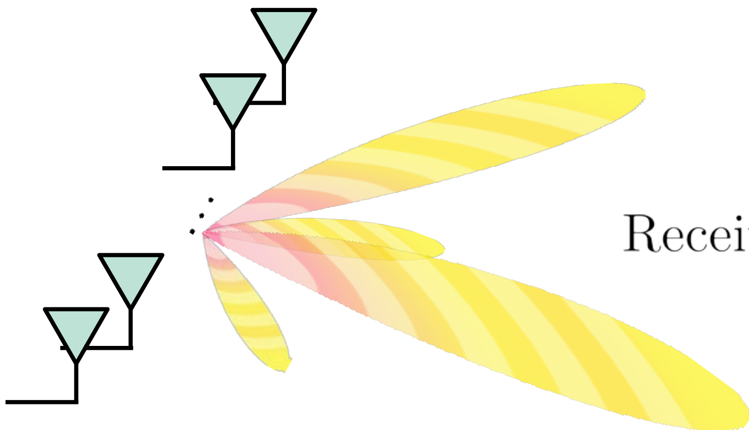
Background and Motivation



Small wavelength → Large-scale antenna arrays

More antennas can be patched in a small area

Background and Motivation



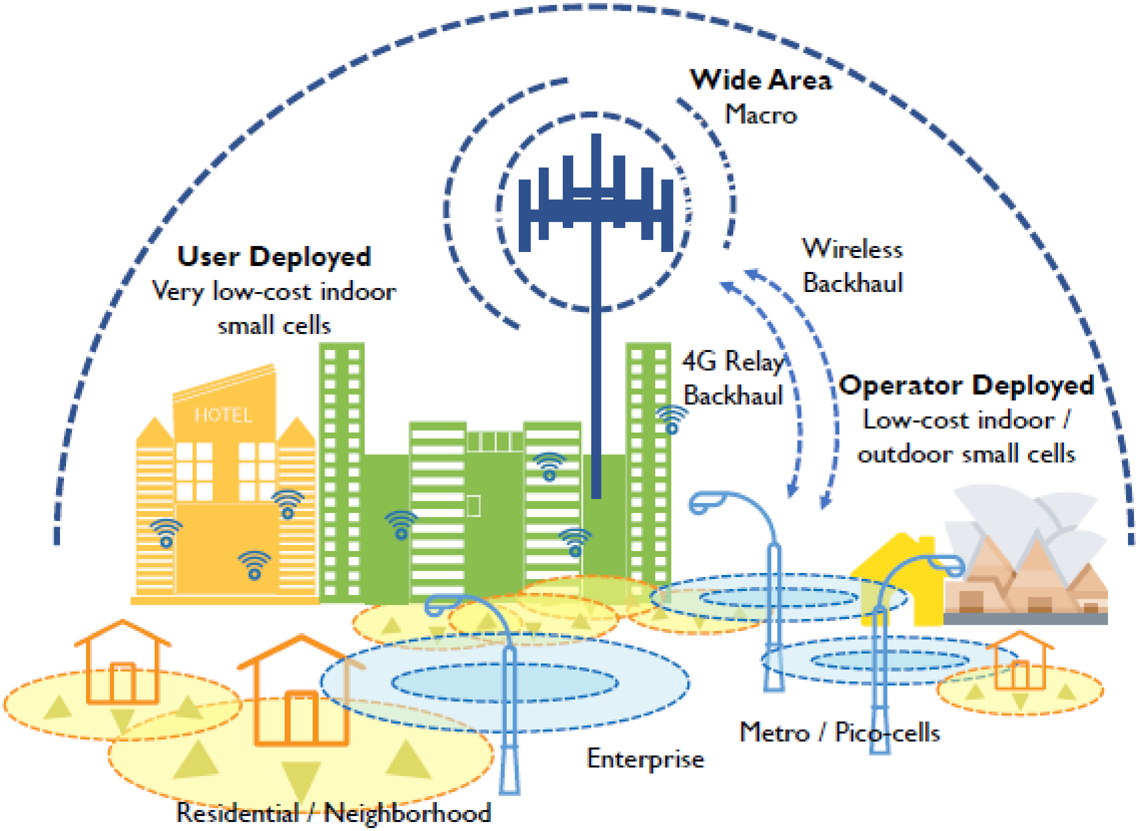
$N_\mathrm{t} > 100$

Receive power: $P_\mathrm{r} = \frac{P_\mathrm{t}}{4\pi d^2} \frac{\lambda^2}{4\pi} G_\mathrm{t} G_\mathrm{r}$

Beamforming!

Higher antenna gains and narrower beams

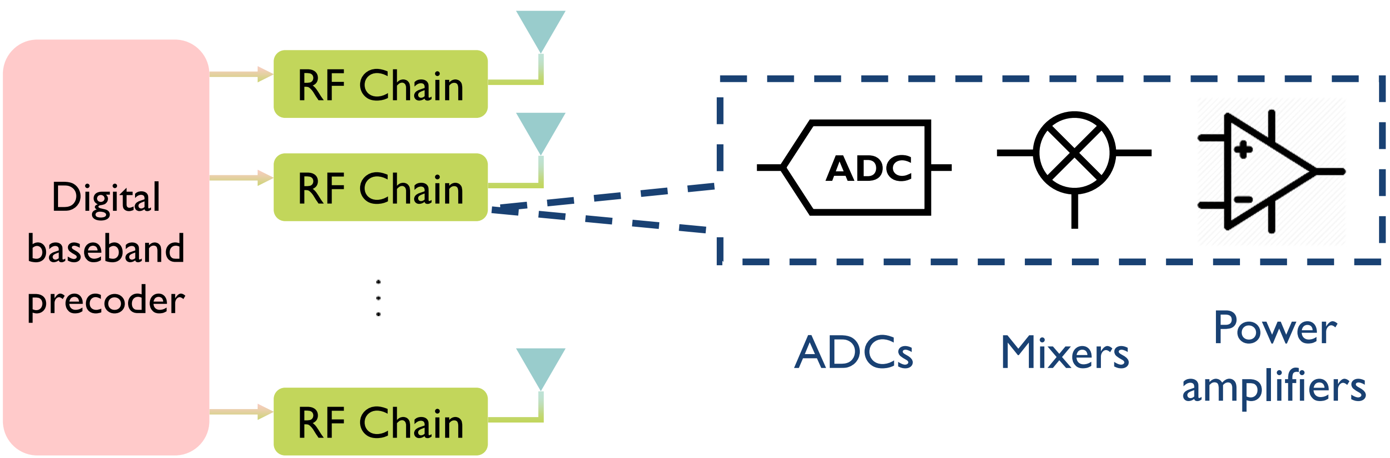
Background and Motivation



Network densification reduces propagation distance

Background and Motivation

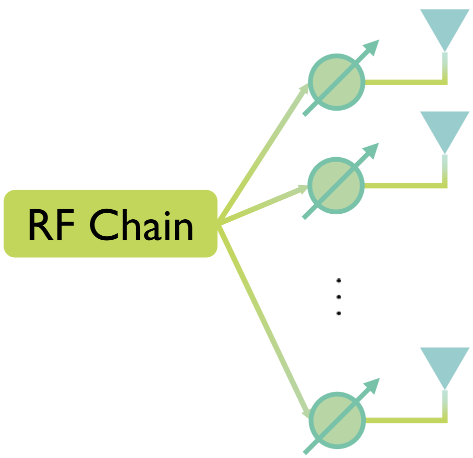
- Conventional beamforming
  - Performed digitally at the baseband
  - Require an RF chain per antenna element



Costly and power hungry for large-scale antenna arrays, especially at mm-wave bands!

Background and Motivation

- Existing solution: Analog beamforming
  - One RF chain only



$$\mathbf{f}(\varphi) = \frac{1}{\sqrt{N_{\mathrm{t}}}}\left[1, \cdots, e^{j2\pi k\varphi}, \cdots, e^{j2\pi(N_{\mathrm{t}}-1)\varphi}\right]^T$$

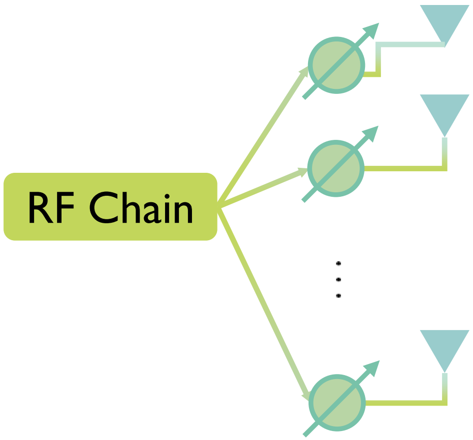
the decisive variable

  - Beams direction readily controlled by a series of phase shifters in the RF domain
  - Low cost and hardware complexity

Background and Motivation

❖ Existing solution: Analog beamforming

➢ Limitations



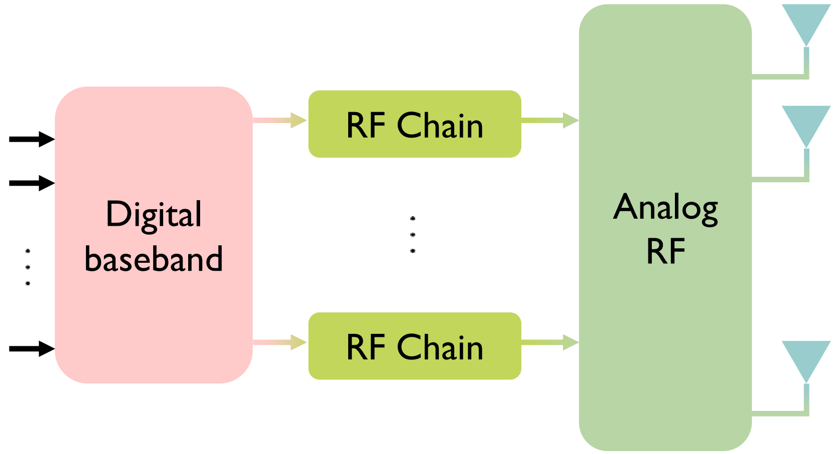
Benefits of MIMO

- Spatial multiplexing
- Support space-division multiple access (SDMA)

Analog beamforming can only support single-stream transmissions

Background and Motivation

❖ Hybrid beamforming



➢ Multi-stream transmission, ability to support SDMA

➢ Multiple RF chains, the number should be very small

➢ Combine the benefits of digital and analog beamforming

Background and Motivation

❖ General references on mm-wave

- T. S. Rappaport *et al*., "Millimeter wave mobile communications for 5G Cellular: It Will Work!," *IEEE Access*, vol. 1, pp. 335-349, 2013.
- Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag*., vol. 49, no. 6, pp. 101-107, June 2011.
- E. Torkildson, U. Madhow, and M. Rodwell, "Indoor millimeter wave MIMO: Feasibility and performance," *IEEE Trans. Wireless Commun*., vol. 10, no. 12, pp. 4150–4160, Dec. 2011.
- M. R. Akdeniz *et al*., "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE J. Sel. Areas Commun.,* vol. 32, no. 6, pp. 1164–1179, Jun. 2014.
- T. S. Rappaport, R. W. Heath, R. C. Daniels, and J. N. Murdock, *Millimeter Wave Wireless Communications*. New York, NY, USA: Pearson Education, 2014.
- P. Wang, Y. Li, L. Song, and B. Vucetic, "Multi-gigabit millimeter wave wireless communications for 5G: From fixed access to cellular networks," *IEEE Commun. Mag*., vol. 53, no. 1, pp. 168–178, Jan. 2015.
- S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter-wave cellular wireless networks: Potentials and challenges," *Proc. IEEE*, vol. 102, no. 3, pp. 366–385, Feb. 2014.

Background and Motivation

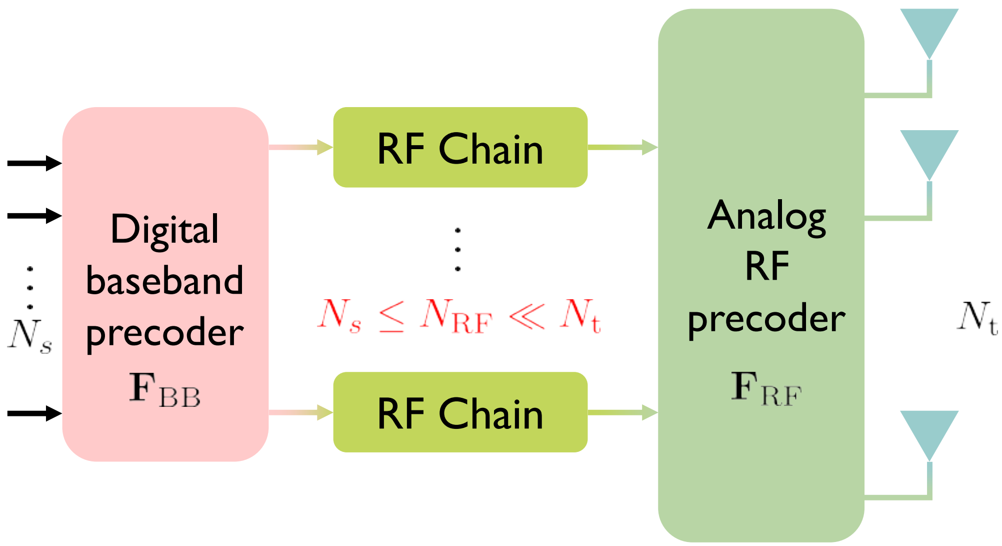
❖ Recognitions on hybrid beamforming

- O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, Jr., "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499-1513, Mar. 2014.
  - **The 2017 Marconi Prize Paper Award in Wireless Communications**
- F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 501-513, Apr. 2016.
  - **The 2017 IEEE Signal Processing Society Best Paper Award**
- A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831-846, Oct. 2014.
  - **The 2016 Signal Processing Society Young Author Best Paper Award**
- X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 485-500, Apr. 2016.
  - **The 2018 Signal Processing Society Young Author Best Paper Award**

Preliminaries of Hybrid Beamforming

Preliminaries of Hybrid Beamforming

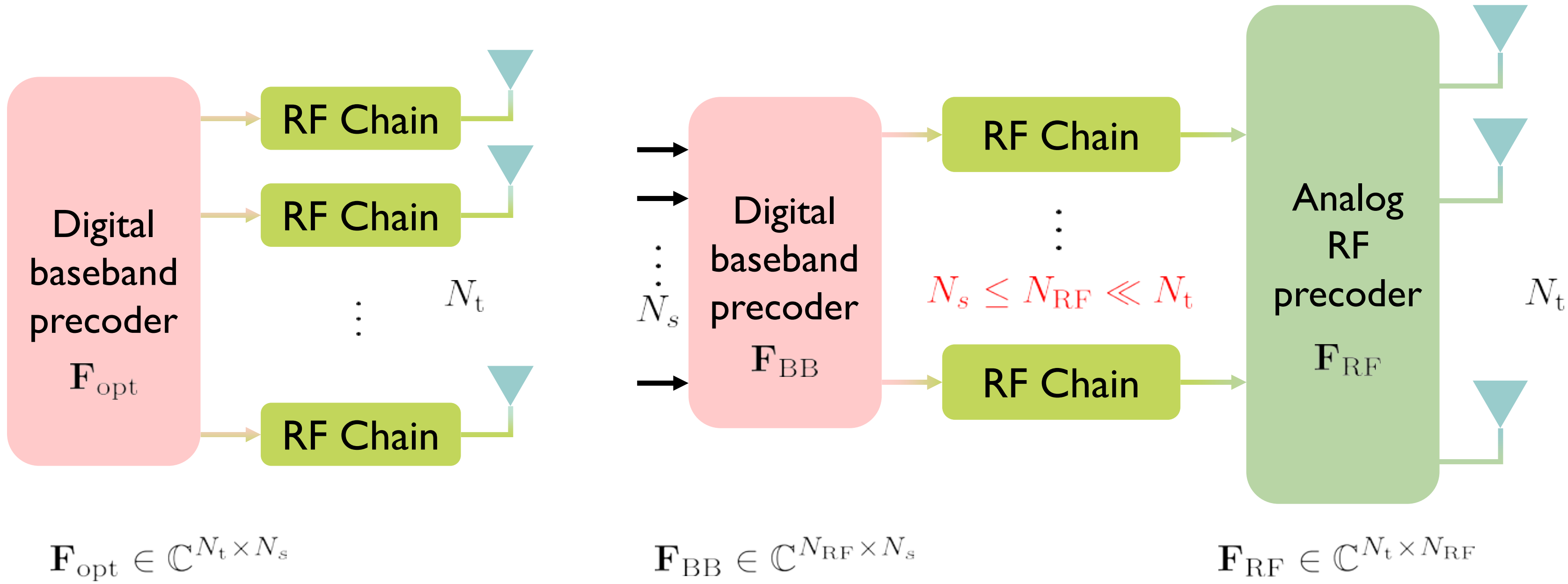
- Hybrid beamforming
  - Also called *Hybrid precoding; Analog/digital precoding*
  - Notations in hybrid beamforming



$\mathbf{F}_{\mathrm{BB}} \in \mathbb{C}^{N_{\mathrm{RF}} \times N_s}$ $\mathbf{F}_{\mathrm{RF}} \in \mathbb{C}^{N_{\mathrm{t}} \times N_{\mathrm{RF}}}$

Preliminaries of Hybrid Beamforming

❖ Fully digital precoding vs. Hybrid precoding



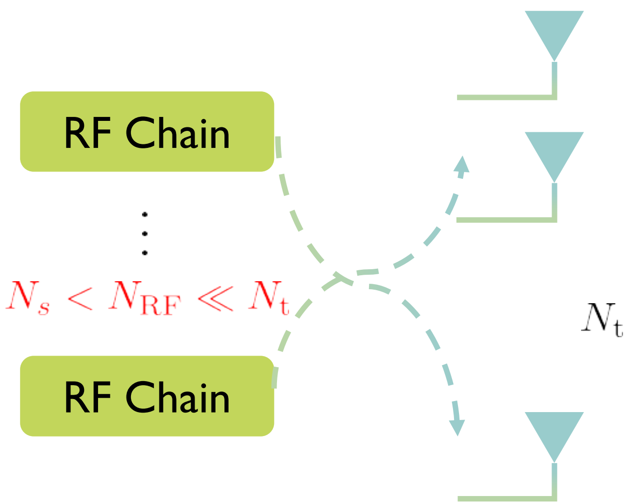
- Main differentiating part: Analog RF precoder
- Mapping from low-dimensional RF chains to high-dimensional antennas, typically implemented by phase shifters

Preliminaries of Hybrid Beamforming

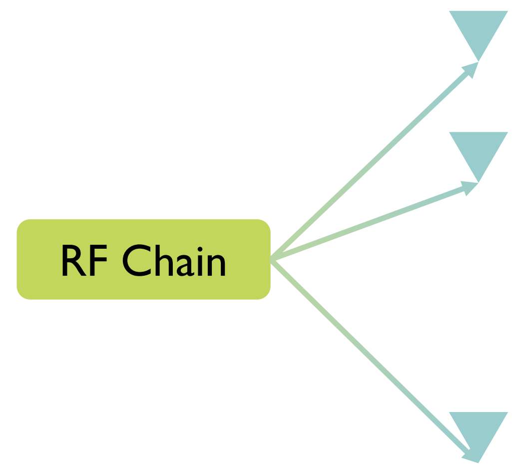
❖ Hybrid precoder structure

(I) Mapping strategy:
Which antennas should be connected to each RF chain?

(II) Hardware implementation:
What kind of hardware should be used to realize each connection?



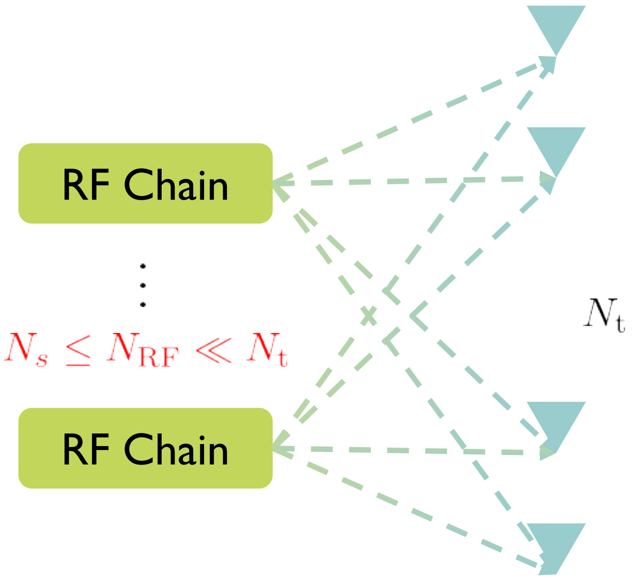
Signal flow



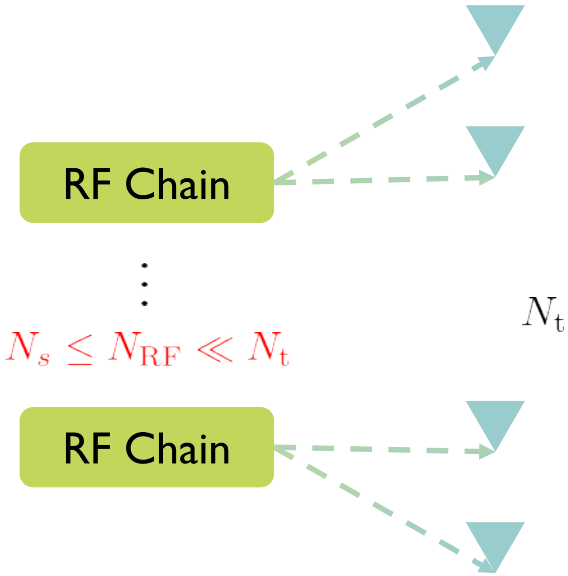
Adopted hardware

Preliminaries of Hybrid Beamforming

- The state-of-the-art hybrid precoder structure
  - Mainly focus on different mapping strategies



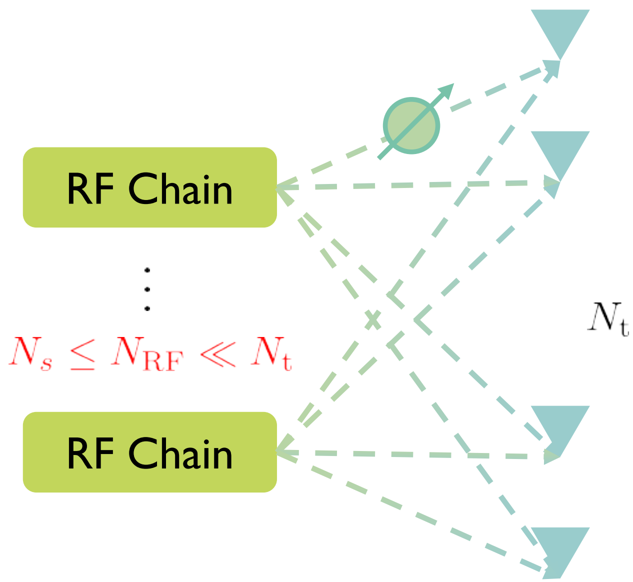
Fully-connected



Partially-connected

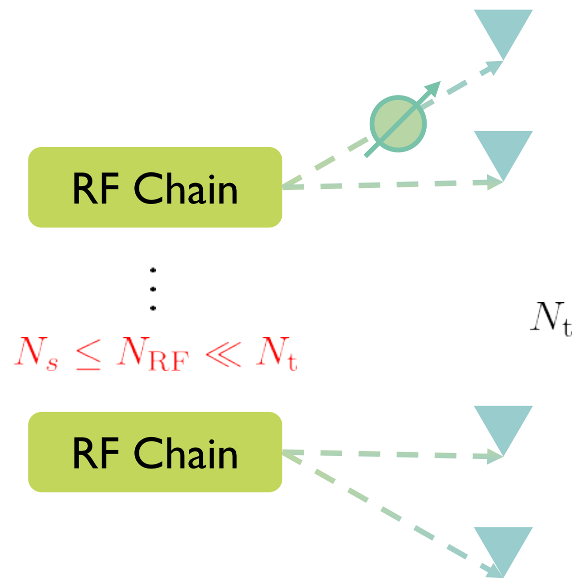
Preliminaries of Hybrid Beamforming

- The state-of-the-art hybrid precoder structure
  - One prevalent hardware implementation: Single phase shifter (SPS)



SPS Fully-connected

$N_{\mathrm{PS}} = N_{\mathrm{t}} N_{\mathrm{RF}}$

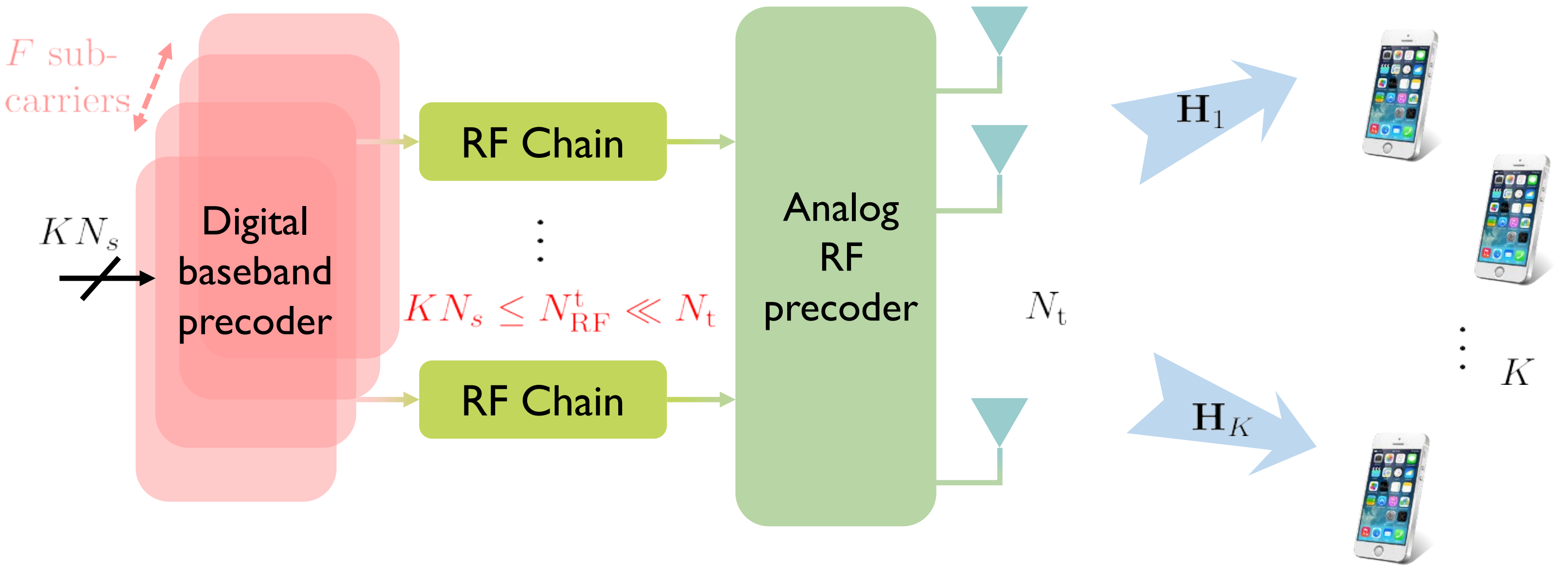


SPS Partially-connected

$N_{\mathrm{PS}} = N_{\mathrm{t}}$

Preliminaries of Hybrid Beamforming

❖ General multiuser multicarrier (MU-MC) systems

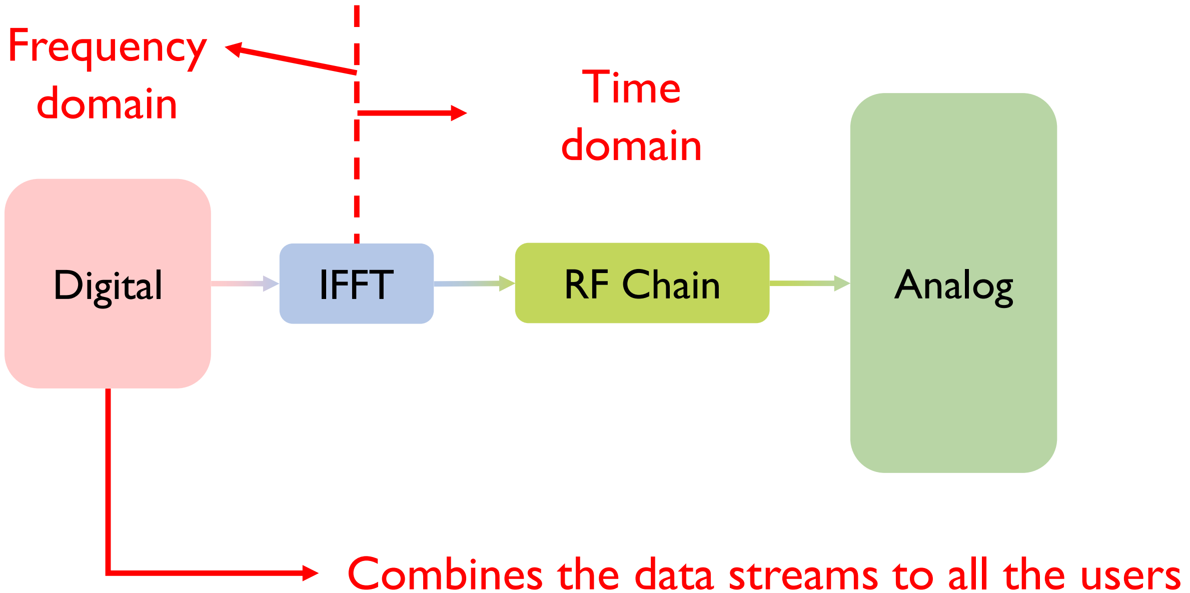


➢ One single digital precoder for each user on each subcarrier

$$\mathbf{F}_{\mathrm{BB}k,f}$$

Preliminaries of Hybrid Beamforming

❖ General multiuser multicarrier (MU-MC) systems



➢ Analog precoder $\mathbf{F}_{RF}$ is shared by all the users and subcarriers

Preliminaries of Hybrid Beamforming

- Generic hybrid precoder design problem
  - Minimize the Euclidean distance between the hybrid precoders and the fully digital precoder [O. El Ayach *et al.*, 2014]

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \le P_{\max} \\ & \mathbf{F}_{\mathrm{RF}} \in \mathcal{A}_x \end{aligned}$$

Main difficulty

$$\mathbf{F}_{\mathrm{opt}} = \left[\mathbf{F}_{\mathrm{opt}\,1,1}, \cdots, \mathbf{F}_{\mathrm{opt}\,k,f}, \cdots, \mathbf{F}_{\mathrm{opt}\,K,F}\right] \in N_{\mathrm{t}} \times KN_sF$$

$$\mathbf{F}_{\mathrm{BB}} = \left[\mathbf{F}_{\mathrm{BB}1,1}, \cdots, \mathbf{F}_{\mathrm{BB}k,f}, \cdots, \mathbf{F}_{\mathrm{BB}K,F}\right] \in N_{\mathrm{RF}}^{\mathrm{t}} \times KN_sF$$

  - $\mathcal{A}_x$ varies according to different hybrid precoder structures, e.g., $|(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1$ for the SPS fully-connected structure

Preliminaries of Hybrid Beamforming

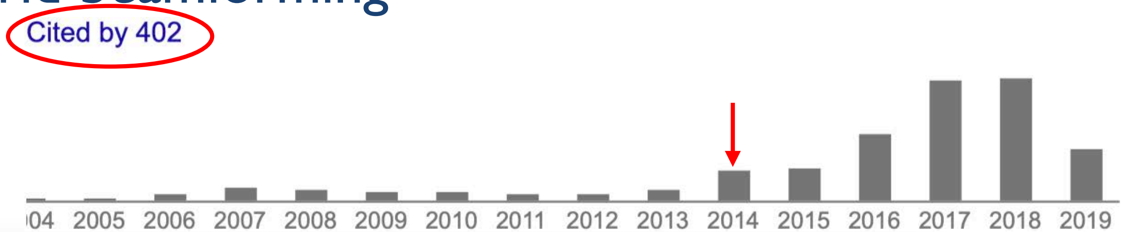
- Generic hybrid precoder design problem

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \le P_{\max} \\ & \mathbf{F}_{\mathrm{RF}} \in \mathcal{A}_x \end{aligned}$$

  - This formulation applies for an arbitrary digital precoder
  - It is applicable for different hybrid beamformer structures
  - It facilitates beamforming algorithm design

Preliminaries of Hybrid Beamforming

- An early work on hybrid beamforming
  - Nov. 2005



IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 53, NO. 11, NOVEMBER 2005 4091

Variable-Phase-Shift-Based RF-Baseband Codesign for MIMO Antenna Selection

Xinying Zhang, Andreas F. Molisch, *Fellow, IEEE*, and Sun-Yuan Kung, *Fellow, IEEE*

  - Phase shifter based RF beamforming
  - N_{RF}=2 is enough for N_s=1 to achieve the performance of the fully digital precoder
  - Have not got too much attention before hybrid beamforming was proposed (cited 75 times before 2014 while 327 times after 2014)

Preliminaries of Hybrid Beamforming

- An extension
  - Sep. 2014

On Achieving Optimal Rate of Digital Precoder by RF-Baseband Codesign for MIMO Systems

Edin Zhang and Chiachi Huang
Department of Communications Engineering
Yuan Ze University
Taoyuan, Taiwan

  - Generalization: $N_{RF}=2N_s$ to achieve the performance of the fully digital precoder
  - The number of RF chains to achieve fully digital will be very large for MU-MC systems

Preliminaries of Hybrid Beamforming

- Questions to be answered in this tutorial
  - **Q1:** Can hybrid precoder provide performance close to the fully digital one with $N_{RF}<2N_s$?
  - **Q2:** How many RF chains are needed?
  - **Q3:** How many phase shifters are needed?
  - **Q4:** How to connect RF chains with antennas?
  - **Q5:** How to efficiently design hybrid precoding algorithms?

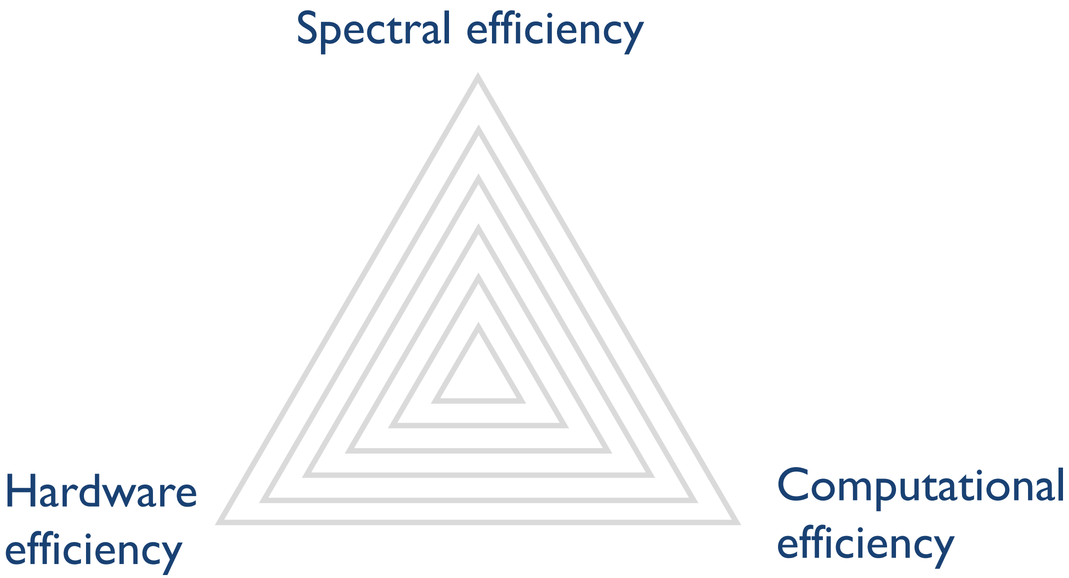
Spectral efficiency

Hardware efficiency

Computational efficiency

Preliminaries of Hybrid Beamforming

- Performance metrics
  - “Scoring triangle”

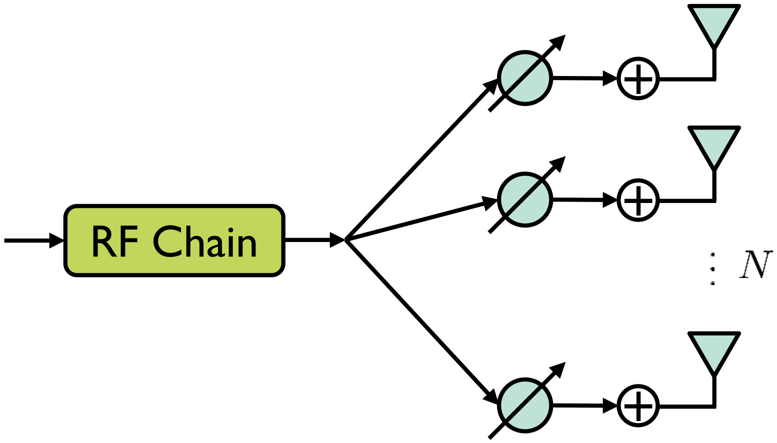


Improve Spectral Efficiency: Approaching the Fully Digital Beamforming

[Ref] X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE J. Sel. Topics Signal Process., Special Issue on Signal Process. for Millimeter Wave Wireless Commun.*, vol. 10, no. 3, pp. 485-500, Apr. 2016. (**The 2018 IEEE Signal Processing Society Young Author Best Paper Award**)

Improve Spectral Efficiency

- Single phase shifter (SPS) implementation

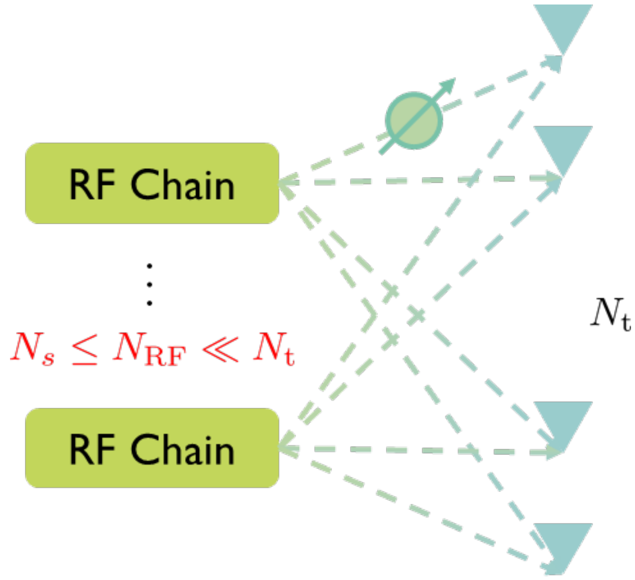


$$N = \begin{cases} N_\mathrm{t} & \text{fully-connected} \\ N_\mathrm{t}/N_\mathrm{RF}^\mathrm{t} & \text{partially-connected} \end{cases}$$

- Fully digital achieving condition: $N_\mathrm{RF}^\mathrm{t} = 2KN_s,\ N_\mathrm{RF}^\mathrm{r} = 2N_s$

Q: Can we further reduce the number of RF chains?

(I) Fully-Connected Mapping



Improve Spectral Efficiency

- Existing work
  - Mar. 2014

Citation >1354

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 13, NO. 3, MARCH 2014 1499

Spatially Sparse Precoding in Millimeter Wave MIMO Systems

Omar El Ayach, *Member, IEEE,* Sridhar Rajagopal, *Senior Member, IEEE,* Shadi Abu-Surra, *Member, IEEE,* Zhouyue Pi, *Senior Member, IEEE,* and Robert W. Heath, Jr., *Fellow, IEEE*

- Orthogonal matching pursuit (OMP) algorithm
- The columns of the analog precoding matrix $\mathbf{F}_{RF}$ is selected from a candidate set *C* (array response vectors)

$$\mathcal{C} = \{\mathbf{f}(\varphi_i)\}_{i=1}^{|\mathcal{C}|} \qquad \mathbf{f}(\varphi_i) = \frac{1}{\sqrt{N_t}}\left[1, \cdots, e^{j2\pi k\varphi_i}, \cdots, e^{j2\pi(N_t-1)\varphi_i}\right]^T$$

Improve Spectral Efficiency

- Existing work
  - OMP Algorithm

Algorithm 1 Spatially Sparse Precoding via Orthogonal Matching Pursuit

---

Require: $\mathbf{F}_{\text{opt}}$

$\mathbf{F}_{\text{RF}} = \text{Empty Matrix}$

$\mathbf{F}_{\text{res}} = \mathbf{F}_{\text{opt}}$

for $i \leq N_{\text{t}}^{\text{RF}}$ **do**

$\quad \mathbf{\Psi} = \mathbf{A}_t^* \mathbf{F}_{\text{res}}$

$\quad k = \arg\max_{\ell=1,\ \dots,\ N_{\text{cl}} N_{\text{ray}}} \left(\mathbf{\Psi}\mathbf{\Psi}^* \right)_{\ell,\ell}$

$\quad \mathbf{F}_{\text{RF}} = \left[\mathbf{F}_{\text{RF}} | \mathbf{A}_t^{(k)} \right]$

$\quad \mathbf{F}_{\text{BB}} = \left(\mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{RF}} \right)^{-1} \mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{opt}}$

$\quad \mathbf{F}_{\text{res}} = \frac{\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}}{\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F}$

end for

$\mathbf{F}_{\text{BB}} = \sqrt{N_{\text{s}}} \frac{\mathbf{F}_{\text{BB}}}{\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F}$

return $\mathbf{F}_{\text{RF}},\ \mathbf{F}_{\text{BB}}$

Find the array response vector along which the optimal precoder has the maximum projection

Appends the selected array response vector to the $\mathbf{F}_{\text{RF}}$

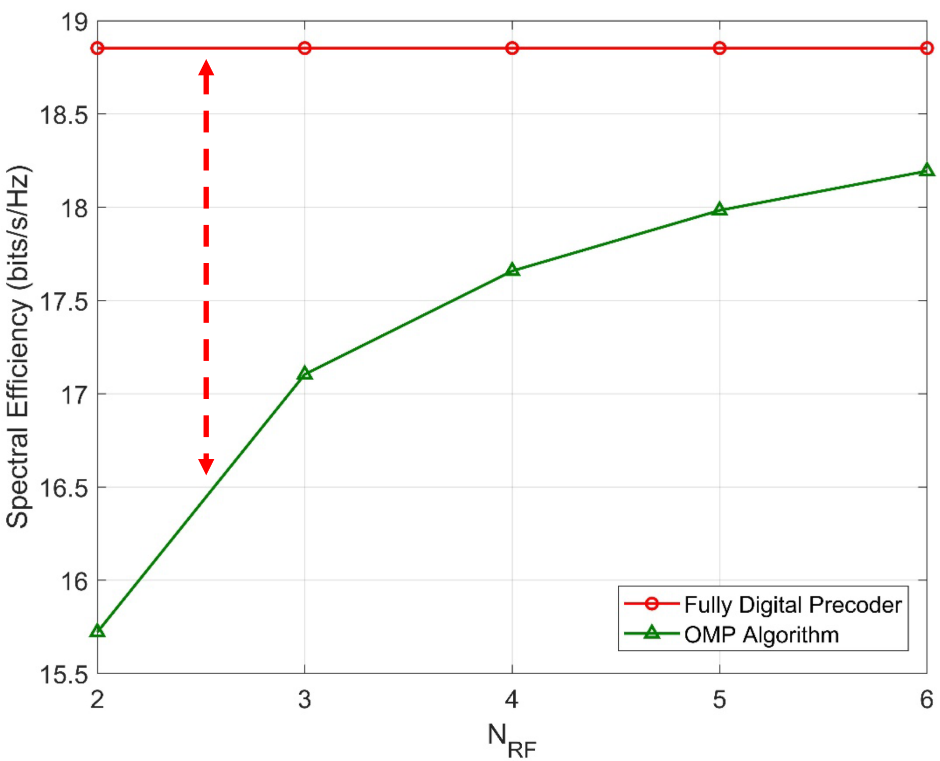
Least squares solution to $\mathbf{F}_{\text{BB}}$

Calculate "residual precoding matrix"

Improve Spectral Efficiency
(I) Fully-Connected Mapping

❖ Simulation result

$$N_\mathrm{t} = 144,\ N_\mathrm{r} = 36,\ N_\mathrm{RF}^\mathrm{t} = N_\mathrm{RF}^\mathrm{r} = N_\mathrm{RF},\ N_s = 2,\ \mathrm{SNR} = 0\,\mathrm{dB}$$

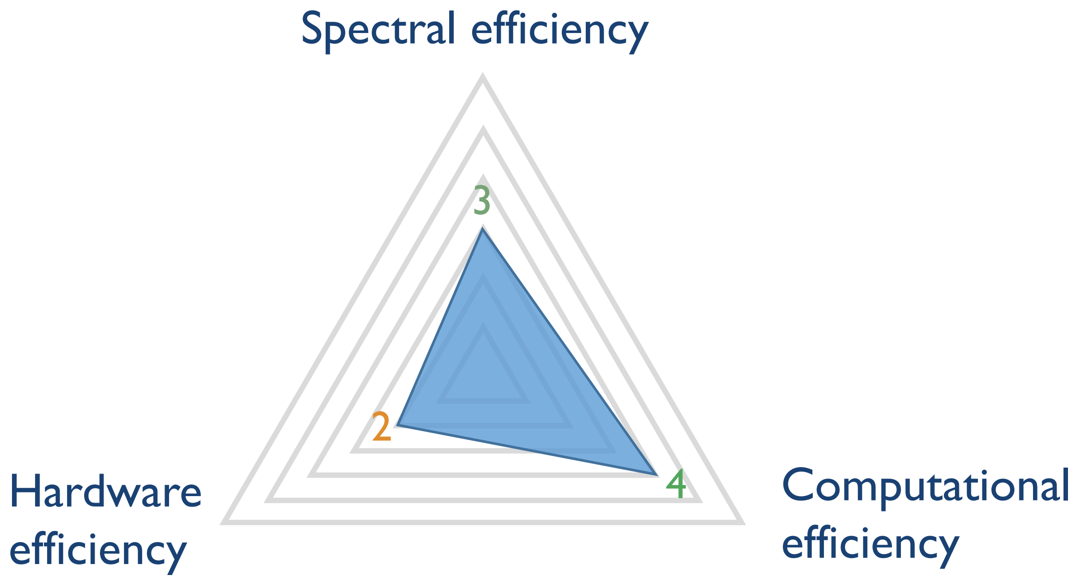


- Prominent performance loss especially with a small number of RF chains

Q: How to improve spectral efficiency with a few RF chains?

Improve Spectral Efficiency
(I) Fully-Connected Mapping

- Performance metrics
  - "Scoring triangle"



Baseline: SPS fully-connected with OMP

Improve Spectral Efficiency

(I) Fully-Connected Mapping

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i,j. \end{aligned}$$

- Start from single-user systems
  - Alternating minimization

$$\underset{\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i,j. \end{aligned}$$

  - Digital precoder: $\mathbf{F}_{\mathrm{BB}} = \mathbf{F}_{\mathrm{RF}}^{\dagger}\mathbf{F}_{\mathrm{opt}}$
  - Difficulty: Analog precoder design with the unit modulus constraints

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i,j. \end{aligned}$$

  - The vector $\mathbf{x} = \mathrm{vec}(\mathbf{F}_{\mathrm{RF}})$ forms a complex circle manifold

$$\mathcal{M}_{cc}^{m} = \{\mathbf{x} \in \mathbb{C}^m : |\mathbf{x}_1| = |\mathbf{x}_2| = \cdots = |\mathbf{x}_m| = 1\}, \quad m = N_{\mathrm{t}}N_{\mathrm{RF}}^{\mathrm{t}}.$$

Improve Spectral Efficiency
(I) Fully-Connected Mapping

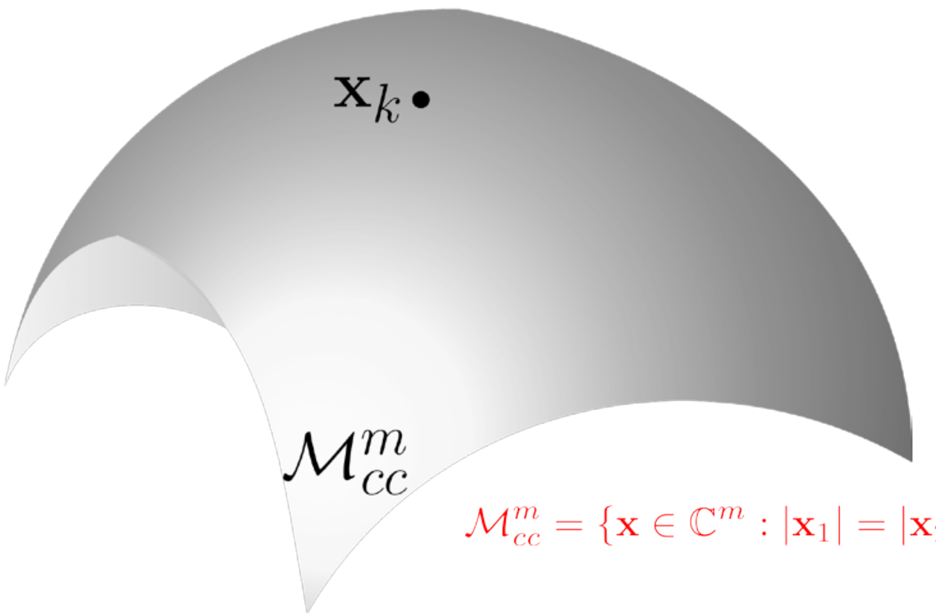
- Manifold optimization
  - What is a manifold?

    - **In mathematics, a manifold is a topological space that locally resembles Euclidean space near each point. More precisely, each point of an *n*-dimensional manifold has a neighborhood that is homeomorphic to the Euclidean space of dimension *n*.**

  - How to optimize on manifolds?

Improve Spectral Efficiency
(I) Fully-Connected Mapping

- Manifold optimization (cont.)
  - Euclidean space: gradient descent
  - Similar approaches on manifolds?



Q: For any given point $\mathbf{x}_k$ on the manifold, where to go to further decrease the objective?

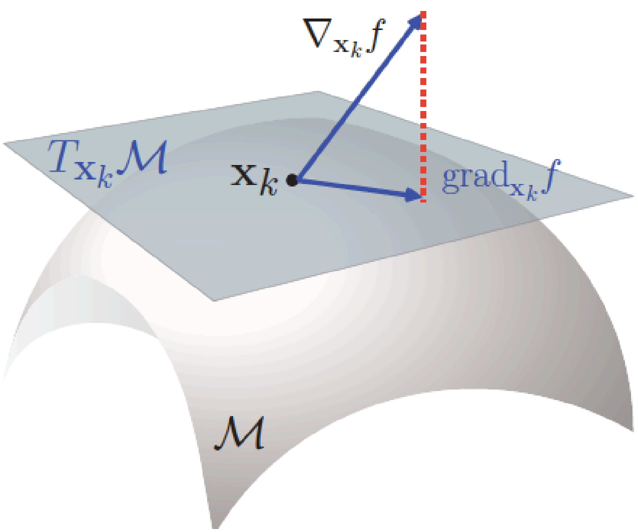
$$\mathcal{M}_{cc}^m = \{\mathbf{x} \in \mathbb{C}^m : |\mathbf{x}_1| = |\mathbf{x}_2| = \cdots = |\mathbf{x}_m| = 1\}, \quad m = N_{\mathrm{t}} N_{\mathrm{RF}}^{\mathrm{t}}.$$

Improve Spectral Efficiency

(I) Fully-Connected Mapping

- Manifold optimization (cont.)

(I) Tangent space and Riemannian gradient



- **Tangent space**:

$$T_{\mathbf{x}_k}\mathcal{M} = \left\{\mathbf{z} \in \mathbb{C}^M : \Re\left\{\mathbf{z} \circ \mathbf{x}_k^*\right\} = \mathbf{0}_M\right\},$$

where $\circ$ stands for the Hadamard product between two matrices.

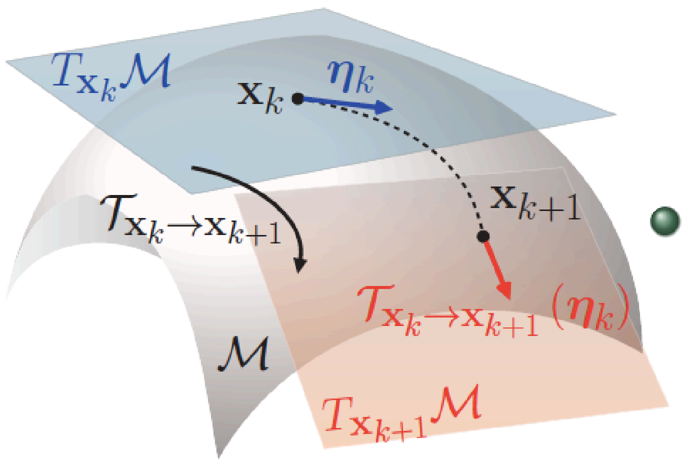
- **Riemannian gradient**: Orthogonal projection of the Euclidean gradient $\nabla_{\mathbf{x}_k} f$ onto the tangent space $T_{\mathbf{x}_k}\mathcal{M}$

$$\mathrm{grad}_{\mathbf{x}_k} f = \nabla_{\mathbf{x}_k} f - \Re\{\nabla_{\mathbf{x}_k} f \circ \mathbf{x}_k^*\} \circ \mathbf{x}_k,$$

Improve Spectral Efficiency
(I) Fully-Connected Mapping

❖ Manifold optimization (cont.)

(II) Vector transport



- Conjugate gradient (CG) method in the Euclidean space

$$\boldsymbol{\eta}_{k+1} = -\nabla_{\mathbf{x}_{k+1}} f + \beta_k \boldsymbol{\eta}_k,$$

where $\boldsymbol{\eta}_k$ is the search direction at $\mathbf{x}_k$.

- **Transport**: Mapping of a tangent vector from one tangent space to another

$$\mathcal{T}_{\mathbf{x}_k \to \mathbf{x}_{k+1}}(\boldsymbol{\eta}_k) \triangleq T_{\mathbf{x}_k}\mathcal{M} \mapsto T_{\mathbf{x}_{k+1}}\mathcal{M}:$$

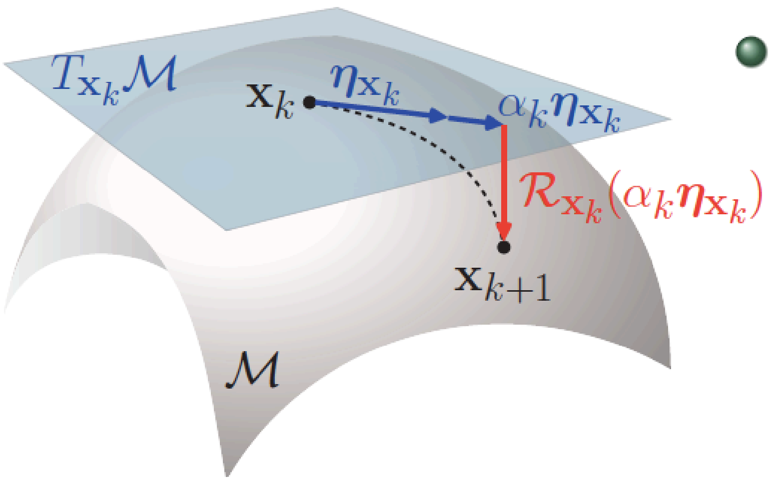
$$\boldsymbol{\eta}_k \mapsto \boldsymbol{\eta}_k - \Re\{\boldsymbol{\eta}_k \circ \mathbf{x}_{k+1}^*\} \circ \mathbf{x}_{k+1}.$$

CG on the manifold: $\eta_{k+1} = -\mathrm{grad}_{\mathbf{x}_{k+1}} f + \beta_k \mathcal{T}_{\mathbf{x}_k \to \mathbf{x}_{k+1}}(\eta_k)$

Improve Spectral Efficiency
(I) Fully-Connected Mapping

- Manifold optimization (cont.)

(III) Retraction



- **Retraction**: Mapping from the tangent space to the manifold itself to find the destination on the manifold

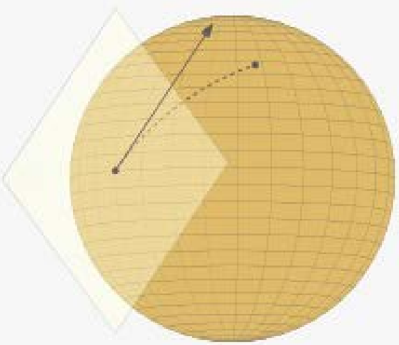
$$\mathscr{R}_{\mathbf{x}_k}(\alpha_k\boldsymbol{\eta}_k) \triangleq T_{\mathbf{x}_k}\mathscr{M} \mapsto \mathscr{M}:$$
$$\alpha_k\boldsymbol{\eta}_k \mapsto \mathrm{unt}(\alpha_k\boldsymbol{\eta}_k)$$

Optimality and complexity

- The CG method based manifold optimization converges to a critical point

Improve Spectral Efficiency
(I) Fully-Connected Mapping

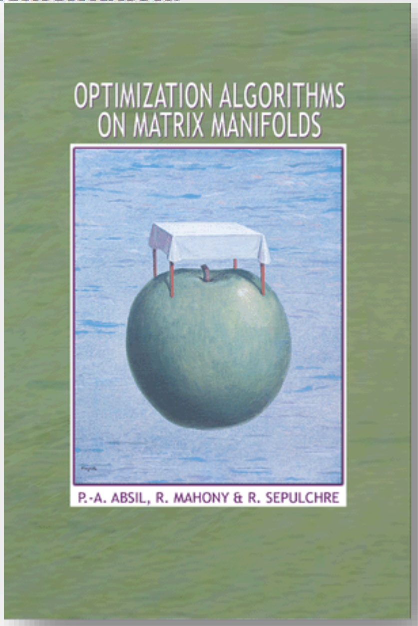
- Manifold optimization (cont.)



Manopt: a Matlab toolbox for optimization on Manifolds

Manopt, available at manopt.org, is a user-friendly, open source and **documented** Matlab toolbox which can be used to leverage the power of modern Riemannian optimization algorithms with ease. Manopt won the ORBEL Wolsey Award 2014 for best open source operational research implementation.

Tell me more/less



https://www.manopt.org/

ORBEL Wolsey Award 2014

Improve Spectral Efficiency

(I) Fully-Connected Mapping

❖ MO-AltMin Algorithm

MO-AltMin Algorithm: Manifold Optimization Based Hybrid Precoding for the Fully-connected Structure

Input: $\mathbf{F}_{\mathrm{opt}}$

1: Construct $\mathbf{F}_{\mathrm{RF}}^{(0)}$ with random phases and set $k = 0$;
2: **repeat**
3: Fix $\mathbf{F}_{\mathrm{RF}}^{(k)}$, and $\mathbf{F}_{\mathrm{BB}}^{(k)} = \mathbf{F}_{\mathrm{RF}}^{(k)\dagger}\mathbf{F}_{\mathrm{opt}}$;
4: Optimize $\mathbf{F}_{\mathrm{RF}}^{(k+1)}$ using Algorithm 1 when $\mathbf{F}_{\mathrm{BB}}^{(k)}$ is fixed;
5: $k \leftarrow k + 1$;
6: **until** a stopping criterion triggers;
7: For the digital precoder at the transmit end, normalize $\widehat{\mathbf{F}}_{\mathrm{BB}} = \frac{\sqrt{N_s}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F}\mathbf{F}_{\mathrm{BB}}$.

Manifold optimization for analog precoder

Improve Spectral Efficiency
(I) Fully-Connected Mapping

- SPS fully-connected (cont.)
  - A low-complexity algorithm
  - Enforce a semi-orthogonal constraint on $\mathbf{F}_{\mathrm{BB}}$

$$\mathbf{F}_{\mathrm{BB}}^{H}\mathbf{F}_{\mathrm{BB}} = \alpha^{2}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{F}_{\mathrm{DD}} = \alpha^{2}\mathbf{I}_{KN_s}$$

$$\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \leq \|\mathbf{F}_{\mathrm{opt}}\|_F^2 - 2\alpha\Re\,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}}\mathbf{F}_{\mathrm{opt}}^{H}\mathbf{F}_{\mathrm{RF}}\right) + \alpha^2\left\|\mathbf{F}_{\mathrm{RF}}\right\|_F^2$$

  - Digital precoder design

$$\begin{aligned} &\underset{\mathbf{F}_{\mathrm{DD}}}{\text{maximize}} && \Re\,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}}\mathbf{F}_{\mathrm{opt}}^{H}\mathbf{F}_{\mathrm{RF}}\right) \\ &\text{subject to} && \mathbf{F}_{\mathrm{DD}}^{H}\mathbf{F}_{\mathrm{DD}} = \mathbf{I}_{KN_s} \end{aligned}$$

  - Semi-orthogonal Procrustes solution $\mathbf{F}_{\mathrm{DD}} = \mathbf{V}_1\mathbf{U}^H$

Improve Spectral Efficiency

(I) Fully-Connected Mapping

- SPS fully-connected (cont.)
  - Analog precoder design

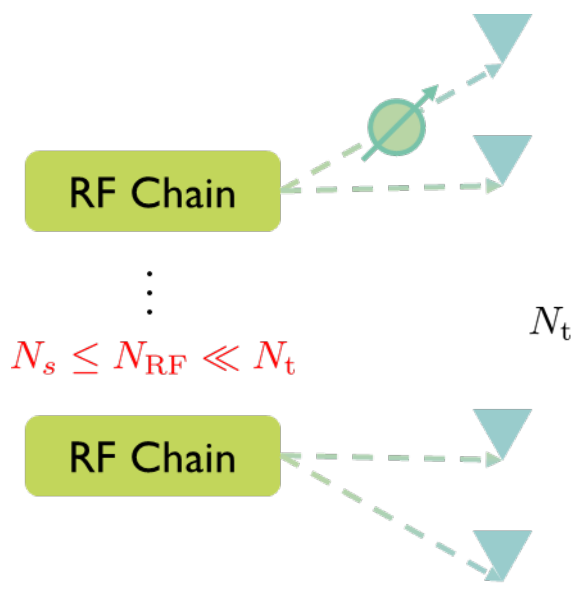
$$\begin{aligned} \underset{\alpha, \mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \left\| \Re\left(\mathbf{F}_{\mathrm{opt}} \mathbf{F}_{\mathrm{DD}}^{H}\right) - \alpha \mathbf{F}_{\mathrm{RF}} \right\|_F^2 \\ \text{subject to} \quad & \left|(\mathbf{F}_{\mathrm{RF}})_{i,j}\right| = 1, \forall i, j. \end{aligned}$$

  - Phase extraction (PE-AltMin)

$$\arg\left(\mathbf{F}_{\mathrm{RF}}\right) = \arg\left(\mathbf{F}_{\mathrm{opt}} \mathbf{F}_{\mathrm{DD}}^{H}\right)$$

  - When $N_{RF}=N_s$, the upper bound is tight, the only approximation is the additional semi-orthogonal constraint

(II) Partially-Connected Mapping



Improve Spectral Efficiency
(II) Partially-Connected Mapping

- Existing work
  - Apr. 2016

Citation > 350

998 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 34, NO. 4, APRIL 2016

Energy-Efficient Hybrid Analog and Digital Precoding for MmWave MIMO Systems With Large Antenna Arrays

Xinyu Gao, *Student Member, IEEE*, Linglong Dai, *Senior Member, IEEE*, Shuangfeng Han, *Member, IEEE*, Chih-Lin I, *Senior Member, IEEE*, and Robert W. Heath Jr., *Fellow, IEEE*

  - SPS partially-connected structure: Energy efficiency
  - Concept of successive interference cancellation (SIC) was transplanted to design the precoding algorithm

Improve Spectral Efficiency
(II) Partially-Connected Mapping

- Existing work
  - Apr. 2016

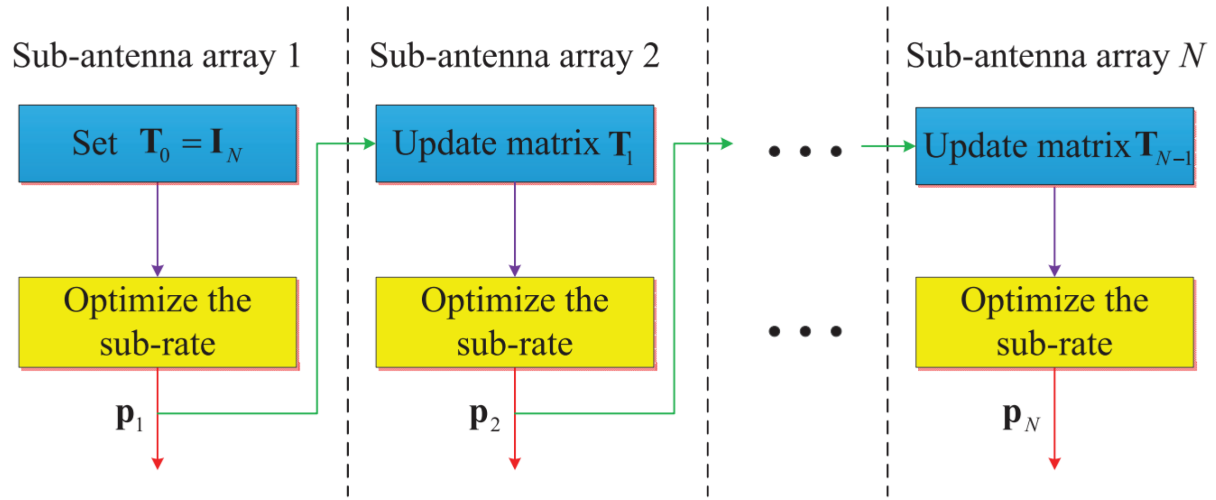


Fig. 2. Diagram of the proposed SIC-based hybrid precoding.

  - Q: How to directly design hybrid beamforming with the partially-connected mapping?

Improve Spectral Efficiency
(II) Partially-Connected Mapping

- SPS partially-connected
  - $\mathcal{A}_x$: Block diagonal $\mathbf{F}_{\mathrm{RF}}$ with unit modulus non-zero elements

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\mathrm{RF}}^{\mathrm{t}}} \end{bmatrix} \qquad \mathbf{p}_i = \left[\exp\left(\jmath\theta_{(i-1)\frac{N_t}{N_{\mathrm{RF}}^t}+1}\right), \cdots, \exp\left(\jmath\theta_{i\frac{N_t}{N_{\mathrm{RF}}^t}}\right)\right]^T$$

phase shifters connected to the *i*-th RF chain

  - Problem decoupled for each RF chain
  - Closed-form solution for $\mathbf{F}_{\mathrm{RF}}$

$$\arg\{(\mathbf{F}_{\mathrm{RF}})_{i,l}\} = \arg\left\{(\mathbf{F}_{\mathrm{opt}})_{i,:}(\mathbf{F}_{\mathrm{BB}})_{l,:}{}^H\right\}, \quad 1 \le i \le N_t,\ l = \left\lceil i\frac{N_{\mathrm{RF}}^t}{N_t}\right\rceil$$

Improve Spectral Efficiency

(II) Partially-Connected Mapping

- SPS partially-connected (cont.)
  - Optimization of $\mathbf{F}_{\mathrm{BB}}$

$$\begin{aligned}\underset{\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N_{\mathrm{RF}}^{\mathrm{t}} N_s}{N_{\mathrm{t}}}.\end{aligned}$$

  - Reformulate as a non-convex problem

$$\begin{aligned}\underset{\mathbf{Y}\in\mathbb{H}^n}{\text{minimize}} \quad & \mathrm{Tr}(\mathbf{C}\mathbf{Y}) \\ \text{subject to} \quad & \begin{cases}\mathrm{Tr}(\mathbf{A}_1\mathbf{Y}) = \frac{N_{\mathrm{RF}}^{\mathrm{t}} N_s}{N_{\mathrm{t}}} \\ \mathrm{Tr}(\mathbf{A}_2\mathbf{Y}) = 1 \\ \mathbf{Y} \succeq 0, \ \mathrm{rank}(\mathbf{Y}) = 1\end{cases}\end{aligned}$$

convex

$$n = N_{\mathrm{RF}}^t N_s + 1, \ \mathbf{y} = \begin{bmatrix}\mathrm{vec}(\mathbf{F}_{\mathrm{BB}}) & t\end{bmatrix}^T,$$

$$\mathbf{Y} = \mathbf{y}\mathbf{y}^H, \ \mathbf{f} = \mathrm{vec}(\mathbf{F}_{\mathrm{opt}}),$$

$$\mathbf{A}_1 = \begin{bmatrix}\mathbf{I}_{n-1} & \mathbf{0} \\ \mathbf{0} & 0\end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix}\mathbf{0}_{n-1} & \mathbf{0} \\ \mathbf{0} & 1\end{bmatrix},$$

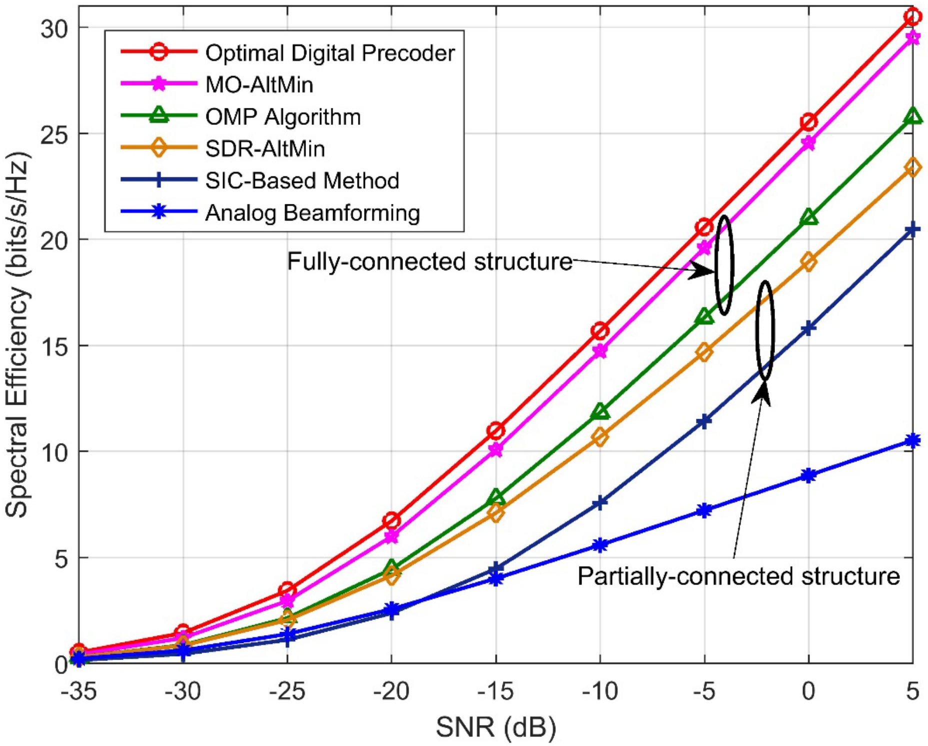
$$\mathbf{C} = \begin{bmatrix}(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}})^H(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}}) & -(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}})^H \mathbf{f} \\ -\mathbf{f}^H(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}}) & \mathbf{f}^H\mathbf{f}\end{bmatrix}.$$

  - Semidefinite relaxation (SDR) is tight for this case so globally optimal solution is obtained [Z.-Q. Luo *et al.*, 2010]

Improve Spectral Efficiency

❖ Simulation results

$$N_\mathrm{t} = 144,\ N_\mathrm{r} = 36,\ N_\mathrm{RF}^\mathrm{t} = N_\mathrm{RF}^\mathrm{r} = N_s = 3$$

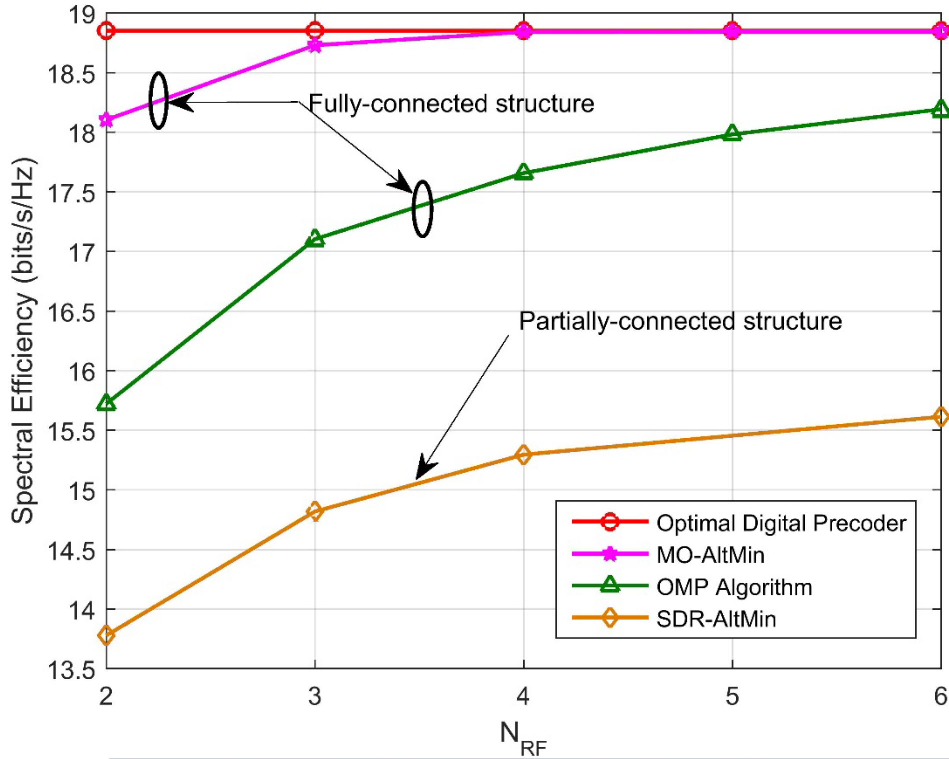


- ➢ Effectiveness of the proposed AltMin algorithms
- ➢ The fully-connected mapping can easily approach the performance of the fully digital precoding

Improve Spectral Efficiency

❖ Simulation results

$N_t = 144,\ N_r = 36,\ N_{RF}^t = N_{RF}^r = N_{RF},\ N_s = 2,\ \mathrm{SNR} = 0\,\mathrm{dB}$



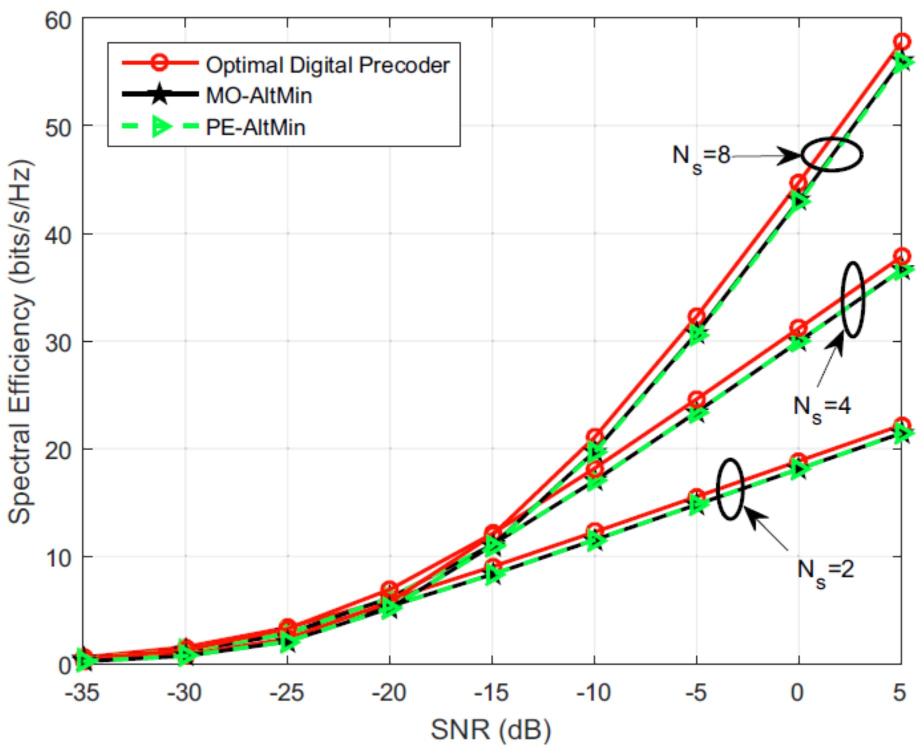
- ~N_s RF chains are enough for the fully-connected mapping
- Employing fewer PSs, the partially-connected mapping needs more RF chains

Limitation: Computational efficiency of the MO-AltMin is not good, thus difficult to extend to MU-MC settings

Improve Spectral Efficiency

❖ Simulation results

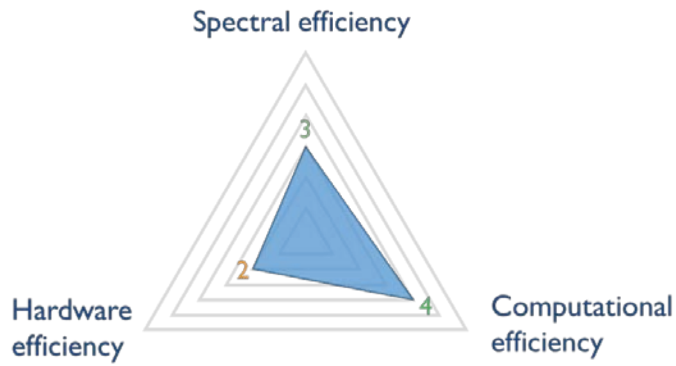
$$N_t = 144,\ N_r = 36,\ N_{RF}^t = N_{RF}^r = N_{RF}$$



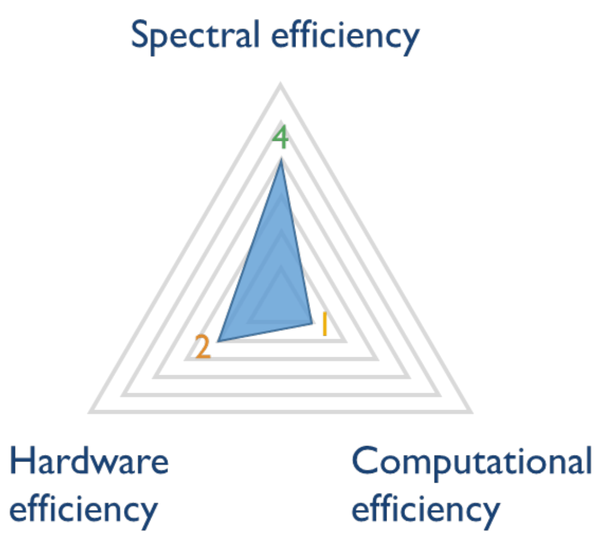
- PE-AltMin algorithm serves as an excellent low-complexity algorithm for hybrid beamforming when $N_{RF}=N_s$

Improve Spectral Efficiency

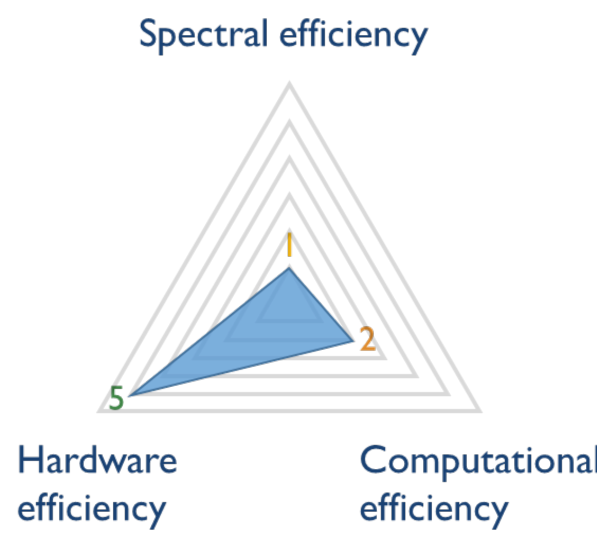
- Conclusions



Baseline: SPS fully-connected with OMP



SPS fully-connected



SPS partially-connected

Improve Spectral Efficiency

- Other approaches
  - Apr. 2016

Citation > 366

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 10, NO. 3, APRIL 2016 501

Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

Foad Sohrabi, *Student Member, IEEE*, and Wei Yu, *Fellow, IEEE*

  - Mainly focus on the special case $N_{RF}=N_s$
  - Directly maximize the spectral efficiency with the semi-orthogonal constraint on the digital precoding matrix $\mathbf{F}_{BB}$
  - Element-wise alternating minimization for the matrix $\mathbf{F}_{RF}$

Improve Spectral Efficiency

- Other approaches
  - Apr. 2016

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 10, NO. 3, APRIL 2016 501

Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

Foad Sohrabi, *Student Member, IEEE*, and Wei Yu, *Fellow, IEEE*

$$\mathbf{F}_1 = \mathbf{H}\mathbf{H}^H$$

$$\mathbf{G}_j = \frac{\gamma^2}{\sigma^2}\mathbf{F}_1 - \frac{\gamma^4}{\sigma^4}\mathbf{F}_1\bar{\mathbf{V}}_{\text{RF}}^j\mathbf{C}_j^{-1}(\bar{\mathbf{V}}_{\text{RF}}^j)^H\mathbf{F}_1$$

$$\zeta_{ij} = \mathbf{G}_j(i,i) + 2\,\text{Re}\left\{\sum_{m\neq i, n\neq i}\mathbf{V}_{\text{RF}}^*(m,j)\mathbf{G}_j(m,n)\mathbf{V}_{\text{RF}}(n,j)\right\}$$

$$\eta_{ij} = \sum_{\ell\neq i}\mathbf{G}_j(i,\ell)\mathbf{V}_{\text{RF}}(\ell,j)$$

$$\mathbf{F}_{\text{RF}}(i,j) = \begin{cases}\frac{\eta_{ij}}{|\eta_{ij}|} & \eta_{ij}\neq 0,\\ 1 & \eta_{ij} = 0\end{cases}$$

Boost Computational Efficiency: Convex Relaxation

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “Alternating minimization for hybrid precoding in multiuser OFDM mmWave Systems,” in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2016. **(Invited Paper)**

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “Doubling phase shifters for efficient hybrid precoding in millimeter-wave multiuser OFDM systems,” *J. Commun. Inf. Netw.*, vol. 4, no. 2, pp. 51-67, Jul. 2019.

Boost Computational Efficiency

- Existing works
  - Jan. 2015

Citation > 93

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 63, NO. 2, JANUARY 15, 2015 305

A Hybrid RF/Baseband Precoding Processor Based on Parallel-Index-Selection Matrix-Inversion-Bypass Simultaneous Orthogonal Matching Pursuit for Millimeter Wave MIMO Systems

Yun-Yueh Lee, Ching-Hung Wang, and Yuan-Hao Huang, *Member, IEEE*

6: $\mathbf{F}_{\mathrm{RF}} = \left[\mathbf{F}_{\mathrm{RF}} | \mathbf{A}_t^{(k)}\right]$

7: $\mathbf{F}_{\mathrm{BB}} = \left(\mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{RF}}\right)^{-1} \mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{opt}}$

6: $\mathbf{A} = \mathbf{G}_{k,\mathcal{I}_{i-1}} \mathbf{G}^{-1}_{\mathcal{I}_{i-1},\mathcal{I}_{i-1}}$

7: $V = 1/(\mathbf{G}_{k,k} - \mathbf{A}\mathbf{G}_{\mathcal{I}_{i-1},k})$

8: $\mathbf{M} = \mathbf{A}\mathbf{\Psi}_0(\mathcal{I}_{i-1},:) - \mathbf{\Psi}_0(k,:)$

9: $\mathcal{I}_i = [\mathcal{I}_{i-1}|k],\ \overline{\mathcal{I}}_i = \overline{\mathcal{I}}_{i-1} - \{k\}$

10: $\mathbf{G}^{-1}_{\mathcal{I}_i,\mathcal{I}_i} = \begin{bmatrix} \mathbf{G}^{-1}_{\mathcal{I}_{i-1},\mathcal{I}_{i-1}} + V\mathbf{A}^H\mathbf{A} & -V\mathbf{A}^H \\ -V\mathbf{A} & V \end{bmatrix}$

11: $\mathbf{X}_i = \begin{bmatrix} \mathbf{X}_{i-1} + V\mathbf{A}^H\mathbf{M} \\ -V\mathbf{M} \end{bmatrix}$

12: $\mathbf{\Psi}_i = \mathbf{\Psi}_{i-1}(\overline{\mathcal{I}}_i,:) - \mathbf{G}_{\overline{\mathcal{I}}_i,\mathcal{I}_i} \begin{bmatrix} V\mathbf{A}^H\mathbf{M} \\ -V\mathbf{M} \end{bmatrix}$

Boost Computational Efficiency

- Existing works
  - Dec. 2014

Citation > 342

IEEE WIRELESS COMMUNICATIONS LETTERS, VOL. 3, NO. 6, DECEMBER 2014 653

Low-Complexity Hybrid Precoding in Massive Multiuser MIMO Systems

Le Liang, *Student Member, IEEE*, Wei Xu, *Member, IEEE*, and Xiaodai Dong, *Senior Member, IEEE*

  - Low-complexity algorithm based on channel phase extraction

$$\mathbf{F}_{\mathrm{RF}} = \exp\{\jmath\angle(\mathbf{H})\}$$

  - Enables asymptotic performance analysis with Rayleigh fading
  - Can only deal with single-antenna multiuser MIMO and $N_{RF}=K$

Boost Computational Efficiency

- Existing works
  - Jun. 2019

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 67, NO. 12, JUNE 15, 2019 3243

A Family of Hybrid Analog–Digital Beamforming Methods for Massive MIMO Systems

Shahar Stein Ioushua, *Student Member, IEEE*, and Yonina C. Eldar, *Fellow, IEEE*

  - Phase extraction operations for different implementations

$$\underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad \left\| f\left(\mathbf{F}_{\mathrm{opt}}, \mathbf{F}_{\mathrm{BB}}\right) - \mathbf{F}_{\mathrm{RF}} \right\|_F^2$$

$$\mathbf{F}_{\mathrm{RF}} = \exp\{\jmath\angle(f)\}$$

$$(\mathbf{F}_{\mathrm{RF}})_{ij} = \begin{cases} \exp\{\jmath\angle(f_{ij})\} & |f_{ij}| \geq 1/2 \\ 0 & |f_{ij}| < 1/2 \end{cases}$$

Boost Computational Efficiency

- Main approaches to handle the unit modulus constraints
  - Candidate set/codebook based, with unit modulus elements
    - E.g., OMP
  - Manifold optimization – directly tackle unit modulus constraints
    - E.g., MO-AltMin
  - Phase extraction
    - E.g., Liang et al., WCL 14.
  - **Convex relaxation**

Boost Computational Efficiency

(I) Fully-Connected Mapping

- Main difficulty in designing the SPS implementation
  - Analog precoder with the unit modulus constraints

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

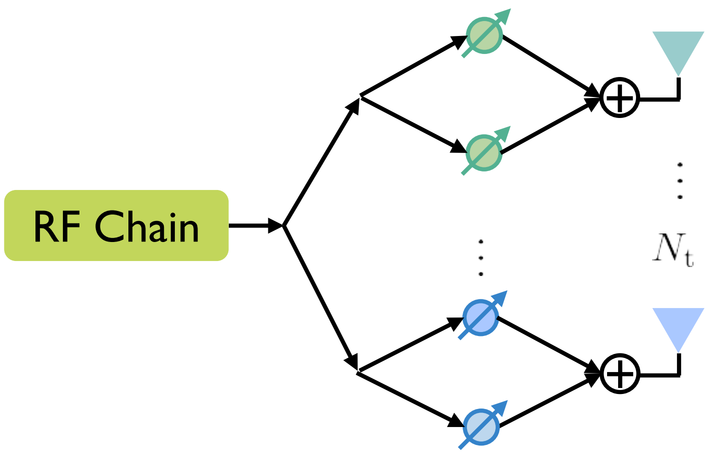
  - An intuitive way to boost computational efficiency is to relax this highly non-convex constraint as a convex one

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| \leq \gamma, \forall i, j. \end{aligned}$$

  - The value of γ does not affect the hybrid beamformer design
  - We shall choose $\gamma=2$ instead of keeping it as 1. Why?

Boost Computational Efficiency

- **Double phase shifter** (DPS) implementation
  - The relaxed solution with γ=2 can be realized by a hardware implementation



- Unit modulus constraint is eliminated
- Sum of two phase shifters

$$|e^{\jmath\theta_1} + e^{\jmath\theta_2}| \leq 2$$

Boost Computational Efficiency

(I) Fully-Connected Mapping

- Fully-connected mapping
  - RF-only precoding

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| \le 2 \end{aligned} \iff \underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + 2\|\mathbf{x}\|_1$$

LASSO

  - Closed-form solution for semi-unitary codebooks $\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H = \mathbf{I}_{N_{\mathrm{RF}}^{\mathrm{t}}}$

$$\mathbf{F}_{\mathrm{RF}}^{\star} = \mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H - \exp\left\{j\angle\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H\right)\right\} \circ \left(\left|\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H\right| - 2\right)^+ .$$

  - Hybrid precoding

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| \le 2 \end{aligned} \Rightarrow \text{Matrix factorization}$$

Redundant

Boost Computational Efficiency

(I) Fully-Connected Mapping

- Fully-connected mapping (cont.)
  - Optimality in single-carrier systems

$$\mathbf{F}_{\mathrm{opt}} = \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} \text{ with } \underline{N_{\mathrm{RF}}^{\mathrm{t}} = KN_s} \text{ and } \underline{N_{\mathrm{RF}}^{\mathrm{r}} = N_s} \text{ when } F = 1$$

Minimum number of RF chains

  - It reduces the required number of RF chains by half for achieving the fully digital precoding

  - Multi-carrier systems

$$\underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

  - Low-rank matrix approximation: SVD, globally optimal solution

Boost Computational Efficiency

(I) Fully-Connected Mapping

- Fully-connected mapping (cont.)
  - Q: How to use this relaxed result for SPS implementation?
  - Optimal solution:

$$\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^H$$

  - Some clues: The unitary matrix $\mathbf{U}_1$ fully extracts the information of the column space of $\mathbf{F}_{RF}\mathbf{F}_{BB}$, whose basis are the orthonormal columns in $\mathbf{F}_{RF}$
  - Phase extraction

$$\mathbf{F}_{\mathrm{RF}} = \exp\{\jmath\angle(\mathbf{U}_1)\}, \quad \mathbf{F}_{\mathrm{BB}} = \mathbf{S}_1\mathbf{V}_1^H$$

unit modulus constraint

Convex relaxation-enabled (CR-enabled) SPS

Boost Computational Efficiency

(II) Partially-Connected Mapping

- Partially-connected mapping
  - Block diagonal structure

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\mathrm{RF}}^{\mathrm{t}}} \end{bmatrix} \qquad \mathbf{p}_j = \left[a_{(j-1)\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}}+1}, \cdots, a_{j\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}}} \right]^T$$

  - Decoupled for each RF chain

$$\mathcal{P}_j: \quad \underset{\{a_i\},\mathbf{x}_j}{\text{minimize}} \sum_{i\in\mathcal{F}_j} \|\mathbf{y}_i - a_i\mathbf{x}_j\|_2^2,$$

$$\mathcal{F}_j = \left\{ i \in \mathbb{Z} \,\middle|\, (j-1)\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}} + 1 \leq i \leq j\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}} \right\}, \mathbf{y}_i = \mathbf{F}_{\mathrm{opt}}^T(i,:), \text{ and } \mathbf{x}_j = \mathbf{F}_{\mathrm{BB}}^T(j,:)$$

  - Eigenvalue problem

$$\mathbf{x}_j^\star = \boldsymbol{\lambda}_1 \left(\sum_{i\in\mathcal{F}_j} \mathbf{y}_i\mathbf{y}_i^H \right), \quad a_i^\star = \frac{\mathbf{x}_j^H \mathbf{y}_i}{\|\mathbf{x}_j\|_2^2}$$

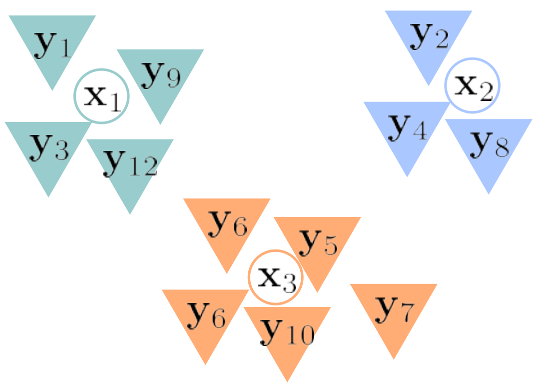
Boost Computational Efficiency

(II) Partially-Connected Mapping

- DPS partially-connected mapping (cont.)
  - Not much performance gain obtained by simply adopting the DPS implementation
  - Dynamic mapping:

    Adaptively separate all N_{t} antennas into N_{RF} groups

$$\underset{\{\mathcal{D}_j\}_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}}}{\text{maximize}} \quad \sum_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}} \lambda_1 \left(\sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right)$$



  - Modified K-means algorithm
    - Centroid: $\mathbf{x}_j^\star = \boldsymbol{\lambda}_1 \left(\sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right)$
    - Clustering: $j^\star = \arg\max_j \quad \left| \mathbf{y}_i^H \mathbf{x}_j \right|^2$
  - Convergence guarantee

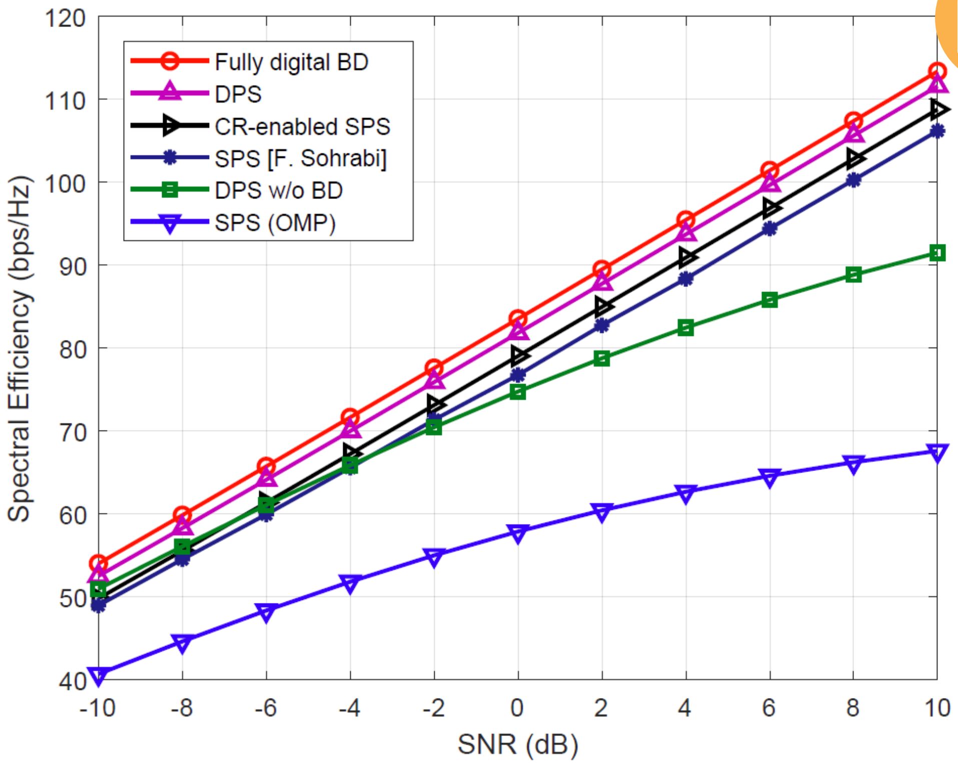
Boost Computational Efficiency

- MU-MC systems: Inter-user interference
  - Approximating the fully digital precoder leads to near-optimal performance in single-user single-carrier, single-user multicarrier, and multiuser single-carrier mm-wave MIMO systems
  - Inter-user interference will be more prominent in multiuser multicarrier systems as the analog precoder is shared by a large number of subcarriers
    - **Additional care is needed**
  - Cascade an additional block diagonalization (BD) precoder
    - Effective channel: $\hat{\mathbf{H}}_{k,f} = \mathbf{W}_{\mathrm{BB}\,k,f}^{H} \mathbf{W}_{\mathrm{RF}\,k}^{H} \mathbf{H}_{k,f} \mathbf{F}_{\mathrm{RF}} \mathbf{F}_{\mathrm{BB}\,f}$
    - BD: $\hat{\mathbf{H}}_{j,f} \mathbf{F}_{\mathrm{BD}\,k,f} = \mathbf{0}, \quad k \neq j$

Boost Computational Efficiency

❖ Simulation results (Fully-connected)

$N_t = 256,\ N_r = 16,\ K = 3,\ F = 128,\ N_s = 3,\ N_{RF}^{t} = 9$, and $N_{RF}^{r} = 3$



- Achieve near-optimal spectral efficiency and optimal multiplexing gain with low-complexity algorithms

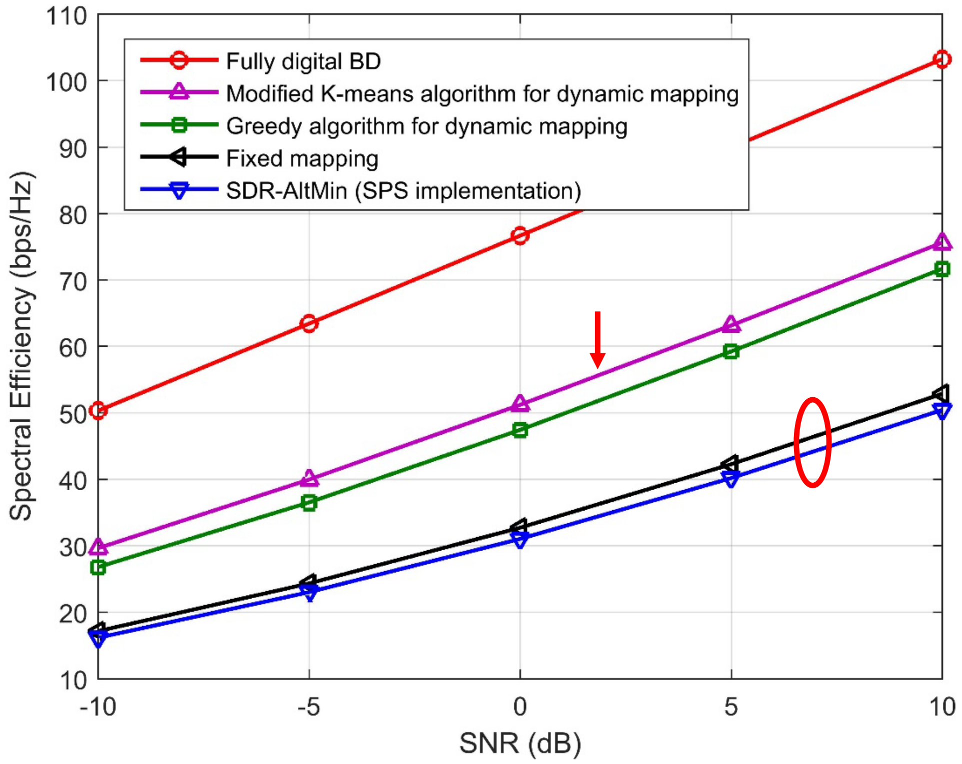
- Effectiveness of the proposed CR-enabled SPS method

[Ref] F. Sohrabi and W. Yu, "Hybrid Analog and Digital Beamforming for mmWave OFDM Large-Scale Antenna Arrays," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 7, pp. 1432-1443, July 2017.

Boost Computational Efficiency

Simulation results (Partially-connected)

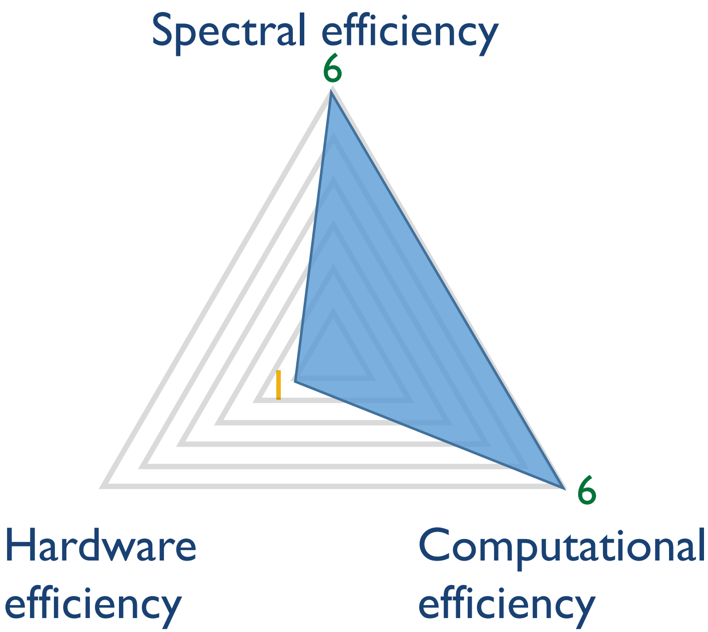
$N_\mathrm{t} = 256,\ N_\mathrm{r} = 16,\ K = 4,\ F = 128,\ N_s = 2$ $\quad N_\mathrm{RF}^\mathrm{t} = KN_s,$ and $N_\mathrm{RF}^\mathrm{r} = N_s$



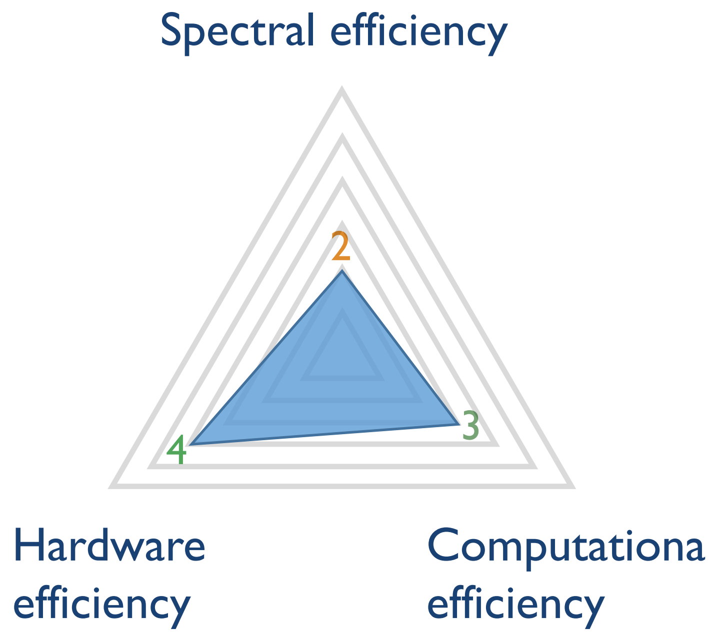
- Simply doubling PSs in the partially-connected mapping is far from satisfactory
- Superiority of the modified K-means algorithm with lower computational complexity than the greedy algorithm

Boost Computational Efficiency

❖ Conclusions



DPS fully-connected



DPS partially-connected

Boost Computational Efficiency

- Discussions
  - Comparison of computational complexity

Imple-mentation	Structure	Design approach	Hardware complexity (No. of phase shifters)	Computational complexity	Performance
SPS	Fully-connected	MO-AltMin	$N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	Extremely high	✓✓✓
	Partially-connected	SDR-AltMin	N_{t}	High	✓
DPS	Fully-connected	Matrix decomposition	$2N_{\mathrm{RF}}^{\mathrm{t}}(N_{\mathrm{t}} - N_{\mathrm{RF}}^{\mathrm{t}})$	$\mathcal{O}\left({N_{\mathrm{RF}}^{\mathrm{t}}}^2 N_{\mathrm{t}} F\right)$	✓✓✓✓
	Partially-connected	Modified K-means	$2N_{\mathrm{t}}$	$\mathcal{O}\left(N {N_{\mathrm{RF}}^{\mathrm{t}}}^2 N_{\mathrm{t}} F\right)$	✓✓

  - The proposed DPS implementation enables low complexity design for hybrid beamforming

Boost Computational Efficiency

- Discussions
  - The number of RF chains has been reduced to the minimum

$$N_{\mathrm{RF}}^{\mathrm{t}} = KN_{\mathrm{s}}$$

  - A large number of high-precision phase shifters are still needed

	Fully-connected	Partially-connected
SPS	$N_t N_{RF}$	N_t
DPS	$2N_t N_{RF}$	$2N_t$

  - Need to adapt the phases to channel states

- Practical phase shifters are typically with coarsely quantized phases

How to reduce # phase shifters?

Fight for Hardware Efficiency: How Many Phase Shifters Are Needed?

[Ref] X. Yu, J. Zhang, and K. B. Letaief, "Hybrid precoding in millimeter wave systems: How many phase shifters are needed?" in *Proc. IEEE Global Commun. Conf. (Globecom)*, Singapore, Dec. 2017. **(Best Paper Award)**

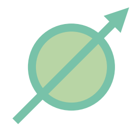
[Ref] X. Yu, J. Zhang, and K. B. Letaief, "A hardware-efficient analog network structure for hybrid precoding in millimeter wave systems," *IEEE J. Sel. Topics Signal Process., Special Issue on Hybrid Analog-Digital Signal Processing for Hardware-Efficient Large Scale Antenna Arrays*, vol. 12, no. 2, pp. 282-297, May 2018.

Fight for Hardware Efficiency

- Commonly-used hardware in hybrid beamforming

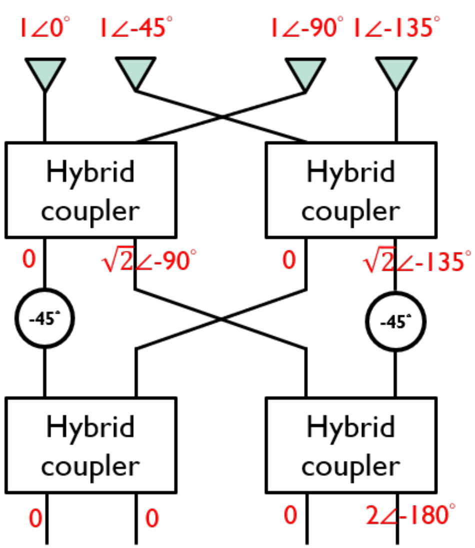
Switch ~ binary

Phase shifter ~ unit modulus



- Adaptive
- Quantized with fixed phases

Butler matrix ~ FFT matrix



Generate fixed phase difference between antenna elements

$$\mathbf{B} = \mathbf{T}\mathbf{F}\mathbf{T}$$

$$\mathbf{F} = \mathrm{FFT}(N_t) \qquad \mathbf{T} = \mathrm{diag}\left[e^{\jmath 0}, e^{-\jmath \frac{\pi}{N_t}}, \cdots, e^{-\jmath\left(\pi + \frac{\pi}{N_t}\right)}\right]$$

Fight for Hardware Efficiency

❖ Different implementations

TABLE I
COMPARISONS OF HARDWARE COMPONENTS IN THE ANALOG NETWORK FOR DIFFERENT HYBRID PRECODER STRUCTURES

		Phase shifter			Other hardware components		
		Number N_{PS}	**Type**	**Power P_{PS}**	**Hardware**	**Number N_{OC}**	**Power P_{OC}**
SPS	**Fully-connected**	$N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	Adaptive	50 mW	N/A	N/A	N/A
	Partially-connected	N_{t}					
SPS with Butlter matrices	**Fully-connected**	$\frac{N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}}{2}(\log_2 N_{\mathrm{t}} - 1)$	Fixed	20 mW	Coupler	$\frac{N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}}{2} \log_2 N_{\mathrm{t}}$	10 mW
	Partially-connected	$\frac{N_{\mathrm{t}}}{2}\left(\log_2 \frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}} - 1\right)$				$\frac{N_{\mathrm{t}}}{2} \log_2 \frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}}$	
DPS	**Fully-connected**	$2N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	Adaptive	50 mW	N/A	N/A	N/A
	Partially-connected	$2N_{\mathrm{t}}$					
FPS	**Fully-connected**	$N_c \ll N_{\mathrm{t}}$	Multi-channel	20 mW	Switch	$N_c N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	5 mW
	Group-connected		Fixed			$\frac{1}{\eta} N_c N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	

➢ How to reduce the overall hardware complexity while maintaining good performance?

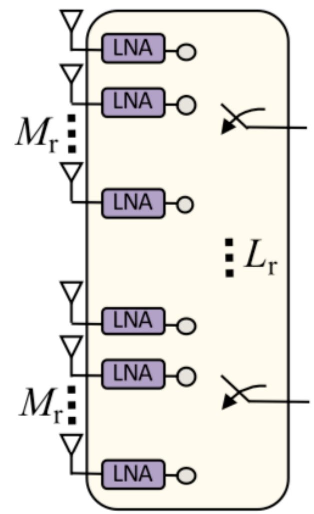
Fight for Hardware Efficiency

❖ Existing works with switches

Hybrid MIMO Architectures for Millimeter Wave Communications: Phase Shifters or Switches?

ROI MÉNDEZ-RIAL[1], CRISTIAN RUSU[1], NURIA GONZÁLEZ-PRELCIC[1],
AHMED ALKHATEEB[2], (Student Member, IEEE), AND ROBERT W. HEATH, JR.[2], (Fellow, IEEE)

- Switches with a lower dimension analog precoder: Antenna selection
- Performance loss



Fight for Hardware Efficiency

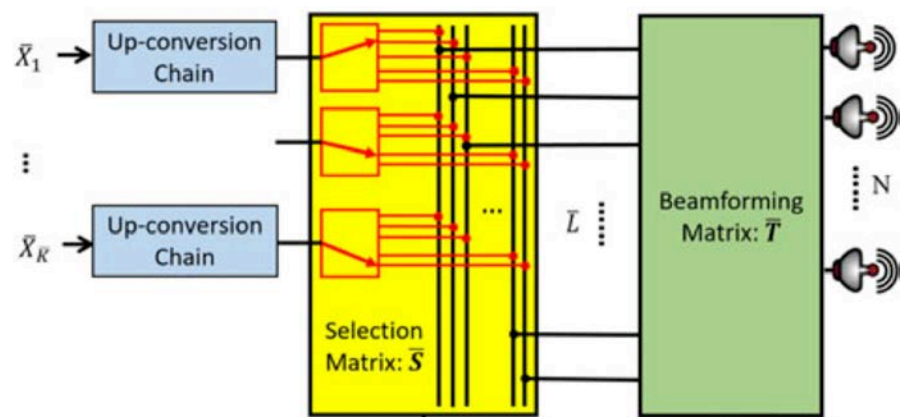
- Existing works with switches

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 66, NO. 15, AUGUST 1, 2018 4105

Hybrid Beamforming With Selection for Multiuser Massive MIMO Systems

Vishnu V. Ratnam, *Student Member, IEEE*, Andreas F. Molisch, *Fellow, IEEE*,
Ozgun Y. Bursalioglu, *Member, IEEE*, and Haralabos C. Papadopoulos, *Member, IEEE*

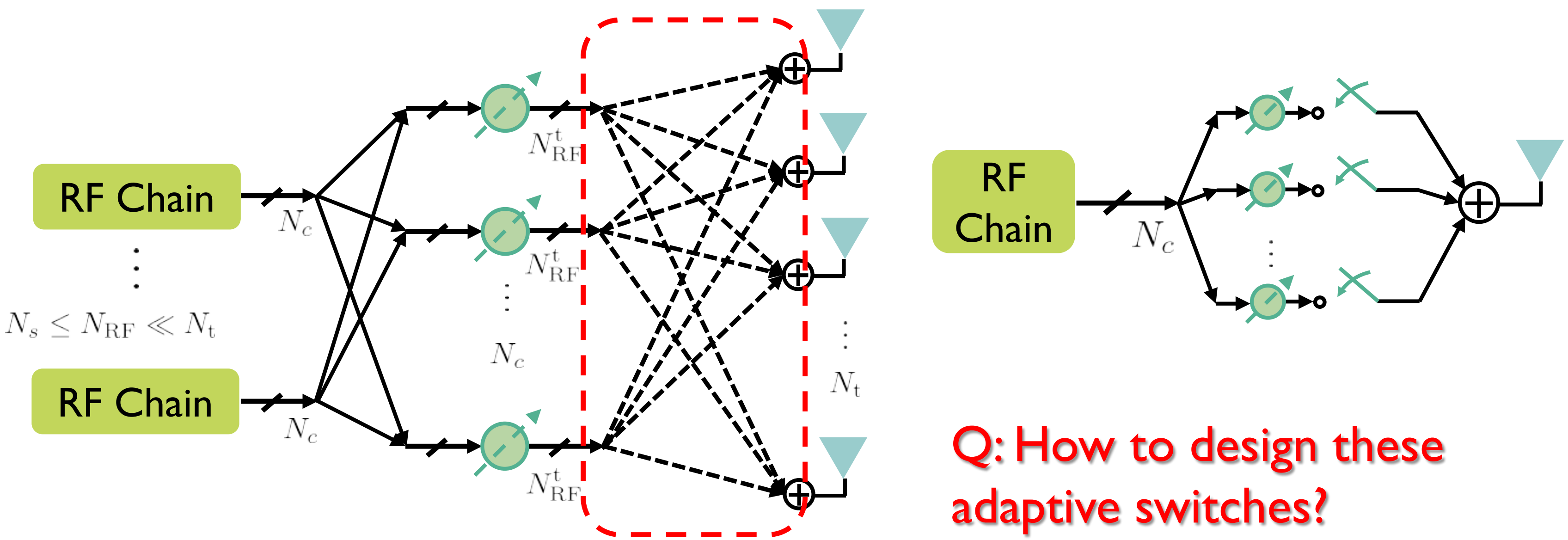
- Switches only with a higher dimension analog precoder
- Sub-matrix structure



Fight for Hardware Efficiency

(I) Fixed phase shifter implementation

- Fixed phase shifter (FPS) implementation



Q: How to design these adaptive switches?

- N_c multi-channel fixed PSs [Z. Feng *et al.*, 2014]

Fight for Hardware Efficiency
(I) Fixed phase shifter implementation

- Problem formulation
  - $\mathcal{A}_x$: $\mathbf{F}_{\mathrm{RF}} = \mathbf{SC}$
  - FPS matrix $\mathbf{C} = \mathrm{diag}(\overbrace{\mathbf{c}, \mathbf{c}, \cdots, \mathbf{c}}^{N_{\mathrm{RF}}^{\mathrm{t}}}),\quad \mathbf{c} = \frac{1}{\sqrt{N_c}}\left[e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_{N_c}}\right]^T$
  - Binary switch matrix $\mathbf{S} \in \{0,1\}^{N_{\mathrm{t}} \times N_c N_{\mathrm{RF}}^{\mathrm{t}}}$

Phases are fixed

$$\begin{aligned} \underset{\mathbf{S}, \mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{SCF}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \mathbf{S} \in \{0,1\}^{N_{\mathrm{t}} \times N_c N_{\mathrm{RF}}^{\mathrm{t}}} \end{aligned}$$

NP-hard

- An objective upper bound enables a low-complexity algorithm
  - Enforce a semi-orthogonal constraint on $\mathbf{F}_{\mathrm{BB}}$ [X.Yu *et al.*, 2016]

$$\mathbf{F}_{\mathrm{BB}}^H \mathbf{F}_{\mathrm{BB}} = \alpha^2 \mathbf{F}_{\mathrm{DD}}^H \mathbf{F}_{\mathrm{DD}} = \alpha^2 \mathbf{I}_{KN_s}$$

$$\|\mathbf{F}_{\mathrm{opt}} - \mathbf{SCF}_{\mathrm{BB}}\|_F^2 \le \|\mathbf{F}_{\mathrm{opt}}\|_F^2 - 2\alpha \Re \,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}} \mathbf{F}_{\mathrm{opt}}^H \mathbf{SC}\right) + \alpha^2 \|\mathbf{S}\|_F^2$$

Fight for Hardware Efficiency

(I) Fixed phase shifter implementation

- Alternating minimization
  - Digital precoder

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{DD}}}{\text{maximize}} \quad & \Re\,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}}\mathbf{F}_{\mathrm{opt}}^{H}\mathbf{SC}\right) \\ \text{subject to} \quad & \mathbf{F}_{\mathrm{DD}}^{H}\mathbf{F}_{\mathrm{DD}} = \mathbf{I}_{KN_s} \end{aligned}$$

  - Semi-orthogonal Procrustes solution $\mathbf{F}_{\mathrm{DD}} = \mathbf{V}_1\mathbf{U}^H$

$$\boxed{\alpha\mathbf{F}_{opt}^{H}\mathbf{SC} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H}$$

  - Switch matrix optimization

$$\begin{aligned} \underset{\alpha,\mathbf{S}}{\text{minimize}} \quad & \left\|\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) - \alpha\mathbf{S}\right\|_F^2 \\ \text{subject to} \quad & \mathbf{S} \in \{0,1\}^{N_{\mathrm{t}}\times N_c N_{\mathrm{RF}}^{\mathrm{t}}} \end{aligned}$$

  - Once α is optimized, the optimal $\mathbf{S}$ is determined correspondingly

$$\mathbf{S}^{\star} = \begin{cases} \mathbb{1}\left\{\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) > \frac{\alpha}{2}\mathbf{1}_{N_{\mathrm{t}}\times N_c N_{\mathrm{RF}}^{\mathrm{t}}}\right\} & \alpha > 0 \\ \mathbb{1}\left\{\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) < \frac{\alpha}{2}\mathbf{1}_{N_{\mathrm{t}}\times N_c N_{\mathrm{RF}}^{\mathrm{t}}}\right\} & \alpha < 0 \end{cases}$$

Fight for Hardware Efficiency

(I) Fixed phase shifter implementation

- Alternating minimization (cont.)
  - Optimization of α

$$\alpha^\star = \arg \min_{\{\tilde{x}_i, \bar{x}_i\}_{i=1}^n} \{f(\tilde{x}_i), f(\bar{x}_i)\}$$

$$\tilde{\mathbf{x}} = \mathrm{vec}(\Re(\mathbf{F}_{\rm opt}\mathbf{F}_{\rm DD}^H\mathbf{C}^H))$$

$$\tilde{\mathbf{x}} \in \mathbb{R}^n, \quad n = N_{\rm t} N_{\rm RF}^{\rm t} N_c$$

$$\bar{x}_i \triangleq \begin{cases} \frac{\sum_{j=1}^i \tilde{x}_j}{i} & \alpha < 0 \text{ and } \frac{\sum_{j=1}^i \tilde{x}_j}{i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ \frac{\sum_{j=i+1}^n \tilde{x}_j}{n-i} & \alpha > 0 \text{ and } \frac{\sum_{j=i+1}^n \tilde{x}_j}{n-i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ +\infty & \text{otherwiese} \end{cases}$$

  - Search dimension: $|\mathcal{X}| = 2N_{\rm t} N_{\rm RF}^{\rm t} N_c$ ✖
  - Acceleration: Optimal point can only be obtained at $\bar{x}_i$

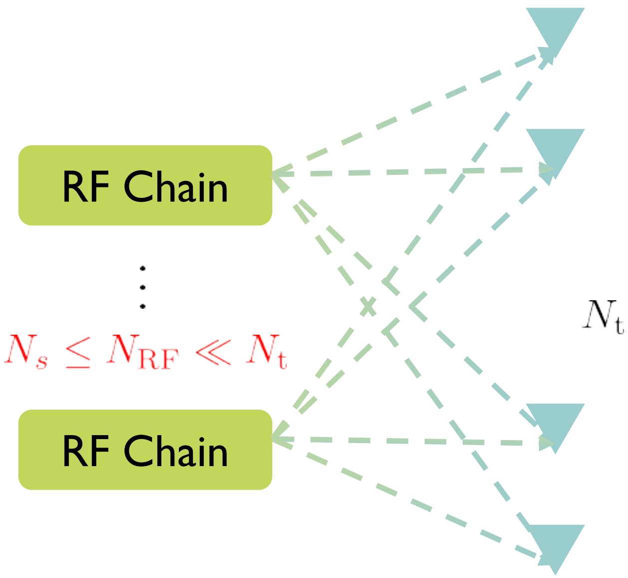
$$\alpha^\star = \arg \min_{\bar{x}_i} \quad f(\bar{x}_i)$$

  - Search dimension $\ll 2N_{\rm t} N_{\rm RF}^{\rm t} N_c$
  - Convergence guarantee

Fight for Hardware Efficiency

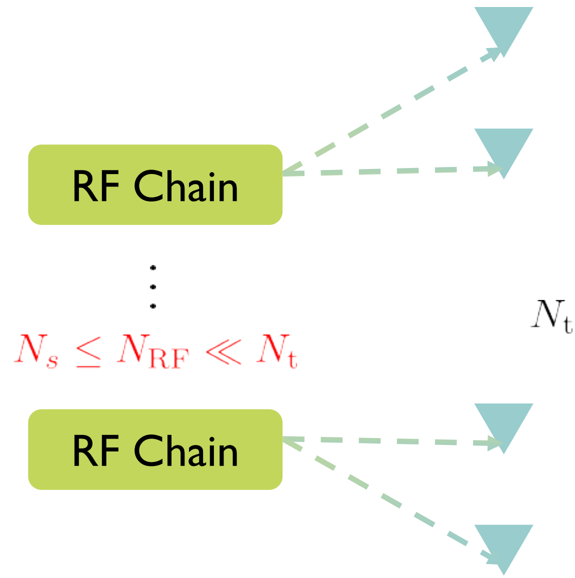
(II) Flexible hardware-performance tradeoff

- Two common mapping strategies



Fully-connected

Performance



Partially-connected

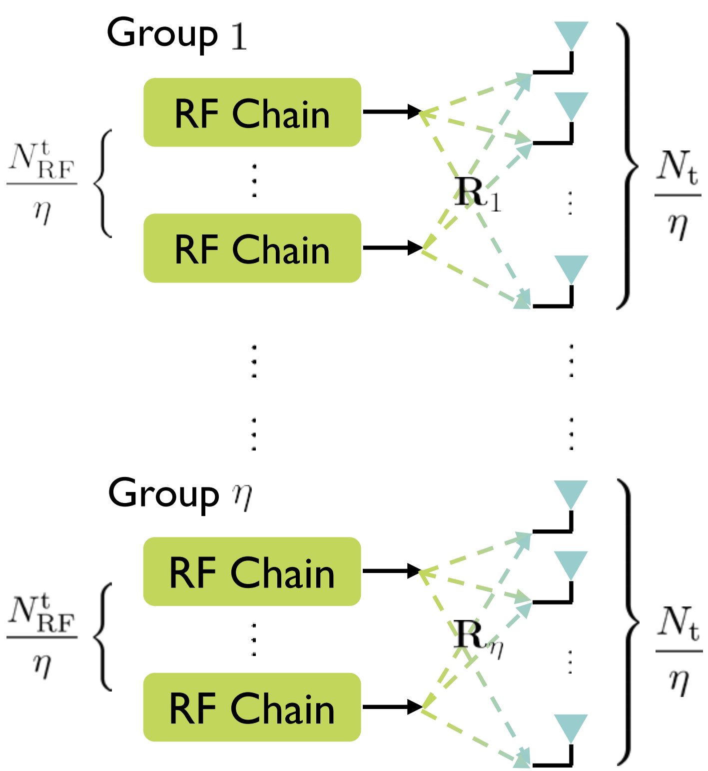
Hardware efficiency

Fight for Hardware Efficiency

(II) Flexible hardware-performance tradeoff

❖ A mapping strategy for flexible hardware-performance tradeoff

- Group-connected mapping



Save hardware by η times

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{R}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{R}_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_\eta \end{bmatrix}$$

- $\eta = 1$: Fully-connected
- $\eta = N_{\mathrm{RF}}$: Partially-connected

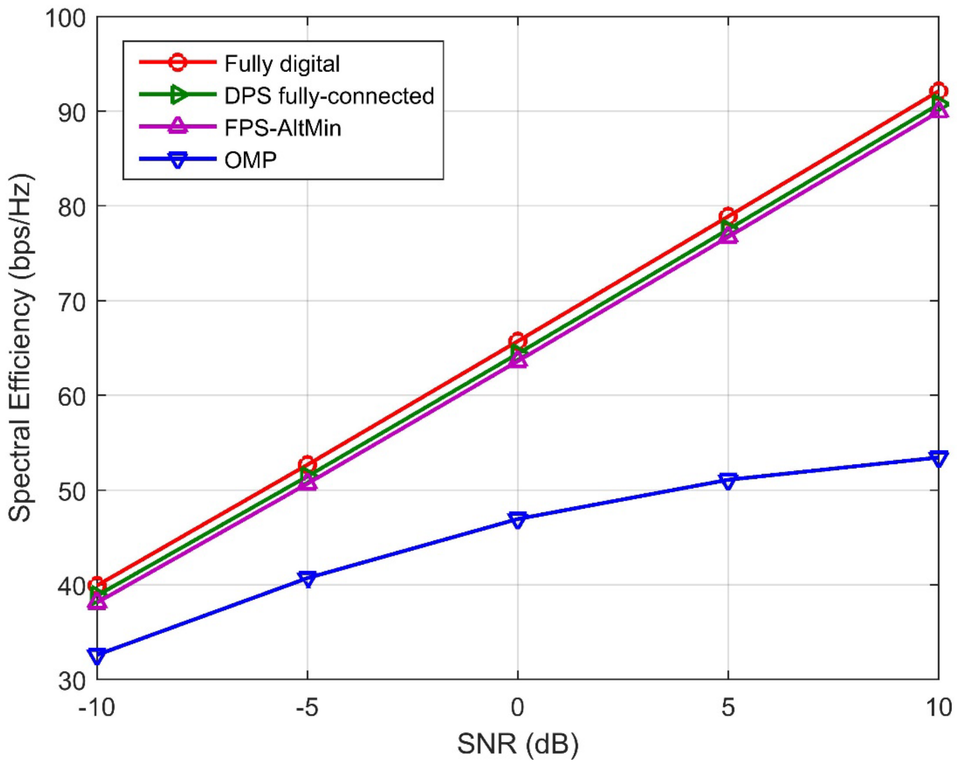
$$\begin{aligned} \underset{\mathbf{R}_i, \mathbf{B}_i}{\text{minimize}} \quad & \|\mathbf{F}_i - \mathbf{R}_i \mathbf{B}_i\|_F^2 \\ \text{subject to} \quad & \mathbf{R}_i \in \mathcal{A}_i \end{aligned}$$

Directly migrate the design for the fully-connected mapping

Fight for Hardware Efficiency

❖ Simulation results: MU-MC systems

$N_\text{t} = 144,\ N_\text{r} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_\text{RF}^\text{t} = 8$, and $N_\text{RF}^\text{r} = 2$

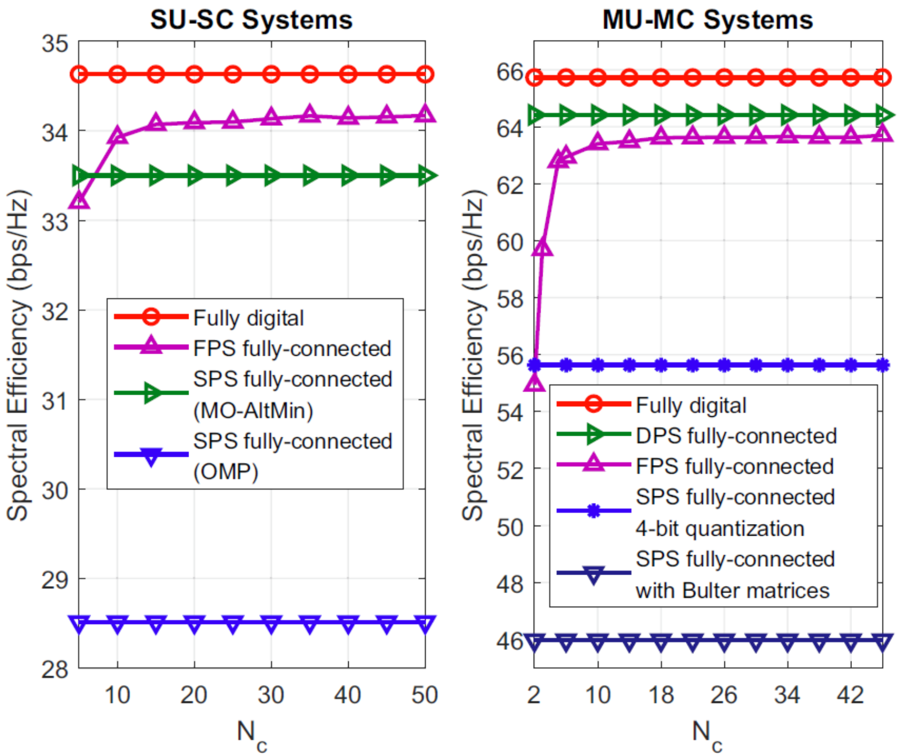


- Slightly inferior to the DPS fully-connected mapping with much fewer PSs
- Significant improvement over the OMP algorithm

Fight for Hardware Efficiency

- Simulation results: How many PSs are needed?

$$N_{\mathrm{t}} = 256,\ N_{\mathrm{r}} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{\mathrm{RF}}^{\mathrm{t}} = 8, \text{ and } N_{\mathrm{RF}}^{\mathrm{r}} = 2$$



- Only ~10 fixed phase shifters are sufficient!
- 200 times reduction compared with the DPS implementation

Fight for Hardware Efficiency

❖ Simulation results: How much power can be saved?

$N_t = 256,\ N_r = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{RF}^{t} = 8,$ and $N_{RF}^{r} = 2$

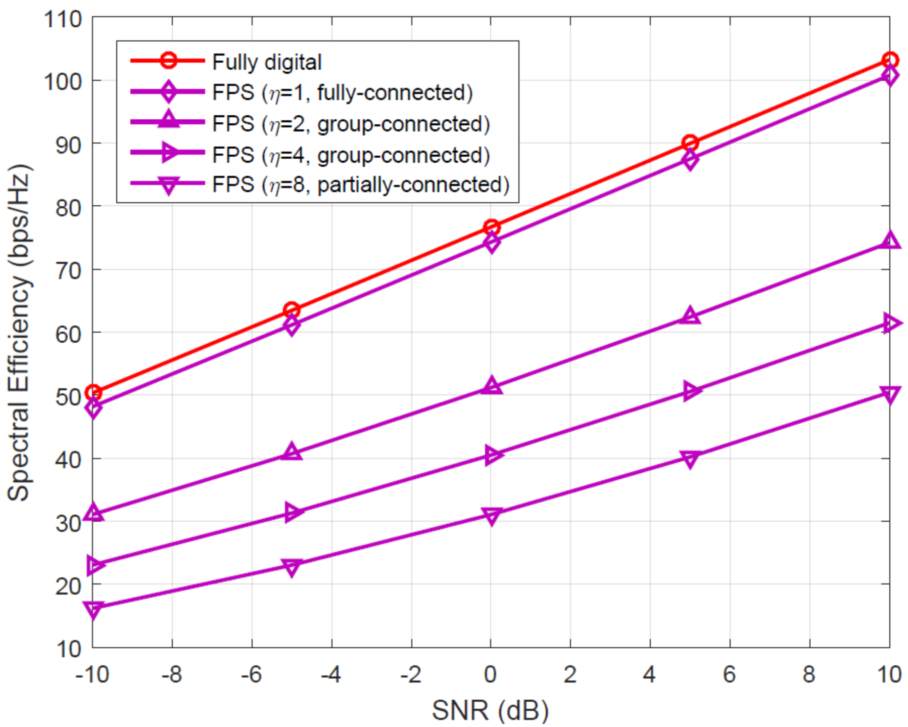
TABLE II
POWER CONSUMPTION OF THE ANALOG NETWORK FOR DIFFERENT HYBRID PRECODER STRUCTURES IN MU-MC SYSTEMS

	Phase shifter		Other hardware		Total power‡
	Number N_{PS}	**Type**	**Hardware**	**Number N_{OC}**	P_{total}
DPS fully-connected	2304	Adaptive	N/A	N/A	115.2 W
FPS fully-connected	10	Fixed§	Switch	11520	59.2 W
SPS fully-connected 4-bit quantization	1152	Adaptive	N/A	N/A	57.6 W
FPS fully-connected	2	Fixed	Switch	2304	11.84 W
SPS fully-connected with Butler matrices	3456	Fixed	Coupler	4032	109.44 W

Fight for Hardware Efficiency

❖ Simulation results

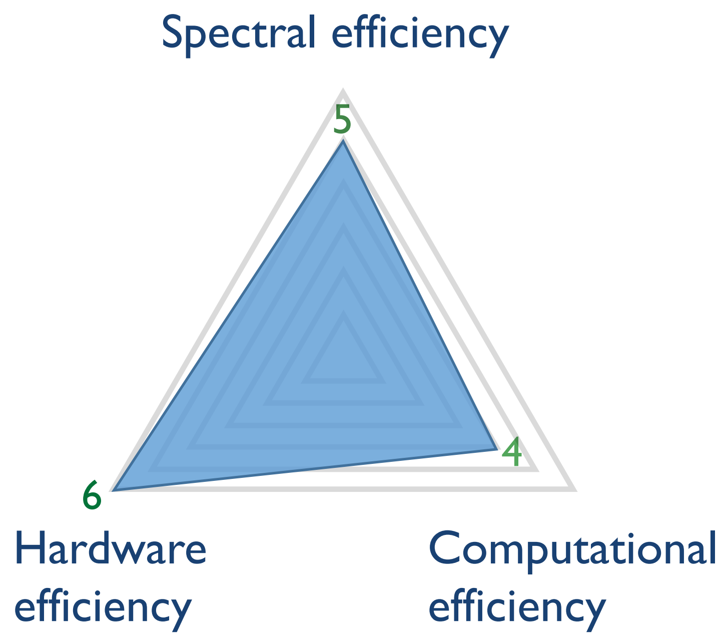
$N_{\mathrm{t}} = 256,\ N_{\mathrm{r}} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{\mathrm{RF}}^{\mathrm{t}} = 8$, and $N_{\mathrm{RF}}^{\mathrm{r}} = 2$



➢ A flexible approach to balance the achievable performance and hardware efficiency

Fight for Hardware Efficiency

- Conclusions



FPS group-connected

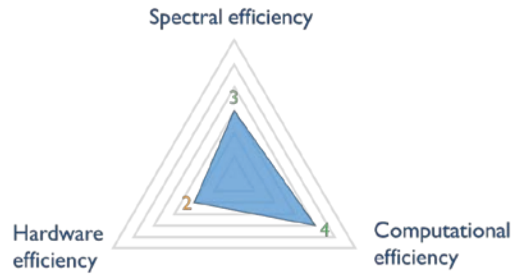
Conclusions

Conclusions

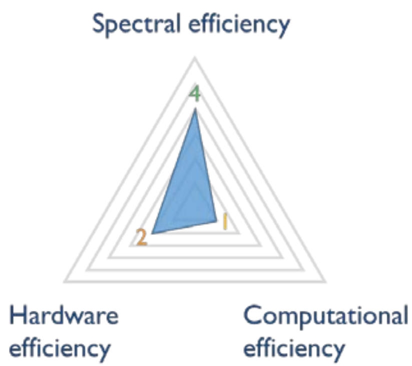
- Questions answered
  - **Q1:** Can hybrid precoder provide performance close to the fully digital one? YES
  - **Q2:** How many RF chains are needed? KN_s
  - **Q3:** How many phase shifters are needed? ~10 FPSs
  - **Q4:** How to connect the RF chains and antennas? Group-connected
  - **Q5:** How to efficiently design hybrid precoding algorithms?

    Alternating minimization provides the basic principle
    Manifold optimization provides good benchmark
    Convex relaxation enables low-complexity algorithms

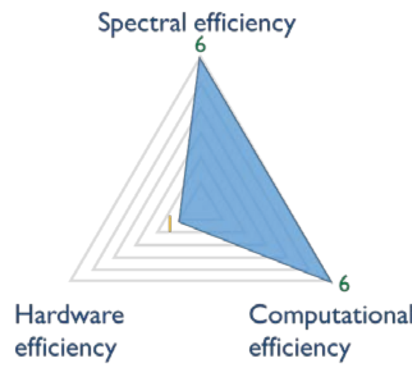
Conclusions



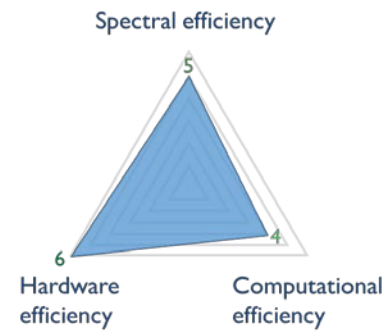
Baseline: SPS fully-connected with OMP



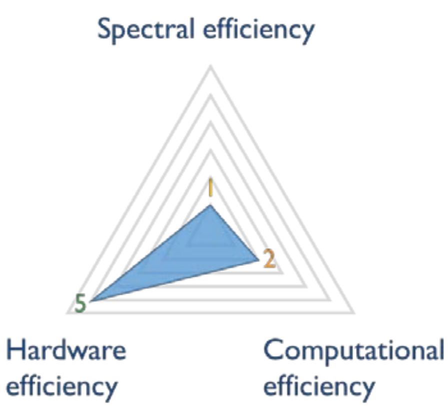
SPS fully-connected



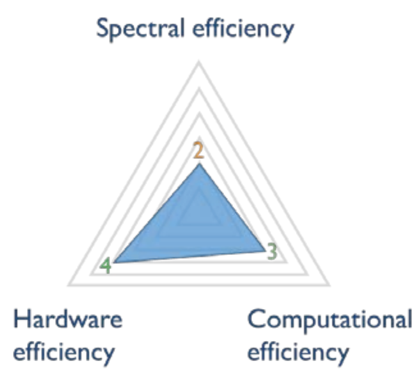
DPS fully-connected



FPS group-connected



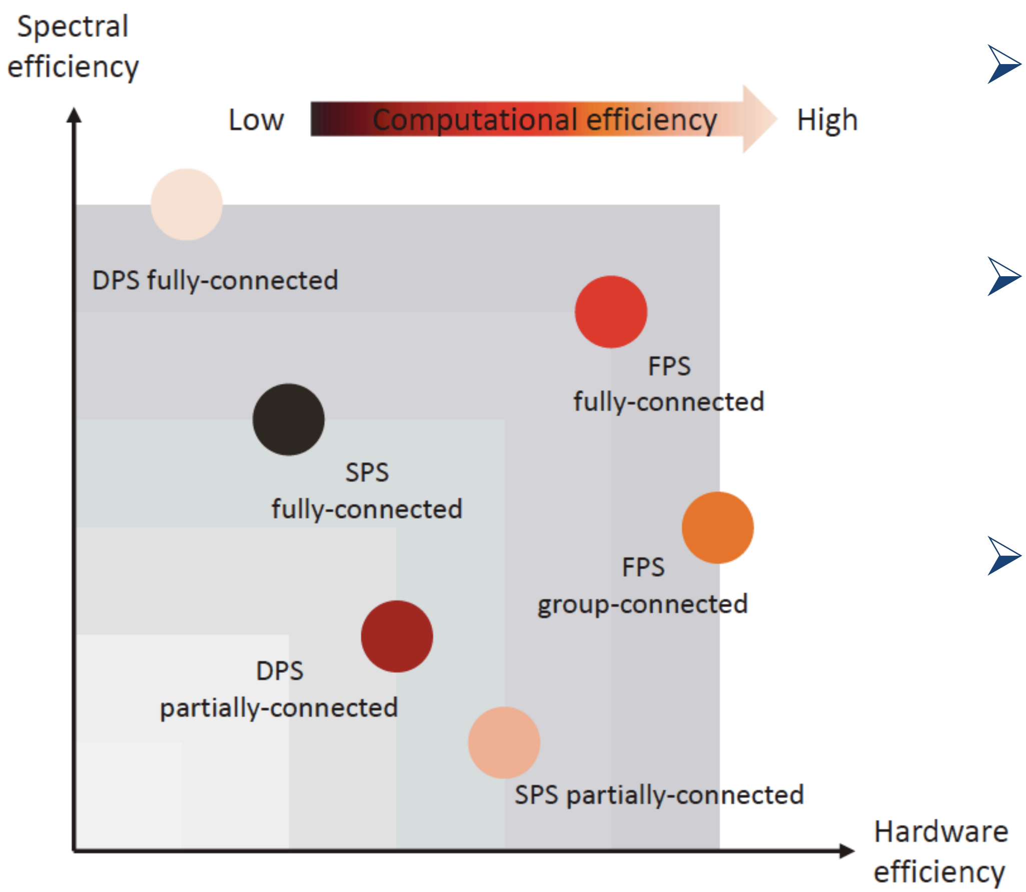
SPS partially-connected



DPS partially-connected

Conclusions

❖ Comparisons between different hybrid precoder structures



- SPS: May not be a good choice
- DPS: An excellent candidate for low-complexity algorithms
- FPS: A trade-off between the hardware and computational complexity, with satisfactory performance

Potential research directions

- Joint design with CSI acquisition and uncertainty

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 8, NO. 5, OCTOBER 2014 831

Channel Estimation and Hybrid Precoding for Millimeter Wave Cellular Systems

Ahmed Alkhateeb, *Student Member, IEEE*, Omar El Ayach, *Member, IEEE*, Geert Leus, and Robert W. Heath, Jr., *Fellow, IEEE*

IEEE COMMUNICATIONS LETTERS, VOL. 20, NO. 6, JUNE 2016 1259

Channel Estimation for Millimeter-Wave Massive MIMO With Hybrid Precoding Over Frequency-Selective Fading Channels

Zhen Gao, Chen Hu, Linglong Dai, and Zhaocheng Wang

Beam design for the training stage with the hybrid structures

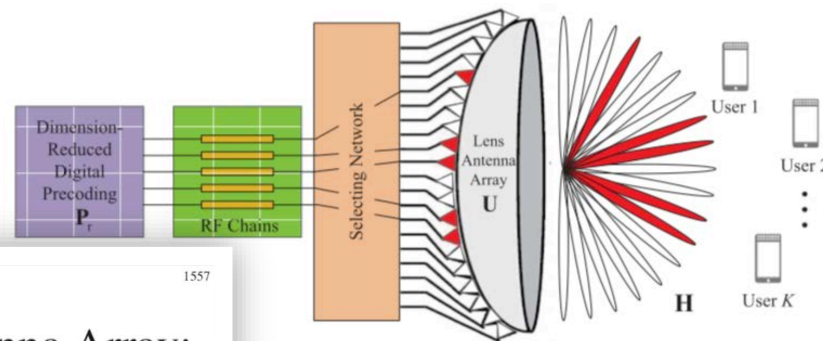
Hybrid Precoding for Millimeter Wave Cellular Systems with Partial Channel Knowledge

Ahmed Alkhateeb[†], Omar El Ayach[†], Geert Leus[‡], and Robert W. Heath Jr.[†]
[†] The University of Texas at Austin, Email: {aalkhateeb, oelayach, rheath},@utexas.edu
[‡] Delft University of Technology, Email: g.j.t.leus@tudelft.nl

Hybrid precoding with partial CSI or covariance info. only

Potential research directions

➢ Comparison between different antenna configurations



IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 64, NO. 4, APRIL 2016 1557

Millimeter Wave MIMO With Lens Antenna Array: A New Path Division Multiplexing Paradigm

Yong Zeng, *Member, IEEE*, and Rui Zhang, *Senior Member, IEEE*

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 16, NO. 9, SEPTEMBER 2017

Reliable Beamspace Channel Estimation for Millimeter-Wave Massive MIMO Systems with Lens Antenna Array

Xinyu Gao, *Student Member, IEEE*, Linglong Dai, *Senior Member, IEEE*, Shuangfeng Han, *Member, IEEE*, Chih-Lin I, *Senior Member, IEEE*, and Xiaodong Wang, *Fellow, IEEE*

Hybrid beamforming and channel estimation with lens antenna arrays

Potential research directions

➢ Hybrid beamforming for THz communications

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 14, NO. 6, JUNE 2015 3097

Indoor Terahertz Communications: How Many Antenna Arrays Are Needed?

Cen Lin and Geoffrey Ye Li, *Fellow, IEEE*

How to use antennas efficiently?

Antenna Subarray Partitioning with Interference Cancellation for Multi-User Indoor Terahertz Communications

Cen Lin and Geoffrey Ye Li
School of ECE, Georgia Institute of Technology, Atlanta, GA 30332, USA
Email: linc@gatech.edu, liye@ece.gatech.edu

Potential research directions

- Performance evaluation

IEEE WIRELESS COMMUNICATIONS LETTERS, VOL. 3, NO. 6, DECEMBER 2014 653

Low-Complexity Hybrid Precoding in Massive Multiuser MIMO Systems

Le Liang, *Student Member, IEEE*, Wei Xu, *Member, IEEE*, and Xiaodai Dong, *Senior Member, IEEE*

Performance characterization of hybrid precoding

952 IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 64, NO. 5, MAY 2016

A Comparison of MIMO Techniques in Downlink Millimeter Wave Cellular Networks With Hybrid Beamforming

Mandar N. Kulkarni, *Student Member, IEEE*, Amitava Ghosh, *Fellow, IEEE*, and Jeffrey G. Andrews, *Fellow, IEEE*

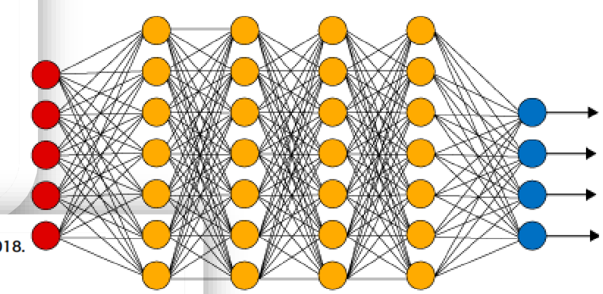
Comparison between MU-MIMO and single user spatial multiplexing

Potential research directions

➢ Further reduction in computational complexity

Machine Learning Inspired Energy-Efficient Hybrid Precoding for MmWave Massive MIMO Systems

Xinyu Gao*, Linglong Dai*, Ying Sun*, Shuangfeng Han†, and Chih-Lin I†
*Tsinghua National Laboratory for Information Science and Technology (TNList), Department of Electronic Engineering, Tsinghua University, Beijing, China
†Green Communication Research Center, China Mobile Research Institute, Beijing 100053, China



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Digital Object Identifier 10.1109/ACCESS.2018.2850226

Deep Learning Coordinated Beamforming for Highly-Mobile Millimeter Wave Systems

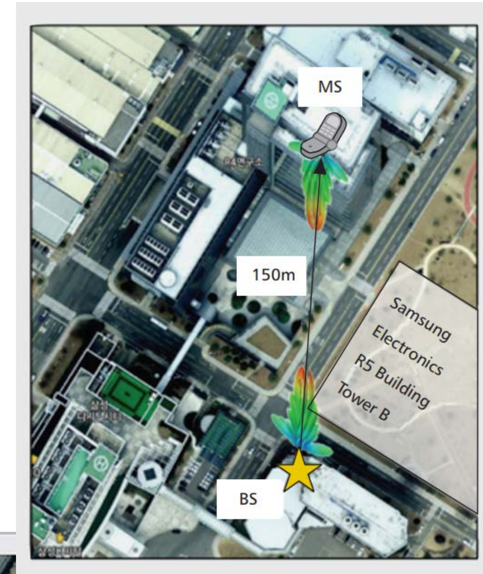
AHMED ALKHATEEB[1], SAM ALEX[2], PAUL VARKEY[2], YING LI[2], QI QU[2], AND DJORDJE TUJKOVIC[2]
[1]School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ 85287, USA
[2]Facebook Inc., Menlo Park, CA 94025, USA
Corresponding author: Ahmed Alkhateeb (alkhateeb@asu.edu)

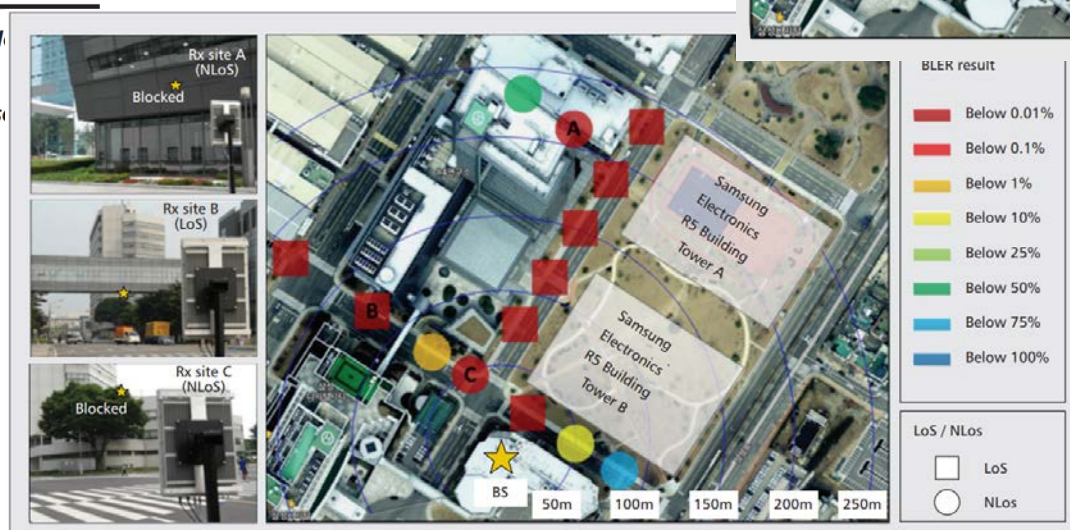
Potential research directions

➢ Hardware implementation and testing

Millimeter-Wave Beamforming as an Enabling Technology for 5G Cellular Communications: Theoretical Feasibility and Prototype Results

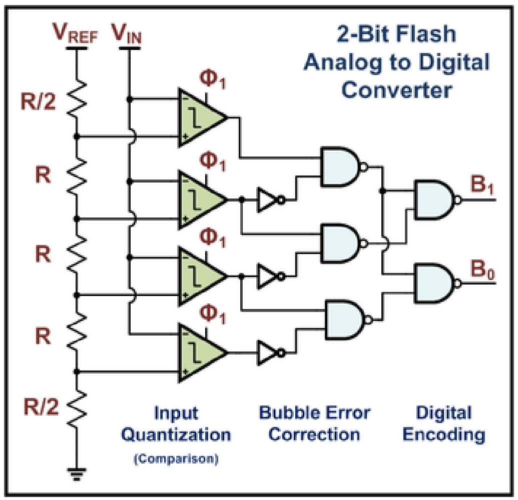
Wonil Roh, Ji-Yun Seol, JeongH
Kyungwhoon Cheun, Samsung
Farshid Aryanfar, Samsung Res

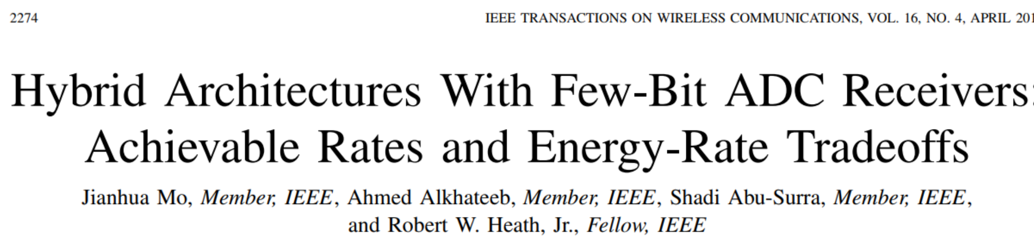




Potential research directions

➢ Hybrid precoding with low-precision ADCs




2274 IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 16, NO. 4, APRIL 2017

Hybrid Architectures With Few-Bit ADC Receivers: Achievable Rates and Energy-Rate Tradeoffs

Jianhua Mo, *Member, IEEE*, Ahmed Alkhateeb, *Member, IEEE*, Shadi Abu-Surra, *Member, IEEE*, and Robert W. Heath, Jr., *Fellow, IEEE*

Performance evaluation with tractable quantization models

High-precision ADCs at mm-wave frequencies are extremely expensive

Conclusions

Our own results

- **X. Yu**, J.-C. Shen, **J. Zhang**, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE J. Sel. Topics Signal Process.*, *Special Issue on Signal Process. for Millimeter Wave Wireless Commun.*, vol. 10, no. 3, pp. 485-500, Apr. 2016. (**The 2018 SPS Young Author Best Paper Award**)
- **X. Yu**, **J. Zhang**, and K. B. Letaief, "Alternating minimization for hybrid precoding in multiuser OFDM mmWave Systems," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2016. **(Invited Paper)**
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Thanks

For more information and Matlab codes:

http://www.eie.polyu.edu.hk/~jeiezhang