Algorithm Design for IRS-Assisted Wireless Communication Systems

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Outline

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- **Background and Introduction**
- **Algorithm Design for IRS-Assisted Systems**
  - **Classic Approaches**
  - **System-Specific Algorithms**
  - **A New IRS Model**
- **Potential Research Directions**

Background and Introduction

Background

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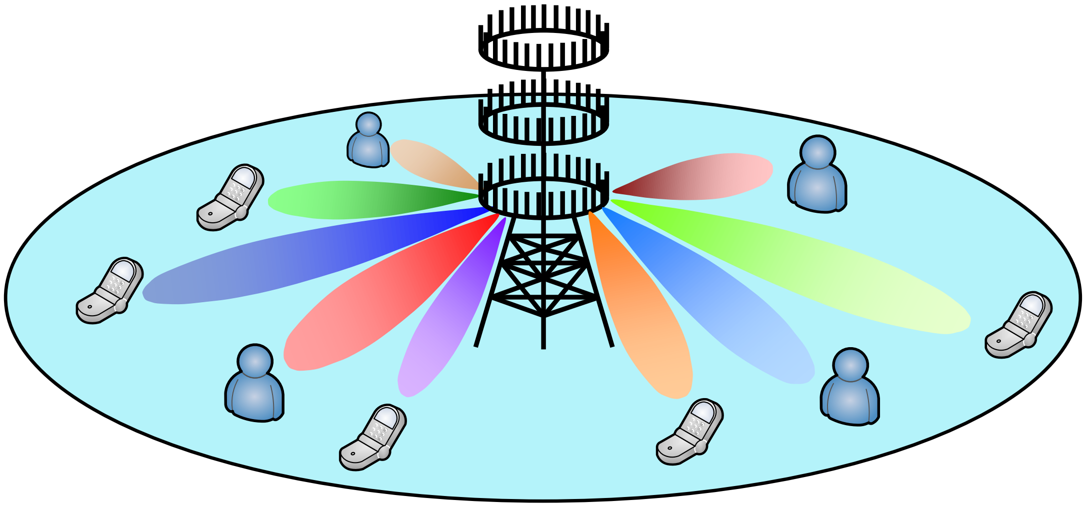
Two Limitations in Current Wireless Systems

- Additional cost and power consumption are inevitably incurred

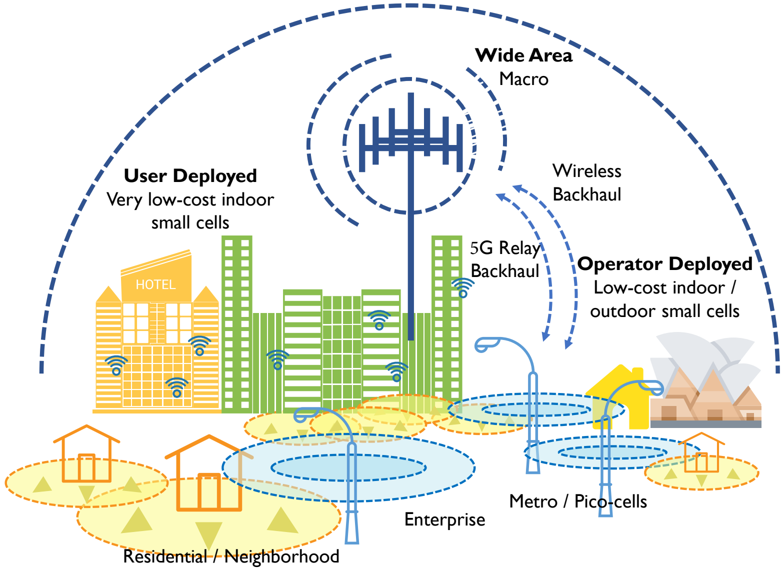


Large-scale antenna arrays for massive MIMO systems

Background

❖ Two Limitations in Current Wireless Systems

➢ Additional cost and power consumption are inevitably incurred



Network densification with a large number of APs in small cells

Background

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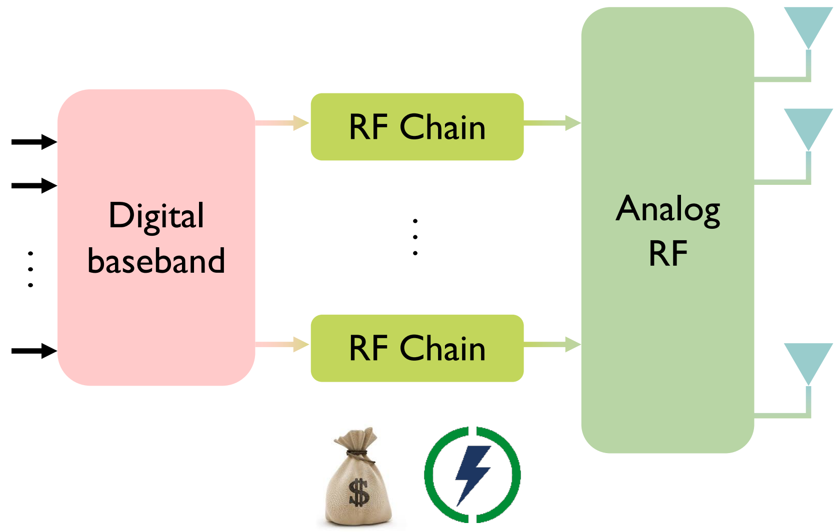
- Two Limitations in Current Wireless Systems
  - Additional cost and power consumption are inevitably incurred



Costly and power-hungry RF chains at mm-wave bands

Background

- Two Limitations in Current Wireless Systems
  - Additional cost and power consumption are inevitably incurred
  - Performance of wireless communication systems is limited by the wireless channels which are treated as a "black box" and cannot be adaptively controlled as would be desired

New paradigms that are <u>energy-efficient</u>, able to <u>customize</u> the wireless environment, and <u>spectral-efficient</u>, are needed for future wireless communication systems

Introduction

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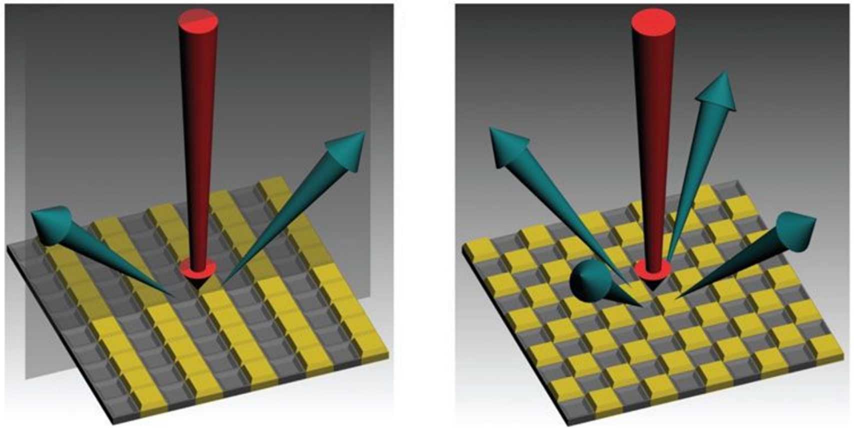
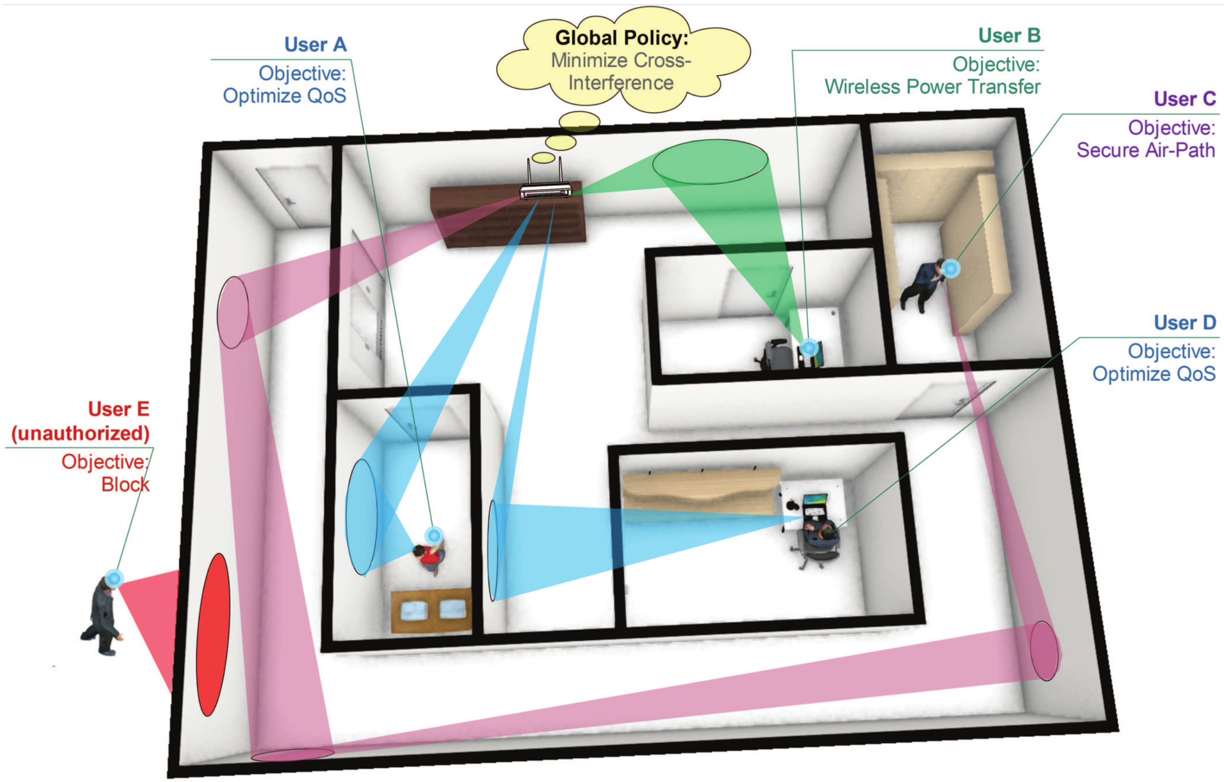
❖ Intelligent Reflecting Surfaces (IRSs)



- IRSs can change the direction of the impinging electromagnetic (EM) waves
- Low-cost passive devices, e.g., dipoles and phase shifters
- Create favorable wireless propagation environments

Introduction

- Intelligent Reflecting Surfaces (IRSs)
  - IRSs can be readily coated on the facades of buildings, which reduces implementation cost and complexity



Introduction

- Intelligent Reflecting Surfaces (IRSs)
  - Conventional communication devices, e.g., relays, receive and retransmit new signals
  - IRSs smartly transform or recycle existing signals
  - Promising key enablers for improving the performance of wireless communications in an economical and energy-efficient manner

Introduction

- Intelligent Reflecting Surfaces (IRSs)
  - IRSs have to be jointly designed with conventional communication techniques, e.g., beamforming at APs

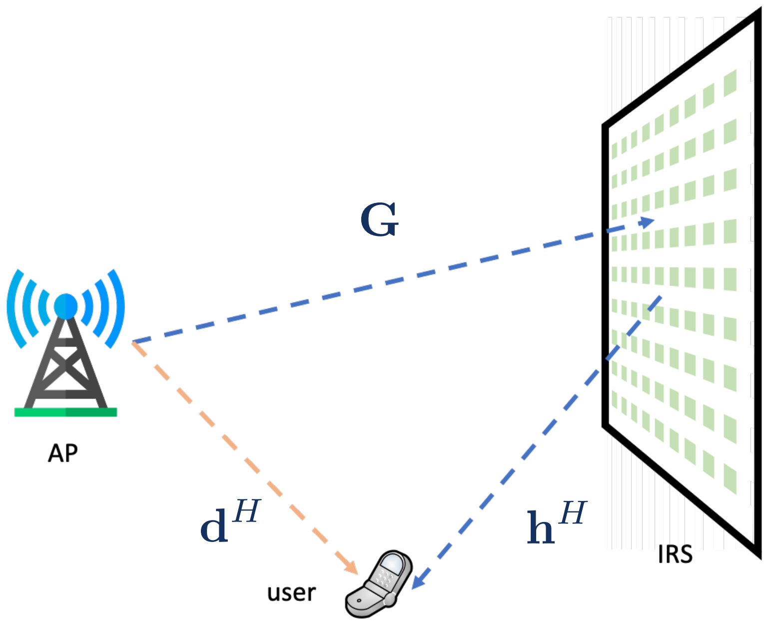
Problem	System		Technique	Paper
Rate maximization	Single-user MISO			
	Secure Wireless Communication	Single user + single eve		
		Multiple users and eves, robust		
	Generic			
Power minimization	Multiuser MISO			
	SWIPT with large scale IRSs			

Algorithm Design for IRS-Assisted Systems

Algorithm Design

Warm-Up: Single-User MISO

- System model and notations



$$\boldsymbol{\Phi} = \mathrm{diag}(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M})$$

- Received signal: $y = \left(\mathbf{h}^H \boldsymbol{\Phi} \mathbf{G} + \mathbf{d}^H\right) \mathbf{w} x + n$

Algorithm Design

❖ Warm-Up: Single-User MISO

➢ Optimal beamformer: MRT

$$\mathbf{w}^{\star} = \sqrt{P}\frac{\mathbf{G}^H \mathbf{\Phi}\mathbf{h} + \mathbf{d}}{\left\|\mathbf{G}^H \mathbf{\Phi}\mathbf{h} + \mathbf{d}\right\|_2}$$

➢ Rate maximization $\mathbf{v} = \left[e^{-\jmath\theta_1}, e^{-\jmath\theta_2}, \cdots, e^{-\jmath\theta_M}, t\right], \quad |t| = 1$

$$\begin{aligned} \underset{\mathbf{v}\in\mathbb{C}^{M+1}}{\text{maximize}} \quad & \mathbf{v}^H \mathbf{R}\mathbf{v} \\ \text{subject to} \quad & |v_i| = 1, \quad i \in \{1, 2, \cdots, M+1\}, \end{aligned}$$

$$\mathbf{R} = \begin{bmatrix} \operatorname{diag}\left(\mathbf{h}^H\right)\mathbf{G}\mathbf{G}^H \operatorname{diag}\left(\mathbf{h}\right) & \operatorname{diag}\left(\mathbf{h}^H\right)\mathbf{G}\mathbf{d} \\ \mathbf{d}^H \mathbf{G}^H \operatorname{diag}\left(\mathbf{h}\right) & 0 \end{bmatrix}$$

➢ The unit modulus constraints are intrinsically non-convex

Algorithm Design

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- Classic Approach
  - Semidefinite relaxation (SDR): $\mathbf{V} = \mathbf{v}\mathbf{v}^H$

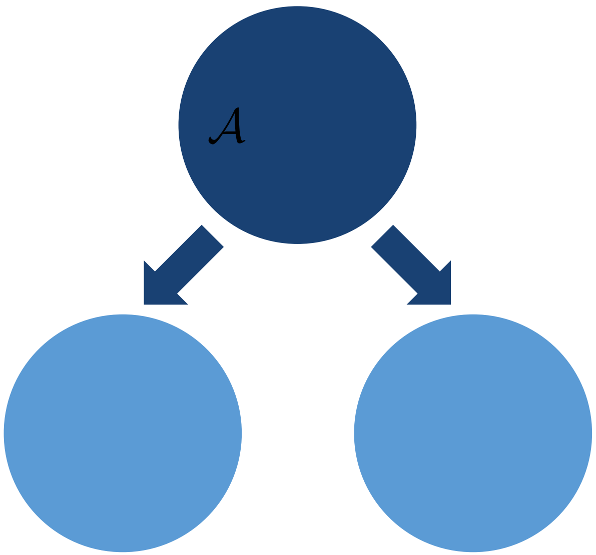
$$\begin{aligned} \underset{\mathbf{V} \in \mathbb{H}^{M+1}}{\text{maximize}} \quad & \operatorname{Tr}(\mathbf{R}\mathbf{V}) \\ \text{subject to} \quad & \operatorname{diag}(\mathbf{V}) = \mathbf{1}_{M+1}, \\ & \cancel{\operatorname{Rank}(\mathbf{V}) = 1.} \end{aligned}$$

  - No guarantee that the solution $\mathbf{V}$ is with unit rank
  - Gaussian randomization $\bar{\mathbf{v}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{V})$
  - Normalization: $\mathbf{v} = \operatorname{unt}(\bar{\mathbf{v}}) \triangleq \left[\frac{v_1}{|v_1|}, \frac{v_2}{|v_2|}, \cdots, \frac{v_{M+1}}{|v_{M+1}|}\right]$
  - Suboptimal solution: only ensures that the value of the objective function is asymptotically at least $\pi/4$ of the optimal value

[Ref] S. Zhang and Y. Huang, "Complex quadratic optimization and semidefinite programming," *SIAM J. Optim.*, vol. 16, no. 3, pp. 871–890, 2006.

Algorithm Design

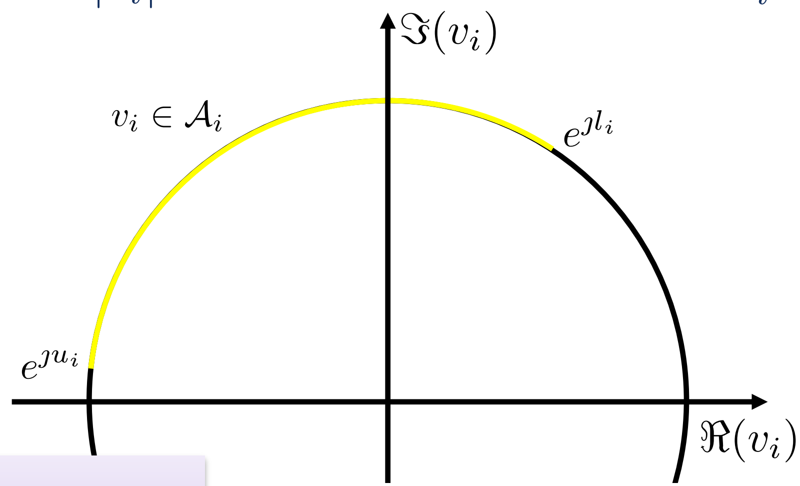
❖ Branch and Bound (BnB)



- **Node:** A subset of the feasible set defined in the problem to be solved

$|v_i| = 1$

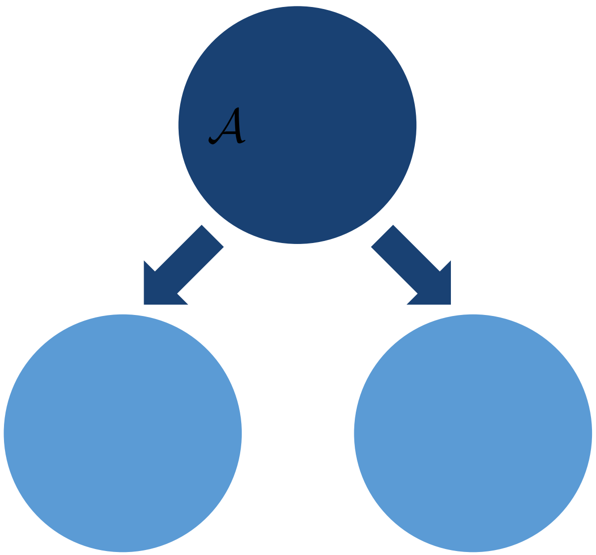
$\mathcal{A}_i$



$$
\begin{aligned}
\underset{\mathbf{v}\in\mathbb{C}^{M+1}}{\text{maximize}} \quad & \mathbf{v}^H\mathbf{R}\mathbf{v} \\
\text{subject to} \quad & |v_i| = 1, \quad i \in \{1, 2, \cdots, M+1\},
\end{aligned}
$$

Algorithm Design

❖ Branch and Bound (BnB)



➢ **Subproblem:** A subset of the feasible set defined in the problem to be solved

$$\begin{aligned} \underset{\mathbf{v}, \mathbf{V} \succeq \mathbf{0}}{\text{minimize}} \quad & \underline{g}(\mathbf{V}) = \operatorname{Tr}(\mathbf{RV}) \\ \text{subject to} \quad & v_i \in \mathcal{A}_i, \quad \forall i, \\ & \operatorname{Diag}(\mathbf{V}) = \mathbf{1}_{M+1}, \\ & \mathbf{V} = \mathbf{v}\mathbf{v}^H. \end{aligned}$$

➢ A lower and an upper bounds are required for each node

Algorithm Design

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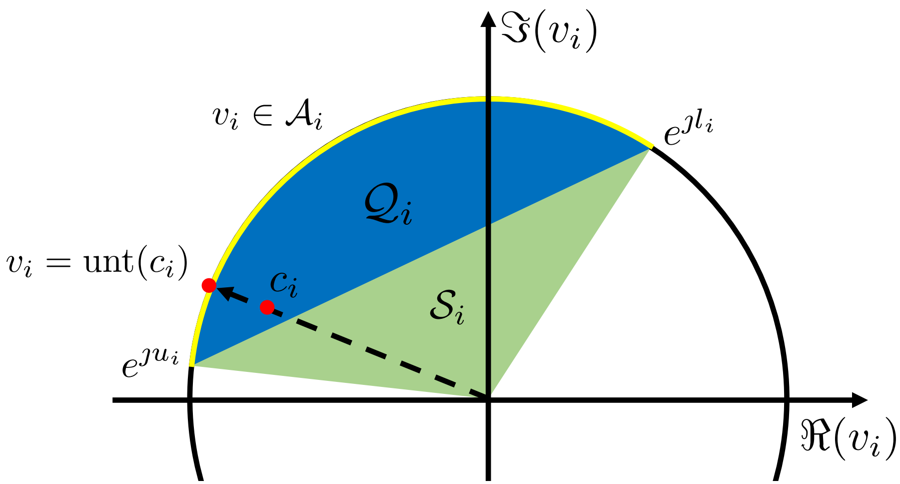
Branch and Bound (BnB)

- **Lower bound**: Relax the feasible set $\mathcal{A}_i$



$$\begin{aligned} \underset{\mathbf{c},\mathbf{C}\succeq\mathbf{0}}{\text{minimize}} \quad & \underline{g}(\mathbf{C}) = \mathrm{Tr}(\mathbf{RC}) \\ \text{subject to} \quad & c_i \in \mathcal{Q}_i \qquad \forall i, \\ & \mathrm{Diag}(\mathbf{C}) = \mathbf{1}_{M+1}, \\ & \mathbf{C} \succeq \mathbf{c}\mathbf{c}^H. \end{aligned}$$

$$c_i \in \mathcal{Q}_i \Leftrightarrow \Re(\mathbf{a}^* \circ \mathbf{c}) \geq \cos\left(\frac{\mathbf{u}-\mathbf{l}}{2}\right)$$

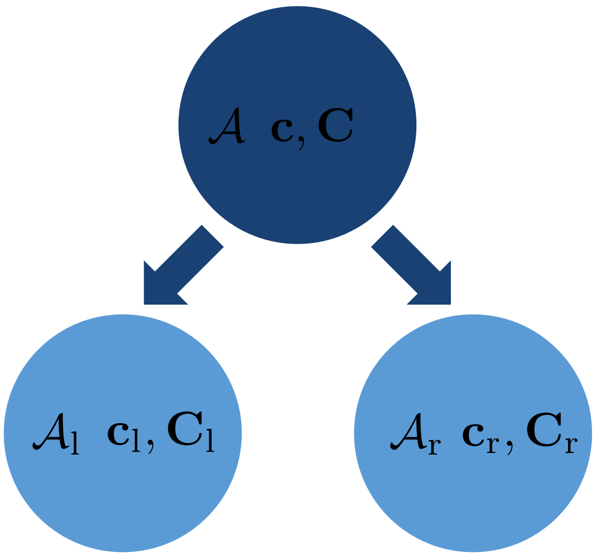
$$a_i = e^{\jmath\frac{u_i+l_i}{2}}$$

- **Upper bound**: Construct a feasible set

$$\mathbf{v} = \mathrm{unt}(\mathbf{c}) \triangleq \left[\frac{c_1}{|c_1|}, \frac{c_2}{|c_2|}, \cdots, \frac{c_{M+1}}{|c_{M+1}|}\right]$$

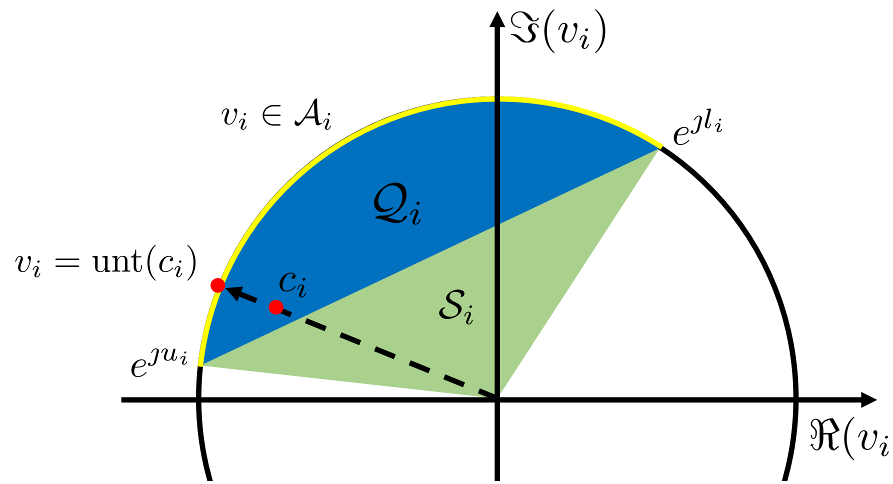
Algorithm Design

- Branch and Bound (BnB)



- **Branch**: Select the node associated with the smallest lower bound
- Equally partition $\mathcal{A}_{i^\star}$

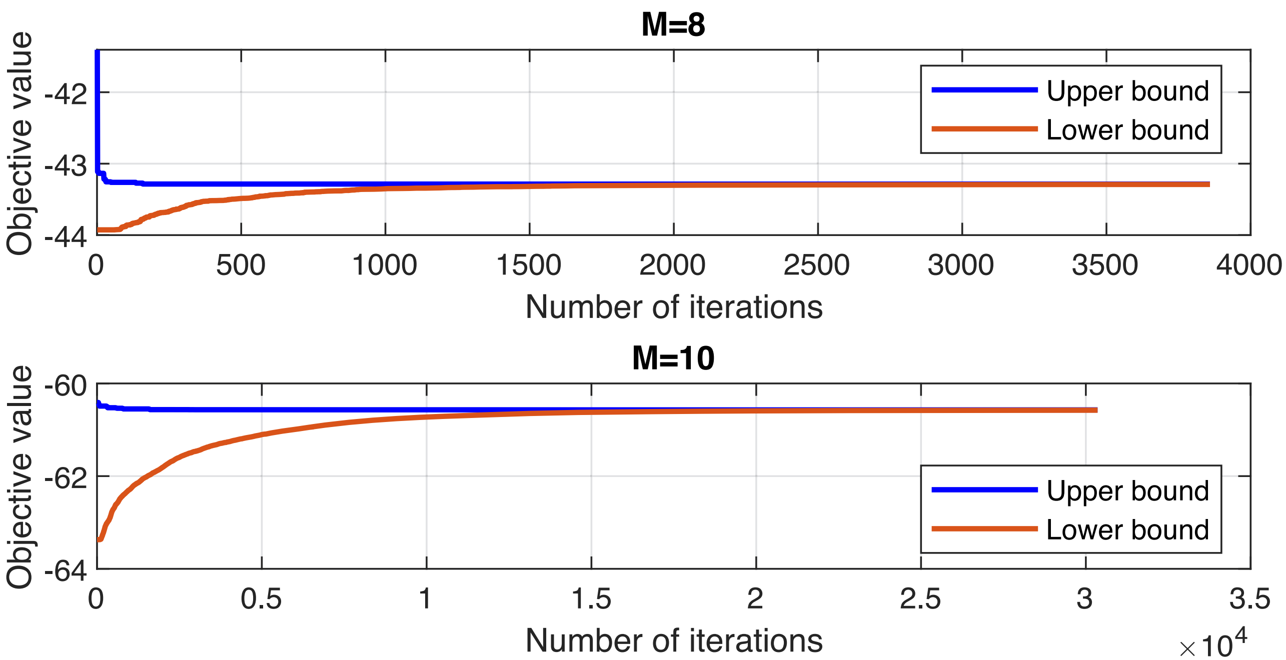
$$i^\star = \arg\max_{i} \quad |c_i - v_i|$$



Algorithm Design

- Branch and Bound (BnB)
  - Simulation result: Convergence

Complexity!

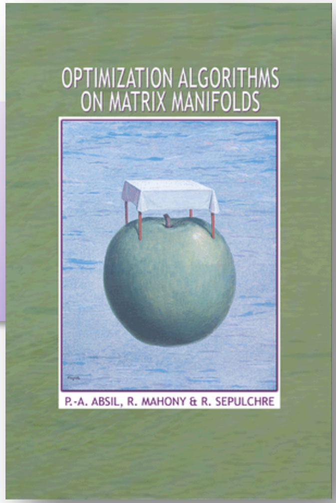


Algorithm Design

Manifold Optimization

- Reformulation

$$
\begin{aligned}
\underset{\mathbf{x}\in\mathbb{C}^M}{\text{minimize}} \quad & f(\mathbf{x}) = -\mathbf{x}^H\mathbf{A}\mathbf{x} - \mathbf{x}^H\mathbf{b} - \mathbf{b}^H\mathbf{x} \\
\text{subject to} \quad & |x_i| = 1, \quad i \in \{1, 2, \cdots, M\},
\end{aligned}
$$



$$
\mathbf{A} = \operatorname{diag}\left(\mathbf{h}_{\mathrm{r}}^H\right)\mathbf{G}\mathbf{G}^H \operatorname{diag}\left(\mathbf{h}_{\mathrm{r}}\right),
$$

$$
\mathbf{b} = \operatorname{diag}\left(\mathbf{h}_{\mathrm{r}}^H\right)\mathbf{G}\mathbf{h}.
$$

- The unit modulus constraints $|x_i| = 1$ form a complex circle manifold
- Manifold optimization

Algorithm Design

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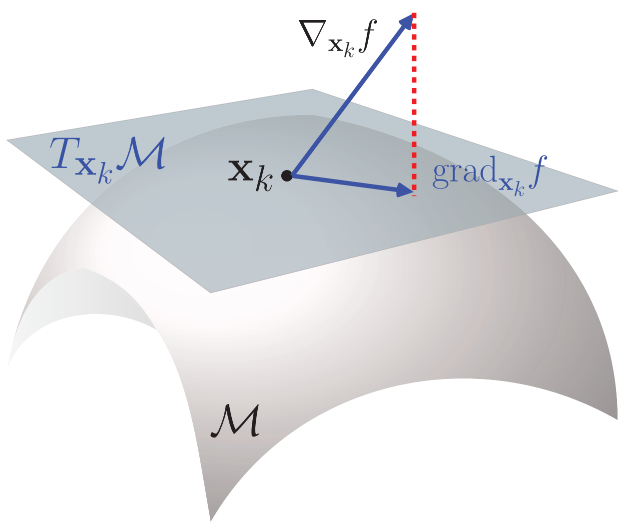
❖ Three Key Elements in Manifold Optimization



- **Tangent space**:

$$T_{\mathbf{x}_k}\mathcal{M} = \left\{\mathbf{z} \in \mathbb{C}^M : \Re\left\{\mathbf{z} \circ \mathbf{x}_k^*\right\} = \mathbf{0}_M\right\}$$

- **Riemannian gradient**: Orthogonal projection of the Euclidean gradient $\nabla_{\mathbf{x}_k} f$ onto the tangent space $T_{\mathbf{x}_k}\mathcal{M}$

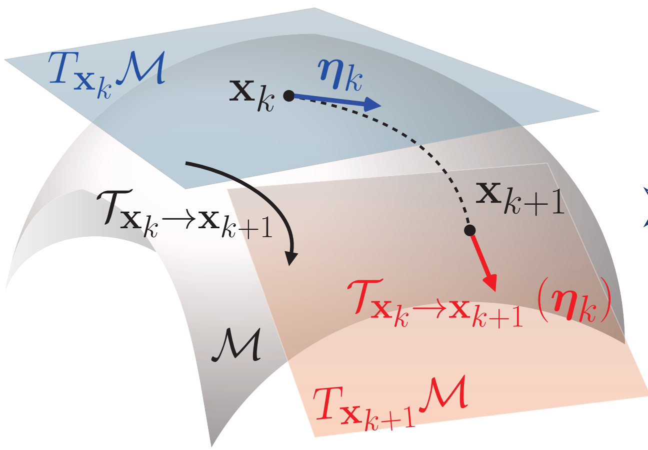
$$\mathrm{grad}_{\mathbf{x}_k} f = \nabla_{\mathbf{x}_k} f - \Re\{\nabla_{\mathbf{x}_k} f \circ \mathbf{x}_k^*\} \circ \mathbf{x}_k,$$

where the Euclidean gradient of the objective function is given by

$$\nabla_{\mathbf{x}_k} f = -2\left(\mathbf{A}\mathbf{x}_k + \mathbf{b}\right).$$

Algorithm Design

❖ Three Key Elements in Manifold Optimization



- Conjugate gradient (CG) method in the Euclidean space

$$\boldsymbol{\eta}_{k+1} = -\nabla_{\mathbf{x}_{k+1}} f + \beta_k \boldsymbol{\eta}_k,$$

where $\boldsymbol{\eta}_k$ is the search direction at $\mathbf{x}_k$.

- **Transport**: Mapping of a tangent vector from one tangent space to another

$$\mathcal{T}_{\mathbf{x}_k \to \mathbf{x}_{k+1}}(\boldsymbol{\eta}_k) \triangleq T_{\mathbf{x}_k}\mathcal{M} \mapsto T_{\mathbf{x}_{k+1}}\mathcal{M} : \quad \boldsymbol{\eta}_k \mapsto \boldsymbol{\eta}_k - \Re\{\boldsymbol{\eta}_k \circ \mathbf{x}_{k+1}^*\} \circ \mathbf{x}_{k+1}.$$

CG on the manifold: $\boldsymbol{\eta}_{k+1} = -\mathrm{grad}_{\mathbf{x}_{k+1}} f + \beta_k \mathcal{T}_{\mathbf{x}_k \to \mathbf{x}_{k+1}}(\boldsymbol{\eta}_k)$

Algorithm Design

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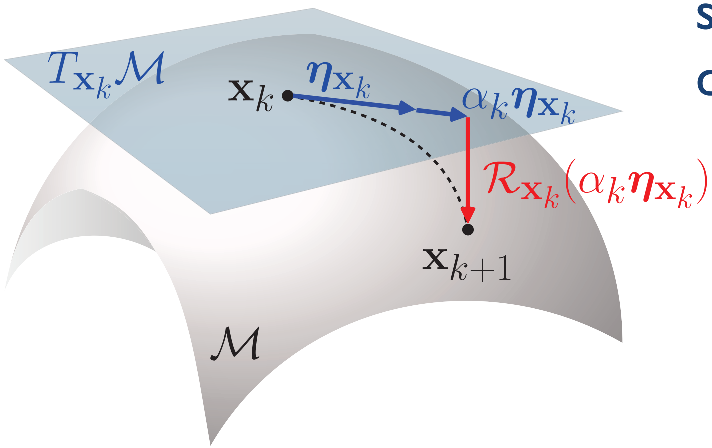
❖ Three Key Elements in Manifold Optimization



- **Retraction**: Mapping from the tangent space to the manifold itself to find the destination on the manifold

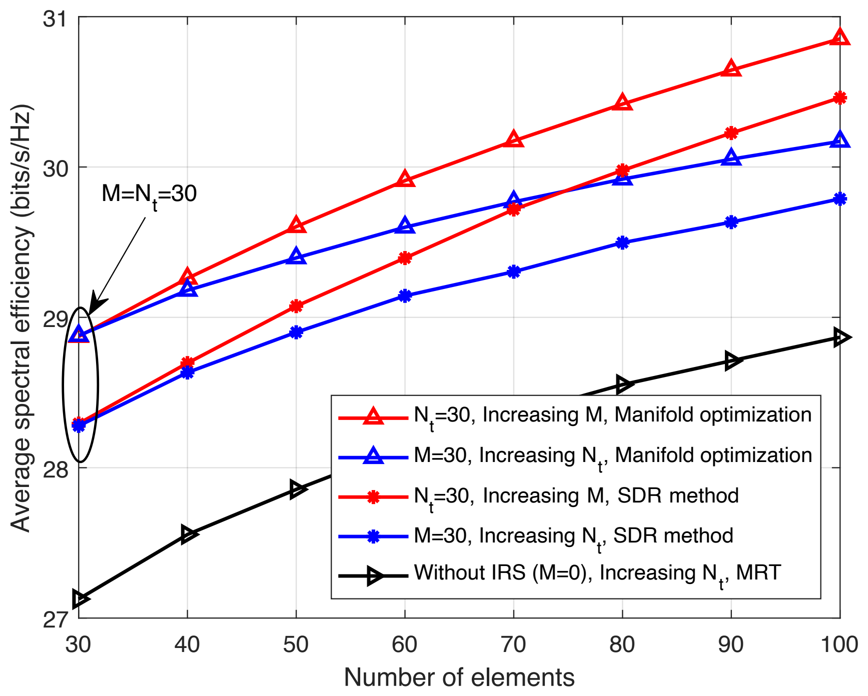
$$\mathcal{R}_{\mathbf{x}_k}\left(\alpha_k\boldsymbol{\eta}_k\right) \triangleq T_{\mathbf{x}_k}\mathcal{M} \mapsto \mathcal{M}: \\ \alpha_k\boldsymbol{\eta}_k \mapsto \mathrm{unt}\left(\alpha_k\boldsymbol{\eta}_k\right)$$

- The CG method based manifold optimization converges to a critical point
- Computational complexity: $\mathcal{O}\left(M^{1.5}\right)$ (SDR: $\mathcal{O}\left((M+1)^6\right)$)

Algorithm Design

❖ Warm-Up: Single-User MISO

➢ Simulation results: Manifold vs. SDR



➢ Superiority over SDR

➢ Effectiveness of incorporating IRSs into wireless communication systems

Algorithm Design

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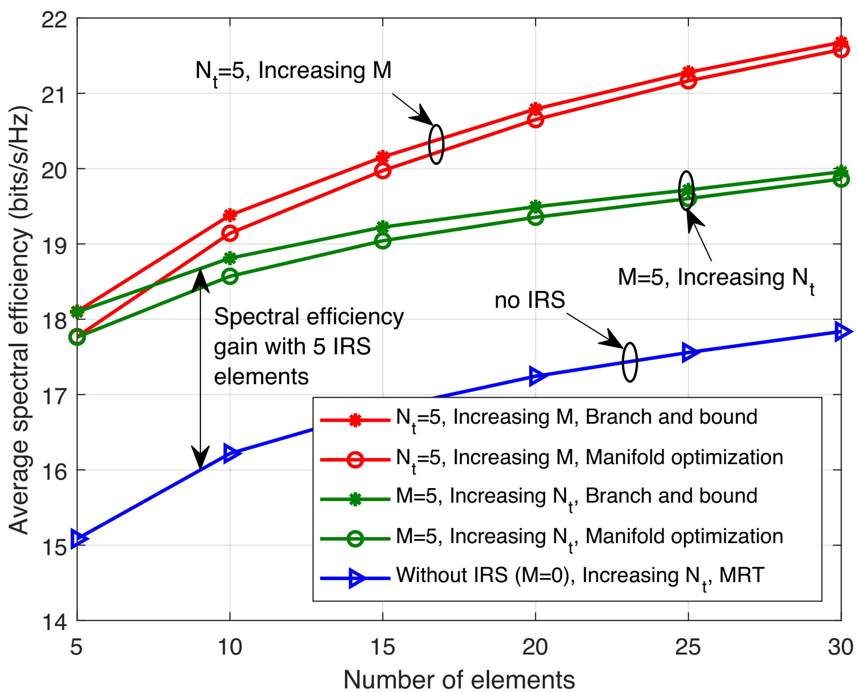
❖ Warm-Up: Single-User MISO

➢ Simulation results: Manifold vs. BnB



- ➢ Manifold optimization-based algorithm achieves a near-optimal performance
- ➢ Increasing the number of IRS elements is more efficient than enlarging the antenna size at the AP in terms of improving spectral efficiency

Algorithm Design

Warm-Up: Single-User MISO

Conclusion

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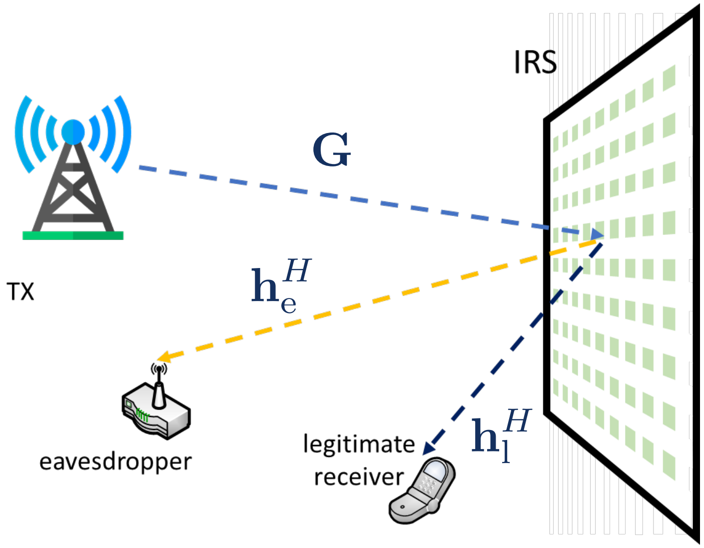
<tr><td rowspan="7">Rate maximization</td><td colspan="2" rowspan="3">Single-user MISO</td><td>SDR</td><td>Introductory</td></tr>
<tr><td>BnB</td><td>SPAWC20</td></tr>
<tr><td>Manifold</td><td>ICCC19</td></tr>
<tr><td rowspan="3">Secure Wireless Communication</td><td rowspan="2">Single user + single eve</td><td></td><td rowspan="2"></td></tr>
<tr><td></td></tr>
<tr><td>Multiple users and eves, robust</td><td></td><td></td></tr>
<tr><td colspan="2">Generic</td><td></td><td></td></tr>
<tr><td rowspan="2">Power minimization</td><td colspan="2">Multiuser MISO</td><td></td><td></td></tr>
<tr><td colspan="2">SWIPT with large scale IRSs</td><td></td><td></td></tr>
</table>

Algorithm Design

Secure Communication: Single-Eavesdropper

- System model



$$\mathbf{\Phi} = \mathrm{diag}(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M})$$

- Received signals:

$$y_{\mathrm{l}} = \mathbf{h}_{\mathrm{l}}^H \mathbf{\Phi} \mathbf{G} \mathbf{w} x + n_{\mathrm{l}},$$

$$y_{\mathrm{e}} = \mathbf{h}_{\mathrm{e}}^H \mathbf{\Phi} \mathbf{G} \mathbf{w} x + n_{\mathrm{e}},$$

Algorithm Design

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Secure Communication: Single-Eavesdropper

- Secrecy rate

$$C = \left[\log\left(1 + \frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \mathbf{\Phi}\mathbf{G}\mathbf{w}\right|^2\right) - \log\left(1 + \frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \mathbf{\Phi}\mathbf{G}\mathbf{w}\right|^2\right)\right]^+$$

- Rate maximization

$$\begin{aligned} \underset{\mathbf{w},\mathbf{\Phi}}{\text{maximize}} \quad & \frac{1 + \frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \mathbf{\Phi}\mathbf{G}\mathbf{w}\right|^2}{1 + \frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \mathbf{\Phi}\mathbf{G}\mathbf{w}\right|^2} \\ \text{subject to} \quad & \|\mathbf{w}\|^2 \le P \\ & \mathbf{\Phi} = \mathrm{diag}\left(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M}\right) \end{aligned}$$

- Difficulty: Coupled variables & unit modulus constraint

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Coupling: Alternately optimizing variables
  - (I) Beamforming optimization

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & \frac{1+\frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \mathbf{\Phi} \mathbf{G} \mathbf{w}\right|^2}{1+\frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \mathbf{\Phi} \mathbf{G} \mathbf{w}\right|^2} \\ \text{subject to} \quad & \|\mathbf{w}\|^2 \leq P \end{aligned}$$

  - Equivalent to a Rayleigh quotient problem

$$\mathbf{w}^{\star}=\sqrt{P} \boldsymbol{\lambda}_{\max }\left(\mathbf{X}_{\mathrm{e}}^{-1} \mathbf{X}_{l}\right),$$

where $\mathbf{X}_i=\mathbf{I}_{N_{\mathrm{t}}}+\frac{P}{\sigma_i^2} \mathbf{G}^H \mathbf{\Phi}^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{\Phi} \mathbf{G}, \quad i \in\{\mathrm{l}, \mathrm{e}\}$

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - (II) IRS optimization

$$\begin{aligned} \underset{\boldsymbol{\Phi}}{\text{maximize}} \quad & \frac{1+\frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2}{1+\frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2} \\ \text{subject to} \quad & \boldsymbol{\Phi} = \operatorname{diag}\left(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M}\right) \end{aligned}$$

SDR?

$f(\mathbf{v})$

Monotonicity

  - Solution 1: Consider each phase θ_k as a block in BCD

$$\frac{1+\frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2}{1+\frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2} = \frac{c_{\mathrm{l},k}+d_{\mathrm{l},k}\cos(\theta_k+p_{\mathrm{l},k})}{c_{\mathrm{e},k}+d_{\mathrm{e},k}\cos(\theta_k+p_{\mathrm{e},k})},$$

$$\theta_k^\star = \begin{cases} \tilde{\theta}_k + \pi & c_{\mathrm{e},k}d_{\mathrm{l},k}\cos\left(p_{\mathrm{l},k}\right) < c_{\mathrm{l},k}d_{\mathrm{e},k}\cos\left(p_{\mathrm{e},k}\right) \\ \tilde{\theta}_k & \text{otherwise} \end{cases}$$

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Solution 1: Consider each phase θ_k as a block in BCD

$$\tilde{\theta}_k = \arctan \frac{c_{\mathrm{e},k} d_{\mathrm{l},k} \cos\left(p_{\mathrm{l},k}\right) - c_{\mathrm{l},k} d_{\mathrm{e},k} \cos\left(p_{\mathrm{e},k}\right)}{c_{\mathrm{e},k} d_{\mathrm{l},k} \sin\left(p_{\mathrm{l},k}\right) - c_{\mathrm{l},k} d_{\mathrm{e},k} \sin\left(p_{\mathrm{e},k}\right)} - \arccos \frac{d_{\mathrm{l},k} d_{\mathrm{e},k} \sin\left(p_{\mathrm{e},k} - p_{\mathrm{l},k}\right)}{\sqrt{c_{\mathrm{e},k}^2 d_{\mathrm{l},k}^2 + c_{\mathrm{l},k}^2 d_{\mathrm{e},k}^2 - 2 c_{\mathrm{l},k} c_{\mathrm{e},k} d_{\mathrm{l},k} d_{\mathrm{e},k} \cos\left(p_{\mathrm{l},k} - p_{\mathrm{e},k}\right)}}$$

$$c_{i,k} = \frac{1}{2}\left(1 + \frac{1}{\sigma_i^2}\left|h_{i,k}^* \mathbf{g}_k^H \mathbf{w}\right|^2 + \frac{1}{\sigma_i^2}\left|\sum_{m \neq k} h_{i,m}^* e^{\jmath\theta_m} \mathbf{g}_m^H \mathbf{w}\right|^2\right),$$

$$d_{i,k} = \frac{1}{\sigma_i^2}\left|h_{i,k}^* \mathbf{g}_k^H \mathbf{w} \sum_{m \neq k} h_{i,m} e^{-\jmath\theta_m} \mathbf{w}^H \mathbf{g}_m\right|,$$

$$p_{i,k} = \angle\left(h_{i,k}^* \mathbf{g}_k^H \mathbf{w} \sum_{m \neq k} h_{i,m} e^{-\jmath\theta_m} \mathbf{w}^H \mathbf{g}_m\right), \quad i \in \{\mathrm{l}, \mathrm{e}\},$$

Algorithm Design

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- Secure Communication: Single-Eavesdropper
  - Solution 2: Minorization maximization (MM) with AO

$$\begin{aligned} \underset{\boldsymbol{\Phi}}{\text{maximize}} \quad & \frac{1+\frac{1}{\sigma_{\mathrm{l}}^2}\left|\mathbf{h}_{\mathrm{l}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2}{1+\frac{1}{\sigma_{\mathrm{e}}^2}\left|\mathbf{h}_{\mathrm{e}}^H \boldsymbol{\Phi} \mathbf{G} \mathbf{w}\right|^2} \\ \text{subject to} \quad & \boldsymbol{\Phi}=\operatorname{diag}\left(e^{\jmath \theta_1}, e^{\jmath \theta_2}, \cdots, e^{\jmath \theta_M}\right) \end{aligned}$$

  - Reformulation

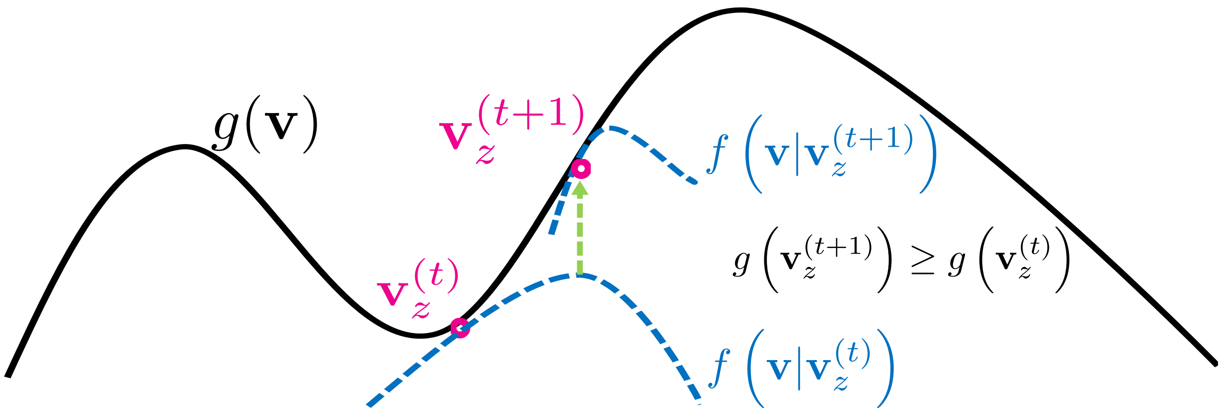
$$\begin{aligned} \underset{\mathbf{v}}{\text{maximize}} \quad & g(\mathbf{v})=\frac{\mathbf{v}^H \mathbf{Y}_{\mathrm{l}} \mathbf{v}}{\mathbf{v}^H \mathbf{Y}_{\mathrm{e}} \mathbf{v}} \\ \text{subject to} \quad & \left|v_k\right|=1, \quad k \in\{1,2, \cdots, M\} \end{aligned}$$

$$\mathbf{v}=\left[e^{\jmath \theta_1}, \ldots, e^{\jmath \theta_M}\right]^H, \quad \mathbf{Y}_i=\frac{1}{M} \mathbf{I}_M+\frac{P}{\sigma_i^2} \operatorname{diag}\left(\mathbf{h}_i^H\right) \mathbf{G} \mathbf{f} \mathbf{f}^H \mathbf{G}^H \operatorname{diag}\left(\mathbf{h}_i\right)$$

Algorithm Design

❖ Secure Communication: Single-Eavesdropper

➢ Solution 2: Minorization maximization (MM)



➢ A lower bound on the objective function $g(\mathbf{v})$ that touches the objective function at point $\mathbf{v}_z^{(t)}$

➢ The key to the success of MM lies in constructing a surrogate objective function $f\left(\mathbf{v}|\mathbf{v}_z^{(t)}\right)$ for which the maximizer $\mathbf{v}_z^{(t+1)}$ is easy to find

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Solution 2: Minorization maximization (MM)

$$g(\mathbf{v})=\frac{\mathbf{v}^H\mathbf{Y}_\mathrm{l}\mathbf{v}}{\mathbf{v}^H\mathbf{Y}_\mathrm{e}\mathbf{v}}\geq f\left(\mathbf{v}|\mathbf{v}_z\right)+\left[g(\mathbf{v}_z)-f\left(\mathbf{v}_z|\mathbf{v}_z\right)\right],$$

where

$$f\left(\mathbf{v}|\mathbf{v}_z\right)=2\frac{\Re\left(\mathbf{v}_z^H\mathbf{Y}_\mathrm{l}\mathbf{v}\right)}{\mathbf{v}_z^H\mathbf{Y}_\mathrm{e}\mathbf{v}_z}-\frac{\mathbf{v}_z^H\mathbf{Y}_\mathrm{l}\mathbf{v}_z}{\left(\mathbf{v}_z^H\mathbf{Y}_\mathrm{e}\mathbf{v}_z\right)^2}\Big\{\lambda_{\max}(\mathbf{Y}_\mathrm{e})M$$
$$+2\Re\left(\mathbf{v}_z^H\left[\mathbf{Y}_\mathrm{e}-\lambda_{\max}(\mathbf{Y}_\mathrm{e})\mathbf{I}_M\right]\mathbf{v}\right)\Big\},$$

and $g(\mathbf{v}_z)-f\left(\mathbf{v}_z|\mathbf{v}_z\right)$ is a term that is irrelevant for optimization.

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Solution 2: Minorization maximization (MM)
  - The phase shift optimization problem in each iteration of the AO is equivalent to

$$\mathbf{v}_z^{(t+1)} = \arg \max_{|v_i|=1} \quad \Re\left[\left(\mathbf{x}^{(t)}\right)^H \mathbf{v}\right],$$

where

$$\mathbf{x}^{(t)} = \frac{\mathbf{Y}_{\mathrm{l}}\mathbf{v}_z^{(t)}}{\left(\mathbf{v}_z^{(t)}\right)^H \mathbf{Y}_{\mathrm{e}}\mathbf{v}_z^{(t)}} - \frac{\left(\mathbf{v}_z^{(t)}\right)^H \mathbf{Y}_{\mathrm{l}}\mathbf{v}_z^{(t)}}{\left[\left(\mathbf{v}_z^{(t)}\right)^H \mathbf{Y}_{\mathrm{e}}\mathbf{v}_z^{(t)}\right]^2} \left[\mathbf{Y}_{\mathrm{e}} - \lambda_{\max}(\mathbf{Y}_{\mathrm{e}})\mathbf{I}_M\right]\mathbf{v}_z^{(t)}.$$

  - The optimal solution is given by $\mathbf{v}_z^{(t+1)} = \mathrm{unt}\left(\mathbf{x}^{(t)}\right)$

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Solution 1: Element-wise BCD

$$\mathbf{w}^{\star} = \sqrt{P}\boldsymbol{\lambda}_{\max}\left(\mathbf{X}_{\mathrm{e}}^{-1}\mathbf{X}_{l}\right),$$

$$\theta_k^{\star} = \begin{cases} \tilde{\theta}_k + \pi & c_{\mathrm{e},k} d_{\mathrm{l},k} \cos\left(p_{\mathrm{l},k}\right) < c_{\mathrm{l},k} d_{\mathrm{e},k} \cos\left(p_{\mathrm{e},k}\right) \\ \tilde{\theta}_k & \text{otherwise} \end{cases}$$

M+1 blocks, optimal solutions

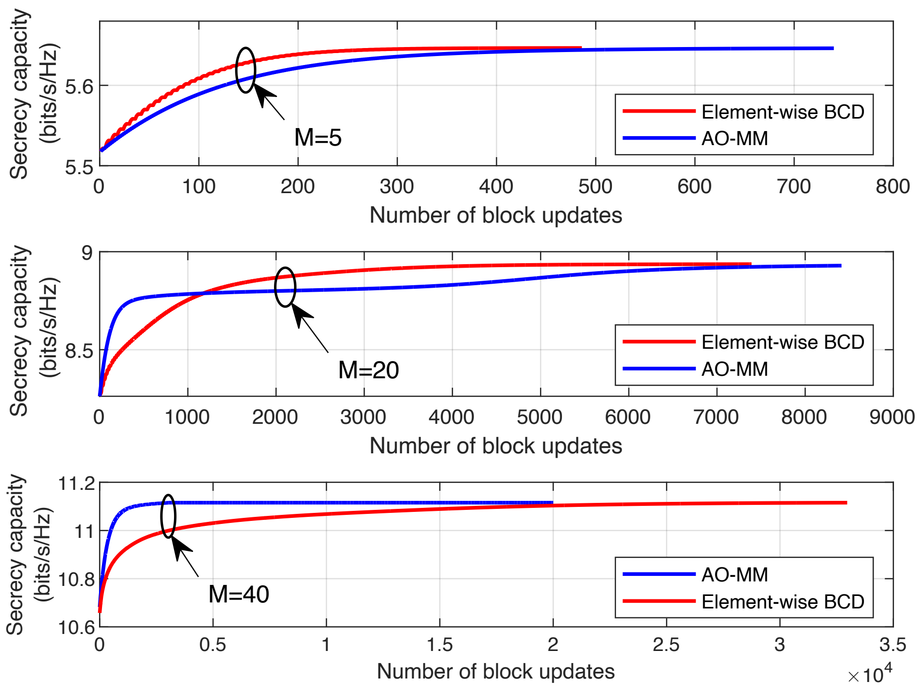
  - Solution 2: AO-MM

$$\mathbf{w}^{\star} = \sqrt{P}\boldsymbol{\lambda}_{\max}\left(\mathbf{X}_{\mathrm{e}}^{-1}\mathbf{X}_{l}\right), \qquad \mathbf{v}_z^{(t+1)} = \mathrm{unt}\left(\mathbf{x}^{(t)}\right)$$

2 blocks, 1 optimal solution + M locally optimal solutions

Algorithm Design

- Secure Communication: Single-Eavesdropper
  - Simulation results

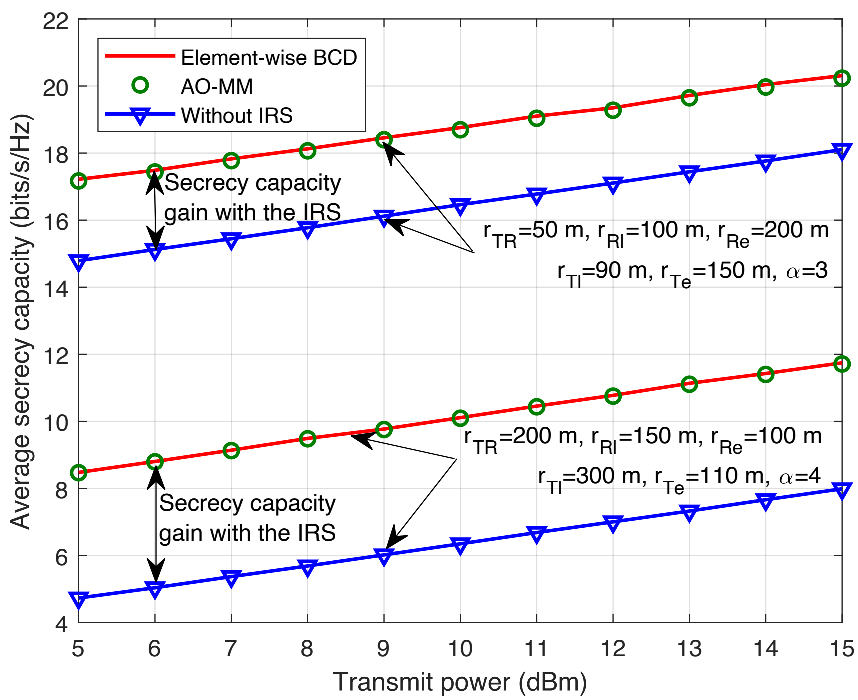


- The element-wise BCD algorithm is suitable for small-scale IRS systems while the AO-MM algorithm is preferable for large-scale IRS

Algorithm Design

Secure Communication: Single-Eavesdropper

Simulation results



- The system with IRS provides a significant performance gain in terms of the secrecy rate
- Deploying IRSs is a promising approach for improving the physical layer security of wireless communications systems

Algorithm Design

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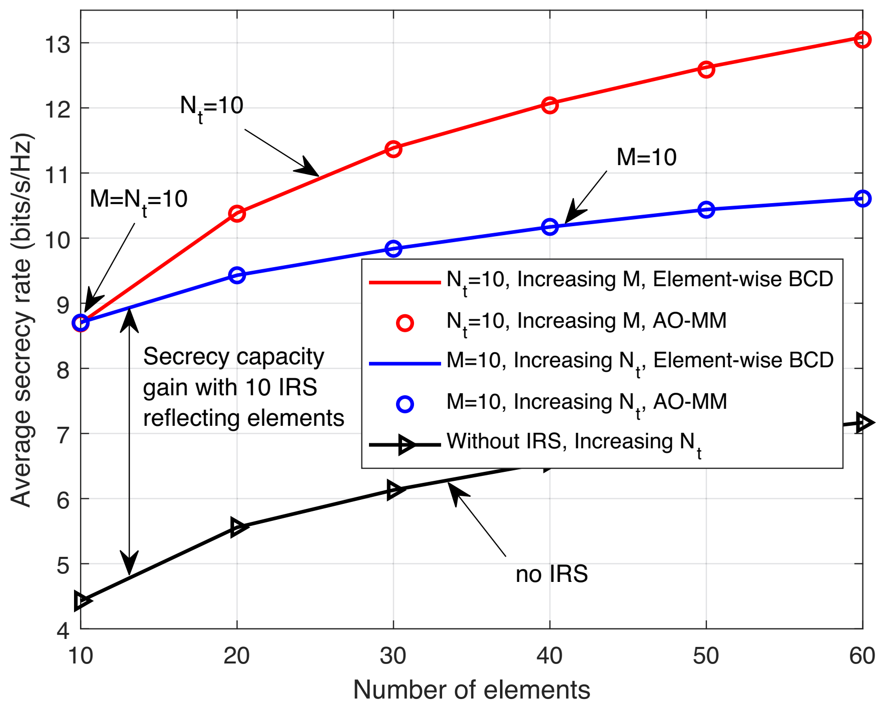
Secure Communication: Single-Eavesdropper

Simulation results



- Increasing the number of IRS reflecting elements is more efficient than enlarging the transmit antenna array size
- More spectral-efficient and energy-efficient than conventional secure wireless systems

Algorithm Design

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Secure Communication: Single-Eavesdropper

Conclusion

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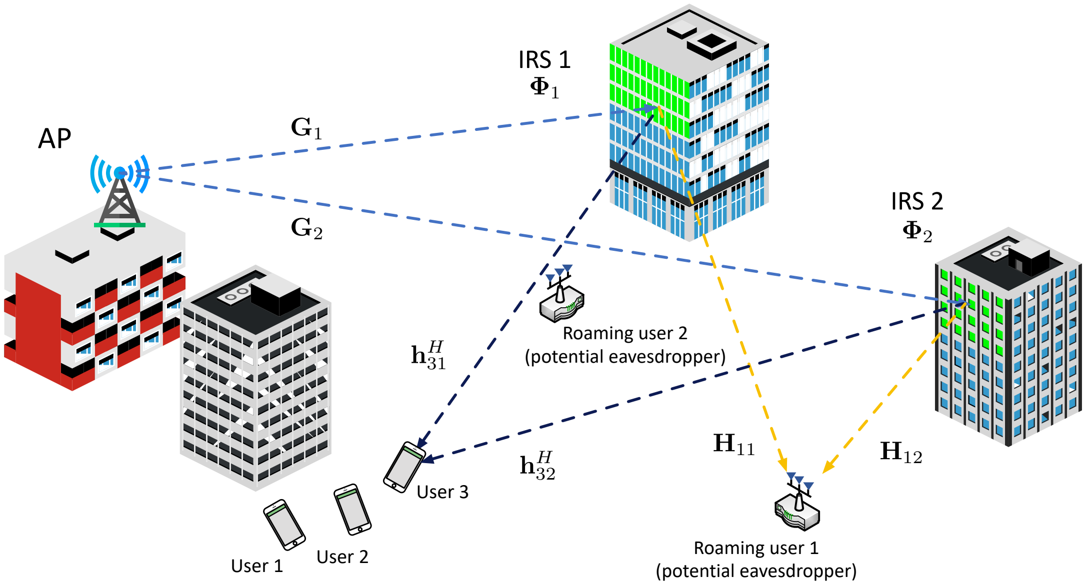
Algorithm Design

Secure Communication: Multiple Users and Eavesdroppers

System model



$$\mathbf{h}_k^H = [\mathbf{h}_{k1}^H, \cdots, \mathbf{h}_{kL}^H]$$

$$\boldsymbol{\Phi} = \mathrm{blkdiag}(\boldsymbol{\Phi}_1, \cdots, \boldsymbol{\Phi}_l)$$

$$\mathbf{G}^H = [\mathbf{G}_1^H, \cdots, \mathbf{G}_L^H]$$

Received signals:

$$y_k = \sum_{l \in \mathcal{L}} \mathbf{h}_{kl}^H \boldsymbol{\Phi}_l \mathbf{G}_l \left(\sum_{i \in \mathcal{K}} \mathbf{w}_i s_i + \mathbf{z} \right) + n_k$$

$$= \mathbf{h}_k^H \boldsymbol{\Phi} \mathbf{G} \left(\sum_{i \in \mathcal{K}} \mathbf{w}_i s_i + \mathbf{z} \right) + n_k, \quad \forall k \in \mathcal{K},$$

Algorithm Design

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❖ Secure Communication: Multiple Users and Eavesdroppers

➢ Worst-case robust sum-rate maximization

$$\begin{aligned} \underset{\mathbf{w}_k, \mathbf{\Phi}, \mathbf{Z} \in \mathbb{H}_{\mathrm{t}}^{N}}{\text{maximize}} \quad & \sum_{k \in \mathcal{K}} R_k \\ \text{subject to} \quad & \text{C1: } \sum_{k \in \mathcal{K}} \left\| \mathbf{w}_k \right\|^2 + \operatorname{Tr}\left(\mathbf{Z}\right) \leq P, \\ & \text{C2: } \mathbf{Z} \succeq \mathbf{0}, \quad \text{C3: } \mathbf{\Phi} = \operatorname{diag}\left(\mathbf{v}\right), \\ & \text{C4: } \max_{\mathbf{\Delta H}_j \in \Omega_j} C_{j,k} \leq \tau_{k,j}, \quad \forall k, j. \end{aligned}$$

Achievable rate of legitimate user k

Artificial noise

$$\mathbf{H}_j = \bar{\mathbf{H}}_j + \mathbf{\Delta H}_j,$$

$$\Omega_j \triangleq \left\{ \mathbf{\Delta H}_j \in \mathbb{C}^{N_{\mathrm{r}} \times M} : ||\mathbf{\Delta H}_j||_F \leq \epsilon_j \right\},$$

Capacity between the AP and potential eavesdropper j for decoding the signal of legitimate user k

➢ Alternating optimization

Algorithm Design

- Secure Communication: Multiple Users and Eavesdroppers
  - AO I: Optimization of beamforming and AN

$$
\begin{aligned}
\underset{\mathbf{w}_k,\mathbf{Z}\in\mathbb{H}_t^N}{\text{maximize}} \quad & \boxed{\sum_{k\in\mathcal{K}} R_k} \qquad \text{SCA} \\
\text{subject to} \quad & \text{C1: } \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|^2 + \mathrm{Tr}(\mathbf{Z}) \le P, \\
& \text{C2: } \mathbf{Z} \succeq \mathbf{0}, \\
& \boxed{\text{C4: } \max_{\Delta\mathbf{H}_j\in\Omega_j} C_{j,k} \le \tau_{k,j}, \quad \forall k, j.}
\end{aligned}
$$

Infinite number of constraints in C4 $\xrightarrow{\text{Generalized S-procedure}}$ A finite number of LMI constraints

  - SOCP or SDP problems

Algorithm Design

- Secure Communication: Multiple Users and Eavesdroppers
  - AO II: Optimization of IRSs

$$\begin{aligned} \underset{p_{k,j}\geq 0,\boldsymbol{\Phi}}{\text{minimize}} \quad & -\sum_{k\in\mathcal{K}} R_k \\ \text{subject to} \quad & \text{C3: } \boldsymbol{\Phi} = \operatorname{diag}(\mathbf{v}), \\ & \overline{\text{C4}}\text{: } \mathbf{P}_{k,j} + \mathbf{S}_j \boldsymbol{\Phi} \mathbf{R}_k \boldsymbol{\Phi}^H \mathbf{S}_j^H \succeq \mathbf{0}, \quad \forall k, j. \end{aligned}$$

  - $\overline{\text{C4}}$: Manifold optimization, BCD, and MM are NOT applicable
  - Reformulation:

$$\begin{aligned} \underset{p_{k,j}\geq 0,\mathbf{V}\in\mathbb{H}^M}{\text{minimize}} \quad & -\sum_{k\in\mathcal{K}} \hat{R}_k \\ \text{subject to} \quad & \overline{\text{C4}}\text{: } \mathbf{P}_{k,j} + \sum_i \mathbf{S}_j \operatorname{diag}(\mathbf{p}_{k,i}) \mathbf{V} \operatorname{diag}(\mathbf{q}_{k,i}^H) \mathbf{S}_j^H \succeq \mathbf{0}, \quad \forall k, j, \\ & \text{C7: } \operatorname{Diag}(\mathbf{V}) = \mathbf{1}_M, \quad \text{C8: } \mathbf{V} \succeq \mathbf{0}, \quad \text{C9: } \operatorname{Rank}(\mathbf{V}) = 1 \end{aligned}$$

Algorithm Design

❖ Secure Communication: Multiple Users and Eavesdroppers

➢ AO II: Optimization of IRSs

➢ An alternative representation of rank-one constraint

$$\mathrm{Rank}\left(\mathbf{V}\right)=1 \Leftrightarrow \left\|\mathbf{V}\right\|_{*}-\left\|\mathbf{V}\right\|_{2}\leq 0$$

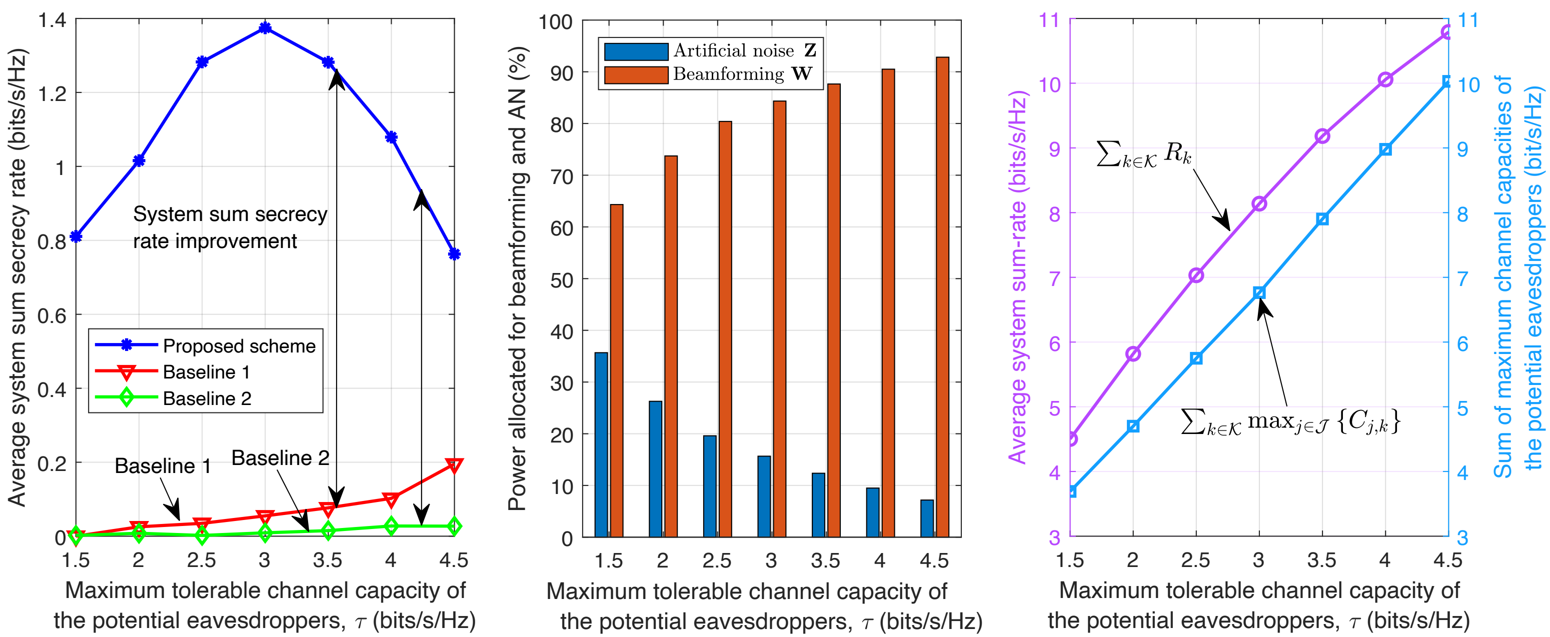
➢ Penalty-based method

$$\begin{aligned}\underset{p_{k,j}\geq 0,\mathbf{V}\in\mathbb{H}^{M}}{\text{minimize}}\quad & -\sum_{k\in\mathcal{K}}\hat{R}_{k}+\frac{1}{2\rho}\left(\left\|\mathbf{V}\right\|_{*}-\left\|\mathbf{V}\right\|_{2}\right) \qquad \text{SCA}\\ \text{subject to}\quad & \overline{\mathrm{C4}}\text{: } \mathbf{P}_{k,j}+\sum_{i}\mathbf{S}_{j}\mathrm{diag}\left(\mathbf{p}_{k,i}\right)\mathbf{V}\mathrm{diag}\left(\mathbf{q}_{k,i}^{H}\right)\mathbf{S}_{j}^{H}\succeq\mathbf{0},\quad \forall k,j,\\ & \mathrm{C7}\text{: } \mathrm{Diag}\left(\mathbf{V}\right)=\mathbf{1}_{M},\quad \mathrm{C8}\text{: } \mathbf{V}\succeq\mathbf{0}\end{aligned}$$

Algorithm Design

❖ Secure Communication: Multiple Users and Eavesdroppers

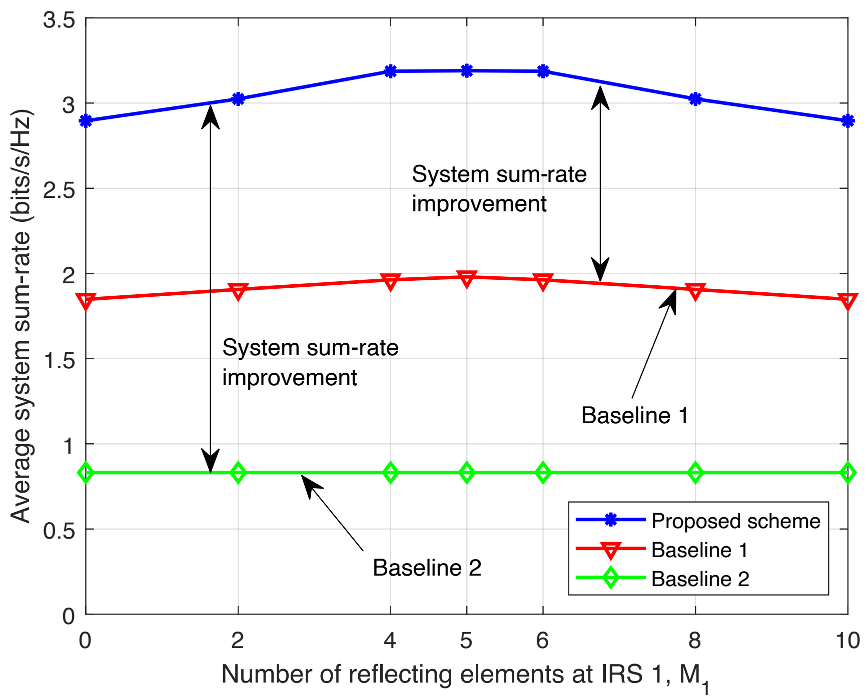
➢ Simulation results



Algorithm Design

Secure Communication: Multiple Users and Eavesdroppers

- Simulation results



- Multiple IRSs create multiple independent propagation paths which introduces macro diversity

- Distance between each legitimate user and its nearest IRS is reduced

Algorithm Design

❖ Secure Communication: Multiple Users and Eavesdroppers

➢ Conclusion

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Algorithm Design

❖ Rate Maximization: A Low-Complexity Framework

➢ Considered optimization problem

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Phi}} \quad & f(\mathbf{W},\mathbf{\Phi}) \\ \text{s.t.} \quad & |\mathbf{\Phi}_{i,i}| = 1, \quad \forall i, \\ & \mathbf{W} \in \mathcal{X}_1 \end{aligned}$$

➢ Alternating optimization (AO)

➢ Optimization of $\mathbf{W}$: Identical to systems without IRSs
Typically, optimal solutions obtained

➢ What about the optimization of $\mathbf{\Phi}$?

Algorithm Design

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Rate Maximization: A Low-Complexity Framework

- Take $\boldsymbol{\Theta} = [\theta_1, \theta_2, \cdots, \theta_M]^T$ as optimization variable rather than $\boldsymbol{\Phi}$

$$\min_{\boldsymbol{\Theta}} \quad f(\boldsymbol{\Theta}|\mathbf{W}^{(t)})$$

- Gradient descent

$$\boldsymbol{\Theta}^{(t+1)} = \boldsymbol{\Theta}^{(t)} - \gamma^{(t)} \nabla_{\boldsymbol{\Theta}} f(\mathbf{X}^{(t)})$$

- Classic Armijo-Goldstein (AG) line search

Block-wise monotonicity

$$f(\boldsymbol{\Theta}^{(t+1)}, \mathbf{W}^{(t)}) \leq f(\boldsymbol{\Theta}^{(t)}, \mathbf{W}^{(t)}) - c\gamma^{(t)} \nabla_{\boldsymbol{\Theta}} f(\mathbf{X}^{(t)})^T \nabla_{\boldsymbol{\Theta}} f(\mathbf{X}^{(t)})$$

- Our proposal

Iteration-wise monotonicity

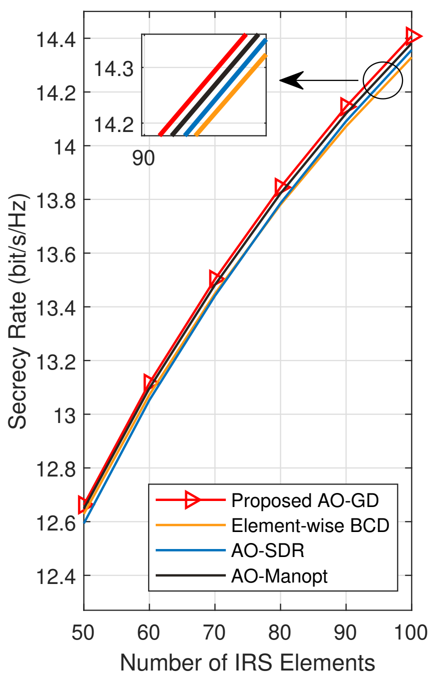
$$f(\boldsymbol{\Theta}^{(t+1)}, \mathbf{W}^{(t+1)}) \leq f(\boldsymbol{\Theta}^{(t)}, \mathbf{W}^{(t)}) - c\gamma^{(t)} \nabla_{\boldsymbol{\Theta}} f(\mathbf{X}^{(t)})^T \nabla_{\boldsymbol{\Theta}} f(\mathbf{X}^{(t)}).$$

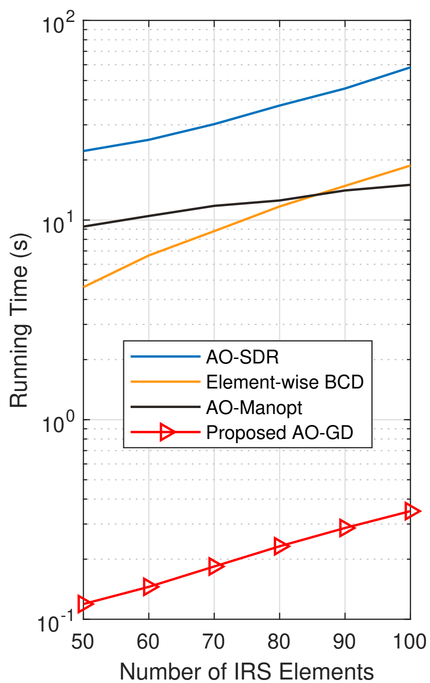
Theoretical convergence result

Algorithm Design

Rate Maximization: A Low-Complexity Framework

- Simulation result





- Support large-scale IRS optimization
- **SDR**: Complexity of SDP
- **Element-wise BCD**: Large number of blocks
- **Manifold**: Block-wise monotonicity

Algorithm Design

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Rate Maximization: A Low-Complexity Framework

Conclusion

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Algorithm Design

- Power Minimization: What is the difference?
  - Limitation of SDR for rate maximization

$$\begin{aligned} \underset{\mathbf{w},\mathbf{v}}{\text{maximize}} \quad & f(\mathbf{w},\mathbf{v}) \\ \text{subject to} \quad & |v_i| = 1, \quad \forall i = 1, 2, \cdots, M+1, \\ & \|\mathbf{w}\|_2 \le P. \end{aligned}$$

  - Optimality cannot be guaranteed when recovering $\mathbf{v}$ from $\mathbf{V}$

$$\begin{aligned} \underset{\mathbf{W},\mathbf{V}}{\text{maximize}} \quad & f(\mathbf{W},\mathbf{V}) \\ \text{subject to} \quad & \operatorname{Diag}(\mathbf{V}) = \mathbf{1}, \\ & \operatorname{Tr}(\mathbf{W}) \le P, \\ & \cancel{\operatorname{Rank}(\mathbf{V}) = 1} \end{aligned}$$

Optimality

Algorithm Design

- Power Minimization: What is the difference?
  - Problem formulation

$$\begin{aligned} \underset{\mathbf{w}_k, \boldsymbol{\Phi}}{\text{minimize}} \quad & f\left(\mathbf{w}_k; \boldsymbol{\Phi}\right) = \sum_{k \in \mathcal{K}} \|\mathbf{w}_k\|_2^2 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \\ & \boldsymbol{\Phi} = \text{diag}\left(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M}\right), \end{aligned}$$

  - What will happen if we apply AO to the above problem?
  - AO I: Beamforming Optimization

$$\begin{aligned} \underset{\mathbf{w}_k}{\text{minimize}} \quad & f\left(\mathbf{w}_k\right) = \sum_{k \in \mathcal{K}} \|\mathbf{w}_k\|_2^2 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \end{aligned}$$

Optimal solution

Algorithm Design

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- Power Minimization: What is the difference?
  - Limitation of SDR for power minimization

$$\begin{aligned} \underset{\mathbf{\Phi}}{\text{minimize}} \quad & 1 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \\ & \mathbf{\Phi} = \operatorname{diag}\left(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M}\right). \end{aligned}$$

$$\begin{aligned} \underset{\mathbf{w}_k}{\text{minimize}} \quad & f\left(\mathbf{w}_k\right) = \sum_{k \in \mathcal{K}} \|\mathbf{w}_k\|_2^2 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k. \end{aligned}$$

  - After the optimization in the *t*-th iteration, we have $\mathbf{w}_k^{(t)}; \mathbf{\Phi}^{(t)}$
  - Then we find a $\mathbf{\Phi}^{(t+1)}$ (if we could)
  - The beamforming vectors are optimized based on $\mathbf{\Phi}^{(t+1)}$

$$f\left(\mathbf{w}_k^{(t)}; \mathbf{\Phi}^{(t+1)}\right) \overset{(a)}{=} f\left(\mathbf{w}_k^{(t)}; \mathbf{\Phi}^{(t)}\right) \overset{(b)}{\ngeq} f\left(\mathbf{w}_k^{(t+1)}; \mathbf{\Phi}^{(t+1)}\right)$$

Monotonicity

Algorithm Design

- Power Minimization: What is the difference?
  - Limitation of SDR for power minimization

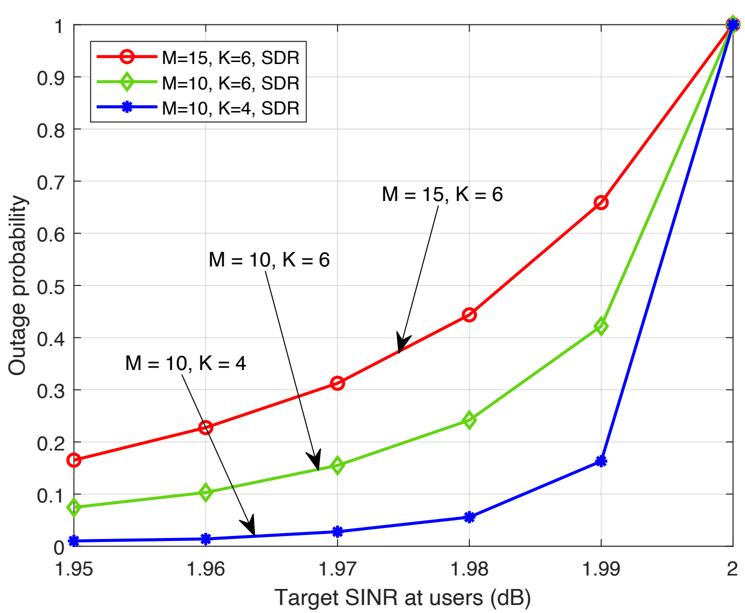
$$\begin{aligned} \underset{\boldsymbol{\Phi}}{\text{minimize}} \quad & 1 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \\ & \boldsymbol{\Phi} = \text{diag}\left(e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_M}\right). \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \underset{\mathbf{V}\in\mathbb{H}^{M+1}}{\text{minimize}} \quad & 1 \\ \text{subject to} \quad & \text{Tr}\left(\mathbf{R}_k\mathbf{V}\right) \leq \hat{\gamma}_k, \quad \forall k, \\ & \text{Diag}\left(\mathbf{V}\right) = \mathbf{1}_{M+1}, \\ & \cancel{\text{Rank}\left(\mathbf{V}\right) = 1}, \\ & \mathbf{V} \succeq \mathbf{0}, \end{aligned}$$

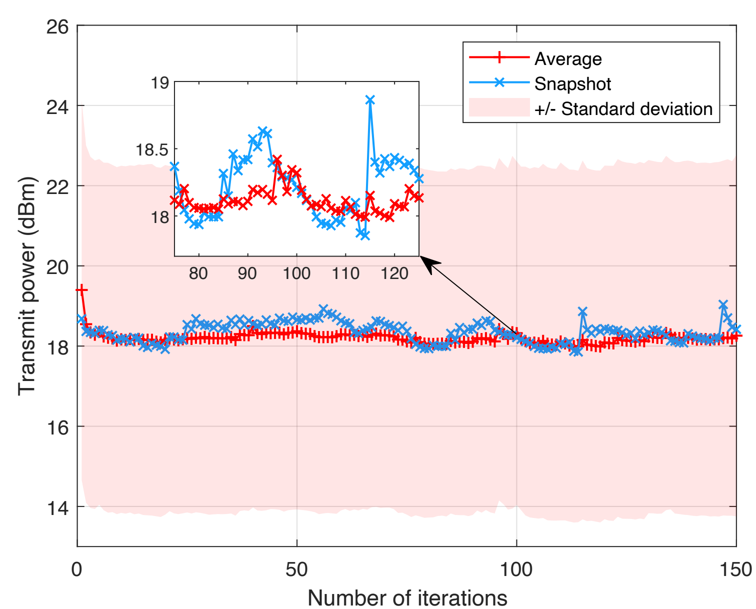
  - Gaussian randomization $\bar{\mathbf{v}} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{V}\right)$
  - Normalization: $\mathbf{v} = \text{unt}\left(\bar{\mathbf{v}}\right) \triangleq \left[\frac{v_1}{|v_1|}, \frac{v_2}{|v_2|}, \cdots, \frac{v_{M+1}}{|v_{M+1}|}\right]$ Feasibility
  - QoS constraint violation – Infeasible! Optimality
  - The convergence of the state-of-the-art SDR-based AO algorithm is NOT guaranteed – Even worse than that for rate maximization

Algorithm Design

❖ Power Minimization: Multiuser MISO

➢ Convergence of the SDR-AltMin algorithm





Algorithm Design

❖ Power Minimization: Multiuser MISO

➢ Problem reformulation

$$
\begin{aligned}
\underset{\mathbf{W}_k \in \mathbb{H}^{N_\mathrm{t}}, \mathbf{V} \in \mathbb{H}^{M+1}}{\text{minimize}} \quad & f(\mathbf{W}_k) = \sum_{k \in \mathcal{K}} \operatorname{Tr}(\mathbf{W}_k) \\
\text{subject to} \quad & \text{C1}: \gamma_k \sigma_k^2 + \gamma_k \sum_{k \in \mathcal{K}} \operatorname{Tr}\left(\mathbf{W}_j \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right) - \operatorname{Tr}\left(\mathbf{W}_k \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right) \le 0, \quad \forall k, \\
& \text{C2}: \operatorname{Diag}(\mathbf{V}) = \mathbf{1}_{M+1}, \\
& \text{C3}: \operatorname{Rank}(\mathbf{V}) = 1, \\
& \text{C4}: \operatorname{Rank}(\mathbf{W}_k) = 1, \quad \forall k, \\
& \text{C5}: \mathbf{V} \succeq \mathbf{0}, \quad \text{C6}: \mathbf{W}_k \succeq \mathbf{0}, \quad \forall k.
\end{aligned}
$$

➢ Coupling of variables

➢ Three non-convex constraints: C1, C3, C4

Algorithm Design

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- Power Minimization: Multiuser MISO
  - Inner Approximation: C1

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The general IA algorithm optimizes a sequence of approximating convex programs. In each iteration of the algorithm, the non-convex constraints are approximated by their convex counterparts

$$\gamma_k \sigma_k^2 + \gamma_k \sum_{k \in \mathcal{K}} \mathrm{Tr}\left(\mathbf{W}_j \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right) - \mathrm{Tr}\left(\mathbf{W}_k \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right) \leq 0$$

$$\mathrm{Tr}\left(\mathbf{W}_j \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right) = \frac{1}{2}\left\|\mathbf{W}_j + \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right\|_F^2 - \frac{1}{2}\mathrm{Tr}\left(\mathbf{W}_j^H \mathbf{W}_j\right) - \frac{1}{2}\mathrm{Tr}\left(\mathbf{G}_k \mathbf{V}^H \mathbf{G}_k^H \mathbf{G}_k \mathbf{V} \mathbf{G}_k^H\right).$$

Algorithm Design

- Power Minimization: Multiuser MISO
  - Inner Approximation: C3

$$\widetilde{\mathrm{C3}}\text{:}\ \|\mathbf{V}\|_* - \|\mathbf{V}\|_2 \le 0$$

$$\overline{\mathrm{C3}}\text{:}\ \|\mathbf{V}\|_* - \mathrm{Tr}\left[\boldsymbol{\lambda}_{\max}\left(\mathbf{V}^{(t)}\right)\boldsymbol{\lambda}_{\max}^H\left(\mathbf{V}^{(t)}\right)\left(\mathbf{V} - \mathbf{V}^{(t)}\right)\right] - \left\|\mathbf{V}^{(t)}\right\|_2 \le 0.$$

  - Resulting optimization to be solved in the *(t+1)*-th iteration of IA

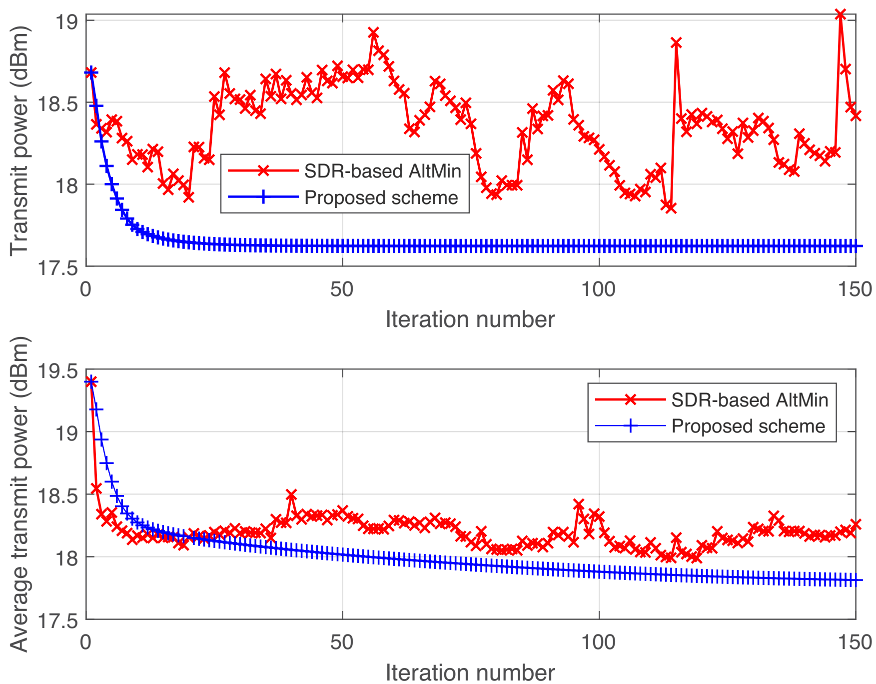
$$\begin{array}{ll} \underset{\mathbf{W}_k \in \mathbb{H}^{N_\mathrm{t}}, \mathbf{V} \in \mathbb{H}^{M+1}}{\text{minimize}} & f(\mathbf{W}_k) = \sum\limits_{k \in \mathcal{K}} \mathrm{Tr}(\mathbf{W}_k) \\ \text{subject to} & \overline{\mathrm{C1}}, \mathrm{C2}, \overline{\mathrm{C3}}, \cancel{\mathrm{C4}}, \mathrm{C5}, \mathrm{C6}. \end{array}$$

  - An optimal beamforming matrix $\mathbf{W}_k$ satisfying $\mathrm{Rank}(\mathbf{W}_k) = 1$ can always be obtained
  - Guaranteed to converged to a KKT point

Algorithm Design

❖ Power Minimization: Multiuser MISO

➢ Simulation results



Algorithm Design

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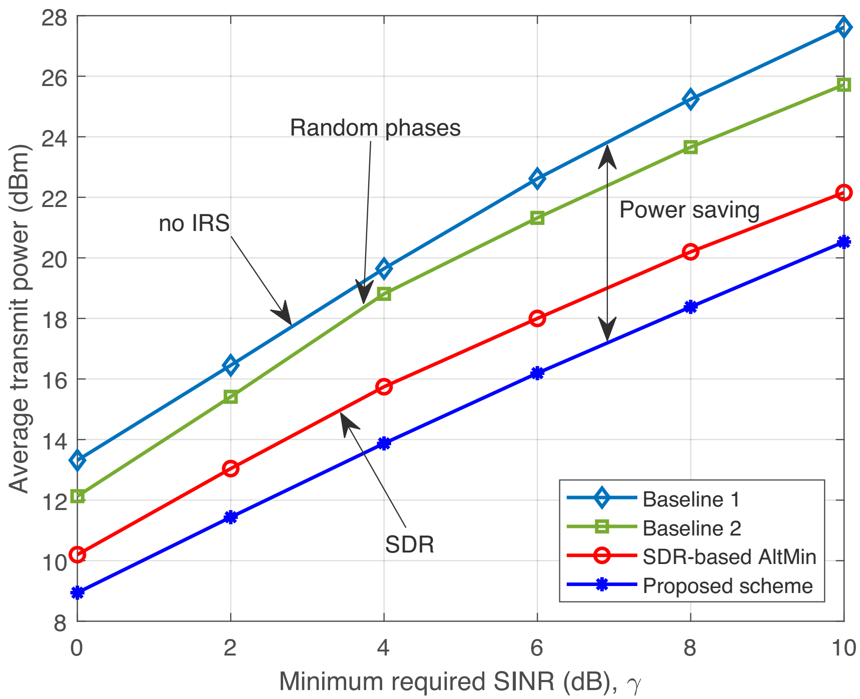
❖ Power Minimization: Multiuser MISO

➢ Simulation results



- ➢ Ability of IRSs to establish **favorable channel conditions**, which facilitates achieving the QoS of the users at lower transmit powers
- ➢ Effectiveness of the proposed optimization methodology

Algorithm Design

❖ Power Minimization: Multiuser MISO

➢ Conclusion

<table>
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</table>

Algorithm Design

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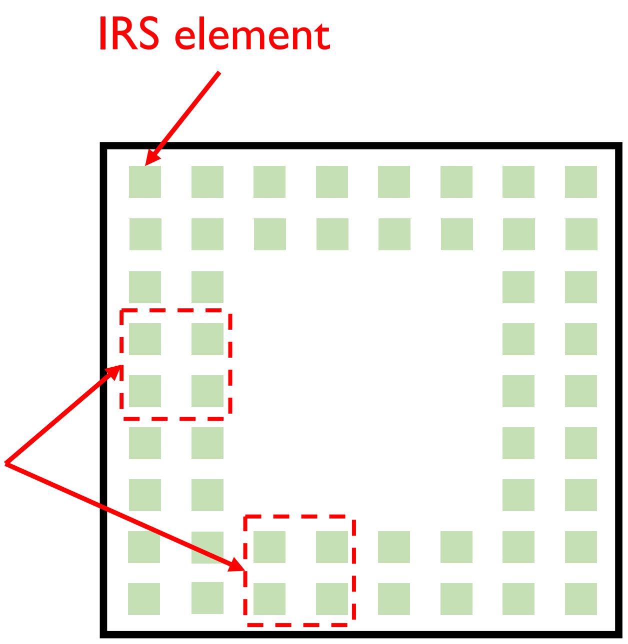
- New IRS Modeling
  - Current IRS modeling
  - Each element is modeled as a phase shifter: $e^{\jmath\theta}$
  - Bottleneck for large-scale optimization
  - A novel physics-based modeling [Ref]
  - Partition the large IRS into several tiles
  - Each tile has a certain configuration of phase shifts



[Ref] M. Najafi, V. Jamali, R. Schober, and H. V. Poor, "Physics-based modeling and scalable optimization of large intelligent reflecting surfaces," under revision in *IEEE Trans. Commun.*, *arXiv:2004.12957*, Sept. 2020.

Algorithm Design

❖ New IRS Modeling

- Example: A codebook-based design
- **Offline design**: Generate a codebook by jointly designing the phase shift elements of each tile for the support of different transmission modes

$$y_k = \sum_{\substack{s\in\mathcal{S},\\ t\in\mathcal{T}\cup\{0\}}} b_{s,t}\mathbf{h}_{s,t,k}^H \sum_{r\in\mathcal{K}} \mathbf{w}_r x_r + n_k$$

- **Online design**: We choose the best transmission mode from the transmission mode set for each tile such that the performance is optimized

$$b_{s,t} = \begin{cases} 1 & \text{tile } t \text{ employs transmission mode } s \\ 0 & \text{otherwise} \end{cases}$$

Algorithm Design

❖ New IRS Modeling

➢ Effective end-to-end channel between the AP and a receiver

$$\mathbf{h}_{s,t,k}^{H} = \mathbf{a}_{\mathrm{R}_k}^{H} \mathbf{C}_{\mathrm{R}_k} \mathbf{R}_{s,t,k} \mathbf{C}_{\mathrm{T}} \mathbf{D}_{\mathrm{T}}$$

Receive steering vector

Response function of tile *t* applying the *s*-th transmission mode [Ref]

Transmit steering vector

[Ref] M. Najafi, V. Jamali, R. Schober, and H. V. Poor, "Physics-based modeling and scalable optimization of large intelligent reflecting surfaces," under revision in *IEEE Trans. Commun.*, *arXiv:2004.12957*, Sept. 2020.

Algorithm Design

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New IRS Modeling

- Example: IRS-Assisted SWIPT

$$
\begin{aligned}
\underset{\mathbf{w}_k,\mathbf{v}_j,b_{s,t}}{\text{maximize}} \quad & \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|^2 + \sum_{j\in\mathcal{J}} \|\mathbf{v}_j\|^2 \\
\text{subject to} \quad & \text{C1: } \mathrm{SINR}_k \geq \gamma_k, \quad \forall k, \\
& \text{C2: } \Upsilon_j^{\mathrm{EH}} \geq E_{\mathrm{req}_j}, \quad \forall j, \\
& \text{C3: } b_{s,t} \in \{0,1\}, \quad \forall s, \ \forall t, \\
& \text{C4: } \sum_{s\in\mathcal{S}} b_{s,t} = 1, \quad \forall t.
\end{aligned}
$$

- Coupling: Big-M reformulation
- Binary: $\text{C3a: } \sum_{s\in\mathcal{S},t\in\widehat{\mathcal{T}}} b_{s,t} - b_{s,t}^2 \leq 0 \text{ and C3b: } 0 \leq b_{s,t} \leq 1, \ \forall s,t.$

Penalty-based method + SCA

Algorithm Design

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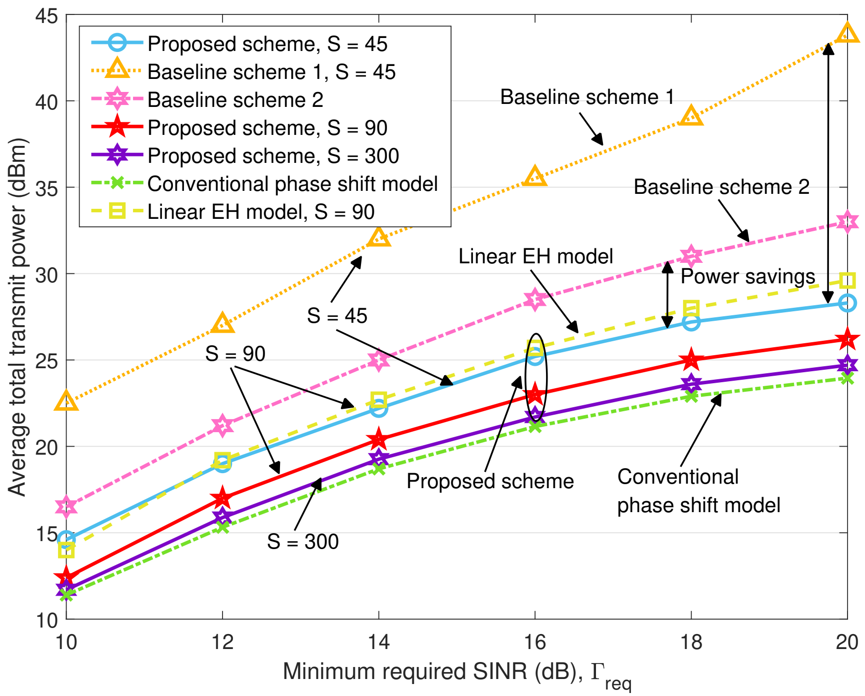
New IRS Modeling

Simulation results



- More transmission modes, less transmit power
- Enable scalable optimization framework [M=480!]
- A flexible tradeoff between computational complexity and performance

Algorithm Design

❖ New IRS Modeling

➢ Conclusion

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Potential Research Directions

Future Research Directions

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Potential Topics

- Other methods to tackle coupling
- Large-scale IRSs optimization for sophisticated systems

$$
\begin{aligned}
\min_{\mathbf{W},\boldsymbol{\Phi}} \quad & f(\mathbf{W}) \\
\text{s.t.} \quad & |\boldsymbol{\Phi}_{i,i}| = 1, \quad \forall i, \\
& \mathbf{W} \in \mathcal{X}_1, \\
& g(\mathbf{W}, \boldsymbol{\Phi}) \leq 0
\end{aligned}
$$

- More tractable IRS/channel model with good accuracy
- IRS deployment optimization
- Channel estimation for IRS-assisted systems
- IRS cooperation
- Stochastic geometry analysis of IRS-assisted networks

[SPAWC20] X. Yu, D. Xu, and R. Schober, "Optimal beamforming for MISO communications via intelligent reflecting surfaces," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Atlanta, GA, USA, May 2020, pp. 1-5. **(Invited Paper)**

[ICCC19] X. Yu, D. Xu, and R. Schober, "MISO wireless communication systems via intelligent reflecting surfaces," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Changchun, China, Aug. 2019, pp. 735-740. **(Invited Paper)**

[GC19] X. Yu, D. Xu, and R. Schober, "Enabling secure wireless communications via intelligent reflecting surfaces," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Waikoloa, HI, USA, Dec. 2019, pp. 1-6. **(Best Paper Award)**

[JSAC20] X. Yu, D. Xu, Y. Sun, D. W. K. Ng, and R. Schober, "Robust and secure wireless communications via intelligent reflecting surfaces," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2637-2652, Nov. 2020.

[GC20Ma] Y. Ma, Y. Shen, X. Yu, J. Zhang, S.H. Song, and K. B. Letaief, "A low-complexity algorithmic framework for large-scale IRS-assisted wireless systems," in *Proc. IEEE Global Commun. Conf. Wkshps. (GLOBECOM Wkshps)*, Taiwan, Dec. 2020.

[GC20Yu] X. Yu, D. Xu, Y. Sun, D. W. K. Ng, and R. Schober, "Power-efficient resource allocation for multiuser MISO systems via intelligent reflecting surfaces," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Taiwan, Dec. 2020.

[TCOM] X. Yu, D. Xu, D. W. K. Ng, and R. Schober, "IRS-assisted green communication systems: Provable convergence and robust optimization," submitted to *IEEE Trans. Commun.*, Nov. 2020.

[WCNC21] D. Xu, X. Yu, V. Jamali, D. W. K. Ng, and R. Schober, "Resource allocation for large IRS-assisted SWIPT systems with non-linear energy harvesting model," submitted to *IEEE Wireless Commun. Netw. Conf. (WCNC)*, Nanjing, China, Mar. 2021.

[1] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun*., vol. 18, no. 8, pp. 4157-4170, Aug. 2019.

[2] Q. Wu and R. Zhang, "Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network," *IEEE Commun. Mag*., vol. 58, no. 1, pp. 106–112, Jan. 2020.

[3] M. Di Renzo et al., "Smart radio environments empowered by reconfigurable AI meta-surfaces: An idea whose time has come," *EURASIP J. Wireless Commun. Netw*., vol. 129, no. 1, pp. 1–20, May 2019.

[4] S. Zhang and R. Zhang, "Capacity characterization for intelligent reflecting surface aided MIMO communication," *IEEE J. Sel. Areas Commun.,* vol. 38, no. 8, pp. 1823–1838, Aug. 2020.

[5] S. Zhang and R. Zhang, "Intelligent reflecting surface aided multi-user communication: Capacity region and deployment strategy," arXiv:2009.02324, Sept. 2020.

[6] T. Lin, X. Yu, Y. Zhu, and R. Schober, "Channel estimation for intelligent reflecting surface-assisted millimeter wave MIMO systems," accepted to IEEE Global Commun. Conf. (GLOBECOM), Taiwan, Dec. 2020.

[7] C. Hu and L. Dai, "Two-timescale channel estimation for reconfigurable intelligent surface aided wireless communications," *arXiv:1912.07990*, Dec. 2019.

[8] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, "A framework of robust transmission design for IRS-aided MISO communications with imperfect cascaded channels," *IEEE Trans. Signal Process*., vol. 68, pp. 5092–5106, Aug. 2020.

[9] H. Shen, W. Xu, S. Gong, Z. He and C. Zhao, "Secrecy rate maximization for intelligent reflecting surface assisted multi-antenna communications," *IEEE Commun. Lett.*, vol. 23, no. 9, pp. 1488-1492, Sept. 2019.

Thanks

For more information:

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