Superoscillatory Antenna Arrays for Sub-Diffraction Focusing at the Multi-Wavelength Range in a Waveguide Environment

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Introduction

In the past decade there has been tremendous interest in focusing electromagnetic waves to subwavelength levels. While most works achieve subwavelength focusing using evanescent waves, [1, 2] recently proposed to achieve subwavelength focusing using propagating waves, through the concept of superoscillation. Superoscillation is a phenomenon whereby the delicate interference of *propagating* electromagnetic waves results in an overall waveform which, within a limited stretch of space, contains variations faster than the highest spatial frequency component of the involved electromagnetic wave. Since only propagating waves are involved, i) the focusing is certainly of a sub-diffraction type; and ii) the subwavelength focusing capability can be extended to much longer imaging distances – to several wavelengths and beyond.

In this work we propose a method to synthesize superoscillatory waveforms by means of adapting the theory of superdirective antennas. We begin by reviewing superdirective antennas and relating them to superoscillation. We then describe our superoscillation antenna array for subwavelength focusing within a waveguide environment. Finally, we show full-wave simulation results verifying subwavelength focusing to 0.6 times the diffraction-limit, at 5 wavelengths away from the source excitation.

A Space-Spectral Perspective on Superdirectivity

Schelkunoff's seminal work in 1943 introduced a useful approach to antenna array design, in which the far-field angular distribution, $f(\theta)$, of an array of isotropic antennas is represented by the array factor [3]:

$$f(\theta) = \sum_{n=0}^{N-1} a_n u^n, \text{ where } u = e^{-jkd\cos\theta}. \quad (1)$$

See Fig. 1 for a diagram of the array. Here N is the number of elements in the array, a_n is the complex excitation coefficient for the n'th element, and u represents the phase shift between adjacent array elements, which depends on the uniform element separation d, the spatial frequency k and the angle of observation θ. The right side of (1) is an analytic function $F(u)$ with $N-1$ zeros on the plane of complex u; $f(\theta)$ is then the value of $F(u)$ on a further restricted domain of real values of θ, which corresponds to an arc on the unit circle joining e^{-jkd} to e^{jkd} through the unity point. In antenna array theory, this restricted domain is called the visible region (VR) of the antenna array. In this perspective, one can design the antenna pattern by appropriately placing zeros to achieve a desired profile for $F(u)$ along the VR. In particular, when $d < \lambda/2$, the VR subtends an angle less than 2π from the origin, thus the zeros of the polynomial can be concentrated along the VR to obtain sharp variations. This gives rise to the concept of a

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superdirective antenna, which enables one to form an arbitrarily sharp beam with a fixed-size antenna array by judiciously exciting it according to weights derived from the appropriate placement of zeros. However, a detailed analysis shows that as the antenna's beam angle sharpens, its distance to the far-field also increases. Thus a superdirective antenna by itself does not lead to subwavelength focusing.

We find it fruitful to look at (1) from an alternate perspective – that the values of $F(u)$ along the unit circle represent the discrete space Fourier transform of $g[n] = a_n$ ($n = 0$ to $N-1$). We shall cast this into continuous space for later convenience. We define x as the array axis and the origin as the x-location of the first array element, so

$$g(x) = \sum_{n=0}^{N-1} a_n \delta(x - nd) \quad (2)$$

is a continuous space function describing the array. Now, the spatial spectrum of $g(x)$ is

$$f(k_x) = \int_{-\infty}^{\infty} g(x) e^{-jk_x x} dx = \sum_{n=0}^{N-1} a_n e^{-jk_x nd}. \quad (3)$$

Comparing with (1) immediately shows that $f(k_x) = f(\theta)$ for $k_x \leq k$ (i.e. within the VR). This is a statement of the fact that the far-field distribution of any antenna equals its spectral profile. Thus one can view $f(k_x)$ as an extension of $f(\theta)$: while $f(\theta)$ only describes propagating waves, $f(k_x)$ describes both propagating and evanescent waves. Furthermore, the design of superdirective antennas can be interpreted as designing weights on an antenna array to fine-tune its propagation spectrum, at the expense of tolerating an uncontrolled, often high-amplitude, evanescent spectrum. This perspective on superdirectivity frees it from far-field approximations, and casts it purely on a Fourier transform basis. This allows for easy extensions to other areas of interest.

Adapting Schelkunoff's Theory to Design Superoscillatory Waveforms

We swap the space and spectral domains along the array axis ($x \leftrightarrow k_x$) to obtain

$$g(k_x) \xrightarrow{F.T.} f(x). \quad (4)$$

That is to say, N uniformly spaced spectral lines with weights $g[n]$ (excitation weights from a superdirective antenna) correspond to a spatial distribution with subwavelength peaks – which is precisely a superoscillatory waveform. Just as the superdirective antenna is a fixed-size antenna which can form an arbitrarily narrow beam, the superoscillatory waveform will be spatially bandlimited to the range of propagating waves; nonetheless it can still focus electromagnetic radiation to an arbitrarily small spatial width, given the presence and proper tuning of sufficiently many closely-spaced spectral lines. The only caveat is that while the high-amplitude evanescent waves generated by superdirective antennas are invisible in the far-field, the high-amplitude sidebands of a superoscillatory waveform appear along with the subwavelength peak. Indeed high-amplitude sidebands have been proven to exist in all superoscillatory waveforms [4]. Notwithstanding their presence, the generated subwavelength superoscillation peak may still prove useful in many applications.

Designing a superoscillatory waveguide

We use the forgoing theoretical formulation to achieve subwavelength focusing within a rectangular waveguide. While in the following proof-of-concept design we render all fields invariant in the y-direction and target 1D electric field focusing, the principle can be straightforwardly extended to 2D focusing. Fig. 2 shows the superoscillatory focusing scheme. The cross-section of the waveguide is 3λ by λ/3; five y-directed line sources, λ/2-separated from each other, are located in the source plane. These sources are excited with weights $h[n]$, such that we excite the TE_{10}, TE_{30} and TE_{50} modes with appropriate weights to form a superoscillation focus at 5λ away from the excitation cross-section.

Clearly, this is very simple way to implement a subwavelength waveform. In the following we complete our design by determining the weights $h[n]$. The three aforementioned modes form $N = 6$ spectral lines spaced $\Delta k = k/3$ apart, so this gives us freedom in placing $N - 1 = 5$ zeros on the complex u plane. We place 4 of the zeros following [5]'s design procedure for superdirective antennas, and the remaining zero at $u_{z5} = -1$ to form nulls at waveguide walls. With these zero locations, we find $\tilde{E}_{img}(k_x)$ – the required spectrum at the image plane – through (5) and (6), then back propagate and divide the spectrum of the antenna element to find the desired the array spectrum at the source plane $g_{scr}(k_x)$. Finally we sample this array spectrum's inverse Fourier transform at half wavelength intervals to obtain $h[n]$. In summary:

$$E_{img}(x) = \prod_{n=1}^{5}(u - u_{zn}) = \sum_{n=0}^{5} a_n u^n, \text{ where } u = e^{-jx\Delta k}; \quad (5)$$

$$\tilde{E}_{img}(k_x) = \sum_{n=0}^{5} a_n \delta(k_x - n\Delta k + 5k/6), (a_n \text{ from (5) above}); \quad (6)$$

$$g_{scr}(k_x) = \frac{\tilde{E}_{img}(k_x) e^{j(5\lambda)k_x}}{\tilde{E}_{elem}(k_x)}; \quad (7)$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g_{scr}(k_x)\, dk_x; \quad (8)$$

$$h[n] = f(x)\delta(x - n\lambda/2), n = 0 \text{ } to \text{ } 4. \quad (9)$$

Design and Simulation Results

We calculate $h[n]$ through the above procedure, and excite the current line sources according to these weightings in an Ansoft HFSS simulation at 3GHz. Fig. 3a) shows a close-up of the simulated subwavelength peak and compares it with the calculated electric field and the diffraction-limit. The simulated electric field profile agrees extremely well with the calculated profile. The simulated E_y FWHM measures 0.37λ, which is greatly improved over the diffraction-limited FWHM of 0.60λ even though the focal plane is located 5λ from the array aperture. Fig. 3b) shows the field distribution across the waveguide. We see that the subwavelength peak has as an electric field about 1/9 of the peak of the side bands. This compares favourably to other proposed superoscillation focusing schemes, such as the one proposed in [1], where a subwavelength peak with a similar focal width contains only 10^{-4} of the total power at the cross-section of the focus. Our result shows it is possible to obtain a clear focusing improvement over the diffraction-limit with sideband amplitudes which are realistic and tolerable; further trade-off between focal width and peak amplitude can be facilitated by adjusting the number of spectral lines and the spacing between them.

Conclusion

We presented a method of designing a superoscillatory waveform via analogy with superdirectivity. Using this method, we designed a superoscillatory antenna array which forms a sub-diffraction focus within a waveguide, 5 wavelengths away from the source. The subwavelength focus formed has a FWHM of 0.6 times that of the diffraction-limit; both the focal width and the image distance can be further improved with power and sensitivity trade-offs.

References

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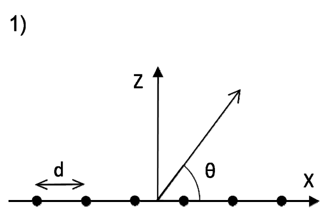

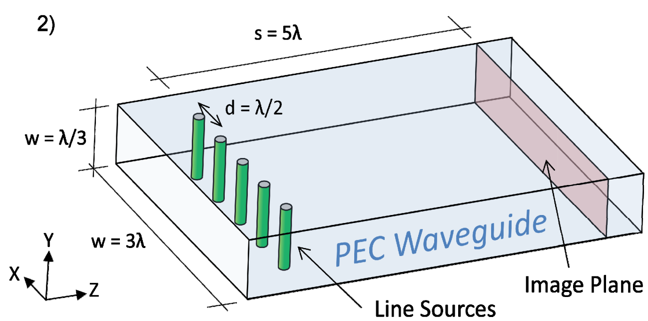

Figure 1. A diagram showing a generic antenna array (elements denoted by black dots).

Figure 2. A diagram of the waveguide superoscillation focusing scheme. 5 line sources, spaced λ/2 apart, are excited with weights $h[n]$, such that a superoscillatory waveform appears 5λ down the rectangular waveguide, thereby generating a subwavelength peak.

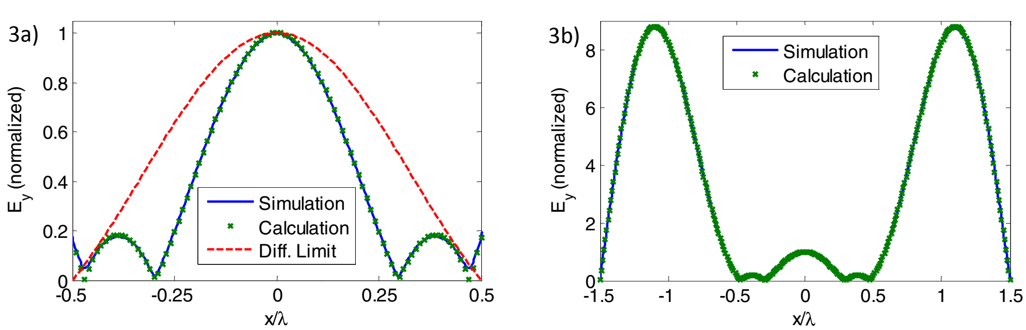

Figure 3. 2 plots comparing simulated and calculated profiles for $E_y(x)$. a) A close up showing the subwavelength peak, compared alongside the diffraction-limited sinc function. b) A comparison of the field profile across the waveguide.