

# Experimental Demonstration of the Huygens' Box: Arbitrary Waveform Generation in a Metallic Cavity

Alex M. H. Wong

Dept. of Electronic Engineering  
City University of Hong Kong  
Hong Kong, China  
alex.mh.wong@cityu.edu.hk

George V. Eleftheriades

Rogers S. Sr. Dept. of Electrical and Computer Engineering  
University of Toronto  
Toronto, Canada  
gelefth@waves.utoronto.ca

**Abstract**—Applying the electromagnetic equivalence principle, we synthesize the necessary electric and magnetic currents along a boundary enclosed by an active Huygens' metasurface, and thereby generate an arbitrary waveform within the area enclosed. We call our contraption the Huygens' box. In this paper, we report the generation of travelling waves inside the Huygens' box: we show calculations, full-wave simulation results and experimental measurements which demonstrate the successful generation of plane waves which travel at an arbitrary direction inside a metallic cavity. These plane waves form unconventional modes in that (i) their existence is forbidden in a typical cavity, and (ii) their linear combination can be used to construct an arbitrary waveform inside the cavity. The ability to construct an arbitrary waveform within an enclosed area by controlling its boundary is a novel concept, with dramatic applications in RF imaging, subwavelength focusing, scattering measurement and waveform control in general.

**Keywords**—metasurface, waveform synthesis, super-resolution, diffraction, antennas.

## I. INTRODUCTION

The past decade has seen the emergence of the metasurface, a ubiquitous tool for shaping an electromagnetic wavefront [1], [2]. In particular, the concept of Huygens' metasurfaces, which exhibit both an electric and a magnetic response to an incident wave, has been employed to design wavefront-engineering metasurfaces offering great flexibility [3]. To date, metasurfaces have been demonstrated which change the amplitude, phase, polarization and propagation direction of an incident electromagnetic wave [4]–[6]. They can operate in reflection and/or transmission mode, and serve as low-profile, low-cost and robust designer-defined components, such as frequency and/or polarization filters, focusing lenses, collimators, anti-reflection surfaces, beam splitters and anomalous angle reflection and/or transmission surfaces to name a few [4]–[6].

Whilst the aforementioned capabilities arise from tuning a surface that is, in most cases, of planar geometry and subwavelength flatness, we expect that much greater waveform manipulation capabilities can be attained if one completely encloses an area of interest with a Huygens' metasurface. The electromagnetic equivalence principle stipulates that the electromagnetic fields in a sourceless region can be perfectly reproduced if one can synthesize its equivalent electric and

magnetic currents along the boundary surrounding the region. Practically, we envision that such a boundary can be synthesized using an active Huygens' metasurface [7]. In the past, we have shown that such sources can be produced using a simple active Huygens' source element, spaced just less than half-wavelength apart to satisfy the Nyquist theorem for propagating electromagnetic waves [8], [9].

In this paper, we report experimental measurements on arbitrary waveform generation in a cavity. We introduce a concept we name the Huygens' box: a rectangular metallic cavity surrounded by a simple active Huygens' metasurface. We experimentally demonstrate that, upon proper excitation, we can generate arbitrary travelling waves that would otherwise not exist within such a cavity environment. Such "unconventional modes" represent building blocks for constructing an arbitrary waveform within the Huygens' box.

## II. MATHEMATICAL FORMULATION

The electromagnetic equivalence principle states that an electromagnetic field in a sourceless free-space environment can be regenerated by electric and magnetic current sources at a closed boundary, which relate to the electromagnetic fields as

$$\begin{aligned} J_s &= \hat{n} \times (H_b - H_a) \\ M_s &= -\hat{n} \times (E_b - E_a) \end{aligned} \quad (1)$$

Fig. 1a gives a pictorial depiction.  $E$  ( $H$ ) denotes the electric (magnetic) field,  $J_s$  ( $M_s$ ) denotes surface electric (magnetic) current, the subscripts 'a' and 'b' denote the interior and exterior regions respectively and  $\hat{n}$  denotes the outward pointing surface normal. In [9], we show that in a 2D TE environment which arises, for example, in a parallel-plate waveguide, the surface currents in (1) can be generated using a simple active Huygens' source comprising a single current source backed by a perfect conductor (PEC) (Fig. 1b). We presented simulation results showing that upon proper excitation one can excite unconventional waveforms within a rectangular cavity, such as travelling waves which terminate at the cavity boundaries. In this paper, we report experimental results which demonstrate this.

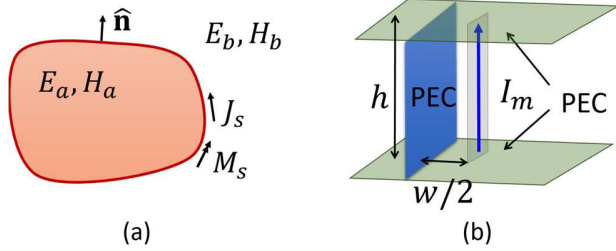


Fig. 1. (a) A diagram showing the equivalence principle. (b) A simple active Huygens' metasurface element, comprising single current source backed by a perfect conductor (PEC).

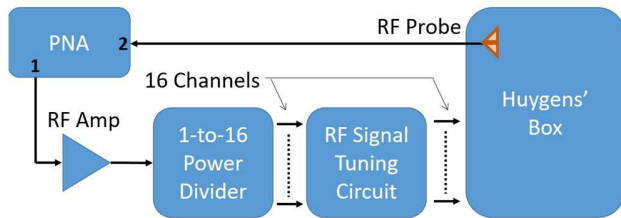


Fig. 2: A schematic describing signal flow in the experimental setup for the Huygens' box.

### III. EXPERIMENTAL MEASUREMENT

Fig. 2 shows a schematic of our experimental apparatus. A Keysight programmable network analyzer (PNA) sends out an RF signal which gets evenly divided into sixteen parts. A custom-built 16-channel RF signal tuning circuit modulates the amplitude and phase of these sixteen signals to the proper complex weighting required to generate the desired waveform. The output channels from the circuit board feed sixteen monopoles within a rectangular cavity  $1\lambda \times 1\lambda$  in size. The cavity and monopoles form the Huygens' box. Fig. 3 shows a diagram of the Huygens' box and a photograph with the top plate removed. The top of the Huygens' box is covered with a perforated aluminum plate, which is electromagnetically equivalent to a metallic plate at the experimental frequency of 1GHz. A monopole test probe connected to the PNA penetrates through the perforated plate to measure the transmission coefficient (S21) at regular locations within the cavity. The monopole couples to/from the z-directed electric field, hence scanning the probe and measuring S21 amounts to measuring the z-directed electric field within the cavity. Fig. 4 compares the experimentally measured electric field (real part, at one phase-point) with full-wave simulation results obtained using Ansys HFSS, for (i) a plane wave travelling in the horizontal (x) direction (Fig. 4a-b) and (ii) a plane wave travelling the diagonal direction (Fig. 4c-d).

### IV. CONCLUSION

Fig. 4 clearly shows the successful experimental generation of travelling plane waves within a metallic cavity using the Huygens' box. Further experimental improvements can be obtained by improving field uniformity amongst the monopole antennas and improving the power throughput using a microwave power amplifier. While we demonstrate the experimental demonstration of plane wave travelling at  $0^\circ$  and

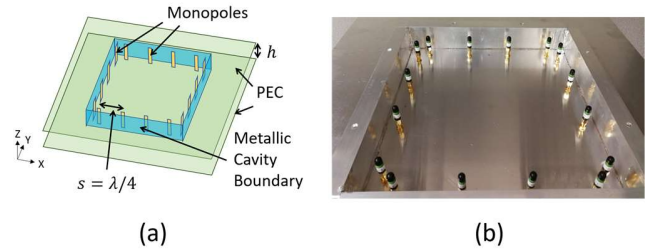


Fig. 3: A diagram (a) and a photo (b) of the Huygens' box, showing the location of the monopole sources within the metallic cavity.

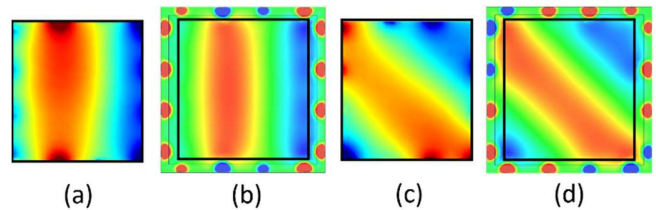


Fig. 4. (a) Measured and (b) simulated electric field (real part, at one phase-point) inside the Huygens' box, showing the generation of a horizontal (x-direction) travelling plane wave. (c-d) The same for a diagonally travelling plane wave. The black boxes indicate regions reachable by the monopole probe.

$45^\circ$  from the x-axis, plane waves travelling in arbitrary directions can be generated. These plane waves form a basis upon which arbitrary waveforms can be generated within a Huygens' box, including standing waves, tight focal spots, and unconventional waveforms such as subwavelength-focused hotspots based on superoscillation [7].

### REFERENCES

- [1] C. L. Holloway, M. A. Mohamed, E. F. Kuester and A. Dienstfrey "Reflection and transmission properties of a metafilm: with an application to controllable surface composed of resonant particles", IEEE Trans. Electromagn. Compat., vol. 47, no. 4, pp. 853-865, Nov. 2005.
- [2] N. Yu et al., "Light propagation with phase discontinuities: generalized laws of reflection and refraction", Science, vol. 334, pp. 333-337, Oct. 2011.
- [3] M. Selvanayagam and G. V. Eleftheriades, "Discontinuous electromagnetic fields using orthogonal electric and magnetic currents for wavefront manipulation", Opt. Express, vol. 21, no. 12, pp. 14409-14429, Jun. 2013.
- [4] C. L. Holloway et al., "An overview of the theory and applications of metasurfaces: the two-dimensional equivalents of metamaterials", IEEE Antennas Propag. Mag., vol. 54, no. 2, pp. 10-35, Apr. 2012.
- [5] N. Yu and F. Capasso, "Flat optics with dedesign metasurfaces", Nat. Mater., vol. 13, pp. 139-150, Jan. 2014.
- [6] M. Chen, M. Kim, A. M. H. Wong and G. V. Eleftheriades, "Huygens' Metasurfaces from Microwaves to Optics: A Review", unpublished.
- [7] A. M. H. Wong and G. V. Eleftheriades, "Superoscillations without sidebands: power efficient sub-diffraction imaging with propagating waves", Sci. Rep., vol. 5, p. 8449, Feb. 2015.
- [8] A. M. H. Wong and G. V. Eleftheriades, "A simple active Huygens source for studying waveform synthesis with Huygens metasurfaces and antenna arrays", IEEE Int. Symp. Antennas Propag., Jul. 2015.
- [9] A. M. H. Wong and G. V. Eleftheriades, "Active Huygens' metasurfaces for RF waveform synthesis in a cavity", IEEE Mediterranean Electrotechnical Conf. (MELECON), Apr. 2016.