Super-resolution Far-field Imaging of Structured Objects Using Superoscillations

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***Abstract*—Superoscillation is a phenomenon where the wave oscillates locally faster than its highest Fourier component. It has been used to achieve imaging resolution beyond the classical diffraction limit. In this paper, superoscillatory point-spread functions (PSF) have been designed using antenna array theory for a 4F imaging system to optically image letter masks which are more complex than objects previously reported. Superoscillatory PSFs with side lobes 4 times lower than the diffraction limit and approximately the same spot width are constructed and used to obtain images with resolution better than the diffraction limit in the far field. This is an important step in the eventual integration of superoscillation into microscopy systems.**

I. Introduction

Superoscillation is a physical phenomenon where a waveform appears to locally oscillate faster than its highest spectral component [1]. As a result, locally the effective wavelength decreases. Normally, propagating waves lose high frequency components carrying small details; the Abbé Diffraction Limit quantifies the smallest resolvable feature as $\Delta x = \lambda/(2NA)$ where NA is the numerical aperture of the imaging system. Any imaging modality capable of resolving even smaller features achieves super-resolution. Through careful wavefront engineering, superoscillating waves can be generated and used to achieve super-resolution at microwave frequencies [2] and in the visible spectrum [3] [4]. No evanescent waves are used; therefore the imaging system has a large working distance.

Despite the promises of superoscillation-based imaging systems, reported results thus far have been restricted to imaging simple aperture structures [3] [4]. Superoscillation is always accompanied by side bands that can be orders of magnitude larger than the superoscillating region [1] lower the image quality for more complex objects.

This paper outlines a new point-spread function (PSF) design approach where, using superoscillation, visible region side lobes around the main beam are reduced to lower than the diffraction limit while keeping the main beam around the same width. Using this approach, we design superoscillation-enabled spatial filters, then experimentally demonstrate their usage in imaging structured objects which lie beyond the diffraction limit.

II. Designing Waveforms for Imaging

In a previous work [5], an antenna-based framework for understanding and designing superoscillating waveforms was developed by formulating it as a dual problem of designing superdirective antenna arrays. A set of incoming spatially bandwidth-limited plane waves can be approximated as

$$\sum_{n=0}^{N} a_n z^n = b_0 \prod_{n=1}^{N} (z - z_n) \tag{1}$$

where $z^n = e^{-j\Delta knx}$ is one plane wave at a particular spatial frequency Δkn and z_n is a zero of the resulting Nth order polynomial. The propagating waves are visualized as zeros on the complex unit circle in the z plane. Similar to superdirectivity, moving the zeros closer than the uniform distribution results in superoscillations in the spatial domain.

For implementation in the optical domain, the key is to design a suitable superoscillating PSF of the imaging system. Since the output image is the convolution of the object and the PSF, super-resolution can be achieved locally by a superoscillating PSF. In past experimental demonstrations [4], superoscillation is used to squeeze the main lobe of the PSF to narrower than the conventional diffraction-limited PSF for an aperture source. However, narrowing the main lobe results in polynomially larger amplitudes in the non-superoscillating outer sidebands in the PSF [1] which could potentially drown out the high-resolution supeorscillating signal. The large outer sidebands could be pushed away from the main lobe by designing for a visible region around the center of the PSF that contains the main lobe and smaller superoscillating side lobes [4]. In practice, this region cannot be arbitrarily large because of increased sensitivity and exponentially larger outer sideband amplitudes [1].

Our new approach is to focus on reducing the amplitudes of the side lobes in the visible region to lower than the diffraction-limited PSF. Using conventional waveform design, reducing side lobes would also widen the main lobe. Using superoscillation, the amplitudes of the visible region side lobes can be reduced to over 4 times lower than that of the diffraction-limit while keeping the main beam half-width to within 5% of the diffraction limit. The trade-off is larger sidebands outside the superoscillating region, but since the main lobe does not need to be squeezed, the outer sidebands are considerably smaller than past results where they are orders of magnitude larger than the main beam.

A design example is shown in Fig. 1. 32 zeros are used, of which 10 are constrained in the visible region. The visible region has a half width of $1.9\lambda/NA$. In our setup this is $139\mu m$; structures with smaller dimensions would not extend into the large outer sideband region, which should result in a clear super-resolved image. As an initial guess of the position of visible region zeros, the Dolph-Chebyshev Method is used to design an antenna pattern where the visible region sidelobes are 30 times lower than the main lobe. An optimization is run with these constraints to find the optimal placement of zeros.

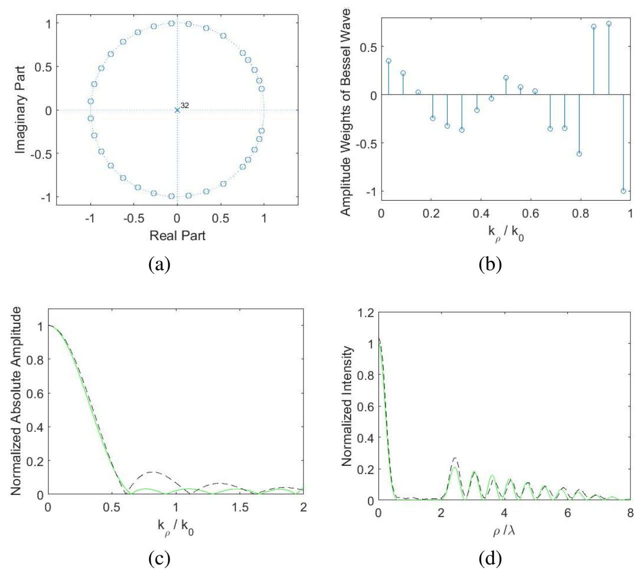

Fig. 1. Superoscillation PSF design using the superdirectivity-inspired method. (a) Distribution of waveform zeros in the complex z-plane for 1D superoscillation design (see Eq. 1). (b) Bessel function weights for the equivalent 2D superoscillation design. The nulls of the superposition of Bessel beams are the same as Fig. 1a but in the radial direction. [4] (c) Zoomed-in view of the visible region of the resulting 2D superoscillation waveform (green solid line), compared to the diffraction limit (black dashed line). The visible region side lobes are 4 times lower, main beam is 5% narrower (Half-width at Half-maximum), and the visible region side lobes are oscillating faster than the diffraction-limited Airy disk. (d) Comparison of cross sections of measured PSF (black dashed line) and designed PSF (green solid line). Good agreement is obtained.

III. Experimental Results

We used a 4F optical imaging system with a numerical aperture of 0.00864 to test our designs. This setup allows easy modification of the system PSF because the Amplitude Transfer Function (the transfer function of optical systems) can be directly accessed on the Fourier Plane [4]. A collimated and polarized HeNe laser beam illuminates the object in the object plane. Modulation is performed by a spatial light modulator (SLM) placed in the Fourier Plane according to designed wave amplitude weights (Fig. 1b). A CMOS camera in the image plane captures the resulting images. The imaging wavelength is $632.8nm$ and the focal length of the 4F system is $40cm$; therefore, imaging is done in the far-field.

A $10\mu m$ aperture was placed in the object plane to obtain an approximation to the PSF in the image plane. This verifies that our system faithfully generates the target PSF, as shown in Fig. 1d. Next, a number of letter masks were imaged. The result for a particular letter is compared in Fig. 2 against the conventional image obtained when illuminated by light without any modulation.

The results show that superoscillatory PSFs with visible region side lobes lower than the diffraction-limited case can achieve super-resolution, even with its main lobe slightly wider than the diffraction-limited case. The outer sideband, while significant, is less intense than the letter illuminated in the visible region. This is an improvement over previous results [4], where the outer sidebands were significantly stronger than the visible region. When larger objects were imaged, the outer sideband regions start to interfere as expected and resolution drops rapidly. In these cases, computational image recovery may be required to retrieve the high resolution components from the noise.

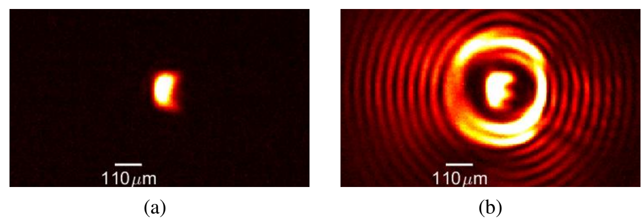

Fig. 2. Experimental results with a letter E of dimension $110\mu m$ x $87\mu m$ as object. Total imaging system numerical aperture is 0.00864. (a) Camera image of object with a diffraction-limited PSF. (b) Camera image after applying the superoscillation optical filter shown in Fig. 1. The three horizontal bars of the letter E have become more visible. The outer rings are due to the PSF side lobes outside the visible region of superoscillation.

IV. Conclusion

Super-resolved images of complex letter masks are obtained using superoscillatory PSFs. In departure from previous works, these PSFs are designed by targeting reduced visible region side lobes while keeping the main beam approximately the same as the diffraction limit. The benefit is reduced sensitivity and smaller non-superoscillating sidebands outside the visible region. This result paves way for utilizing superoscillation for imaging, possibly through integration with conventional microscopy equipment.

References

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