CITYU-LEARNING CLASSROOM

2021年3月5日 下午4:00

Photo by Alex Chung

「差之毫釐，謬以千里」— 淺談蝴蝶效應

謝智剛

香港城市大學 電機工程講座教授

香港城市大學
City University of Hong Kong

蝴蝶效應 Butterfly Effect

「有沒有想過，一隻遠在巴西拍動翅膀的蝴蝶，可能是引發美國德州一場龍捲風暴的元兇？」

"Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"

Edward Lorenz, MIT
(1917–2008)

Peter Howitt 導演

Gwyneth Paltrow 及 John Hannah 主演

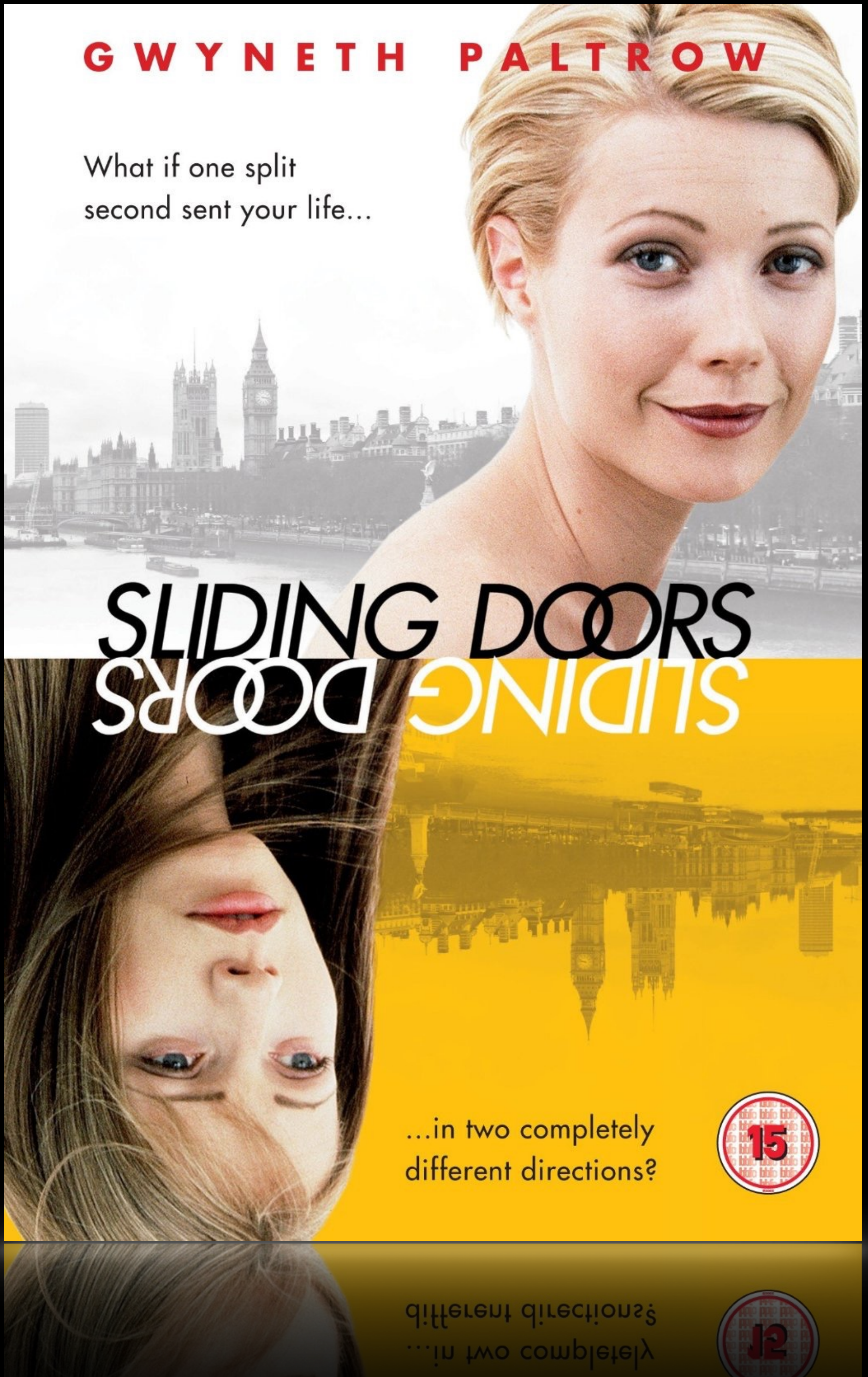
緣份兩面睇

1998英國電影

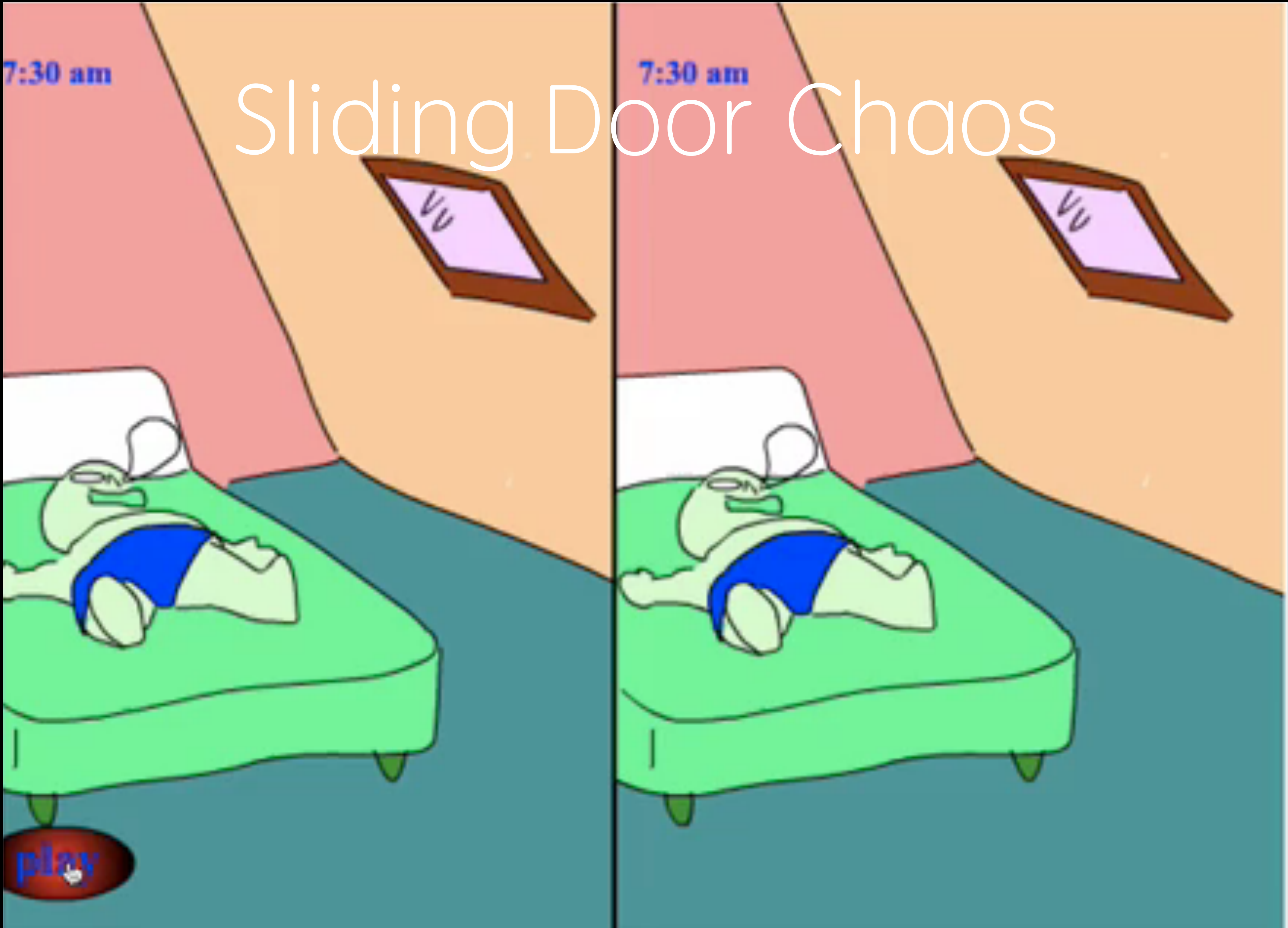
一同進行的兩個時空

同一個人不一樣的命運

http://www.imdb.com/title/tt0120148/



Sliding Door Chaos



aBenz擋路、天雨多意外　網民：
蝶效應釀風暴 (23:53)

(明報製圖)

小時累積而釀成風暴，而觀塘的車龍蔓延開去，小塞車就
車。

停泊在興業街及敬業街交界，
被貨櫃車撞到導致交通癱瘓。
(fb「小心駕駛」群組圖片)

明報 2016年1月22日

觀塘大塞車 Severe Jam

2016.1.22 ～4pm… … 開始塞車是因一輛白色Benz，約下午4時起停泊在興業街及敬業街交界，因被一輛貨櫃車撞到而導致交通癱瘓，其後車龍愈積愈多，未能疏導 … … 今日剛巧天雨路滑，導致沿路出現多宗小車禍，加上天文台預告強勁寒流襲港，市民下班後趕着回家，重重原因交匯，才造成是次的「蝴蝶效應」… …小塞車就變成大塞車。

一子錯滿盤皆落索

AlphaGo beat Lee Sedol

In Two Moves, AlphaGo and Lee Sedol Redefined the Future

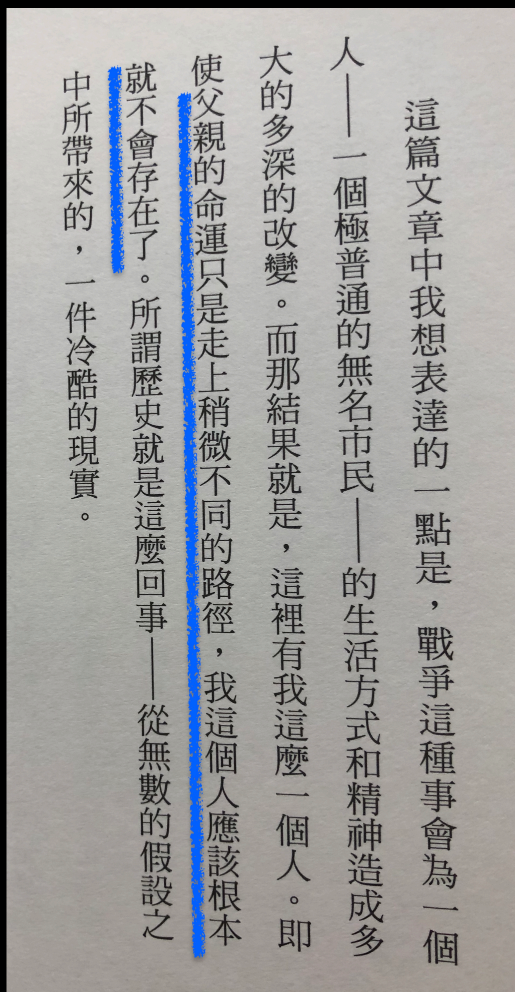
TURE | DESIGN | GEAR | SCIENCE

Move 37

With the 37th move in the match's second game, AlphaGo landed a surprise on the right-hand side of the 19-by-19 board that flummoxed even the world's best Go players, including Lee Sedol. "That's a very strange move," said one commentator, himself a nine dan Go player, the highest rank there is. "I thought it was a mistake," said the other. Lee Sedol, after leaving the match room, took nearly fifteen minutes to formulate a response. Fan Gui—the three-time European Go champion who played AlphaGo during a closed-door match in October, losing five games to none—reacted with incredulity. But then, drawing on his experience with AlphaGo—he has played the machine time and again in the five months since October—Fan Hui saw the beauty in this rather unusual move.

http://www.wired.com/2016/03/two-moves-alphago-lee-sedol-redefined-future/

“稍微不同的路徑”
世上可沒有村上春樹這個人
我們也不可能看到他的小說


這篇文章中我想表達的一點是，戰爭這種事會為一個人——一個極普通的無名市民——的生活方式和精神造成多大的多深的改變。而那結果就是，這裡有我這麼一個人。即使父親的命運只是走上稍微不同的路徑，我這個人應該根本就不會存在了。所謂歷史就是這麼回事——從無數的假設之中所帶來的，一件冷酷的現實。

這些看似隨機的現象其實都是由一個個「因果關係」構建成的。

Our world is made up of many causal events. We just see them as random, having no better way to explain it.

http://wallpaperswide.com/

因果關係 Cause and Effect

因果關係決定一切，但又無法預知！

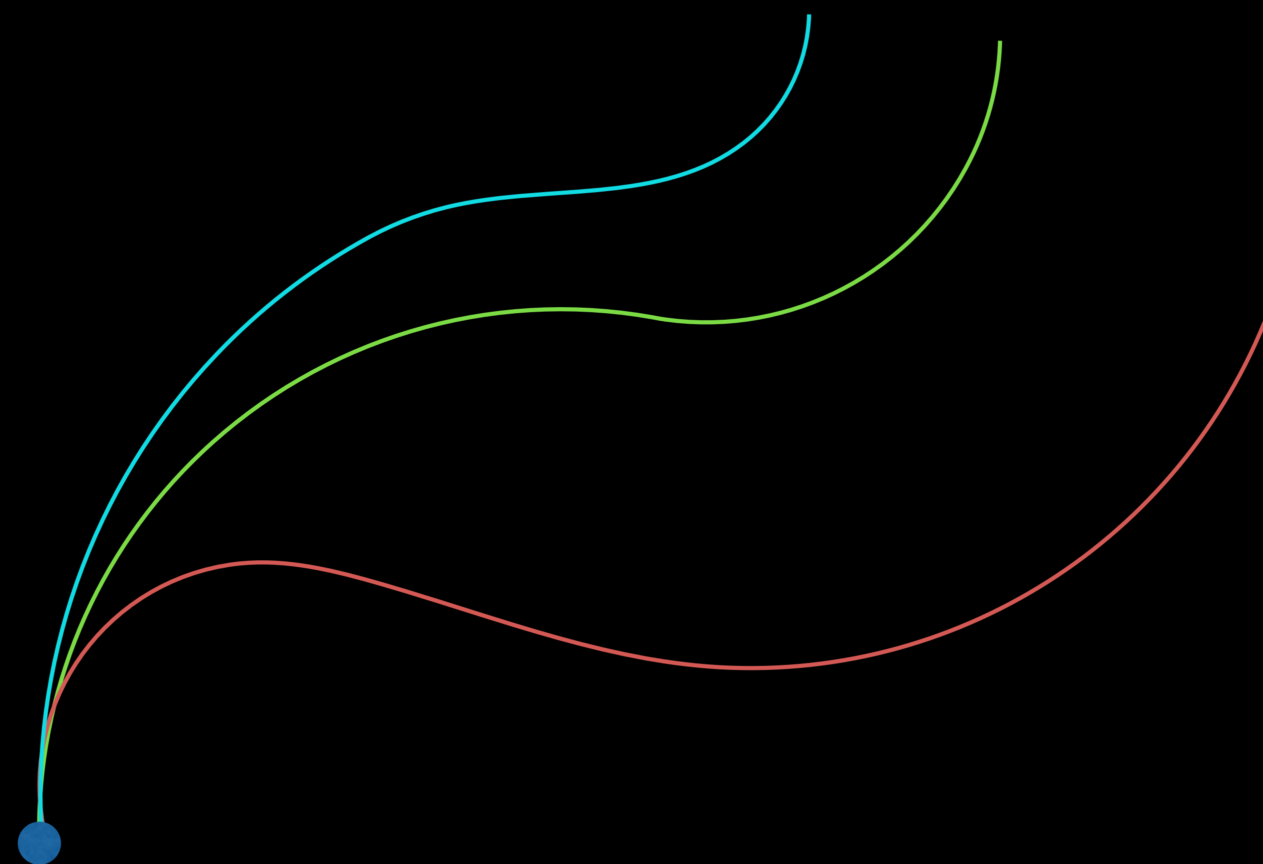
Though we know it is *deterministic*, yet we don’t know what happens next!

http://platformthing.com/

極度敏感的因果關係
Extreme sensitivity to the cause

差之分毫，謬以千里

Small difference causes
hugely different results



起點 start

擲骰子 Let’s play dice

依循物理法則: 重量、慣性矩、引力、碰撞、摩擦力 ⟶ 物理方程

Determined by laws of physics: mass, moment of inertia, gravity, collision, friction ⟶ Physical equations

即使掌握物理方程，又能否估計結果呢？
Is the outcome predictable had we got the exact equation?

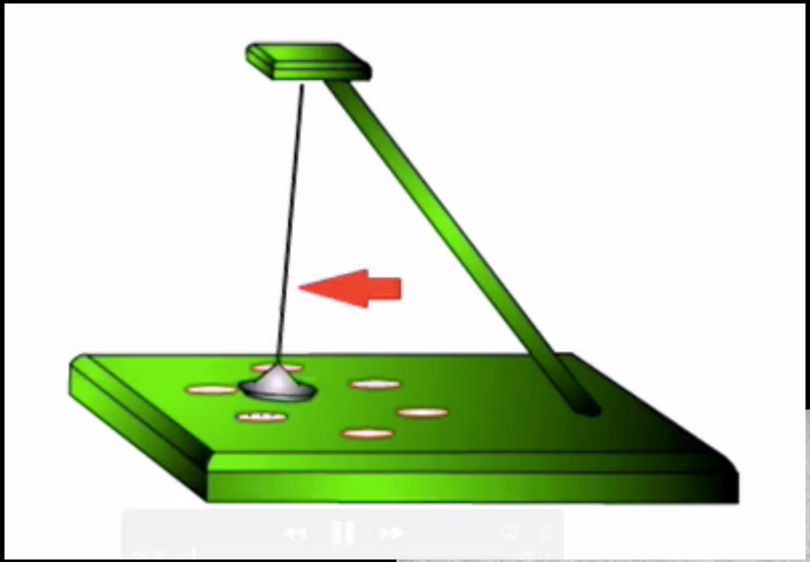
雙截鐘擺 Double Pendulum

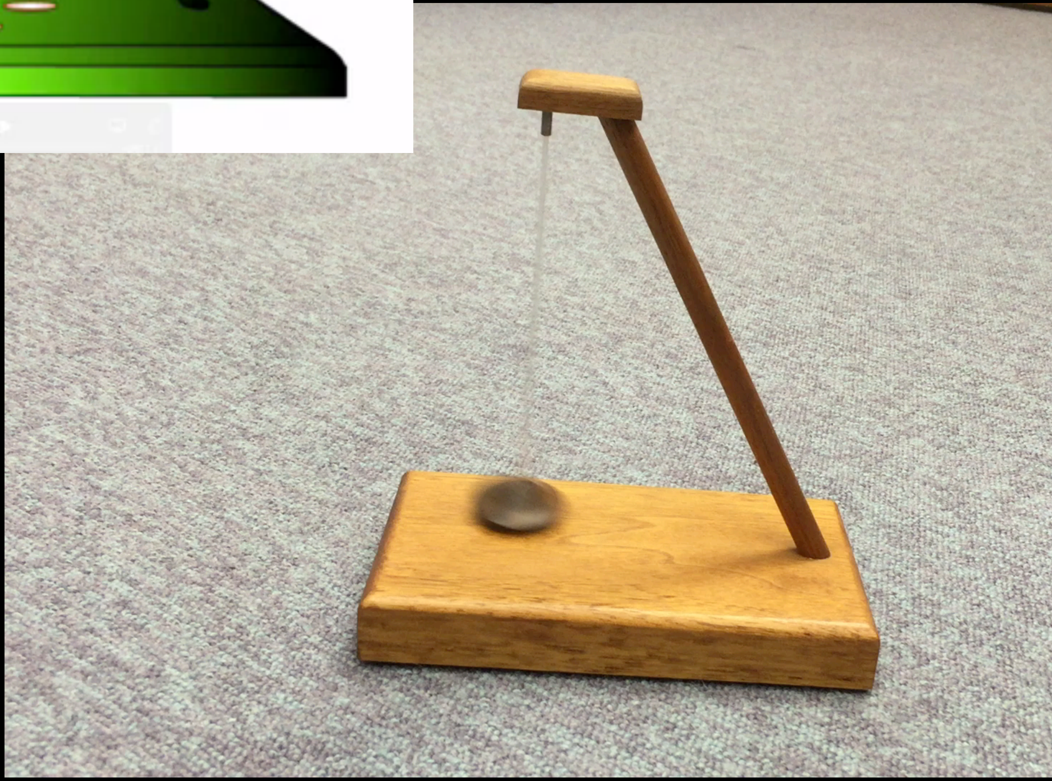
https://www.youtube.com/watch?v=QXf95_EKS6E

磁浮鐘擺 Magnetic Pendulum

同極磁石驅動鐘擺全受物理定律規管，但其軌道永不重複、亦無法預測。

Pendulum driven by repelling magnets fully describable by laws of physics. Trajectory never repeats and not predictable.





The swinging sticks in Iron Man 2

New Scientist

HOME NEWS TECHNOLOGY SPACE PHYSICS HEALTH EARTH HUMANS LIFE TOPICS EVENTS JOBS MAGAZINE

In the 1970s, Doyne Farmer, then a graduate student, used the world's first wearable computer to beat roulette tables in Nevada, but never revealed how he did it.

Now he has decided to break his long silence after a pair of researchers, inspired by his story, developed and published their own method of beating the house.

"I kept silent because I did not want to communicate any information that might prevent anyone from taking the casinos' money," writes Farmer, now at the University of Oxford, in a draft paper that he showed to *New Scientist*. "I see no good reason for staying silent any longer."

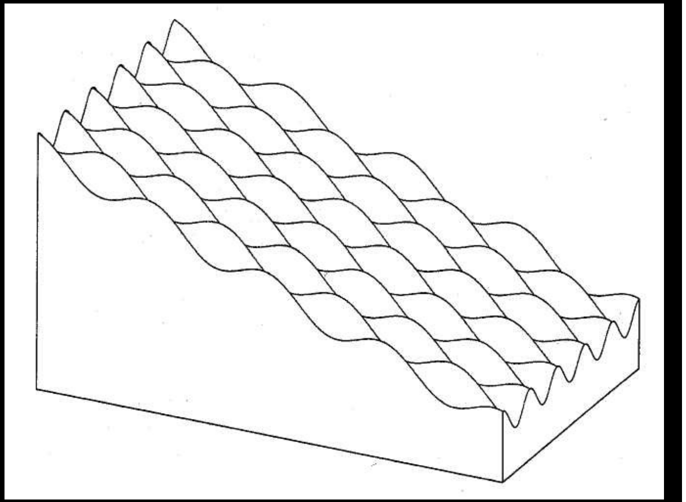
Farmer's paper is a response to recent research by Michael Small from the University of Western Australia in Perth and Michael Tse from Hong Kong Polytechnic University, submitted to the journal *Chaos*. They demonstrate that with a few measurements and a small computer or smartphone, you can indeed tip the odds in your favour. The trick is to record when the ball and a set part of the rotating wheel both pass a chosen point.

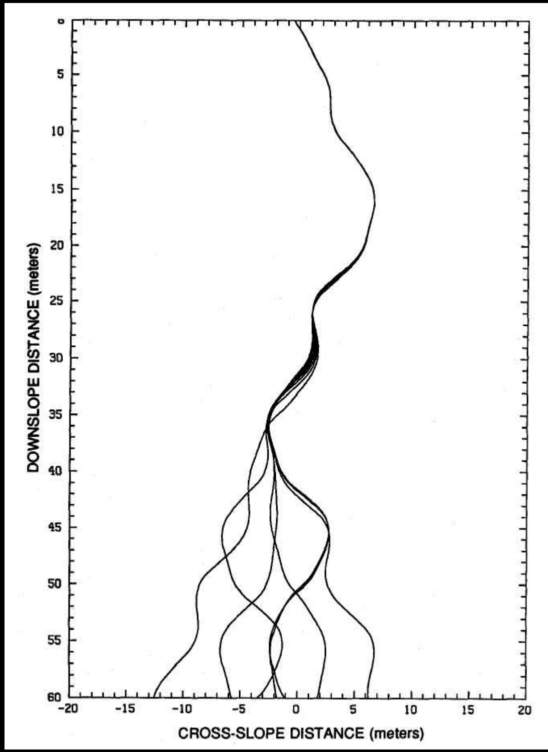
Their model divides the game into two parts: what happens while the ball rolls around the rim of the wheel and then falls, which is highly predictable, and what happens after the ball starts bouncing around, which is chaotic and hard to predict. Because the first part is predictable, Small and Tse were able to calculate roughly where the ball would begin its erratic bouncing and therefore in which part of the wheel it was more likely to land.

輪盤能預測嗎?　Can you win the roulette?

Small and Tse, "Predicting the outcome of roulette" published in Chaos 22, 033150 (2012)

滑雪場 Moguls on ski slope





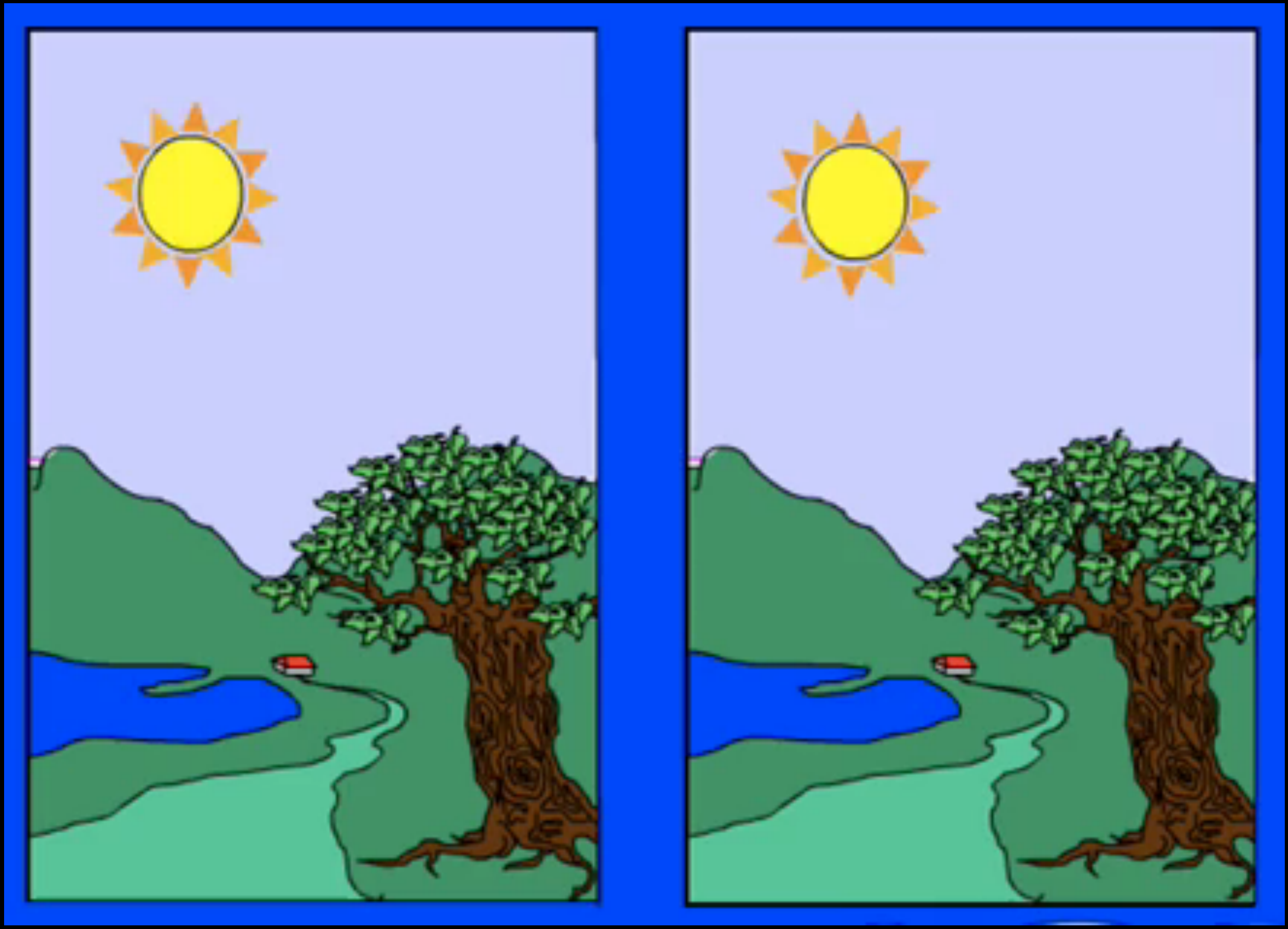
模型實驗證明7個很接近的起點最終走到不同的終點。

Diverging trajectories of 7 boards starting with same speed and spaced at 1mm intervals.

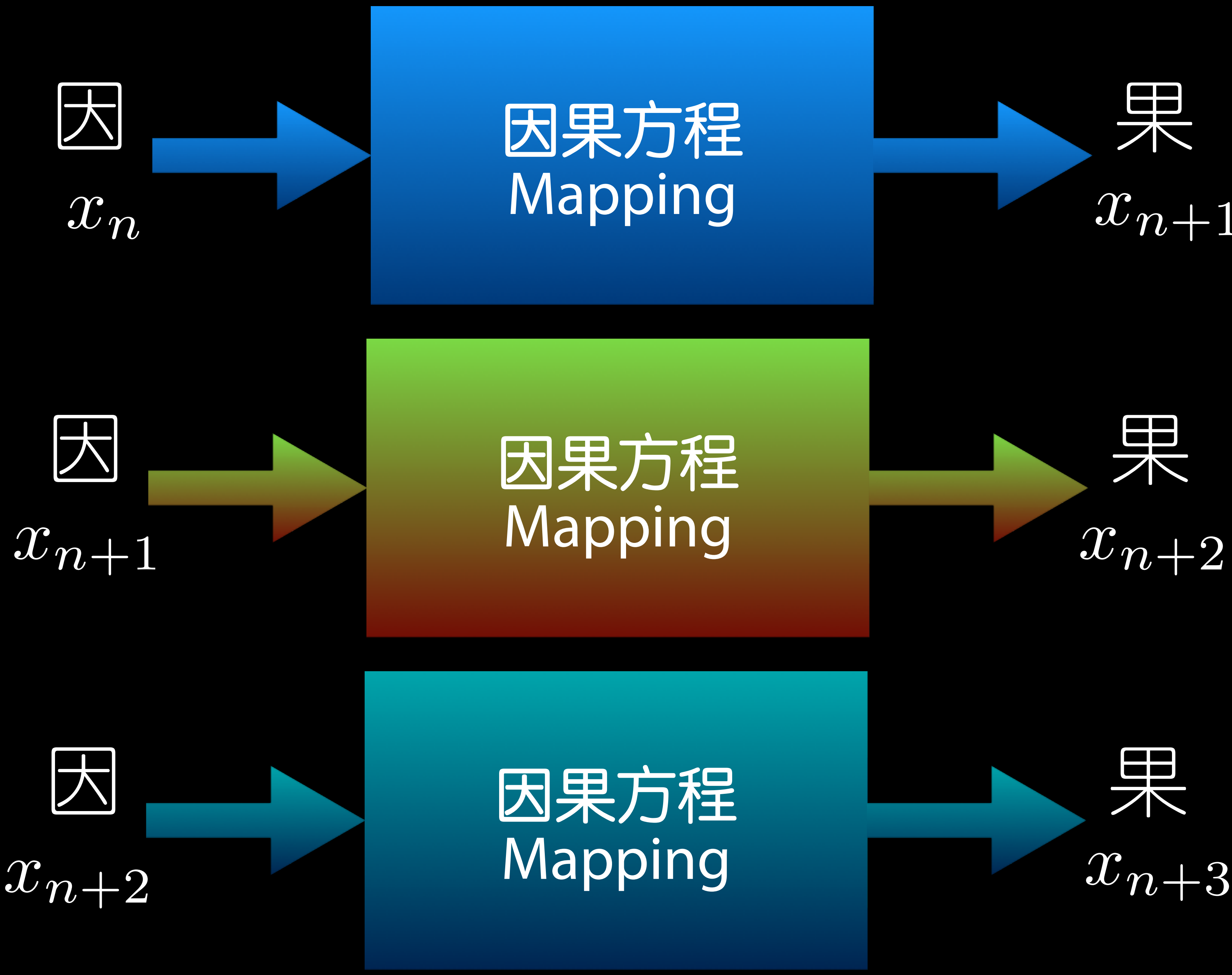
http://desertsnowjunkies.com/

天有不測之風雲 Chaotic weather

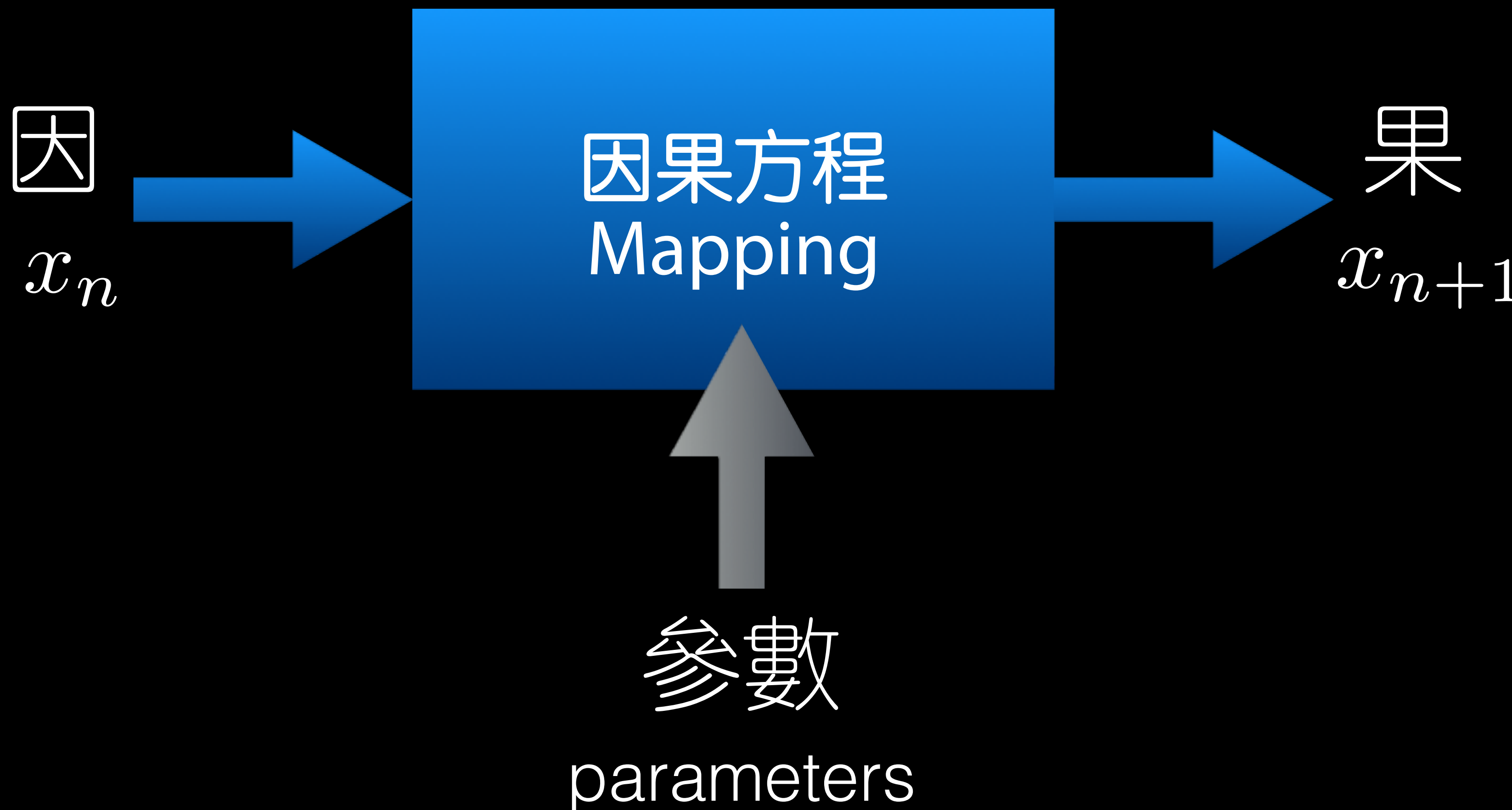
天氣是依循物理定律，理論上可推出因果方程。但由於系統是混沌的，加上參數眾多，蝴蝶效應大大降低長遠天氣預測的有效性。



Weather has a very complex model, though it should theoretically be described by laws of physics. The system is chaotic. When butterfly effect strikes, prediction fails in the long term.


因
x_n
因果方程
Mapping
果
x_{n+1}
因
x_{n+1}
因果方程
Mapping
果
x_{n+2}
因
x_{n+2}
因果方程
Mapping
果
x_{n+3}

極度敏感的因果方程



最簡單的「因果方程」例子
Simplest mapping example

The logistic map (Pierre François Verhulst, 1804-1849)

有限資源環境下的動物繁殖數量
Animal population supported by an environment.

x_n → $$x_{n+1} = \lambda x_n(1 - x_n)$$ → x_{n+1}

λ

重複、循環式因果方程
iterative mapping

從簡單因果看蝴蝶效應
Butterfly effect from simple mapping

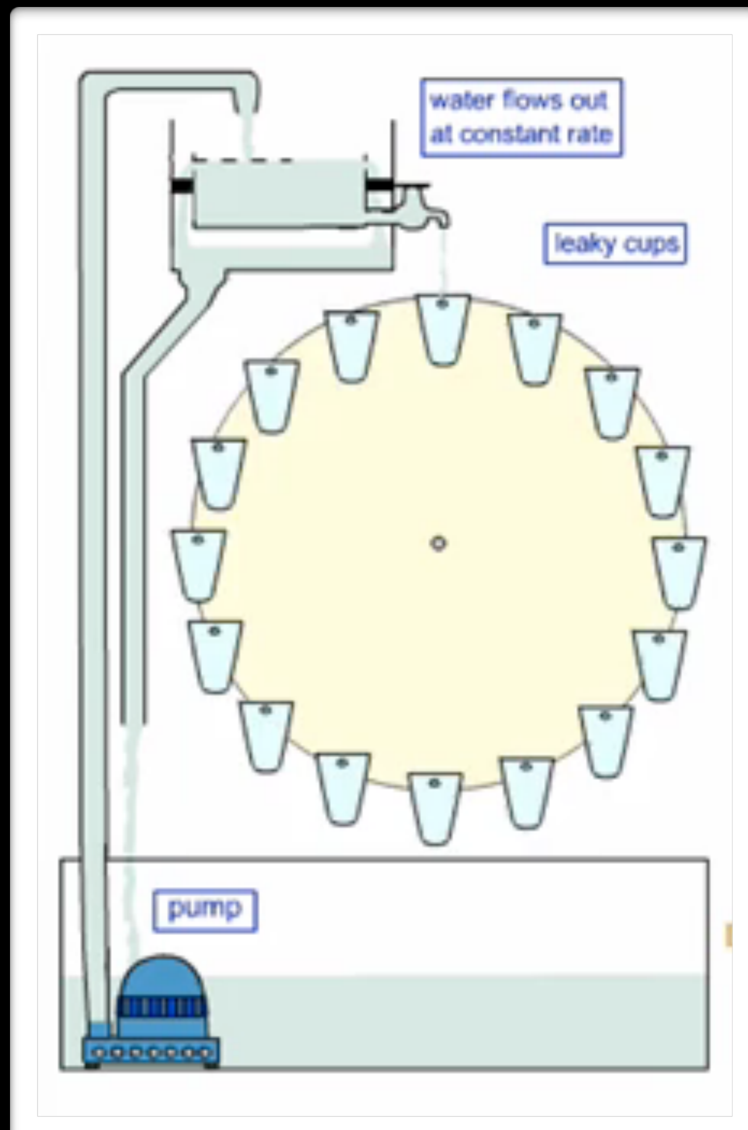
$$x_{n+1} = \lambda x_n (1 - x_n)$$

x_0 = **0.123** and **0.123001**, with λ **= 4**

混沌 DIY 1

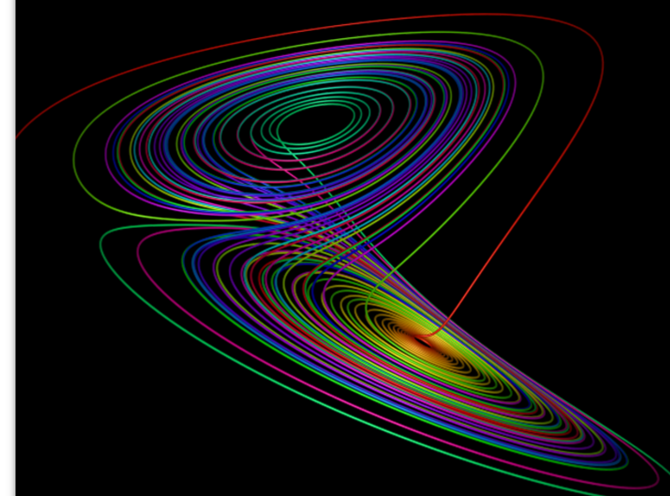
混沌水輪模擬羅倫茲的簡化空氣對流系統。

Chaotic waterwheel for modelling Lorenz's simplified convection model.



$$\begin{cases} \dot{x} & = & p(y-x) \\ \dot{y} & = & rx - y - xz \\ \dot{z} & = & xy - bz \end{cases}$$

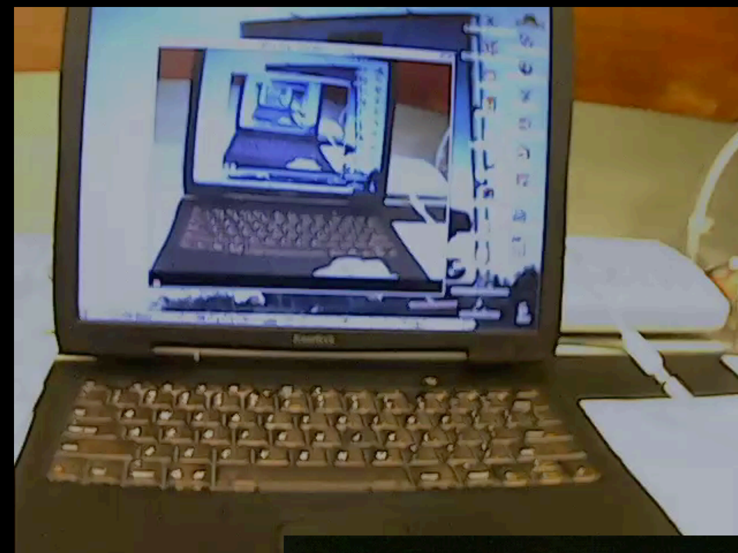
where *p*, *r*, *b* are parameters.



混沌 DIY 2

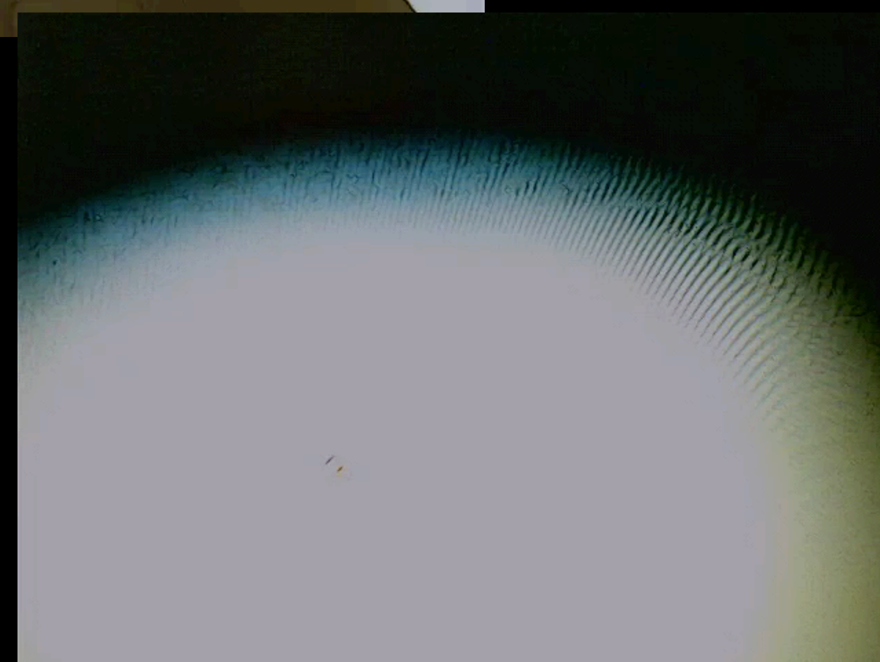
錄像反饋：
鏡頭對著畫面，調教角度、放大比例，輸回同一部電腦，實時播放。

Video feedback:
Take video of the screen, and feed the video back to the same computer that outputs the video continuously.



← delay + reduce

delay + enlarge
↓



2:05pm
2:15pm
2:55pm
3:00pm

Blue Mountain
Sydney
August 26, 2018

蝴蝶效應的啟示
What do we know from butterfly effect?

人生和世事皆屬混沌因果方程，對細微變化極度敏感，蝴蝶效應豐富了事情變化的可能性，讓人生充滿希望。

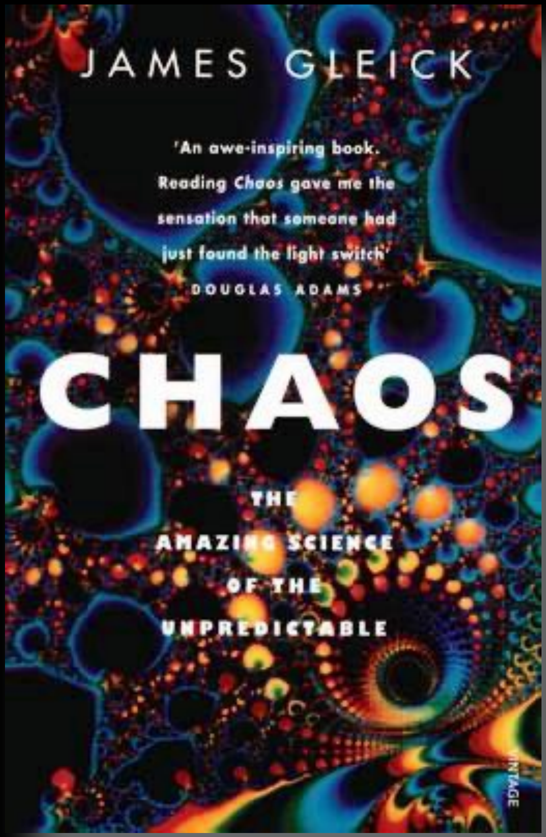
Because we cannot predict future, anything is possible. But cause-and-effect always determines how we will proceed. That’s why there is always hope if you work toward your goal!

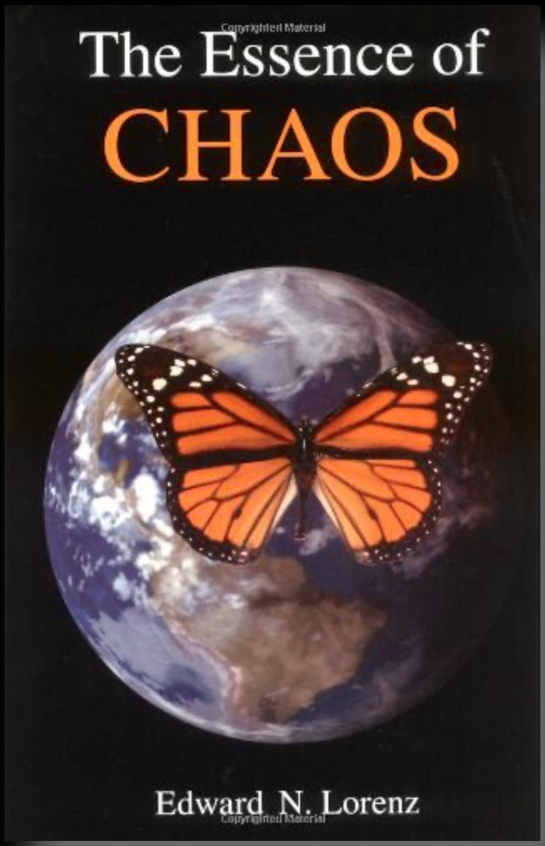
Use of Alex Chung’s photos is acknowledged. http://fotop.net/aaach/

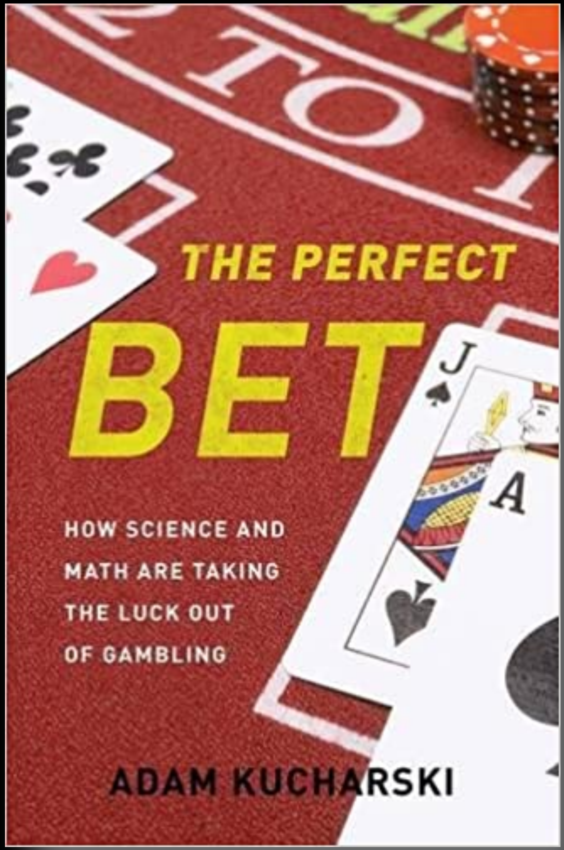
世事常變，永不放棄

CHANGE COURSE,
BUT NEVER GIVE UP

參考 References







Thank you for joining
CityU-Learning Classroom!

Prof. Michael Tse
Dept. of Electrical Engineering
chitse@cityu.edu.hk

香港城市大學
City University of Hong Kong