Part II: Fundamental Theory

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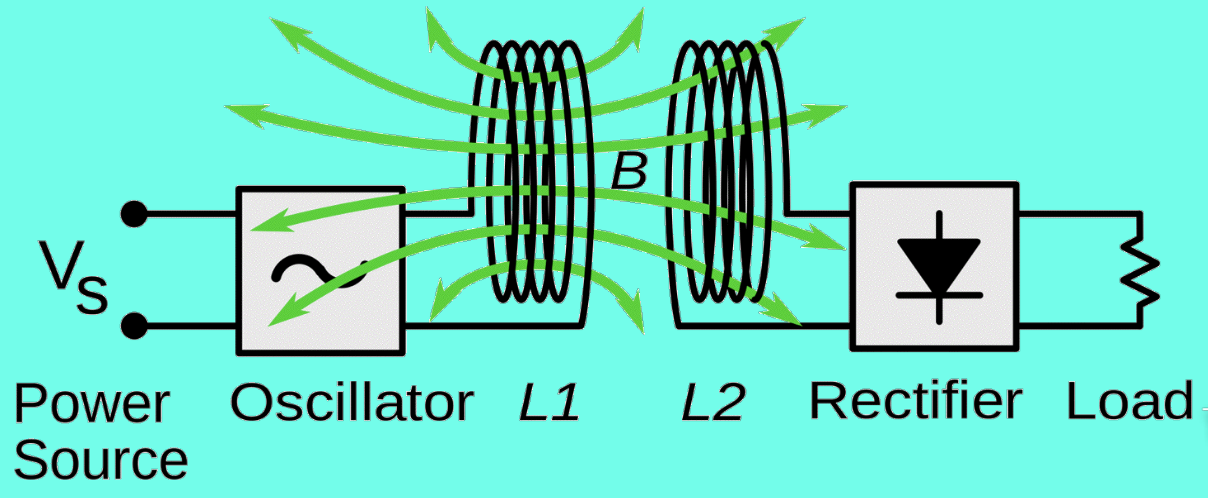
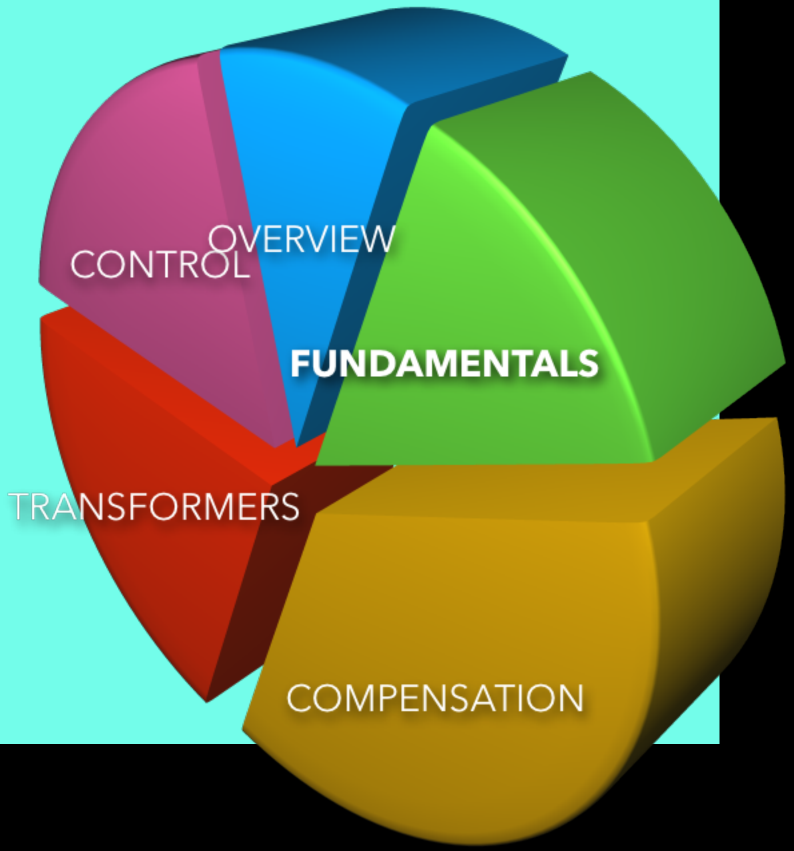




PART II

FUNDAMENTAL THEORY

Some Basic Circuit Theory (Revision)

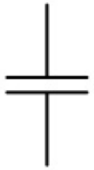
Inductor

Impedance increases with frequency

$j\omega L$

Capacitor

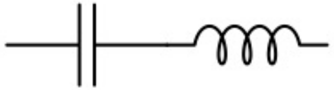
Impedance decreases with frequency



$$\frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Inductor + Capacitor

When a capacitor is **connected in series** with an inductor, their impedances cancel.



$$j\left(\omega L - \frac{1}{\omega C}\right)$$

At low frequency, it is like a capacitor.

At high frequency, it is like an inductor.

At resonant frequency $\omega = 1/\sqrt{LC}$

the impedance is ZERO, which is a **short circuit.**

When **connected in parallel**, their admittances cancel.

At low frequency, it is like an inductor.

At high frequency, it is like a capacitor.

At resonant frequency $\omega = 1/\sqrt{LC}$

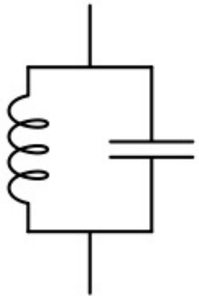
the impedance is INFINITY which is

an **open circuit.**

$$\frac{1}{j\left(\omega C - \frac{1}{\omega L}\right)}$$

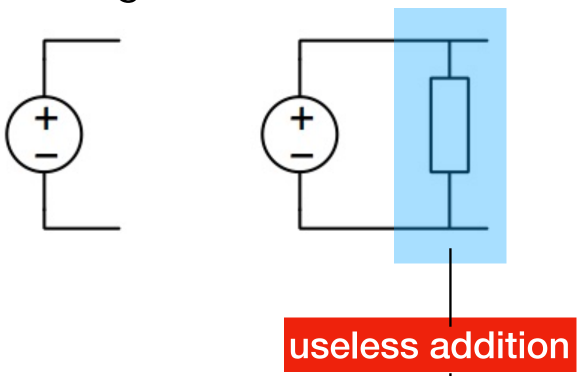
OR

$$\frac{j\omega L}{(1 - \omega^2 LC)}$$

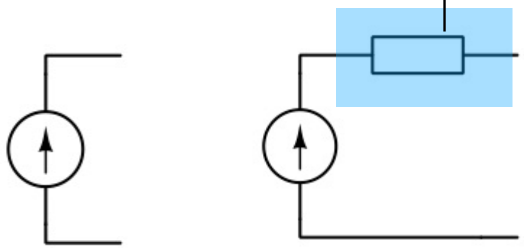


Some Basic Circuit Theory (Revision)

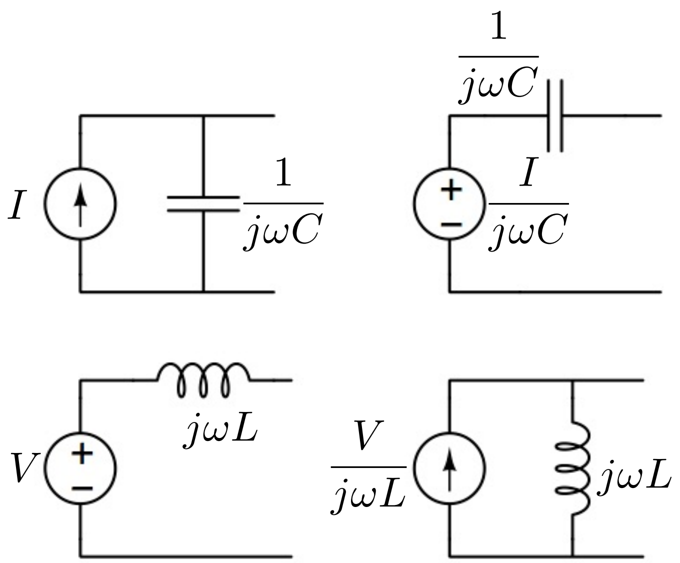
Voltage source



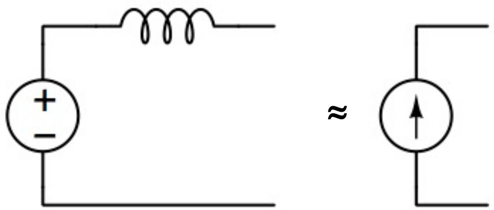
Current source



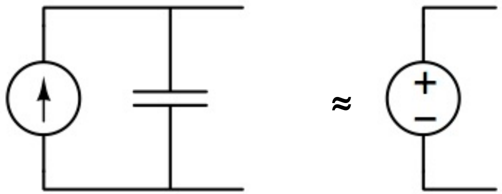
Thévenin & Norton Equivalences



A voltage source behind a sufficiently large inductor is approximately a current source

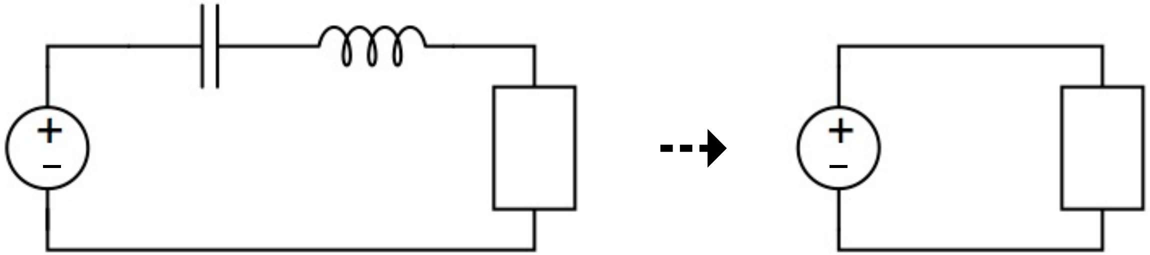


A current source parallel a sufficiently large capacitor is approximately a voltage source

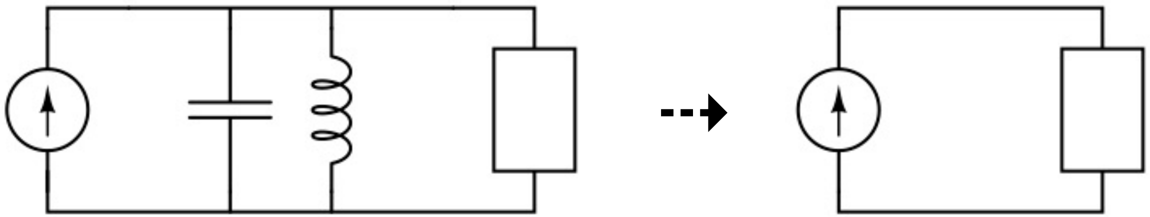


Some Basic Circuit Theory (Revision)

When tuned to the resonant frequency, a series LC can be regarded as short circuit.

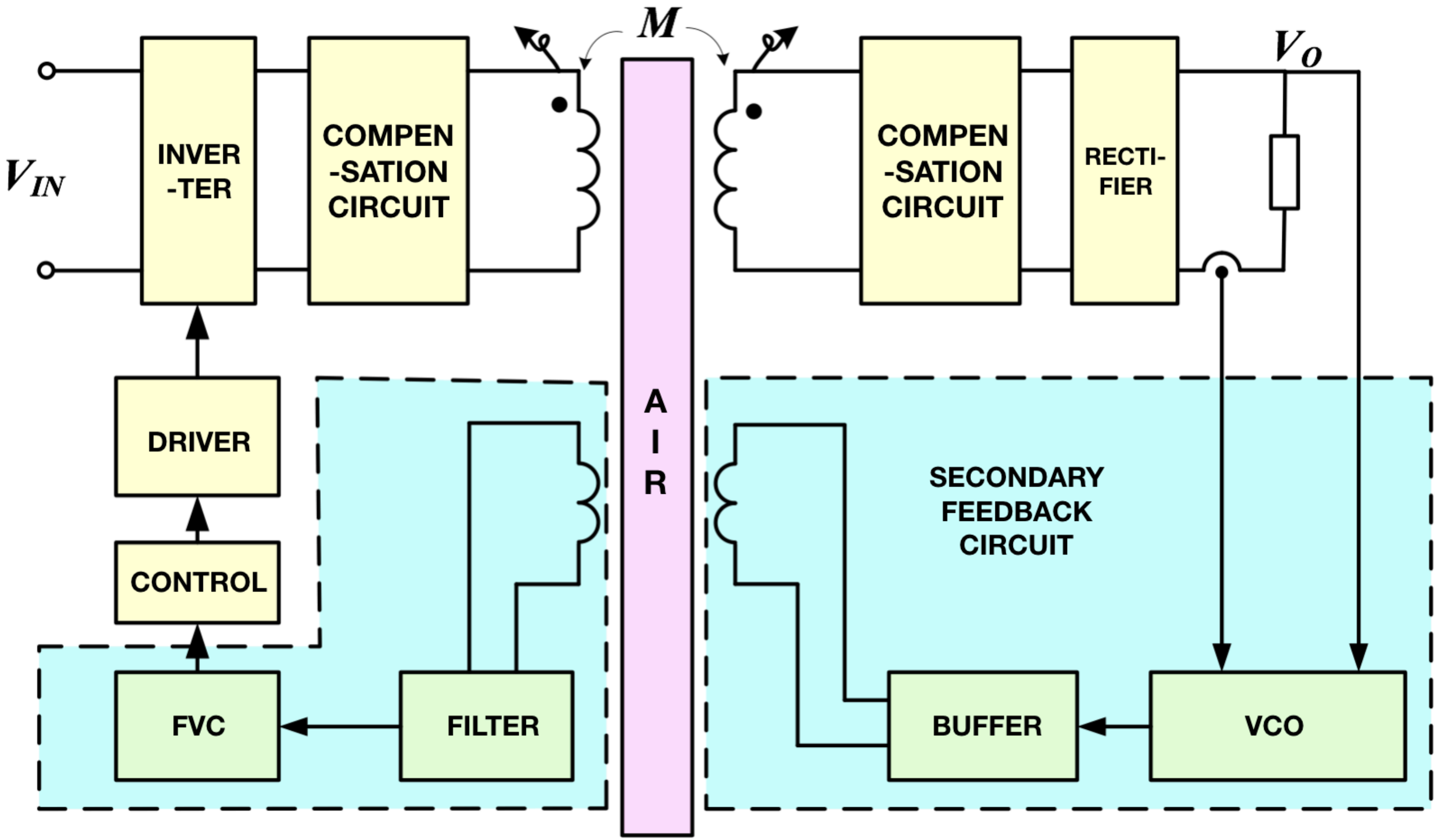


Similarly, when tuned to the resonant frequency, a parallel LC can be treated as open circuit.



共振短路與斷路

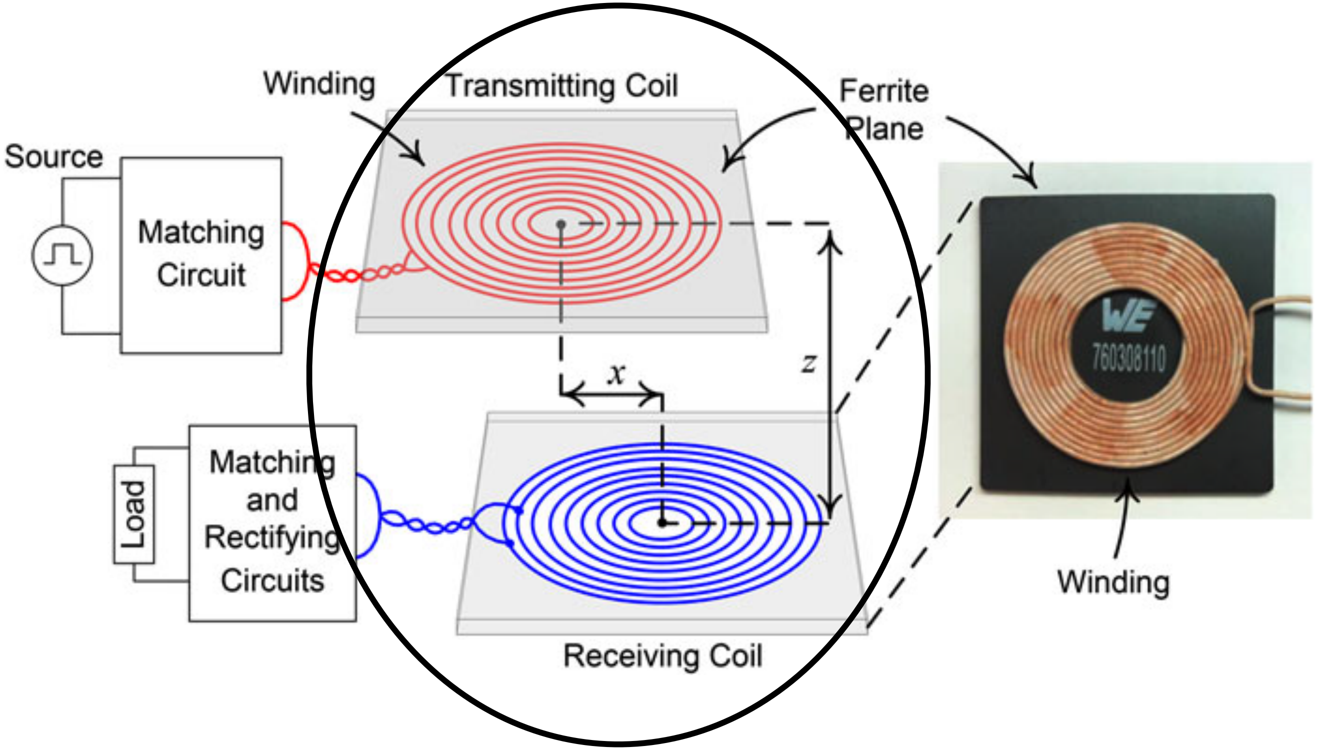
Basic Circuits



The Heart: A Bad Transformer!

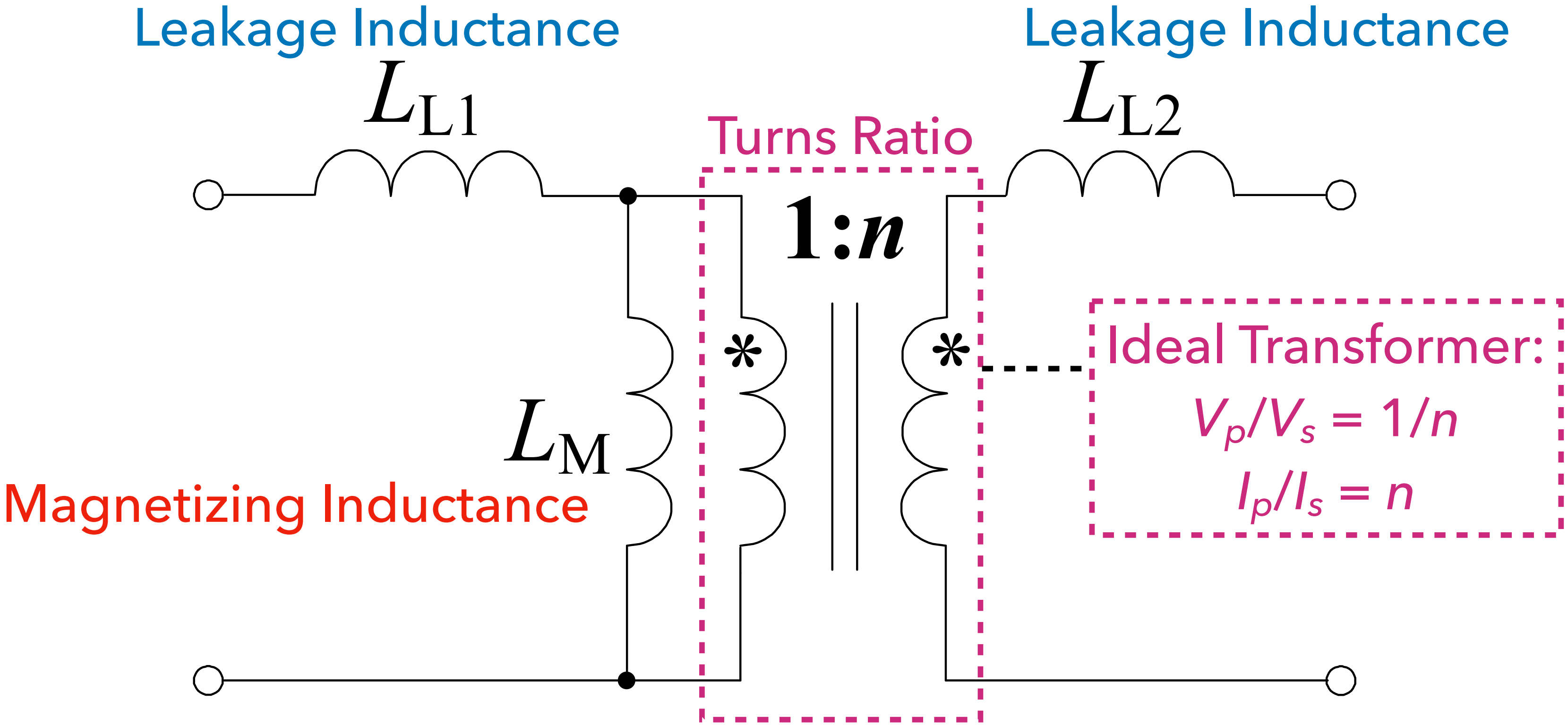
耦合欠緊的變壓器

- Why so difficult to design?
    - Understanding the transformer is MOST important!



Transformer

Usual Physical Model



物理漏感模型

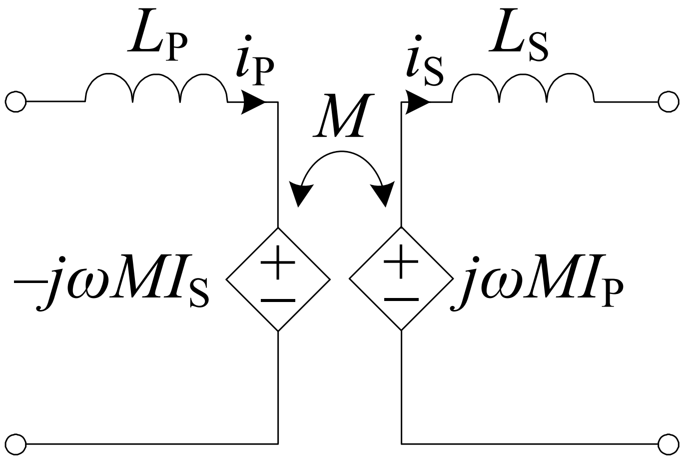
Transformer

互感模型

Basic Coupled Inductors Model

$$\begin{cases} V_P = L_P \dfrac{di_P}{dt} - M \dfrac{di_S}{dt} \\ V_S = M \dfrac{di_P}{dt} - L_S \dfrac{di_S}{dt} \end{cases}$$

$$\Rightarrow \begin{cases} V_P = j\omega L_P I_P - j\omega M I_S \\ V_S = j\omega M I_P - j\omega L_S I_S \end{cases}$$

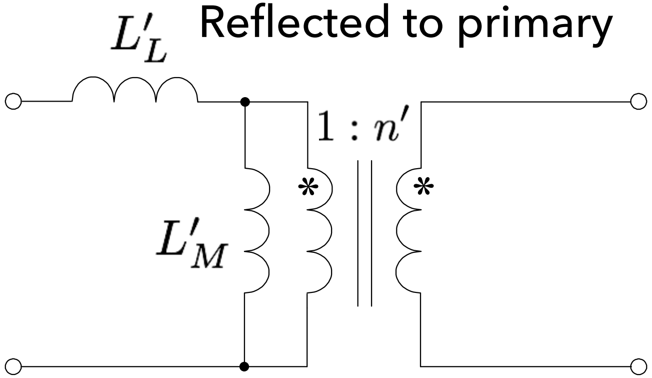


NOTE the direction of I_s

$$\begin{cases} L_P = L_{L1} + L_M \\ L_S = L_{L2} + n^2 L_M \\ M = k\sqrt{L_P L_S} \end{cases}$$

Equivalent Models

Reflected to primary

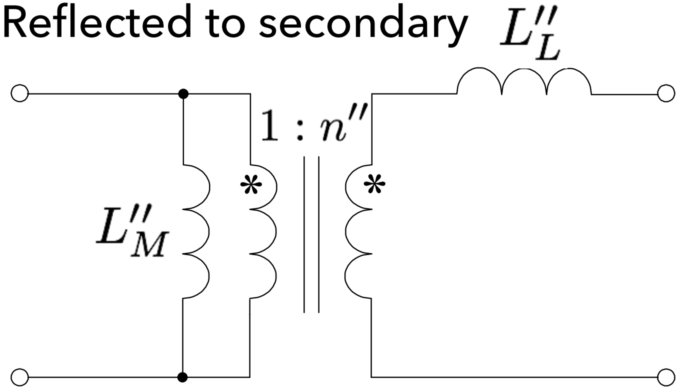


$$n' = \frac{L_S}{M}$$

$$L'_L = \frac{L_{L1}L_S + L_M L_{L2}}{L_S} = \frac{L_P L_S - M^2}{L_S}$$

$$L'_M = \frac{M^2}{L_S}$$

Reflected to secondary



$$n'' = \frac{M}{L_P}$$

$$L''_L = L_S - \frac{M^2}{L_P}$$

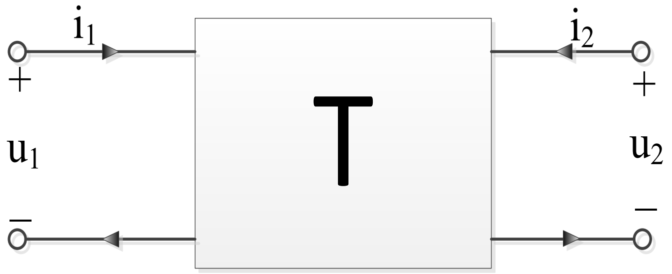
$$L''_M = L_P$$

Two-port Models

Common two-port models for analysis of transfer characteristics, driving point impedance, and output impedance:

- z-parameter model
- y-parameter model
- T-parameter model (ABCD model)
- s-parameter model (Scattering parameters for waves)

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$$

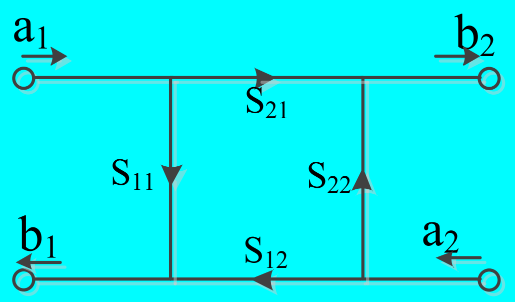
ABCD matrix

Scattering Paremeters

- For high frequency operation, the lumped circuit model fails, and distributed circuit model must be used. Scattering matrix is the appropriate choice.

  a_1, b_1 are the incident and reflected waves at port 1

  a_2, b_2 are the incident and reflected waves at port 2



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Equivalent Representations

	Z-parameter	T-parameter
Z-parameter	$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$	$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \frac{1}{z_{21}} \begin{bmatrix} z_{11} & z_{11}z_{22} - z_{12}z_{21} \\ 1 & z_{22} \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$
T-parameter	$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{1}{c} \begin{bmatrix} a & -(ad - bc) \\ 1 & -d \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$	$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$

Z to S:

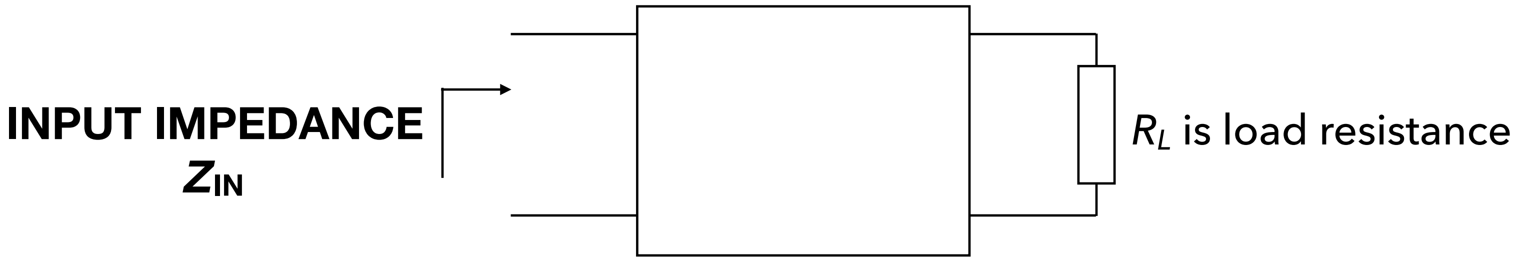
$$S = \frac{1}{(Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}} \begin{bmatrix} (Z_{11}-Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21} & 2Z_{12}Z_0 \\ 2Z_{21}Z_0 & (Z_{11}+Z_0)(Z_{22}-Z_0) - Z_{12}Z_{21} \end{bmatrix}$$

S to Z:

$$Z = \frac{Z_0}{(S_{11}-1)(S_{22}-1) - S_{12}S_{21}} \begin{bmatrix} -(S_{11}+1)(S_{22}-1) + S_{12}S_{21} & 2S_{12} \\ 2S_{21} & -(S_{11}-1)(S_{22}+1) + S_{12}S_{21} \end{bmatrix}$$

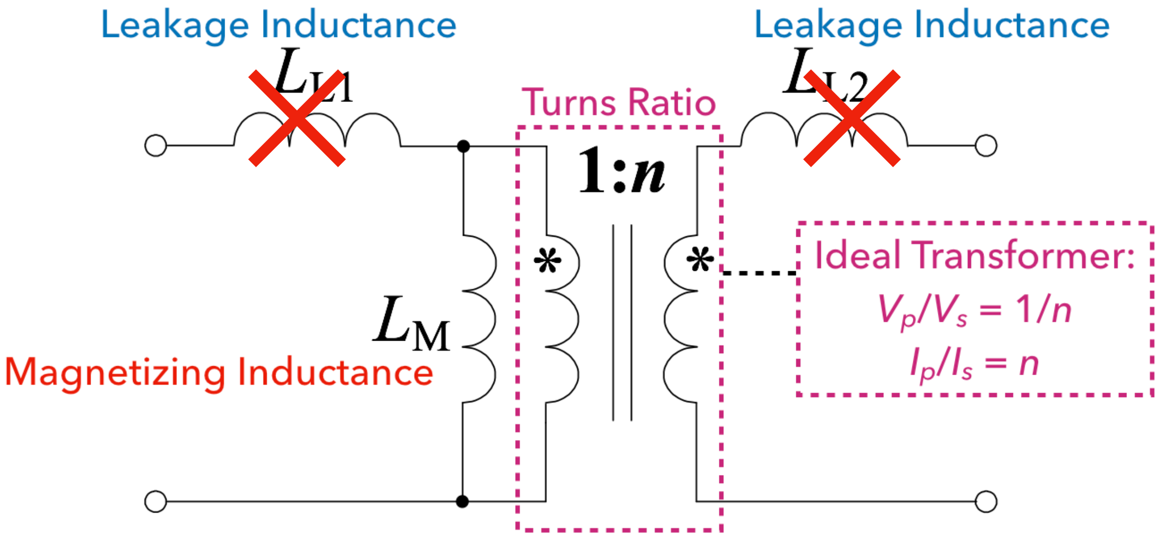
Input and Output Characteristics

	Z-parameter	T-parameter
Input Impedance	$Z_{11} - \dfrac{Z_{12}Z_{21}}{Z_{22}+R_L}$	$\dfrac{aR_L+b}{cR_L+d}$
Voltage Ratio (Gain)	$\dfrac{R_L Z_{21}}{Z_{11}\left(Z_{22}+R_L\right)-Z_{12}Z_{21}}$	$\dfrac{aR_L+b}{R_L}$
Efficiency	$\dfrac{1}{\left(Z_{22}+R_L\right)}\dfrac{R_L {Z_{21}}^2}{Z_{11}\left(Z_{22}+R_L\right)-Z_{12}Z_{21}}$	$\dfrac{R_L}{\left(aR_L+b\right)\left(cR_L+d\right)}$



Compensation

Poorly coupled transformer has large leakage inductance, causing lots of reactive power circulating!
Compensation IS MANDATORY!



What is “compensation”?

補償的作用

Cancel the inductance!

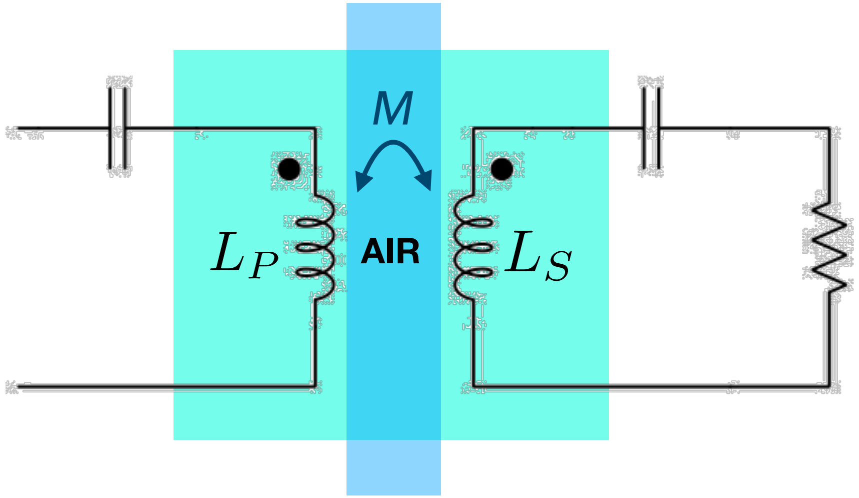
Eliminate reactive power.

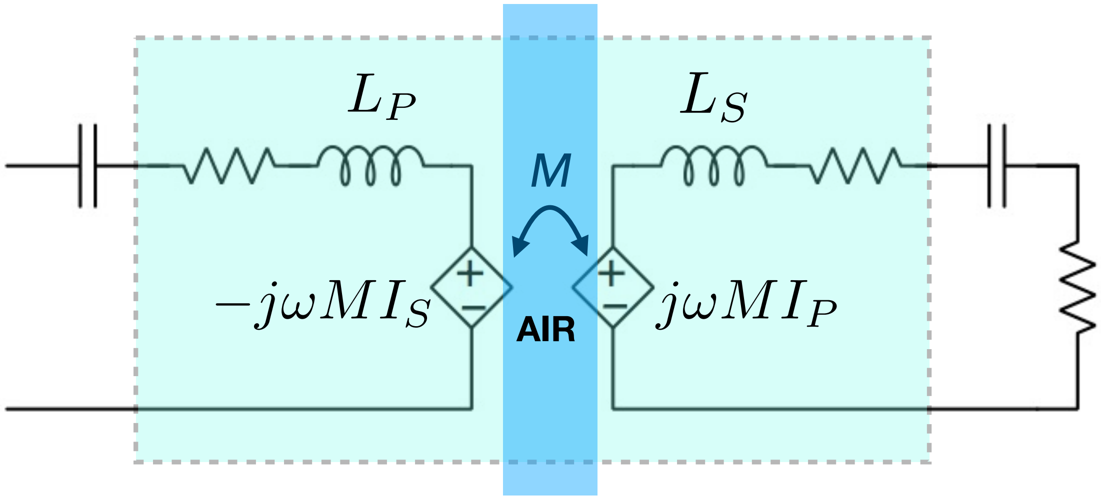
Lower the current magnitude.

Improve efficiency.

Correct input power factor.

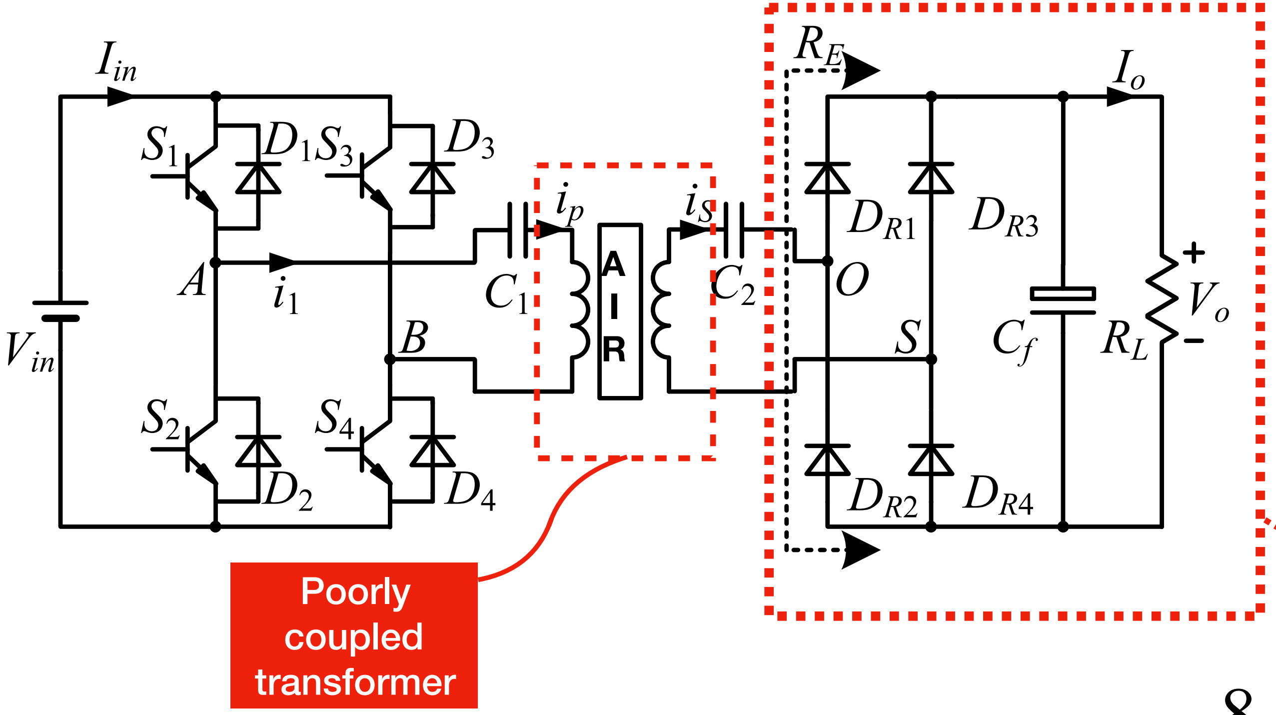
One main technical problem of IPT is compensation



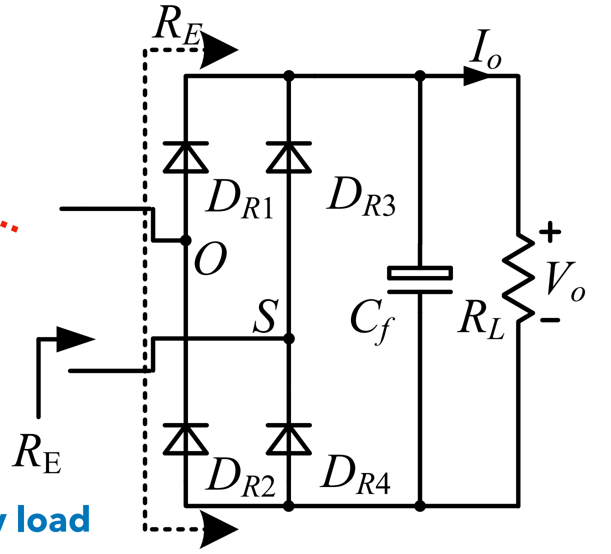


APPROPRIATE MODEL

The General IPT Core Circuit

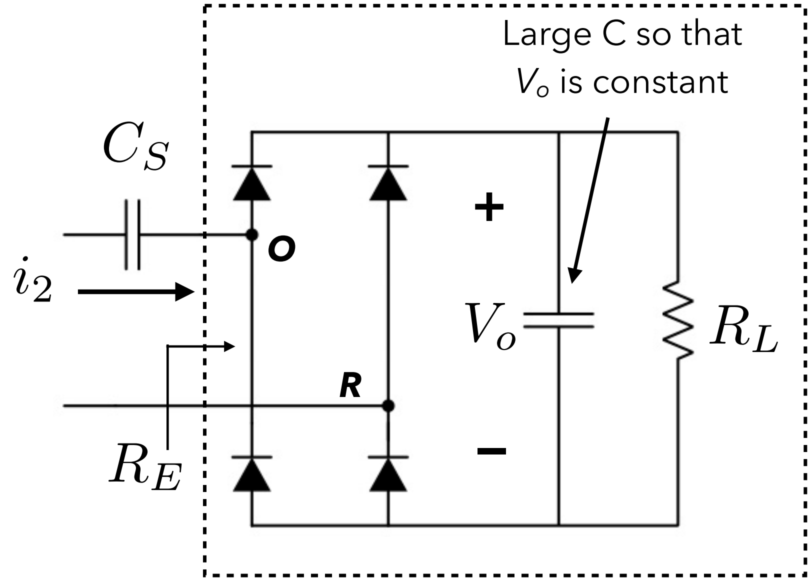


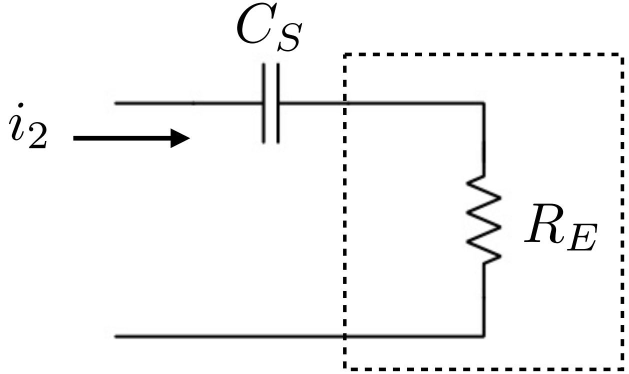
$$R_E = \frac{8}{\pi^2} R_L$$



Equivalent fundamental frequency load

Form factor at load side

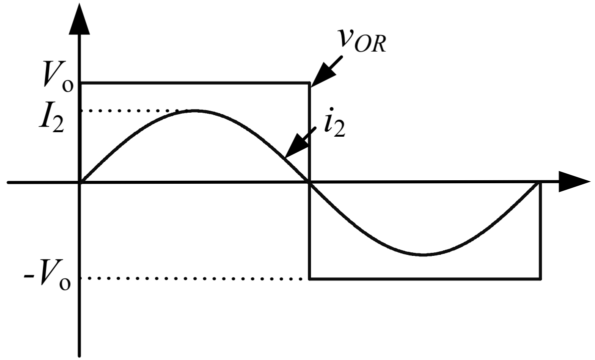




If C is a large, output voltage V_o is constant.

V_{OR} is constant.

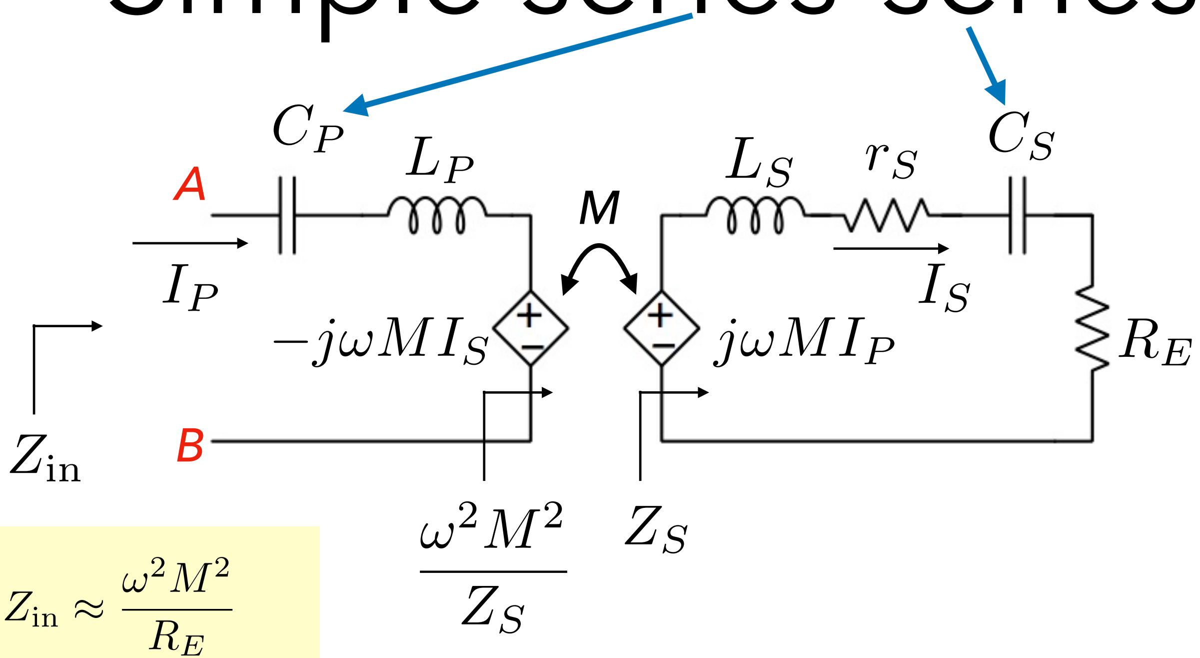
i_2 and V_{OR} are in phase.



$$R_E = \frac{V_{OR}}{i_2} = \frac{\frac{4}{\pi} V_o \sin \omega t}{I_2 \sin \omega t} = \frac{\frac{4}{\pi} V_o}{\frac{\pi}{2} I} = \frac{8 V_o}{\pi^2 I} = \frac{8}{\pi^2} R_L$$

$$R_E = \frac{8}{\pi^2} R_L$$

Simple series-series compensation



Input impedance at secondary

$$Z_S = j\omega L_S + \frac{1}{j\omega C_S} + R_E + r_S$$

Input impedance reflected to primary

$$Z_{in} = j\omega L_P + \frac{1}{j\omega C_P} + \frac{(\omega M)^2}{Z_S}$$

Input impedance angle

$$\theta_{in} = \arctan\frac{\text{Im}(Zin)}{\text{Re}(Zin)}$$

串連補償

$$Z_{\text{in}} \approx \frac{\omega^2 M^2}{R_E}$$

$$I_P \approx \frac{V_{AB} R_E}{\omega^2 M^2}$$

$$I_S \approx \frac{jV_{AB}}{\omega M}$$

$$\frac{I_S}{V_{AB}} \approx \text{const}$$

at resonance

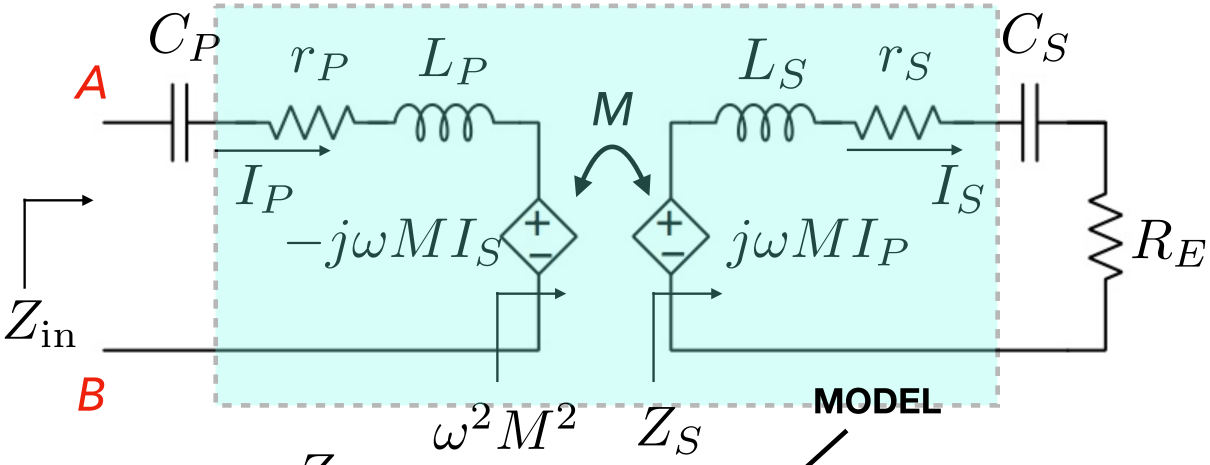
TRANSCONDUCTANCE

$$\frac{I_S}{V_{AB}} = \frac{|j\omega M|}{|Z_{in}|}\frac{1}{|Z_S|} \quad \& \quad \frac{I_O}{V_{in}} = \frac{8}{\pi^2}\frac{I_S}{V_{AB}}$$

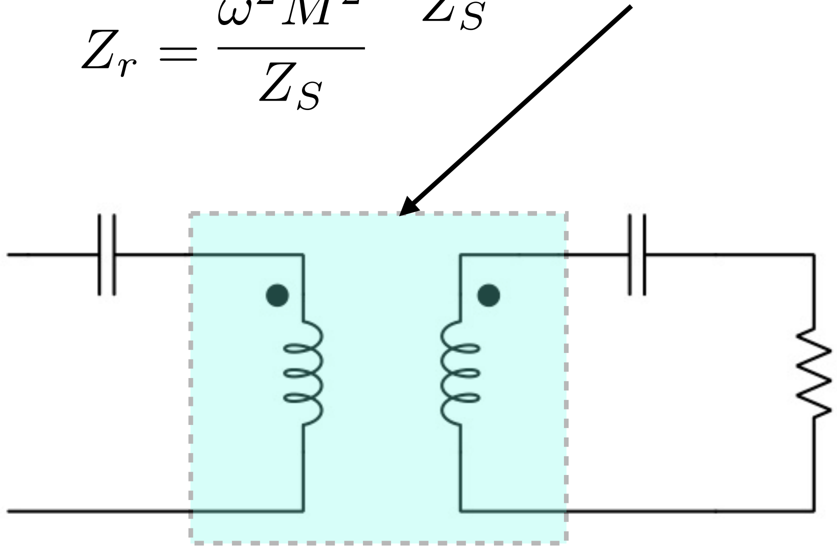
VOLTAGE GAIN

$$\frac{V_{OS}}{V_{AB}} = \frac{|j\omega M|}{|Z_{in}|}\frac{1}{|Z_S|}R_E \quad \& \quad \frac{V_{out}}{V_{in}} = \frac{V_{OS}}{V_{AB}}$$

Efficiency for s/s compensation



$$Z_r = \frac{\omega^2 M^2}{Z_S}$$



Secondary side efficiency

$$\eta_S = \frac{R_E}{\Re[Z_S]} = \frac{R_E}{r_S + R_E}$$

Primary side efficiency

$$\eta_P = \frac{\Re[Z_r]}{\Re[Z_r] + r_P}$$

Total efficiency

$$\eta_T = \eta_P \eta_S$$

效率分析

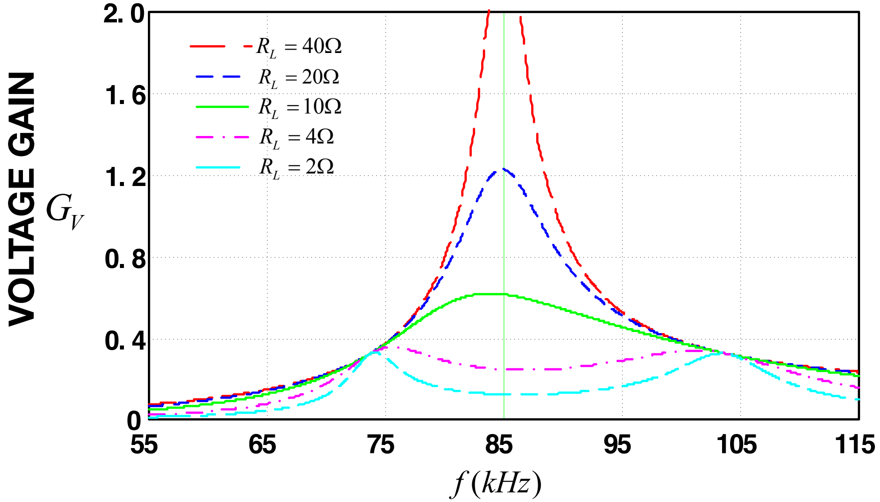
Reflected resistance at primary

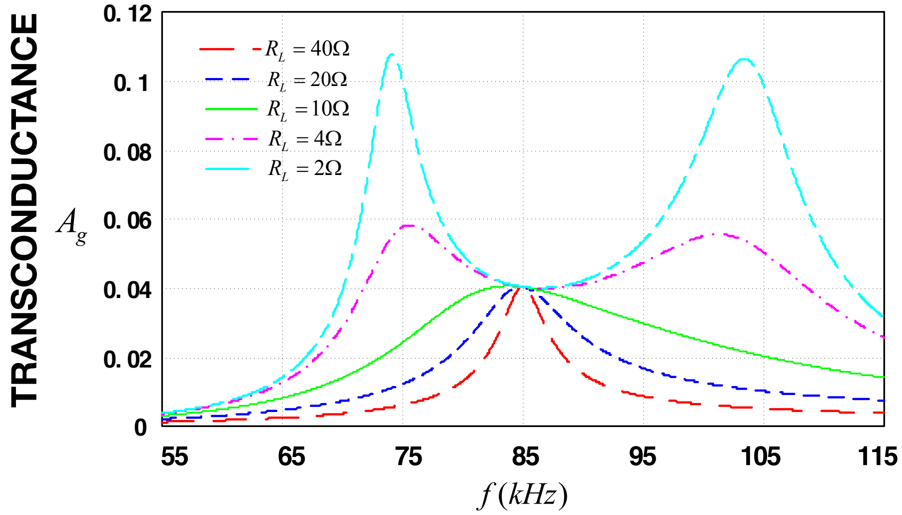
$$\Re[Z_r] = \frac{\omega^2 M^2 (R_E + r_S)}{(R_E + r_S)^2 + \left(\omega L_S - \frac{1}{\omega C_S}\right)^2}$$

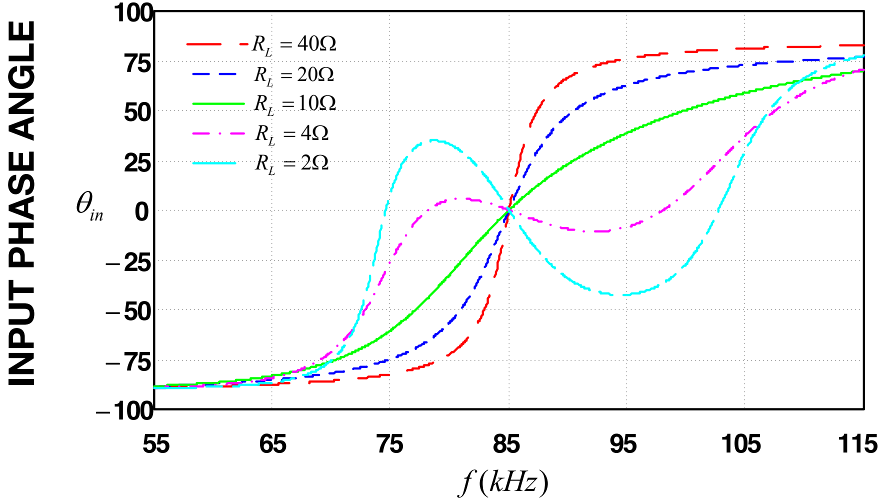
The larger the better for efficiency !!!!!

Some results for s/s compensation

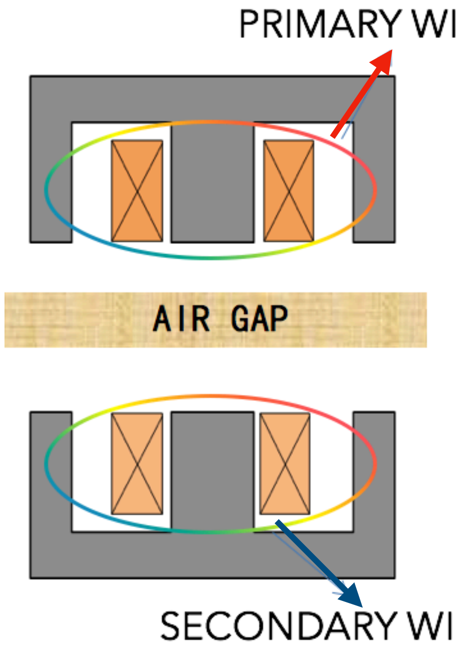
Resonant Frequency f_o	85 kHz
Primary Self Inductance L_P	254.16 µH
Secondary Self Inductance L_S	36.27 µH
Mutual Inductance M	37.65 µH
Primary Compensation Cap C_P	9.899 nF
Secondary Compensation Cap C_S	96.662 nF





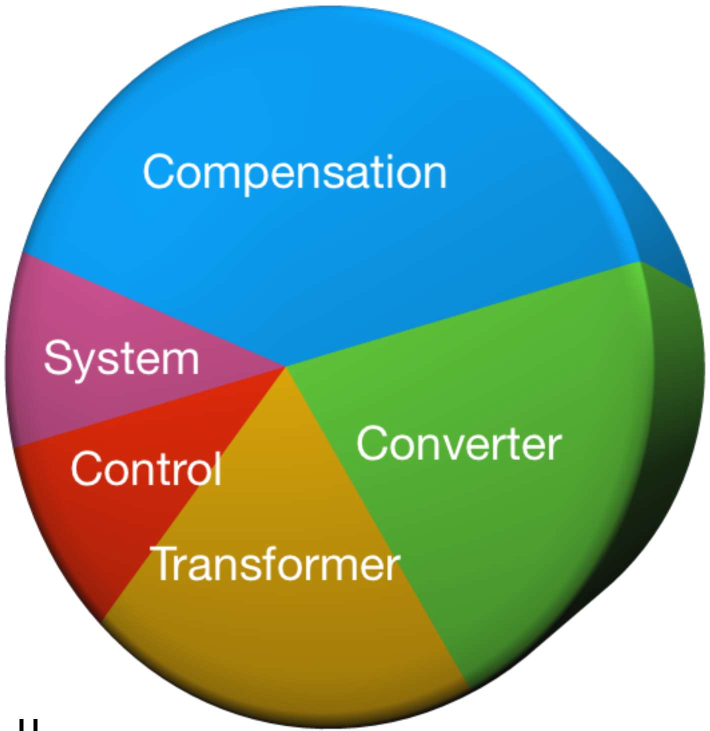


Challenges



Characteristics of transformer

- (Very) loose coupling
- Wide parameter variation



Main challenges

- Compensation of the inductances
- Optimization of the contactless transformer
- Effective control methods

Interim Conclusion

- Although most of the basic theory is well known, specific application to WPT still requires substantial reconsideration and reorganization so as to allow more focused development of relevevant design methods

- Key points:

    ✴ Transformer being leaky, i.e., **high leakage inductance, low coupling**

    ✴ **Appropriate transformer models: physical turns ratio, coupled inductor model**

    ✴ Compensation types: series and parallel for different terminations, with different properties and wide varying parameters

        ✴ In Part III, we will examine compensation in detail.