Assessing Cooperative Behavior in Dynamical Networks

with Applications to Brain Data

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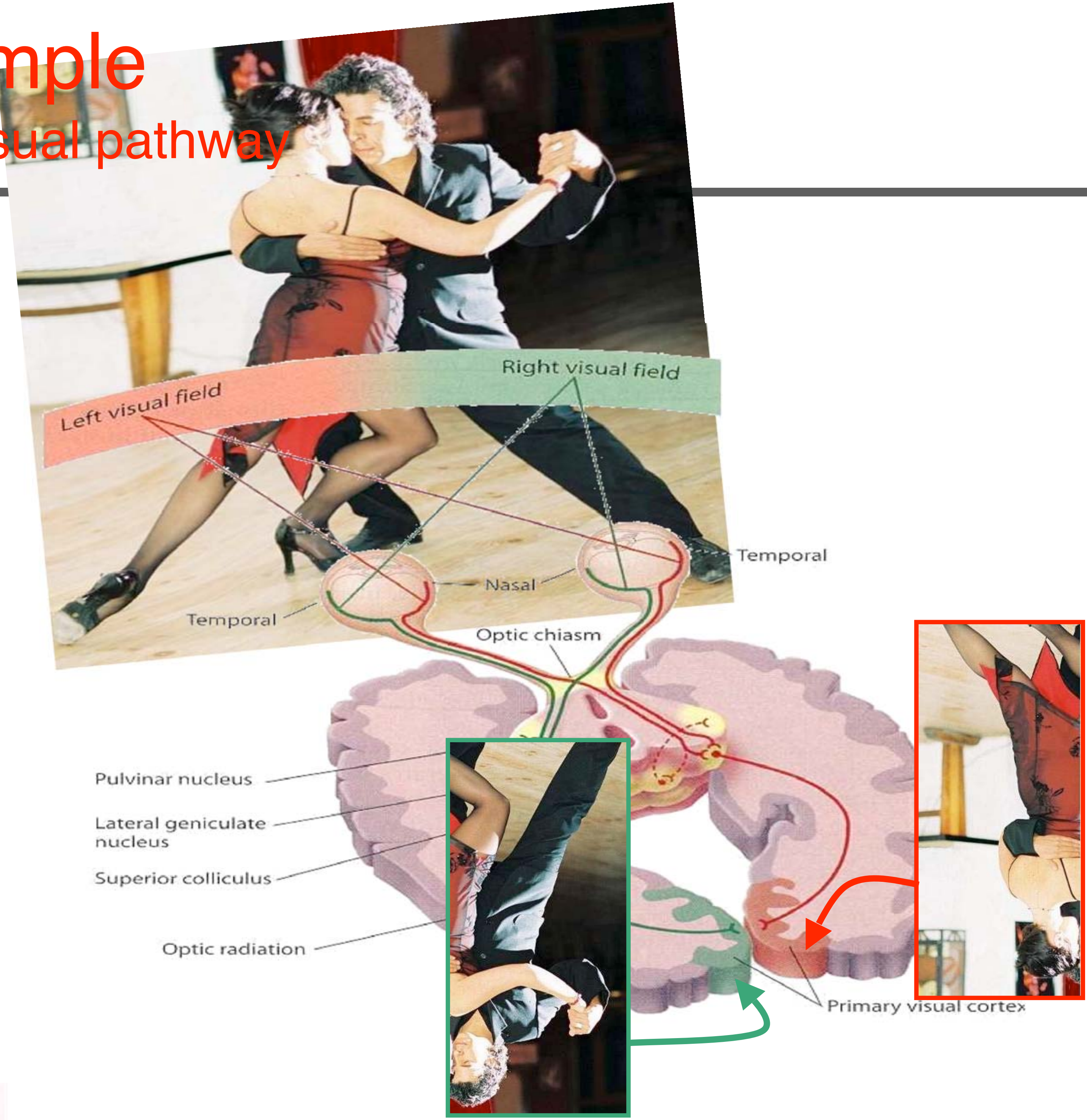
Example

The visual pathway

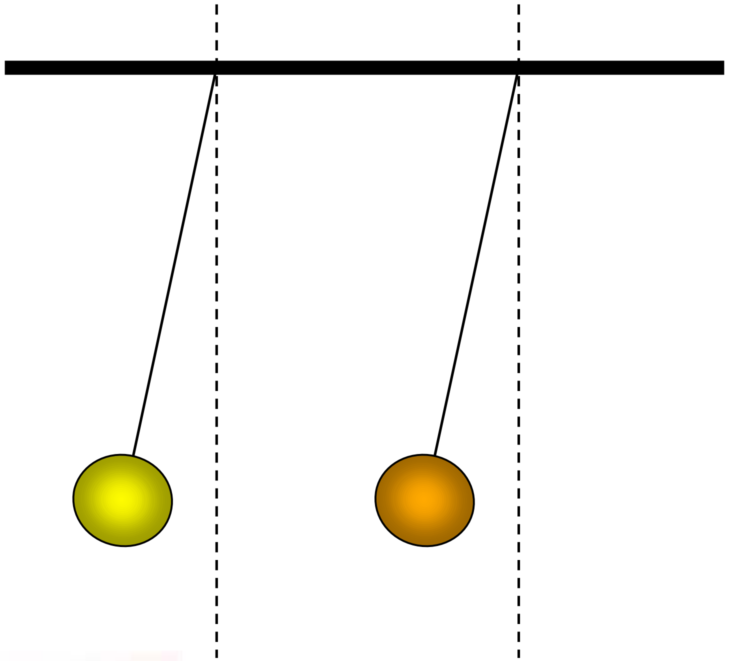


Context

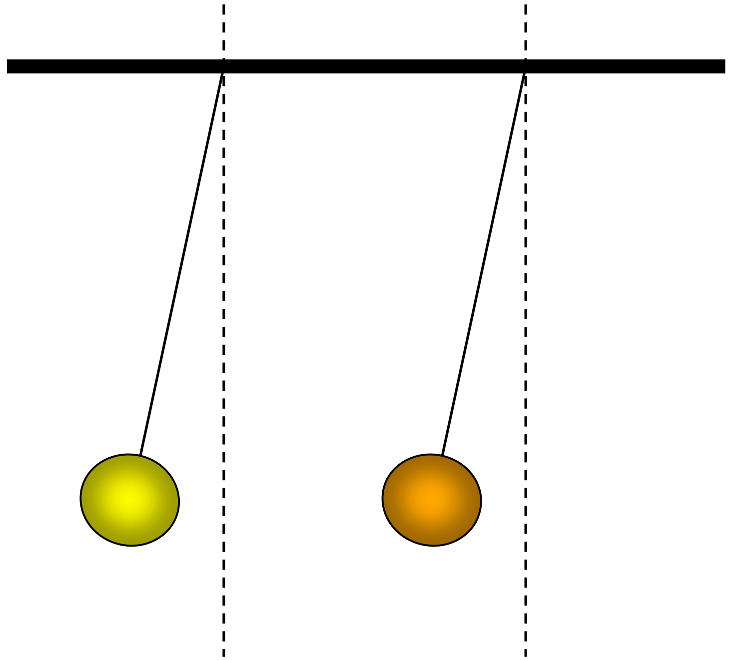
Cooperativeness

- » Weakly correlated motion
- » Strongly correlated motion (synchronization)

Case 1: asynchronous case
(weakly coupled)



Case 2: synchronous case
(strongly coupled)

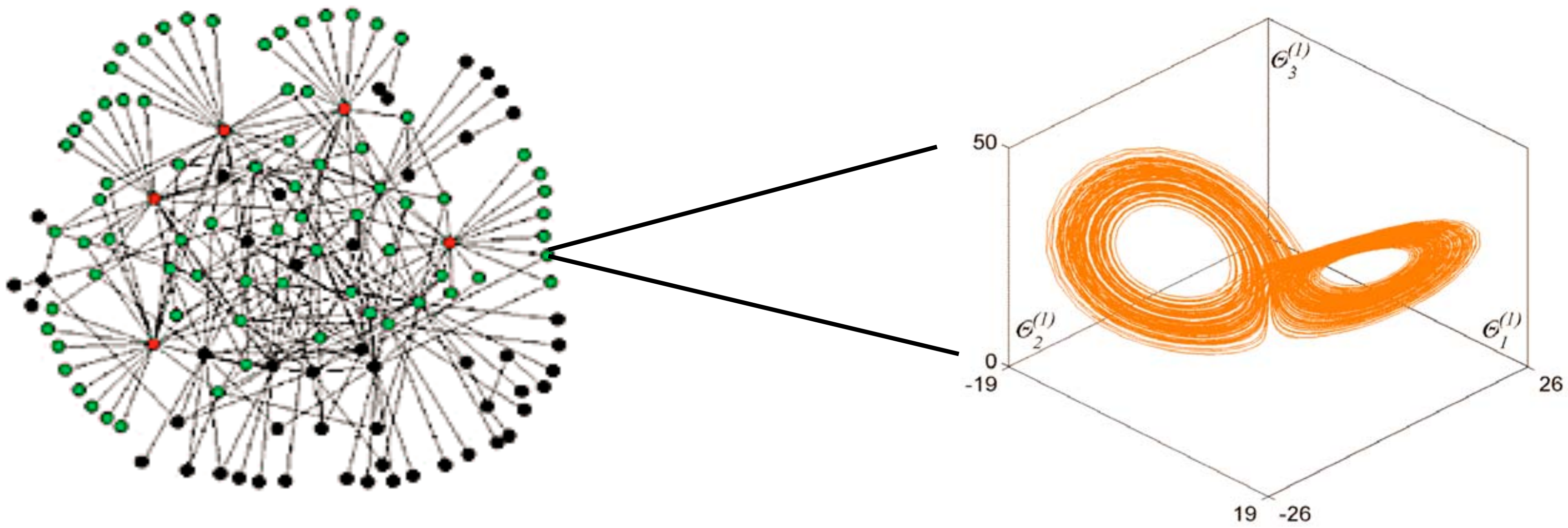


Context

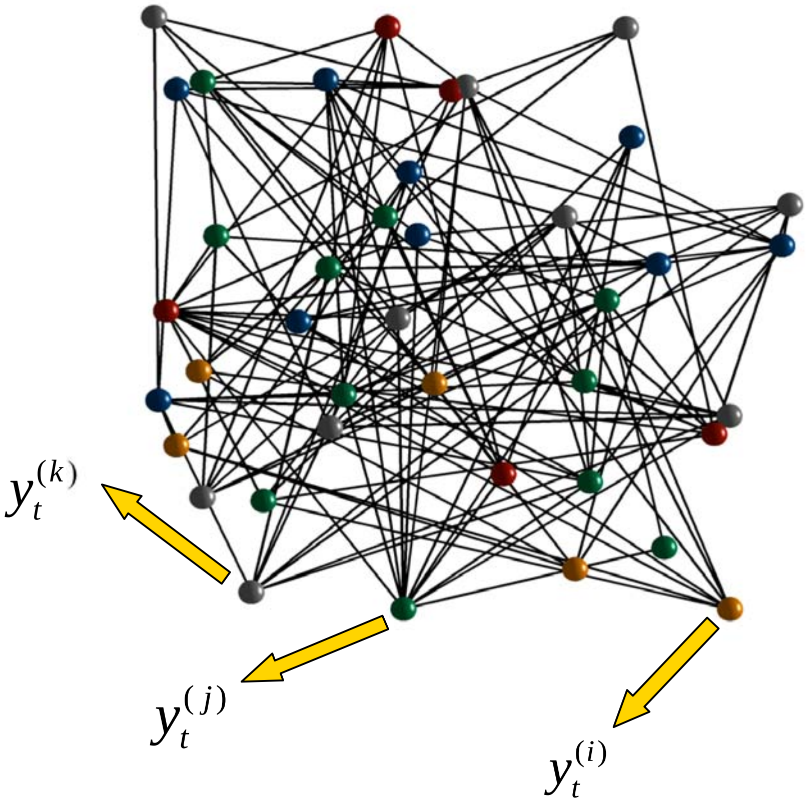
The challenge of Networks

» Graph topology

» Non linear dynamics



Goal



Inferring cooperative behavior from measurements of the network

A generic problem

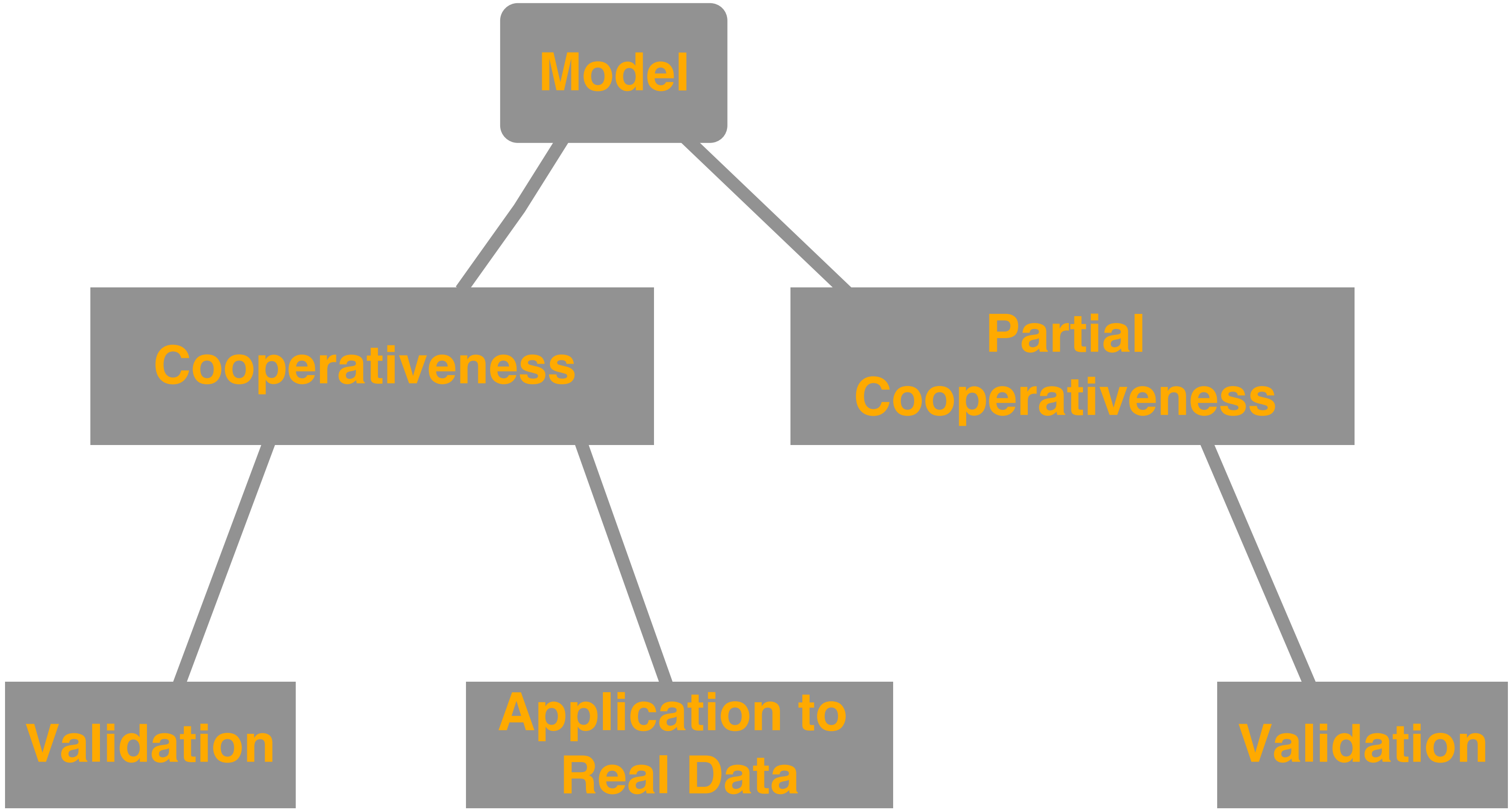
- Neuroscience
  - EEG, MEG, LFP
- Population dynamics
  - migration
- Physiology
  - heart-lungs-brain

Issues when approaching this problem:

» Accessibility of the network

» Amount and quality of data

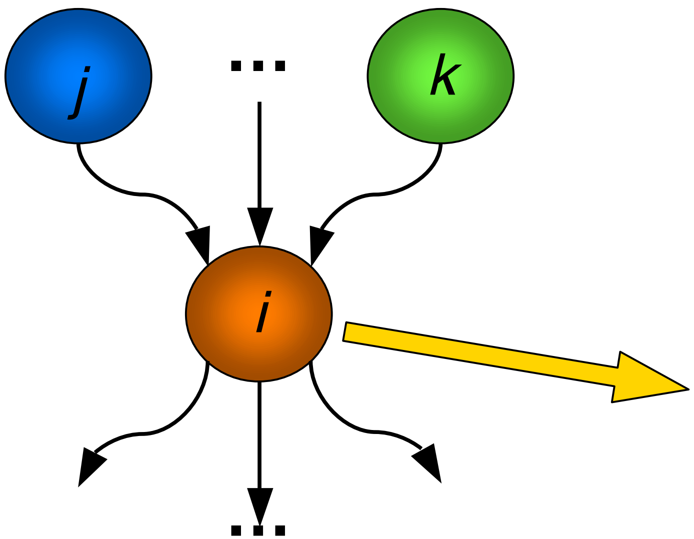
Outline


Model
Cooperativeness
Partial Cooperativeness
Validation
Application to Real Data
Validation

Model & Working Hypotheses

Reference model

- heterogeneous network of dynamical sub-systems



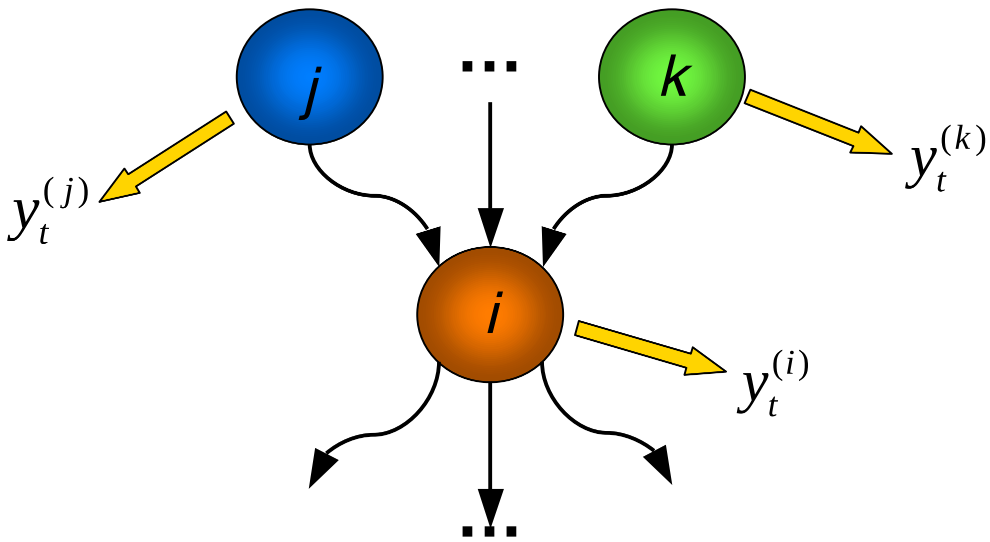
$$\begin{cases} \dfrac{d\Theta}{dt} = F(\Theta) + \eta & \text{(Continuous time)} \\ Y_t = G(\Theta_t) + \nu_t & \text{(Discrete time)} \end{cases}$$

$$y_t^{(i)} = g^{(i)}(\Theta_t^{(i)}) + \nu_t^{(i)}$$

Cooperativeness

Assessment from multivariate measurements

Ex.: cooperativeness of the ensemble



Bivariate time series

» Four approaches
- Linear
- Information Theory
- State-Space
- Phase

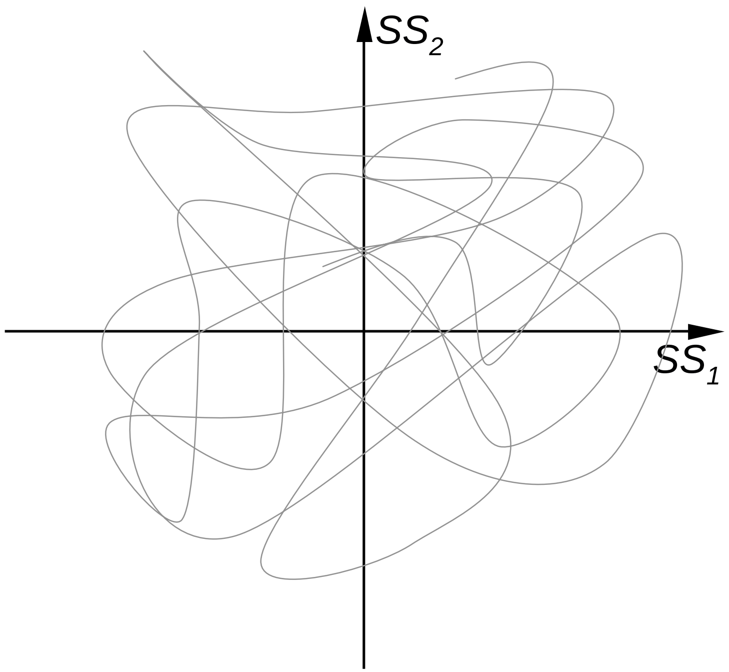
Multivariate time series

- Information theory
- Phase

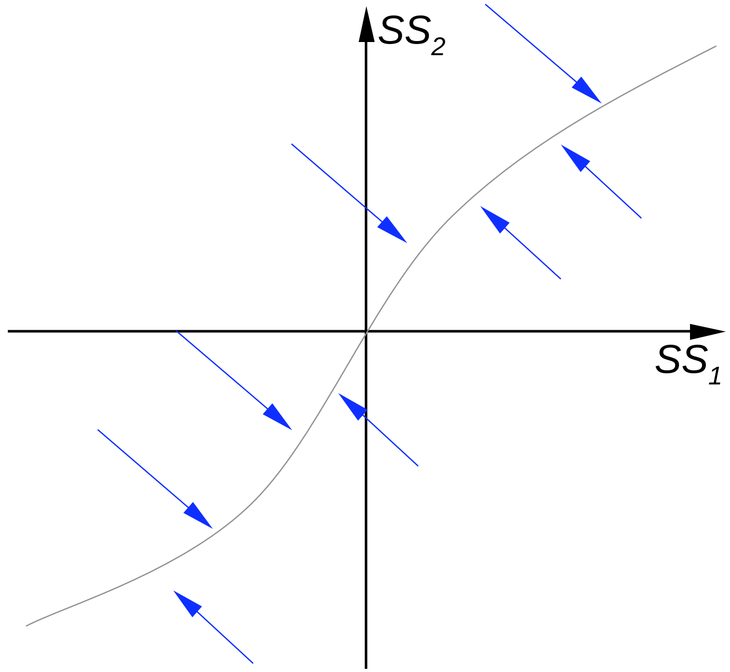
Cooperativeness

I synch therefore I shrink

Case 1: asynchronous case



Case 2: synchronization case



» Assessing the shrinking by measuring the relative importance of the embedded sub-spaces

Assessing Cooperativeness

I synch therefore I shrink

» Collect the measurements

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} \quad t = 1, \ldots, L$$

» Pearson-like correlation matrix

$$C = \frac{1}{L-1} \sum_{t=1}^{L} Y_t^T Y_t$$

λ_1', λ_2' normalized eigenvalues of *C*

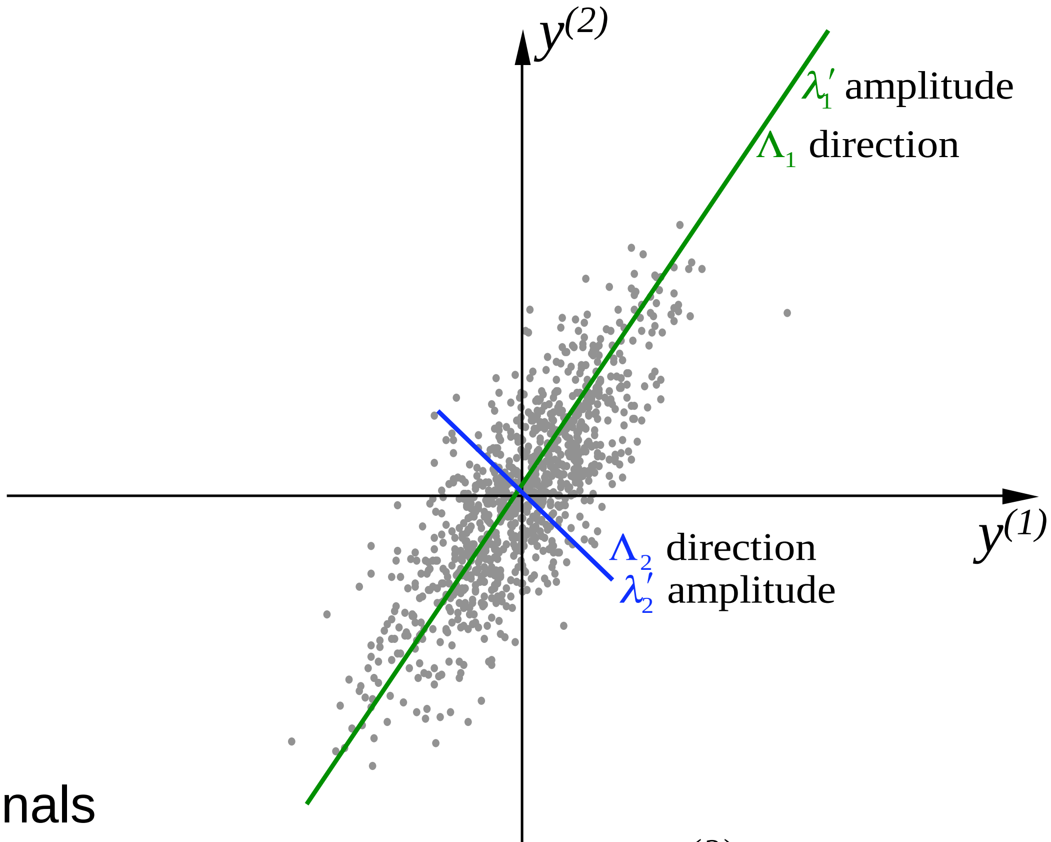
Λ_1, Λ_2 eigenvectors of *C*

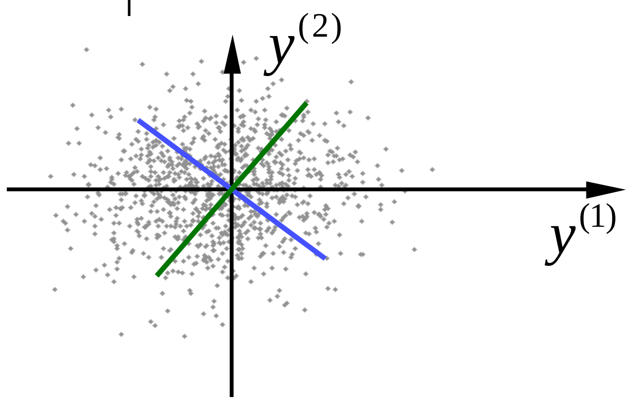
Case 1: completely correlated signals

$$\lambda_1' \to 1 \quad \lambda_2' \to 0$$

Case 2: uncorrelated signals

$$\lambda_1' \to \frac{1}{2} \quad \lambda_2' \to \frac{1}{2}$$





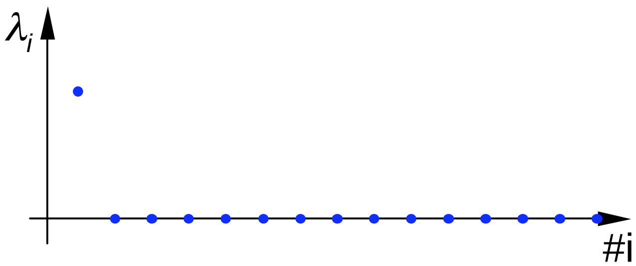
Assessing Cooperativeness

S estimator

In the generic (*M*-dimensional) case

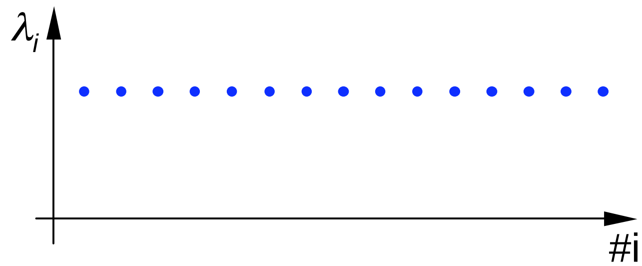
Case 1: completely correlated signals

$$\lambda_i' = \left[1\ 0\ 0 \cdots 0\right]$$



Case 2: uncorrelated signals

$$\lambda_i' = \left[\frac{1}{M}\ \frac{1}{M} \cdots \frac{1}{M}\ \frac{1}{M}\right]$$



The difference is the distribution of eigenvalues

- Entropy to assess it

$$S = 1 + \frac{\sum_{i=1}^{M} \lambda_i' log\left(\lambda_i'\right)}{log\left(M\right)} = \begin{cases} 1 & completely\ correlated\ signals \\ 0 & uncorrelated\ signals \end{cases}$$

Assessing Cooperativeness

S estimator

» A (scalar) signal for each sub-system

$$C = \begin{bmatrix} 1 & \cdots & \rho^{(1,i)} & \cdots & \rho^{(1,M)} \\ \vdots & \ddots & & & \vdots \\ \rho^{(1,i)} & \cdots & 1 & \cdots & \rho^{(i,M)} \\ \vdots & & & \ddots & \vdots \\ \rho^{(1,M)} & \cdots & \rho^{(i,M)} & \cdots & 1 \end{bmatrix}$$

$\rho^{(i,j)}$ is a scalar

» A delay-embedded signal for each sub-system

$$C = \begin{bmatrix} C^{(1,1)} & \cdots & C^{(1,i)} & \cdots & C^{(1,M)} \\ \vdots & \ddots & & & \vdots \\ C^{(1,i)^T} & & C^{(i,i)} & & C^{(i,M)} \\ \vdots & & & \ddots & \vdots \\ C^{(1,M)^T} & \cdots & C^{(i,M)^T} & \cdots & C^{(M,M)} \end{bmatrix} \rightarrow \begin{bmatrix} I & \cdots & R^{(1,i)} & \cdots & R^{(1,M)} \\ \vdots & \ddots & & & \vdots \\ R^{(1,i)^T} & & I & & R^{(i,M)} \\ \vdots & & & \ddots & \vdots \\ R^{(1,M)^T} & \cdots & R^{(i,M)^T} & \cdots & I \end{bmatrix}$$

$C^{(i,j)}$ is a matrix

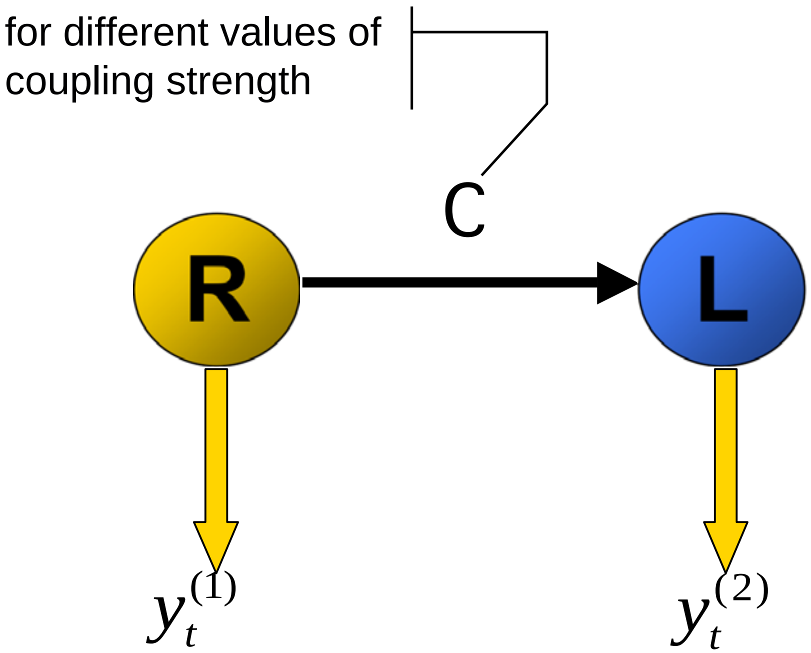
S estimator

Numerical Validation

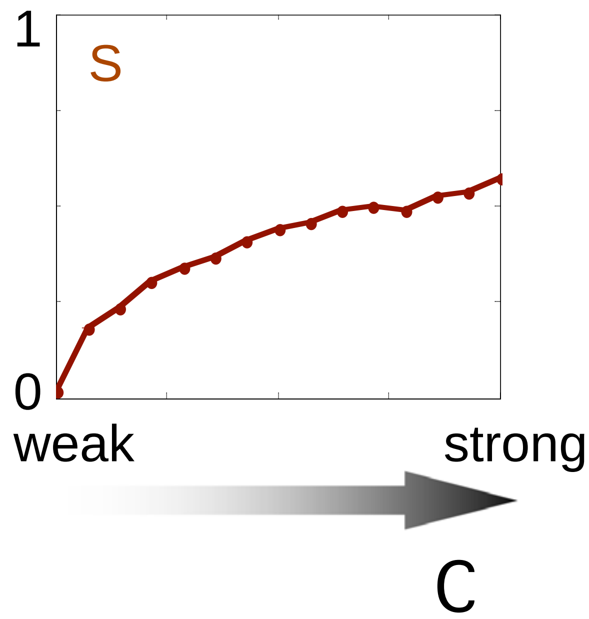
S measures the degree of cooperativeness in a network:

1. Coupling strength
2. Average connectivity degree

1) Setup



1) Result



S estimator

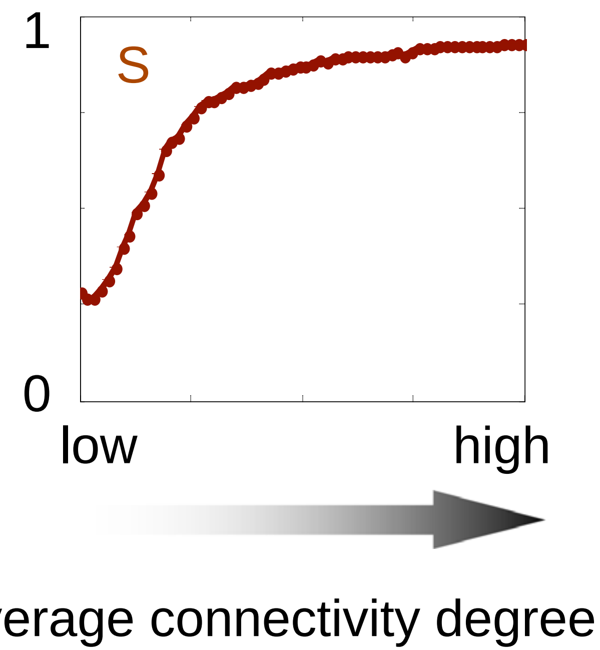
Numerical Validation

2) Setup:

» Network of 128 sub-systems (128 time series)

» For different values of average connectivity degree

2) Result:



» Similar results with different graph topologies

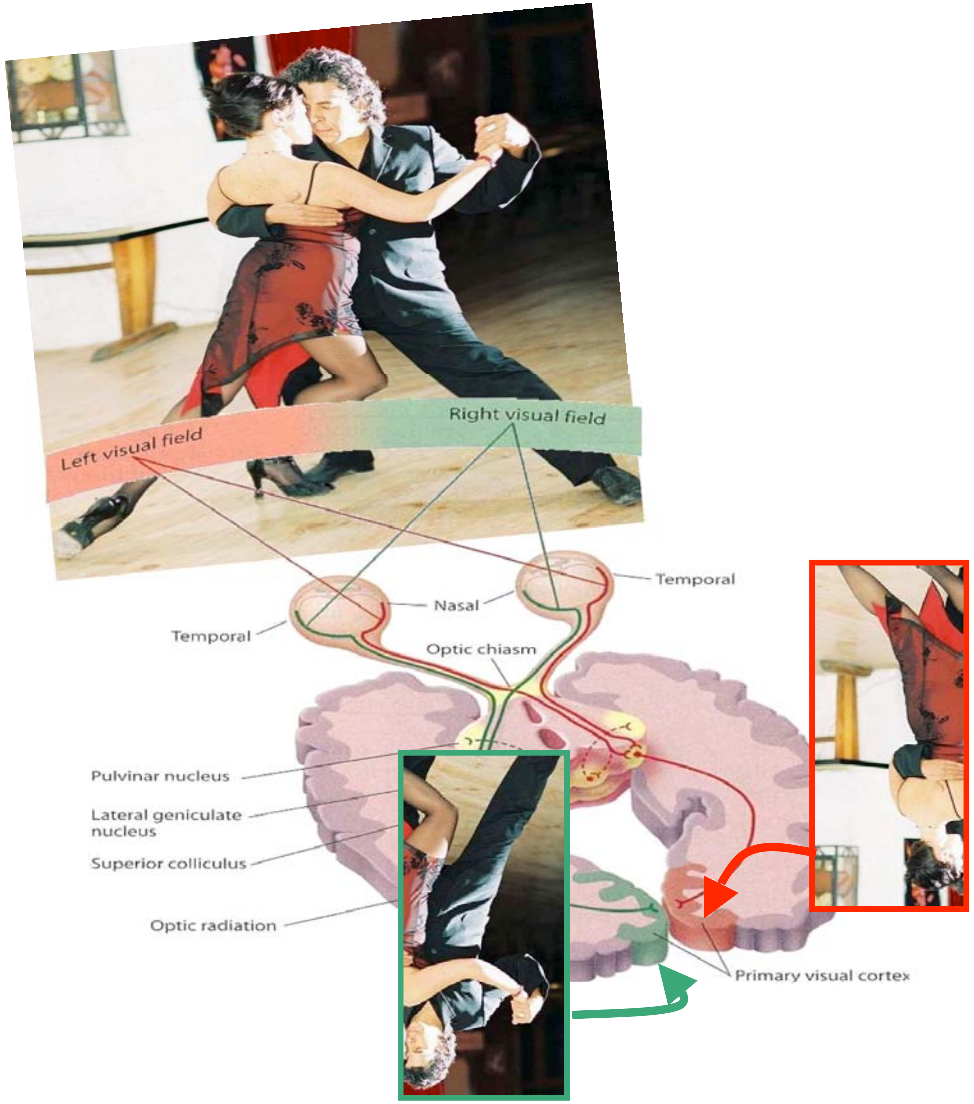
S estimator

Applications to real data

Experimental Paradigm: the split of visual pathway

» Manifestation of neuronal cooperativeness:

- EEGs (macro)
- LFPs (meso)

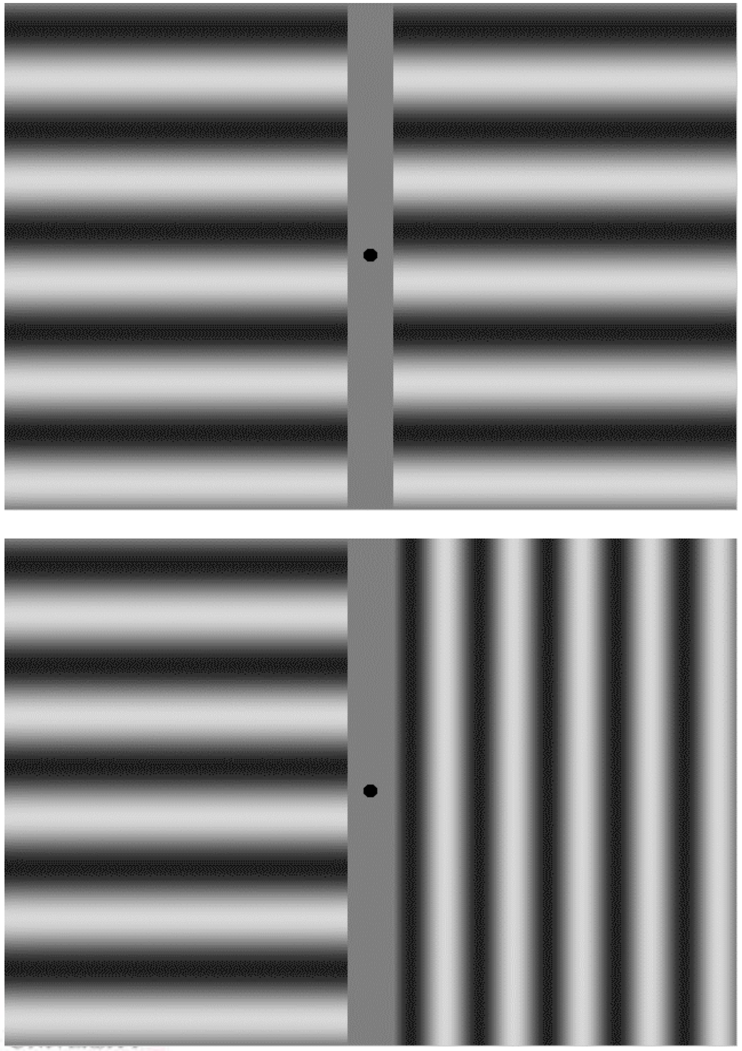


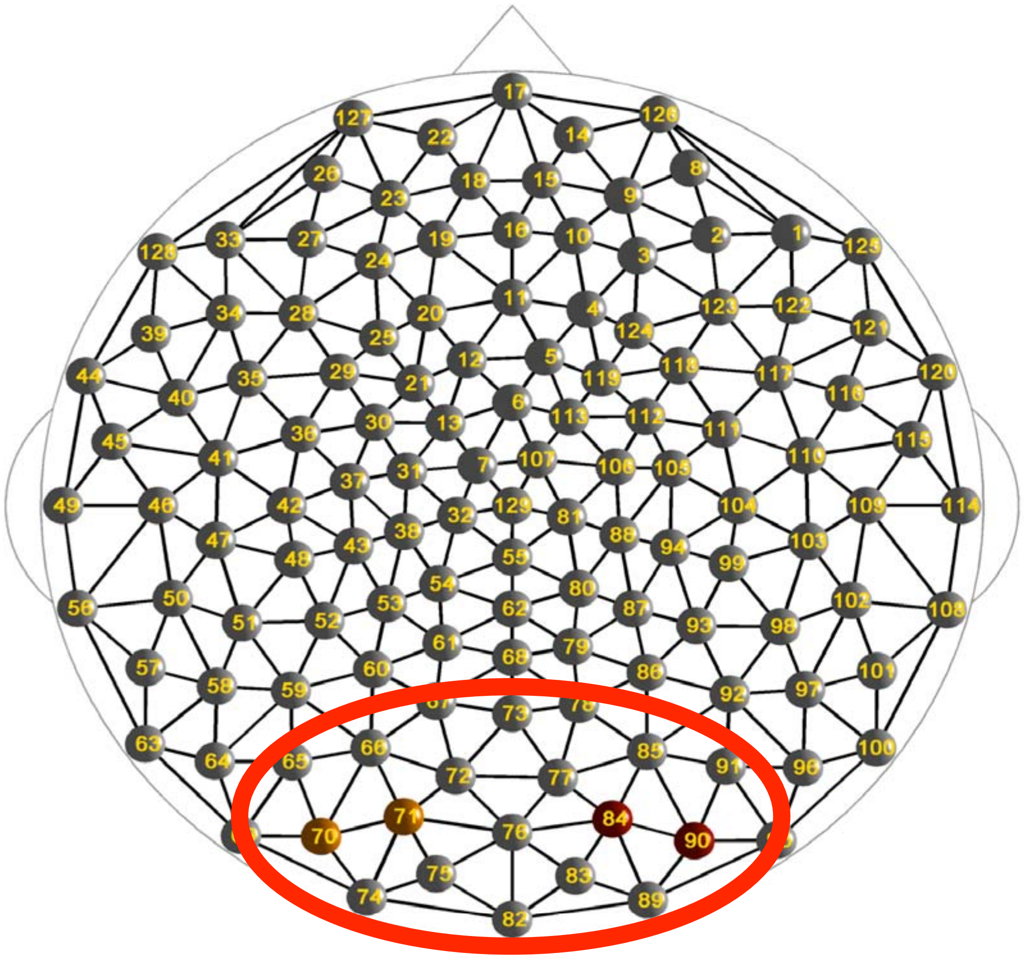
S estimator

Application to real data - EEG

Paradigm: the split of visual pathway

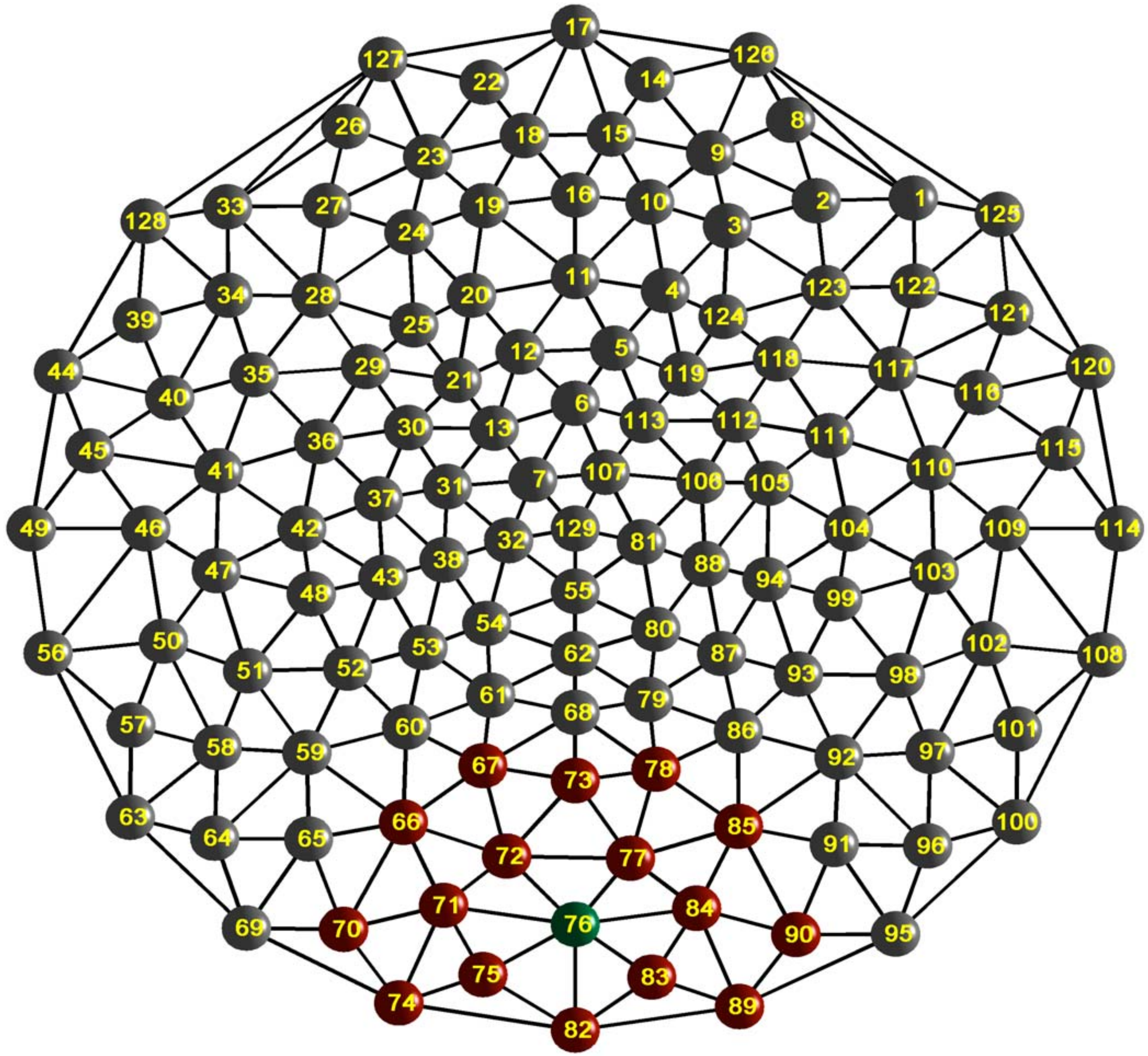
- Assessing inter-hemispheric cooperation





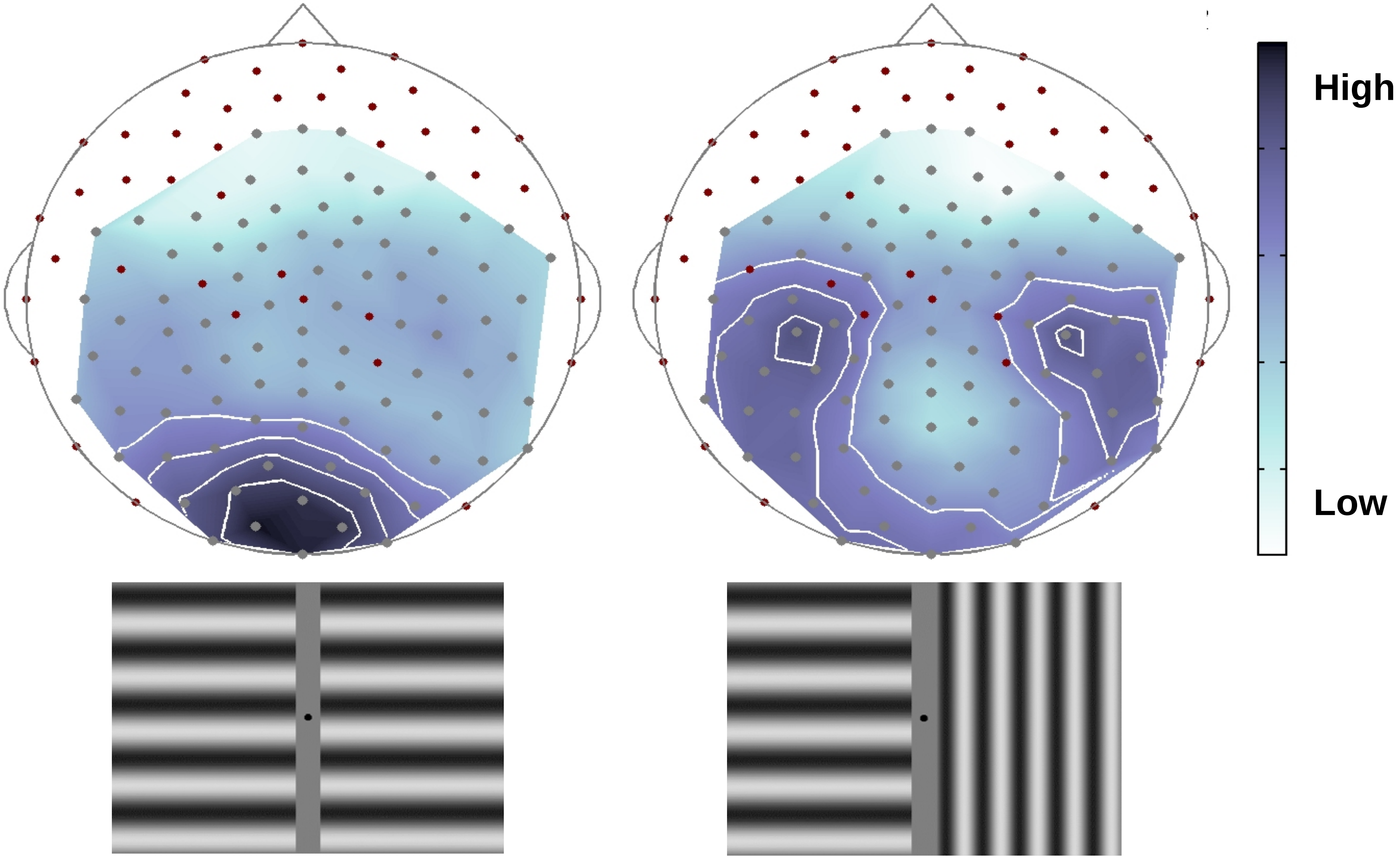
S estimator

Extracting cooperativeness topography in EEG data



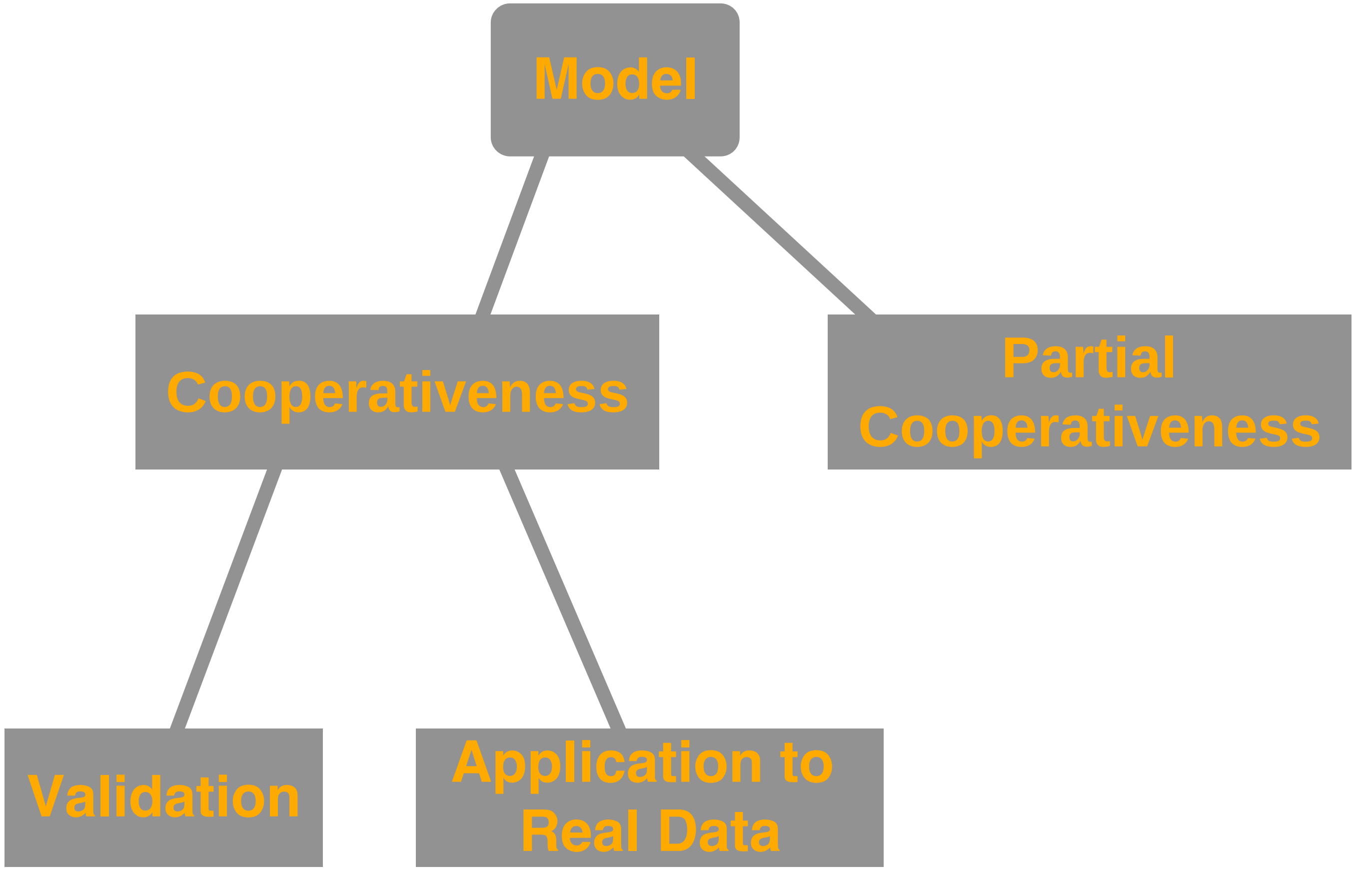
S estimator

Result on human EEG data



Outline

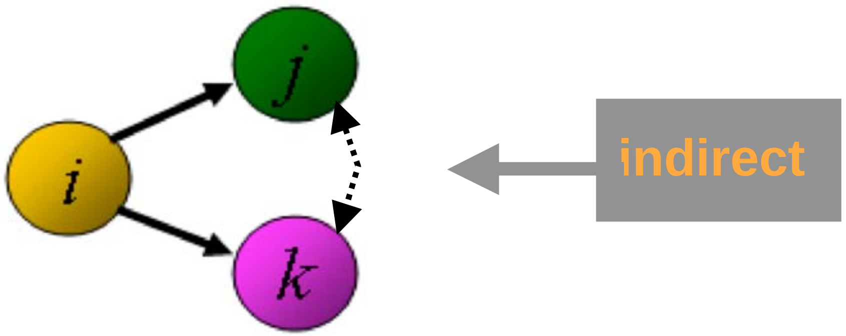
On the way



Partial Cooperativeness

Motivation

» Direct vs. Indirect



Solution:

» Marginalization of the knowledge of thirds

Partial Cooperativeness

Partial S estimator

» S estimator

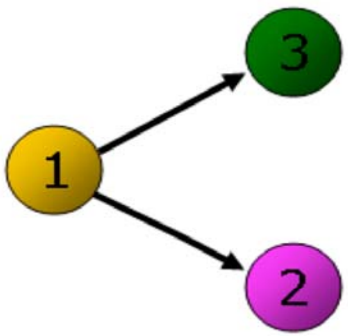
- Correlation matrix

$$C=\begin{bmatrix} 1 & \cdots & \rho^{(1,i)} & \cdots & \rho^{(1,M)} \\ \vdots & \ddots & & & \vdots \\ \rho^{(1,i)} & \cdots & 1 & \cdots & \rho^{(i,M)} \\ \vdots & & & \ddots & \vdots \\ \rho^{(1,M)} & \cdots & \rho^{(i,M)} & \cdots & 1 \end{bmatrix}$$

- Entropy-like statistic

$$S=1+\frac{\sum_{i=1}^{M}\lambda_i' log\left(\lambda_i'\right)}{log\left(M\right)}$$

» Partial S estimator



$$C=\begin{bmatrix} 1 & \rho^{(1,2)} & \rho^{(1,3)} \\ \rho^{(1,2)} & 1 & \rho^{(2,3)} \\ \rho^{(1,3)} & \rho^{(2,3)} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \rho^{(2,3)} \\ \rho^{(2,3)} & 1 \end{bmatrix}-\begin{bmatrix} \rho^{(1,2)} \\ \rho^{(1,3)} \end{bmatrix}\begin{bmatrix} \rho^{(1,2)} & \rho^{(1,3)} \end{bmatrix}$$

Partial Correlation matrix

Scalability to large systems

Inferring "0" – "1" interactions

Setup:

» Network of 128 sub-systems
» 256 interactions

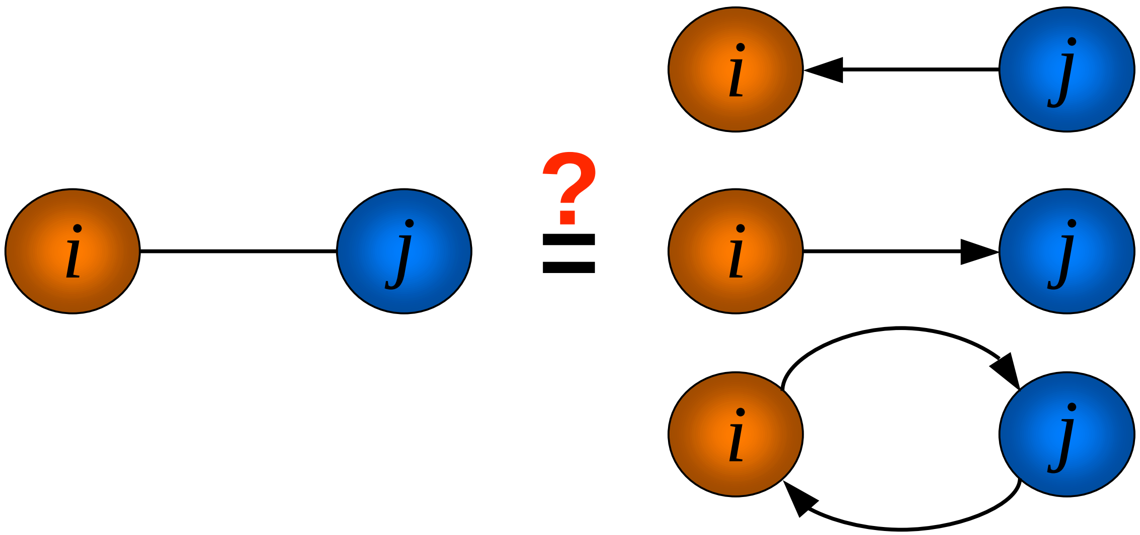
Result:

- 85%   true "1"
- 99%   true "0"

» Sensitiveness upon amount of data and observational noise intensity

Assessing directionality

» Direct vs. Indirect and strength



Addressing:

» Direct vs. Indirect, strength and *direction*

A modeling approach

Further hypothesis:

- weak coupling

Working principle

» identify self model for sub-system i

- only use $y^{(i)}$
- nonlinear model

» cross relates the $y^{(j)}|_{j \neq i}$

- to the modelling residuals at system i
- strength of interaction from j to i, for all $j \neq i$

» Algorithmic setup tuned to the specific application

» Model of local behavior

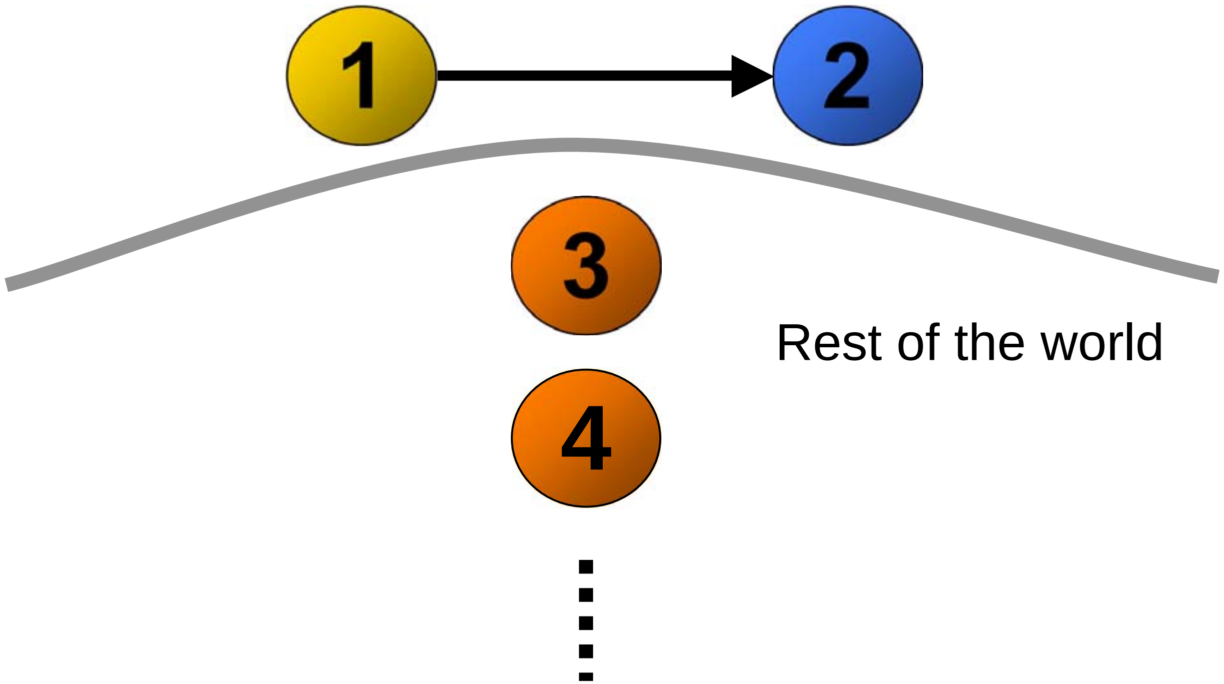
A modeling approach

Numerical Validation

» Proved able to discern direct vs. indirect interactions

» Directionality assessed in 2 sub-systems bi-directionally coupled

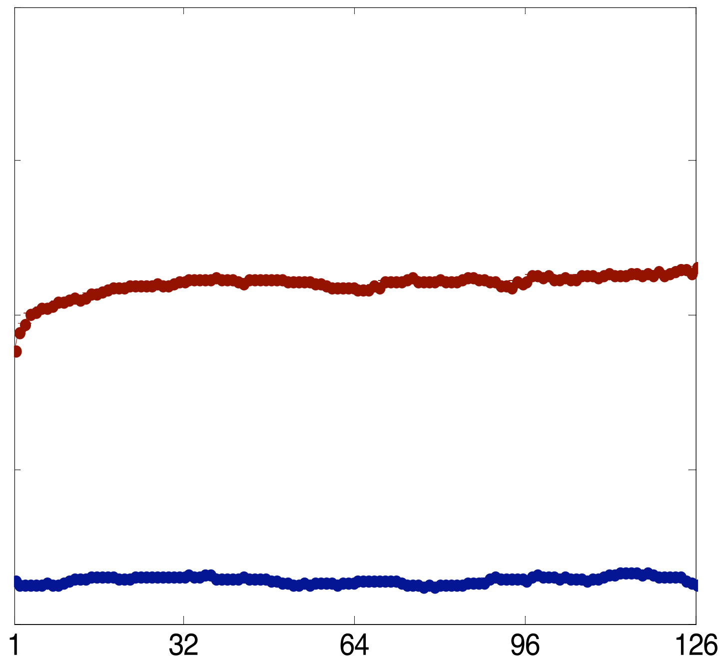
Assessing **saliency**:

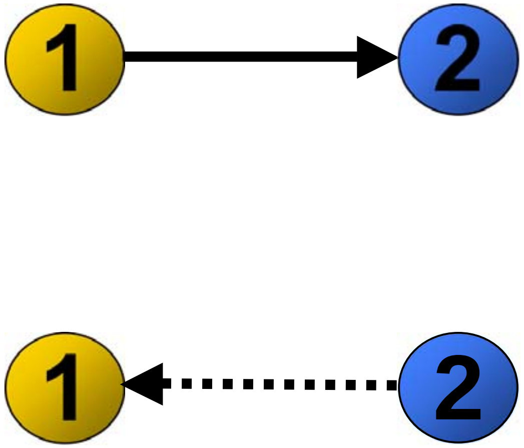


A modeling approach

Numerical Validation - Saliency

Result:





# of third sub-systems

» Sensitiveness upon amount of data

Conclusions

» **Methods to infer cooperativeness from multivariate data**

- S estimator
- Partial S estimator
- Modeling approach
- Numerical validation

» **Application to brain data**

- Stimulus-dependent modulation of neuronal cooperation
- New insight about the role of inter-hemispheric interactions (not shown)

Free download at:
http://library.epfl.ch/theses/?nr=3651

Outlook

Ongoing and future work

» **Clinical Neuroscience**

- Schizophrenia
- Alzheimer

» **Methodological development**

- Decomposing mixed signals (no ICA)