Evolutionary Games on Networks

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Cooperative behavior among selfish individuals

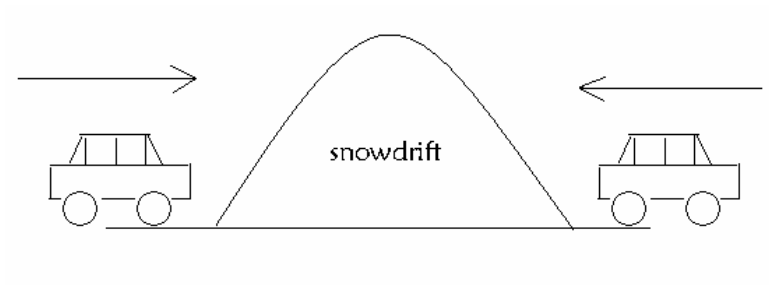
Evolutionary games

- Prisoner's dilemma game (PDG)
- Snowdrift game (SG)
- Repeated games on lattices

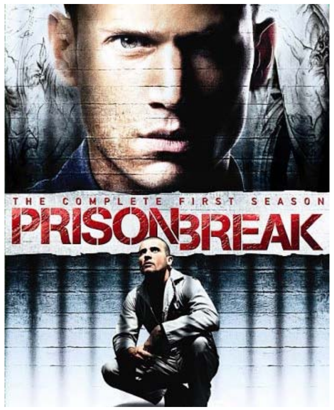
Evolutionary games on complex networks

- Complex networks
- Review of games on complex networks
- Our works

Introduction of PD and SG game



Prisoner's Dilemma game **Snowdrift game**



《Prison Break》

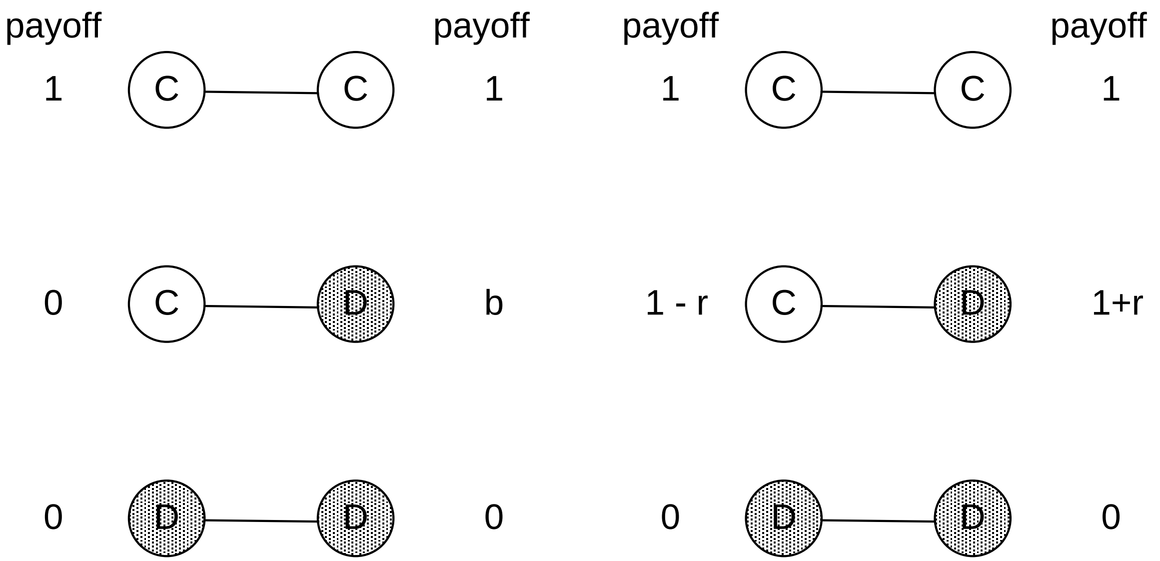
	C	*D*
C	***R***	***S***
D	***T***	***P***

T > R > P > S ***T > R > S > P***

Rescaled Payoff Matrix

Prisoner's Dilemma

Snowdrift Game



1<b<2

0<r<1

Review of some fundamental works

LETTERS TO NATURE

Evolutionary games and spatial chaos

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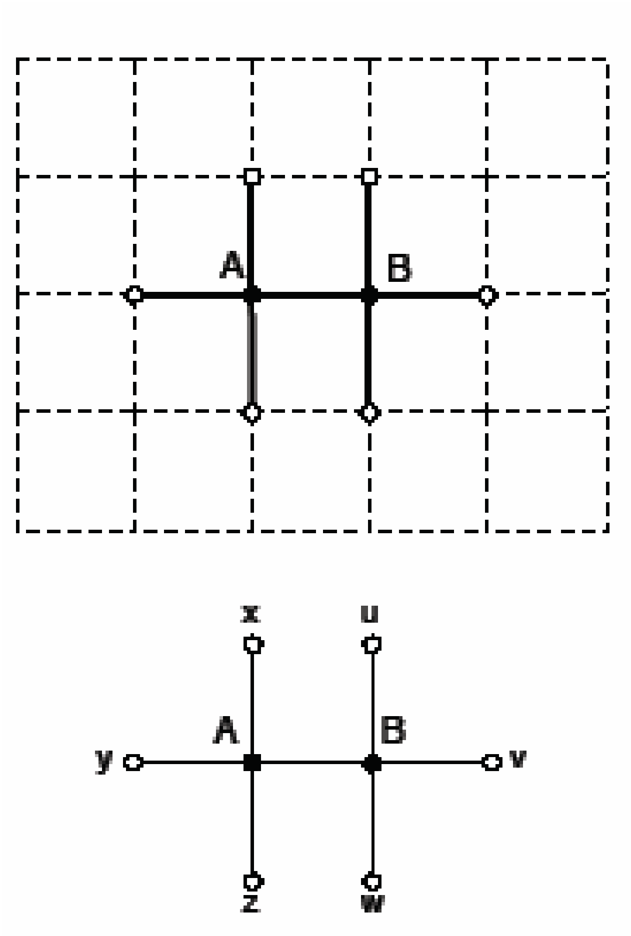
Department of Zoology, University of Oxford, South Parks Road, Oxford OX1 3PS, UK

Prisoner's dilemma game



1 Spatial structure

2 Repeated game

3 Strategy updating

4 Cooperation can emerges

Model rules

- Each pair of connected individuals play the game simultaneously.
- Each node will be occupied by the highest-score individual among its neighbors and itself.
- Repeat above steps.

Results of evolutionary patterns

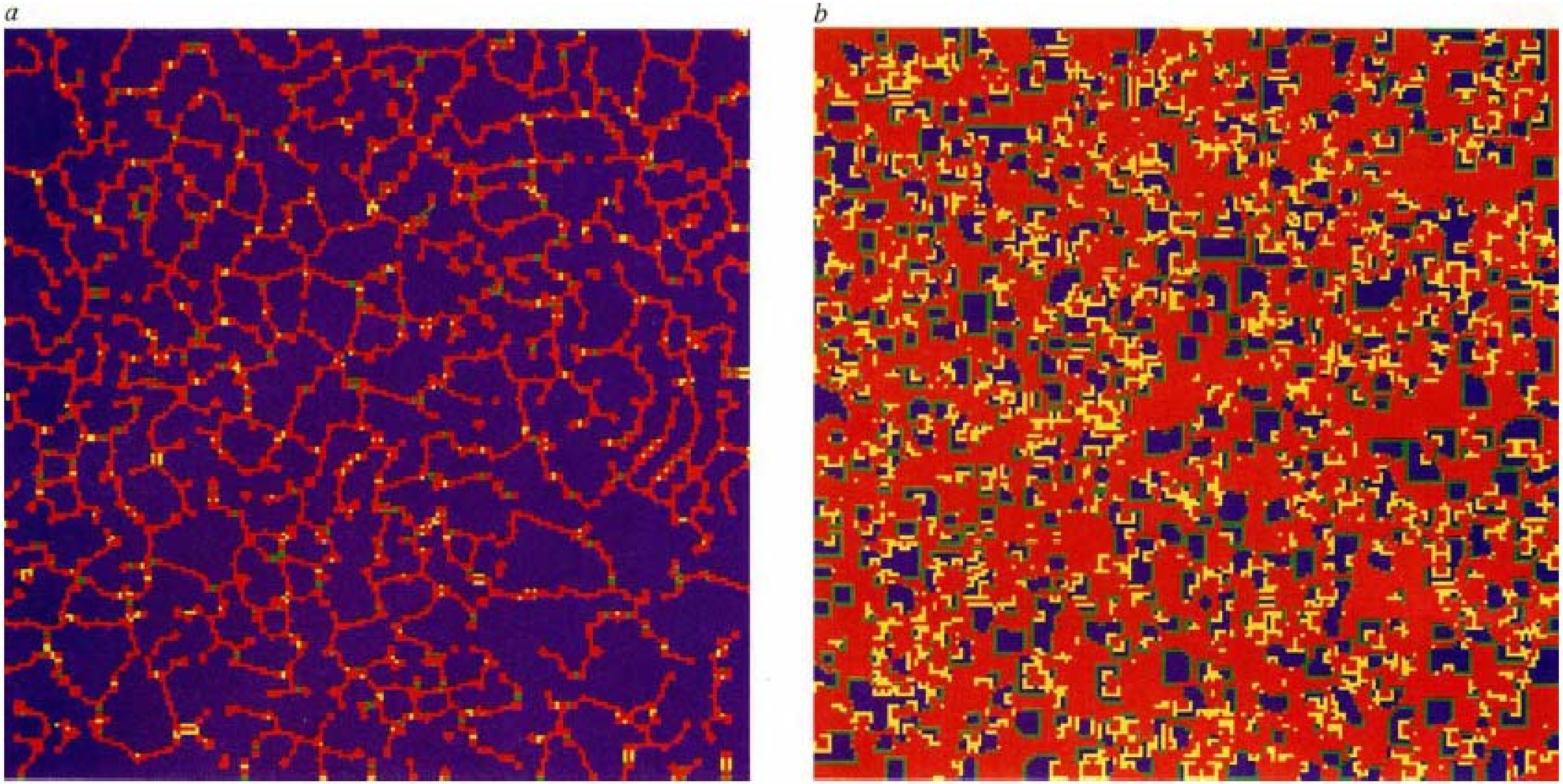


FIG. 1 The spatial Prisoners' Dilemma can generate a large variety of qualitatively different patterns, depending on the magnitude of the parameter, *b*, which represents the advantage for defectors. This figure shows two examples. Both simulations are performed on a 200 × 200 square lattice with fixed boundary conditions, and start with the same random initial configuration with 10% defectors (and 90% cooperators). The asymptotic pattern after 200 generations is shown. The colour coding is as follows: blue represents a cooperator (C) that was already a C in the preceding generation; red is a defector (D) following a D; yellow a D following a C; green a C following a D. *a*, An irregular, but static pattern (mainly of interlaced networks) emerges if $1.75 < b < 1.8$. The equilibrium frequency of C depends on the initial conditions, but is usually between 0.7 and 0.95. For lower *b* values (provided $b > \frac{9}{8}$), D persists as line fragments less connected than shown here, or as scattered small oscillators ('D-blinkers'). *b*, Spatial chaos characterizes the region $1.8 < b < 2$. The large proportion of yellow and green indicates many changes from one generation to the next. Here, as outlined in the text, 2 × 2 or bigger C clusters can invade D regions, and vice versa. C and D coexist indefinitely in a chaotically shifting balance, with the frequency of C being (almost) completely independent of the initial conditions at ~0.318.

Spatial patterns from specific initial state: a central defector in the cooperator sea

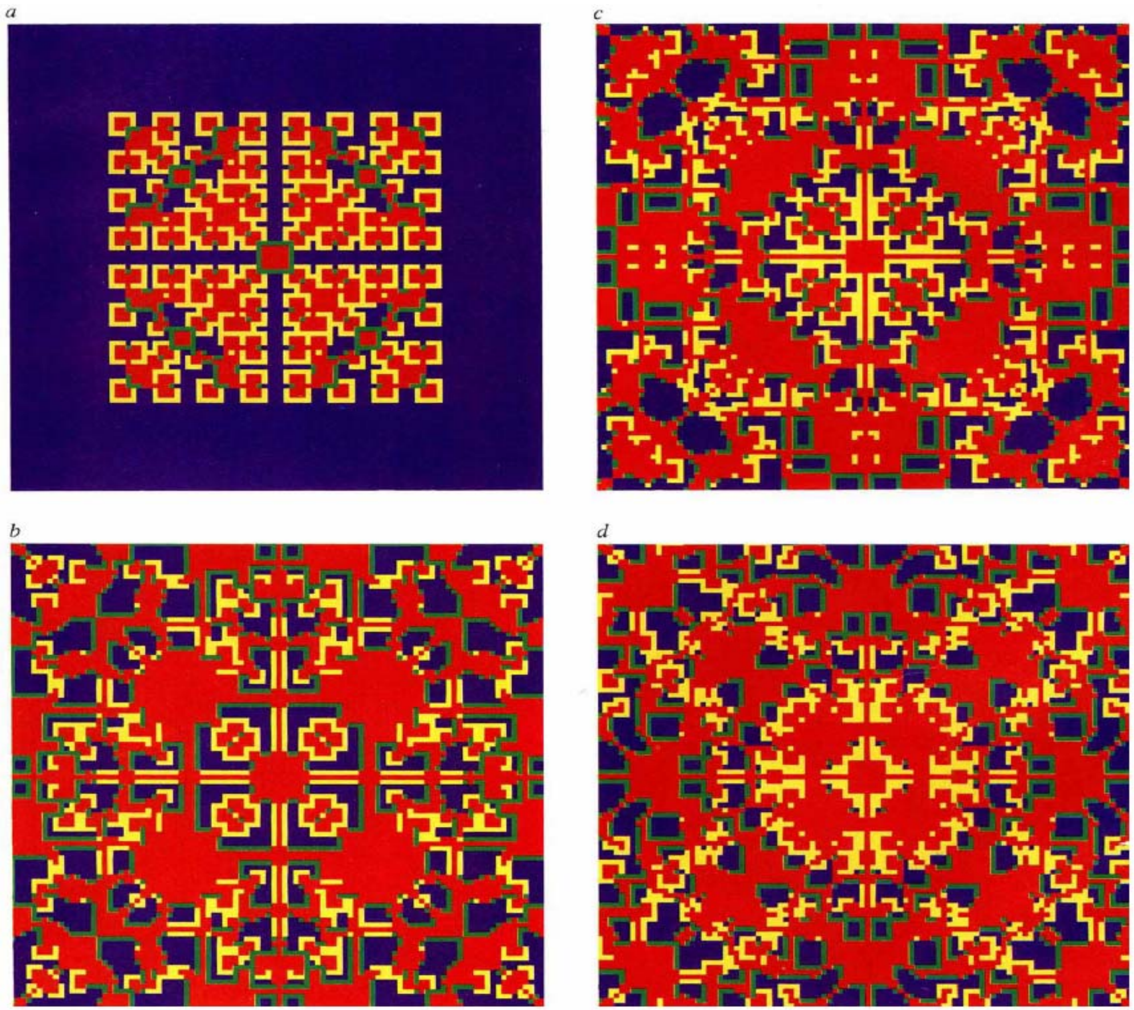


FIG. 3 Spatial games can generate an 'evolutionary kaleidoscope'. This simulation is started with a single D at the centre of a 99×99 square-lattice world of C with fixed boundary conditions. Again $1.8 < b < 2$. This generates an (almost) infinite sequence of different patterns. The initial symmetry is always maintained, because the rules of the game are symmetrical. The frequency of C oscillates (chaotically) around a time average of 12 log 2–8 (of course). *a*, Generation $t = 30$; *b*, $t = 217$; *c*, $t = 219$; *d*, $t = 221$.

letters to nature

Spatial structure often inhibits the evolution of cooperation in the snowdrift game

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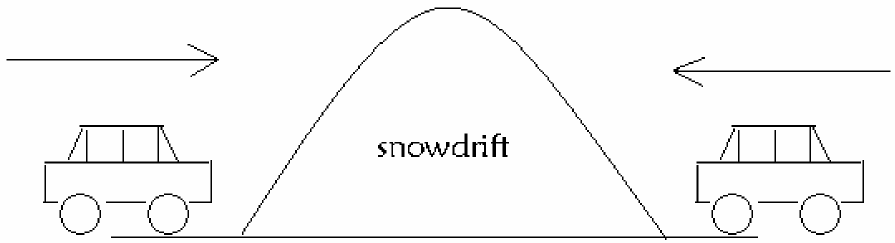


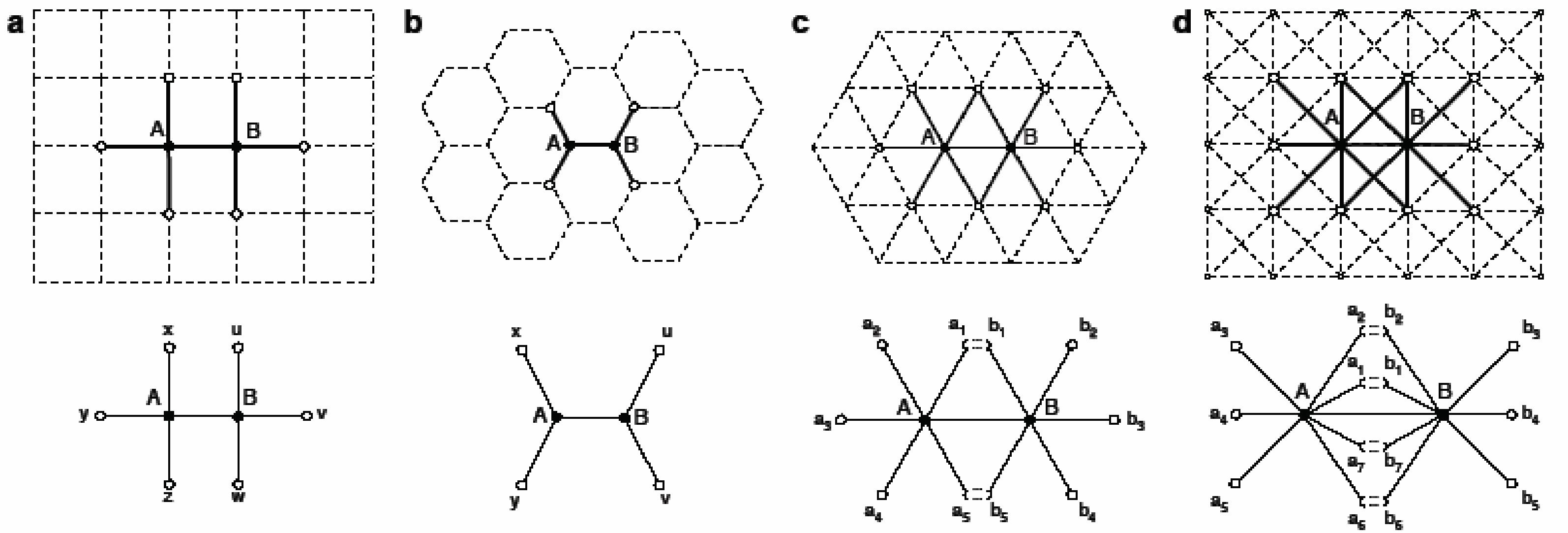
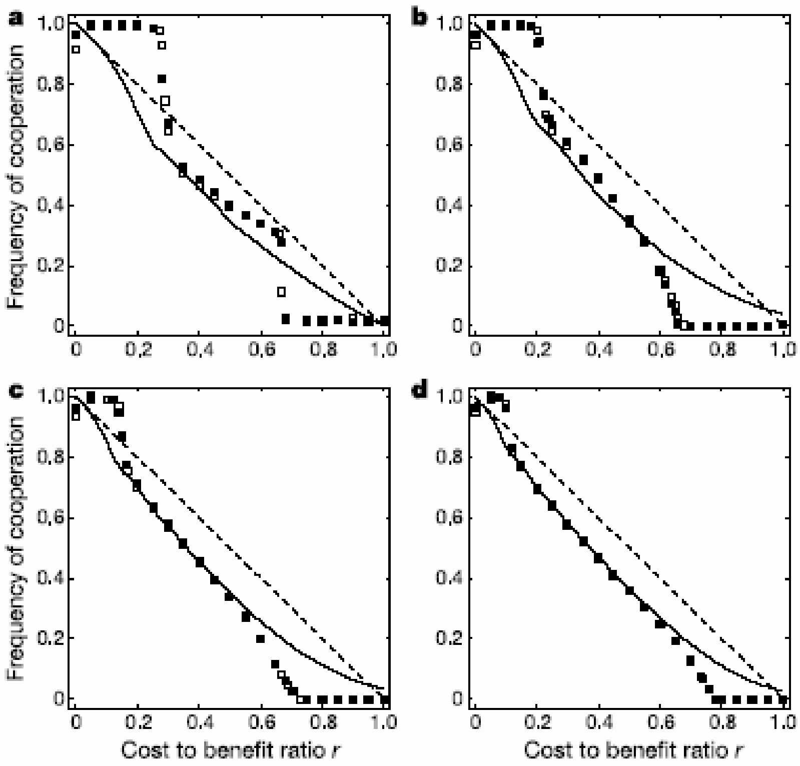


Figure 1: Four lattice configurations (top row) and the corresponding schemes used for the pair approximation with focal sites A and B (bottom row). These schemes are used to determine changes in the pair configuration probabilities $p_{A,B\to B,B}$. **a** square lattice with $N = 4$ neighbours, **b** triangular lattice ($N = 3$), **c** hexagonal lattice ($N = 6$) and **d** square lattice ($N = 8$). Note that on hexagonal and square ($N = 8$) lattices, the edges from A and B to their common neighbours are considered to be independent, i.e., all corrections arising from loops are neglected.



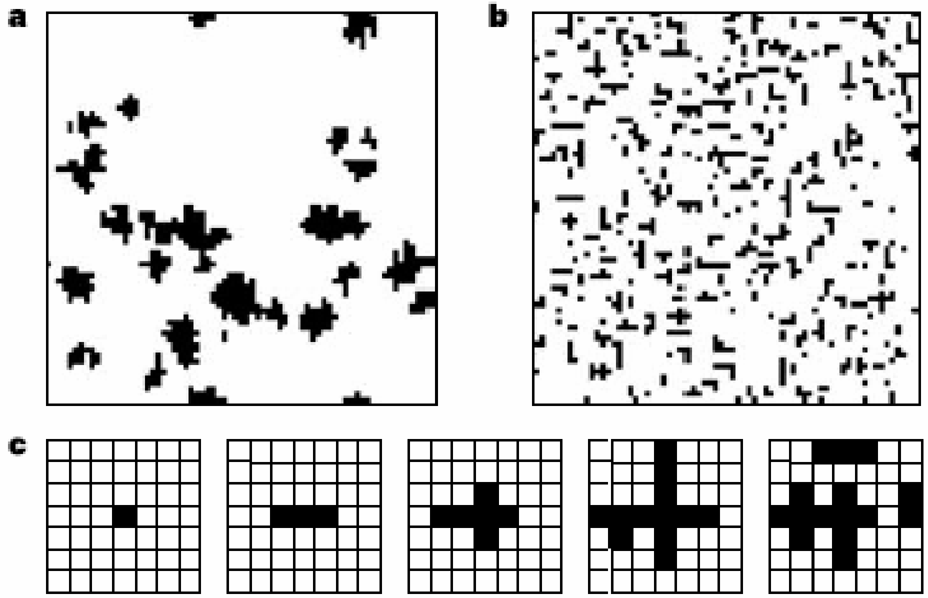


Figure 2 Snapshots of equilibrium configurations of cooperators (black) and defectors (white) in the spatial Prisoner's Dilemma and spatial snowdrift game on a square lattice with $N = 4$ neighbours near the extinction threshold of cooperators. **a**, In the Prisoner's Dilemma, cooperators survive by forming compact clusters ($R = 1$, $T = 1.07$, $S = -0.07$, $P = 0$). **b**, In the corresponding snowdrift game, cooperators are spread out, forming many small and isolated patches ($r = 0.62$; that is, $R = 1$, $T = 1.62$, $S = 0.38$, $P = 0$). This result also holds for other lattice structures (not shown). **c**, Microscopic pattern formation in the spatial snowdrift game. An isolated cooperator can grow into a row of cooperators and then form cross-like structures; however, cooperators cannot expand to compact clusters because the payoff structure protects the defectors in the corners. Eventually, cooperators form a dendritic skeleton. Occasionally, dendrites break off to form new seeds.

Complex networks

Definitions

- Degree: the number of neighbors of a node.
- Average distance:

$$d = \frac{1}{N(N-1)} \sum_{ij} d_{ij}$$

- Clustering coefficient: density of triangular structures

$$C(k) = \frac{1}{NP(k)} \sum_{i/k_i=k} c_i, \quad c_i = \frac{1}{k_i(k_i-1)} \sum_{ij} a_{i,j} a_{i,h} a_{j,h}$$

Common properties of complex networks

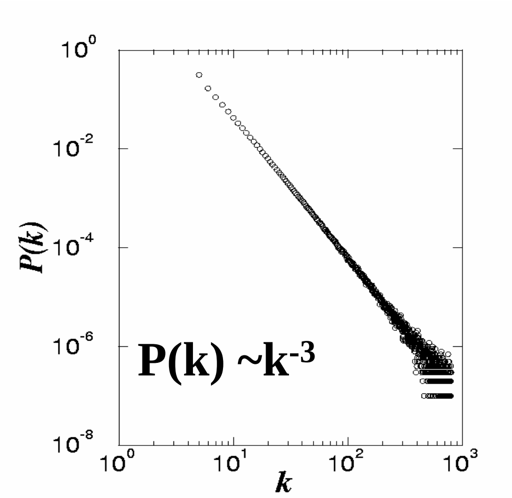
- Small-world property

  1 Short average distance

  2 High clustering coefficient

- Scale-free property

  Power-law degree distribution



Barabási-Albert model

Real networks

- Small-world networks

  Many collaboration networks, power grid networks, train networks…

- Scale-free networks

  The Internet, WWW, airport networks, citation networks, protein-protein interaction networks, brain function networks…

Games on complex networks

PHYSICAL REVIEW E **69**, 036107 (2004)

Cooperation for volunteering and partially random partnerships

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(Received 3 July 2003; published 19 March 2004)

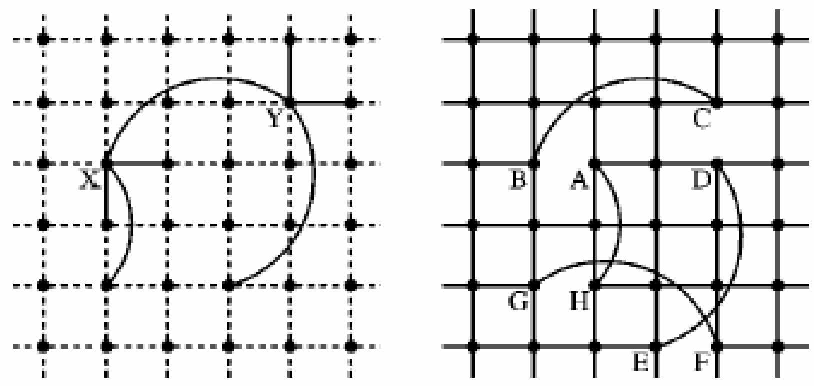
Public goods game

$X\backslash Y$	D	C	L
D	$0\backslash 0$	$b\backslash c$	$\sigma\backslash\sigma$
C	$c\backslash b$	$1\backslash 1$	$\sigma\backslash\sigma$
L	$\sigma\backslash\sigma$	$\sigma\backslash\sigma$	$\sigma\backslash\sigma$

Updating rule

$$W = \frac{1}{1+\exp[(M_X - M_Y)/K]},$$

Small-world networks



Adding links

Rewiring links

Individual *X* randomly selects a neighbor *Y*, then calculates the probability *W*

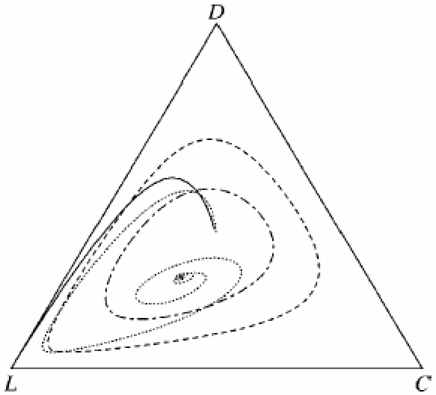


FIG. 2. Trajectories on ternary diagram if the system is started from a random initial state for $b=1.5$. The solid line shows the system where all the coplayers are chosen randomly ($P=1$ or mean-field limit). Evolution on the square lattice ($P=Q=0$) is illustrated by the dotted line. For a weak annealed ($P=0.03$) or quenched ($Q=0.03$) randomness the system tends toward a limit cycle indicated by the dashed and dash-dotted lines.

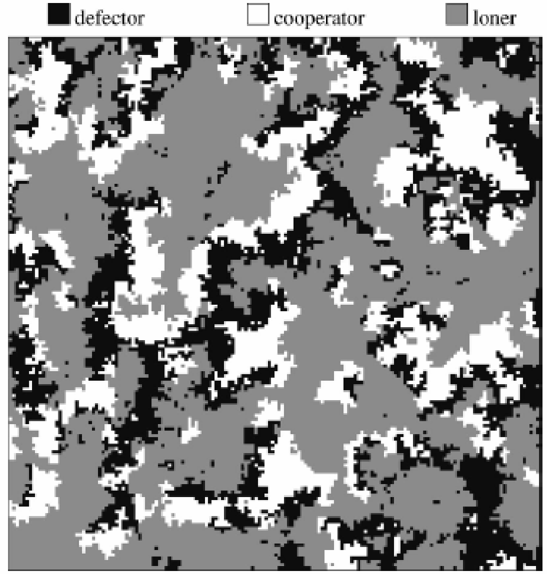


FIG. 3. A typical snapshot on the distribution of the three strategies on a square lattice for $b=1.5$. The different gray scales of

d at the top.

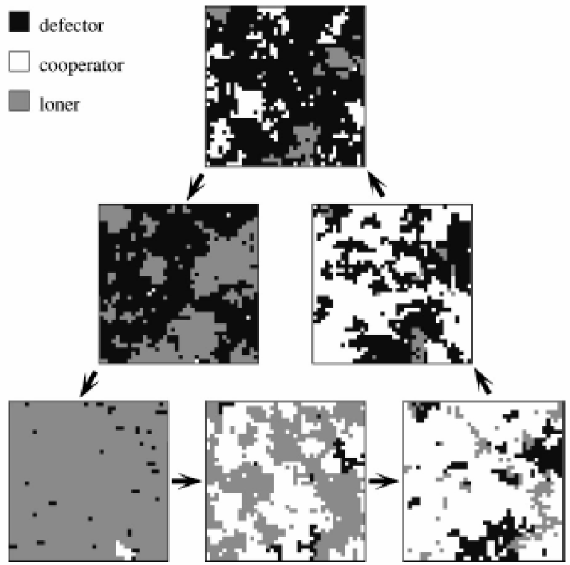


FIG. 4. Typical subsequent patterns occurring along the limit cycles shown in Fig. 2. These snapshots are small parts (40×40 sites) of a larger "homogeneous" phase.

PRL 95, 098104 (2005) PHYSICAL REVIEW LETTERS week ending 26 AUGUST 2005

Scale-Free Networks Provide a Unifying Framework for the Emergence of Cooperation

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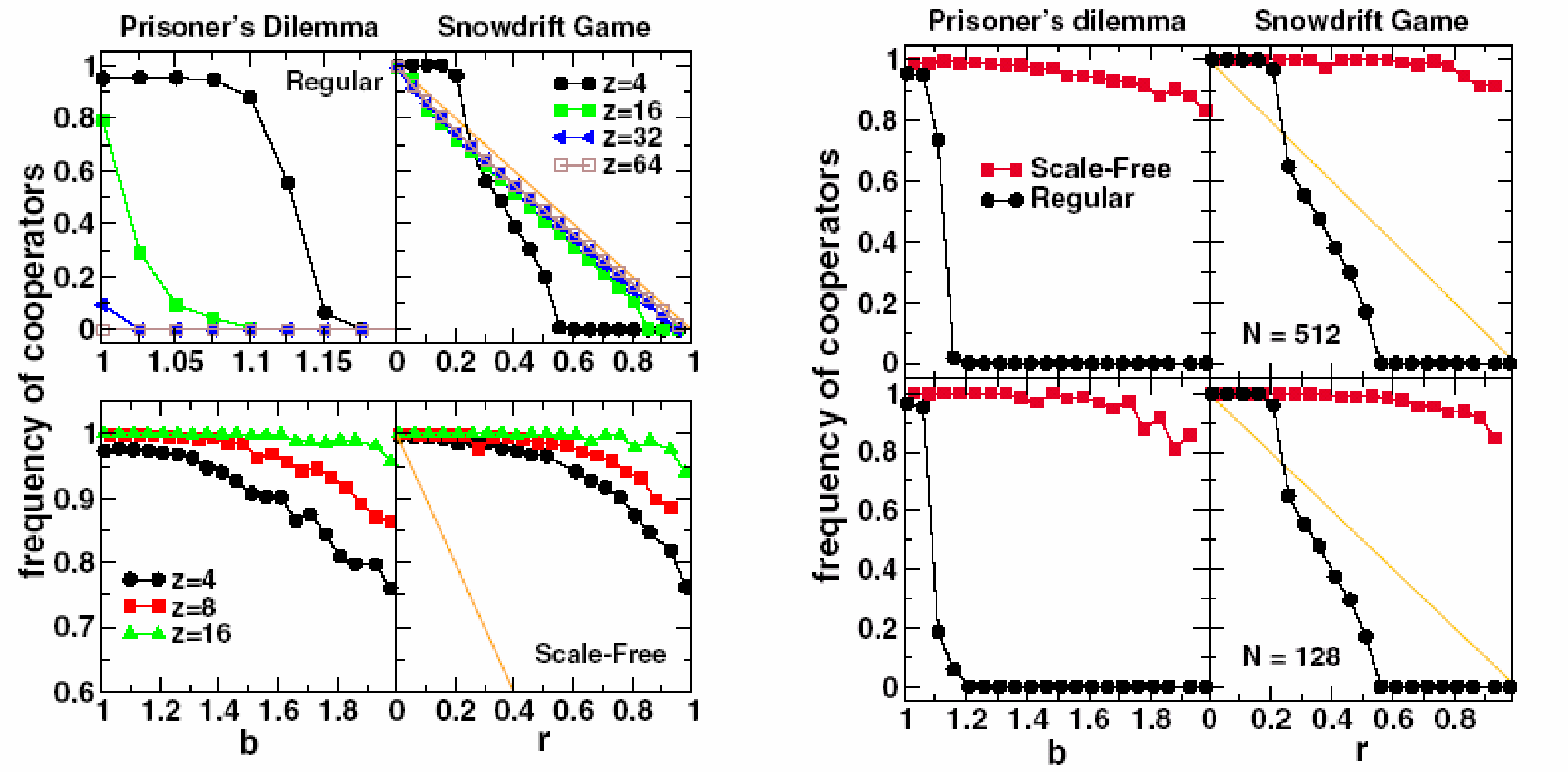

Our works on evolutionary games

- Effects of average degree on cooperation in networked evolutionary game
- Memory-based snowdrift on networks
- Randomness enhances cooperation: a resonance type phenomenon in evolutionary games

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THE EUROPEAN PHYSICAL JOURNAL B

Effects of average degree on cooperation in networked evolutionary game

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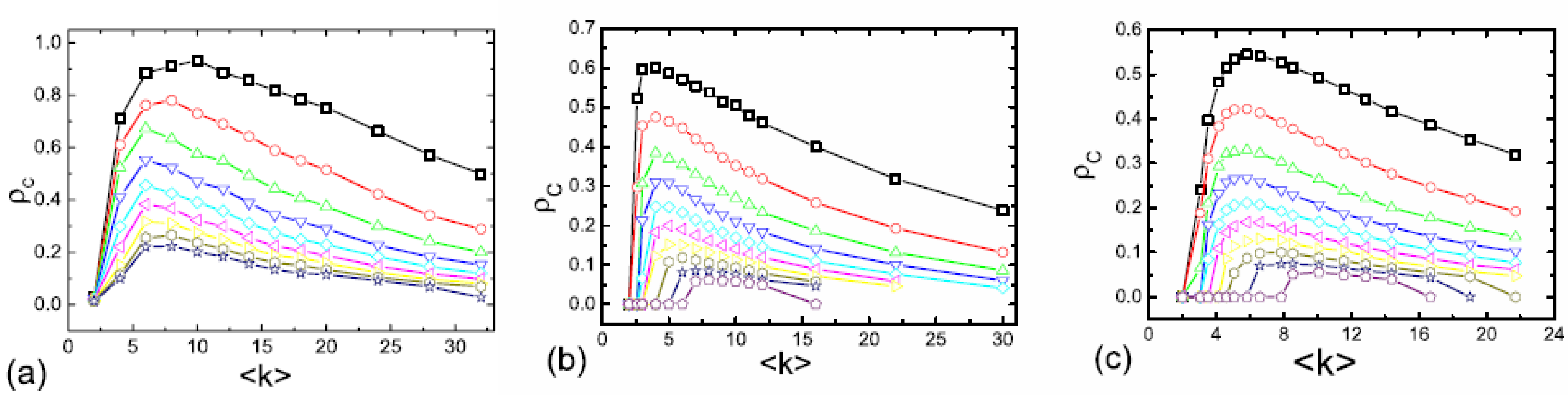
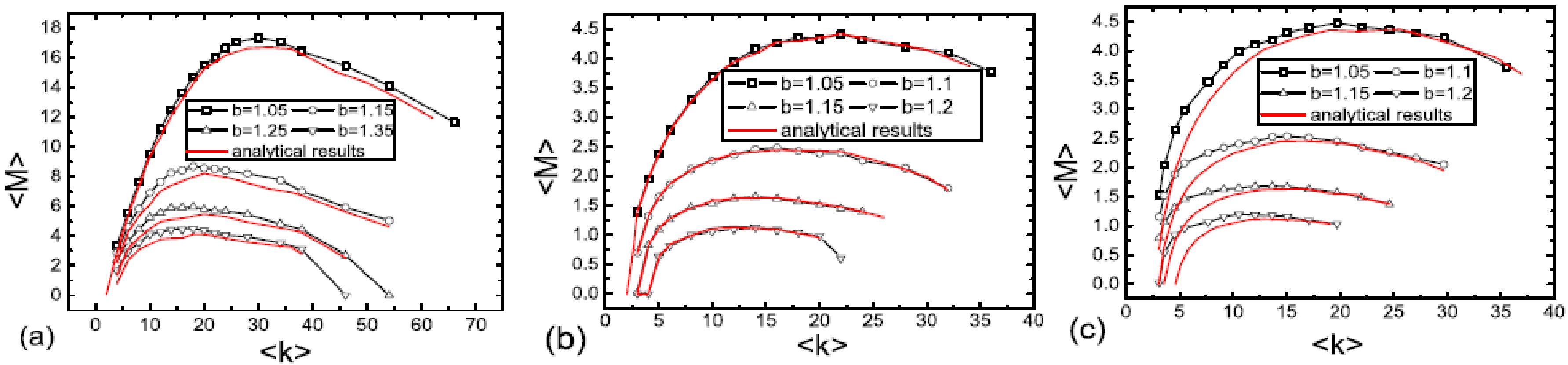
Scale-free networks Small-world networks Random networks



There exist optimal values of <k>, leading to the highest cooperation level

The average payoff <M> as a function of the average degree <k>

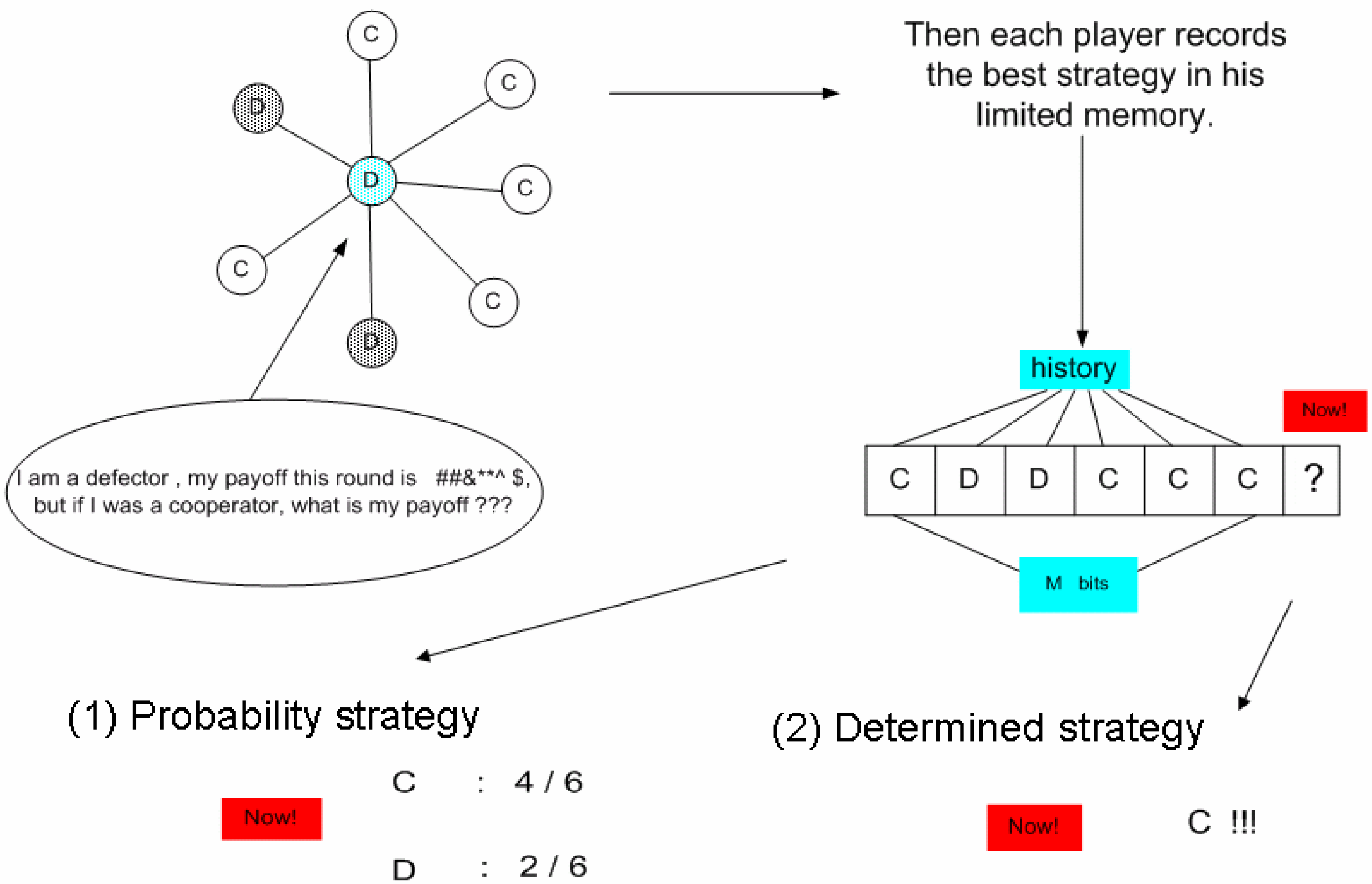


$$\langle M \rangle = (1-\rho_c) \times \langle k \rangle \times \rho_c \times b + \rho_c \times \langle k \rangle \times \rho_c \times 1,$$
$$= \langle k \rangle \times \rho_c \times ((1-\rho_c) \times b + \rho_c).$$

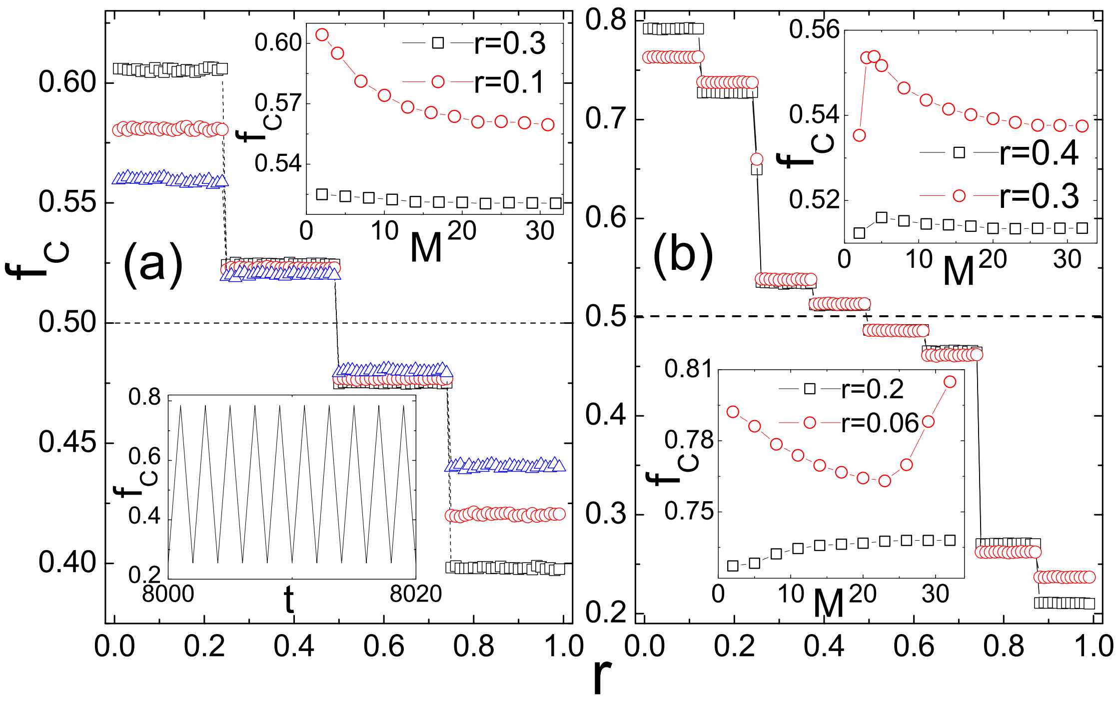
PHYSICAL REVIEW E 74, 056113 (2006)

Memory-based snowdrift game on networks

Wen-Xu Wang,[1,2] Jie Ren,[2] Guanrong Chen,[1,*] and Bing-Hong Wang[2]



Cooperation on lattices with 4 and 8 neighbors



Local stability analyses

$$m+(K-m)(1-r_c)=(1+r_c)m$$

$$\Rightarrow r_c=(K-m)/K$$

m: C neighbors of a given node

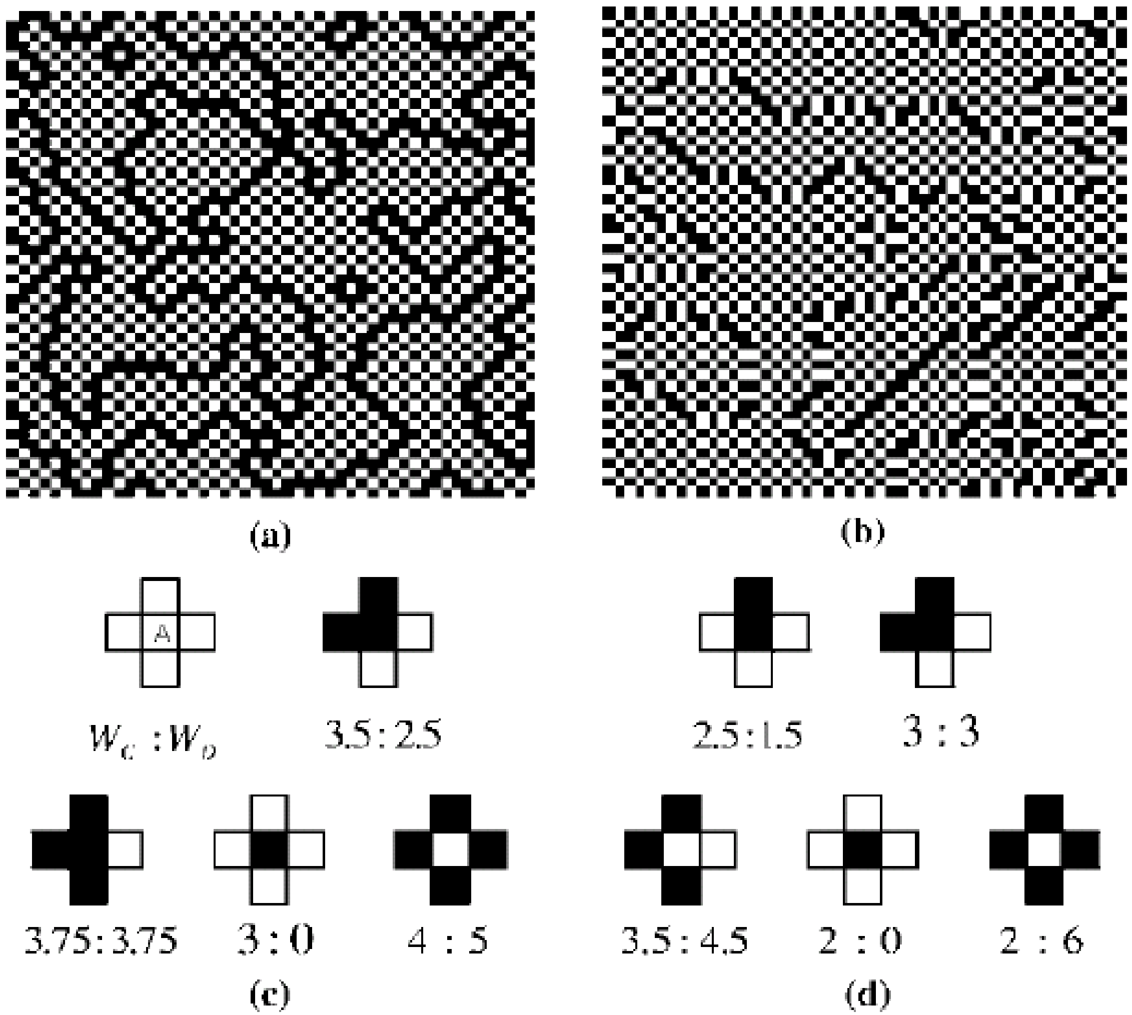
$$4-lattice: 1/4, 2/4, 3/4,$$

K-m: D neighbors of a given node

$$8-lattice: 1/8, 2/8, \cdots, 7/8$$

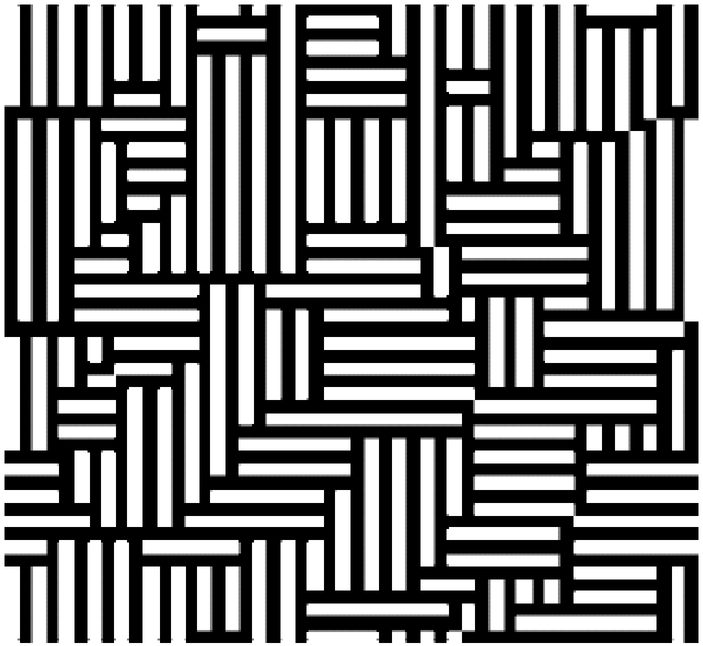
Spatial patterns on lattices with 4 neighbors

For the first two cooperation levels

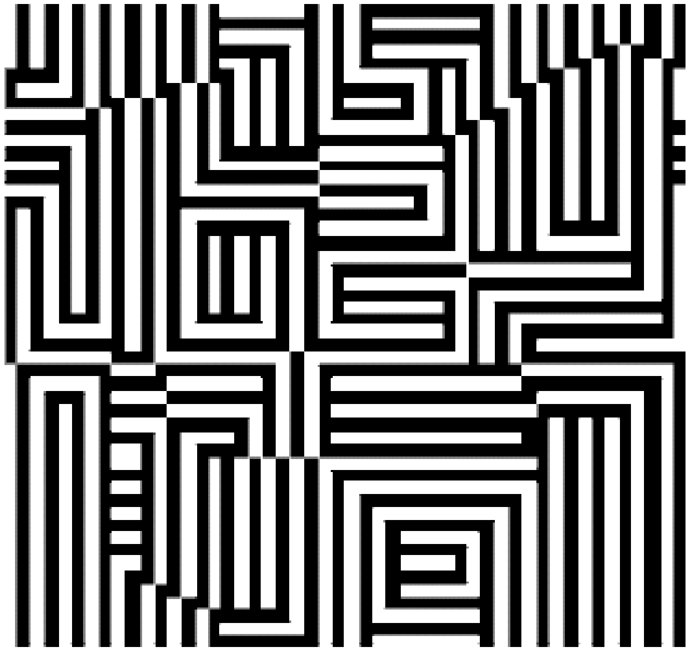


Spatial patterns on lattices with 8 neighbors

For the third and fourth cooperation levels



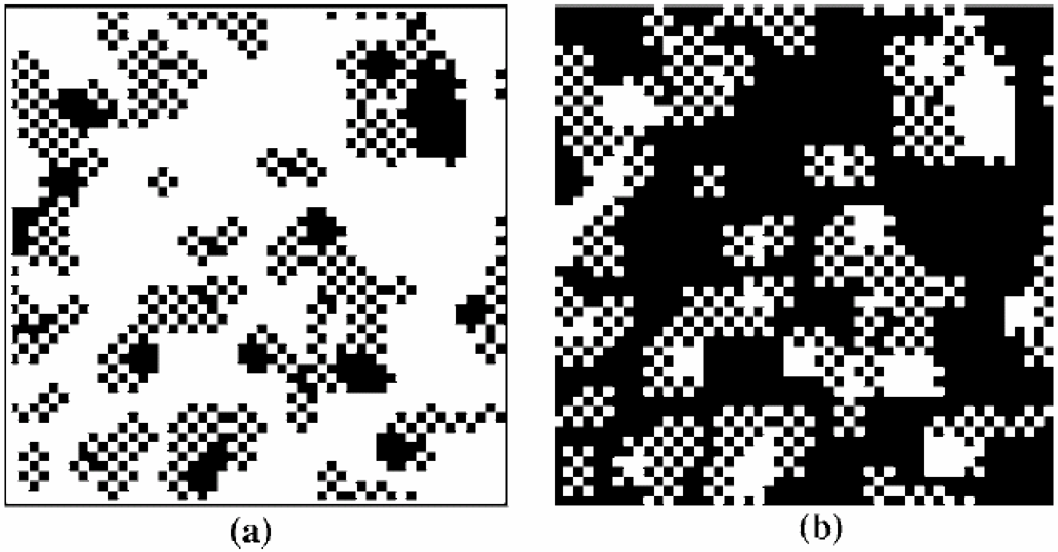
(a)

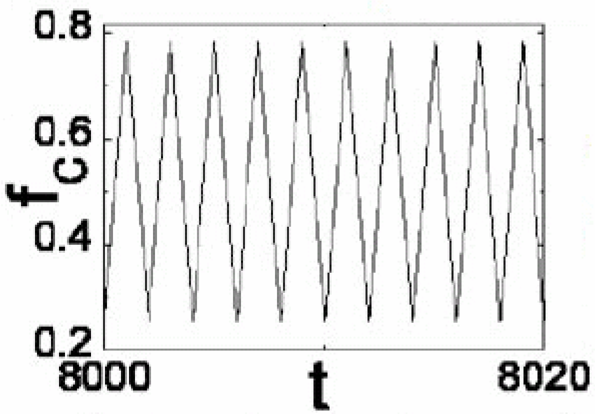


(b)

Oscillation of cooperation

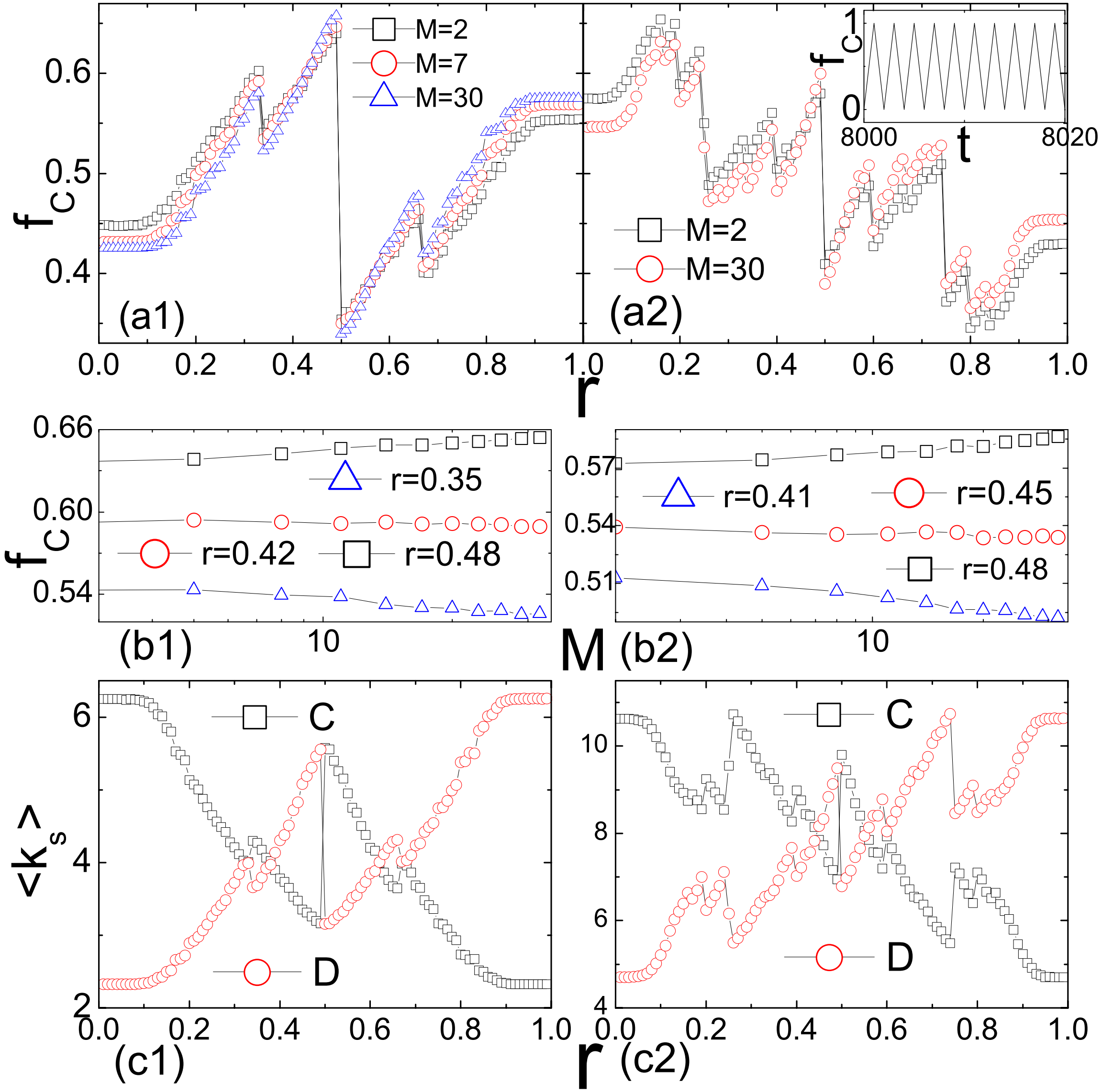
In the case of M=1 (memory lenght)



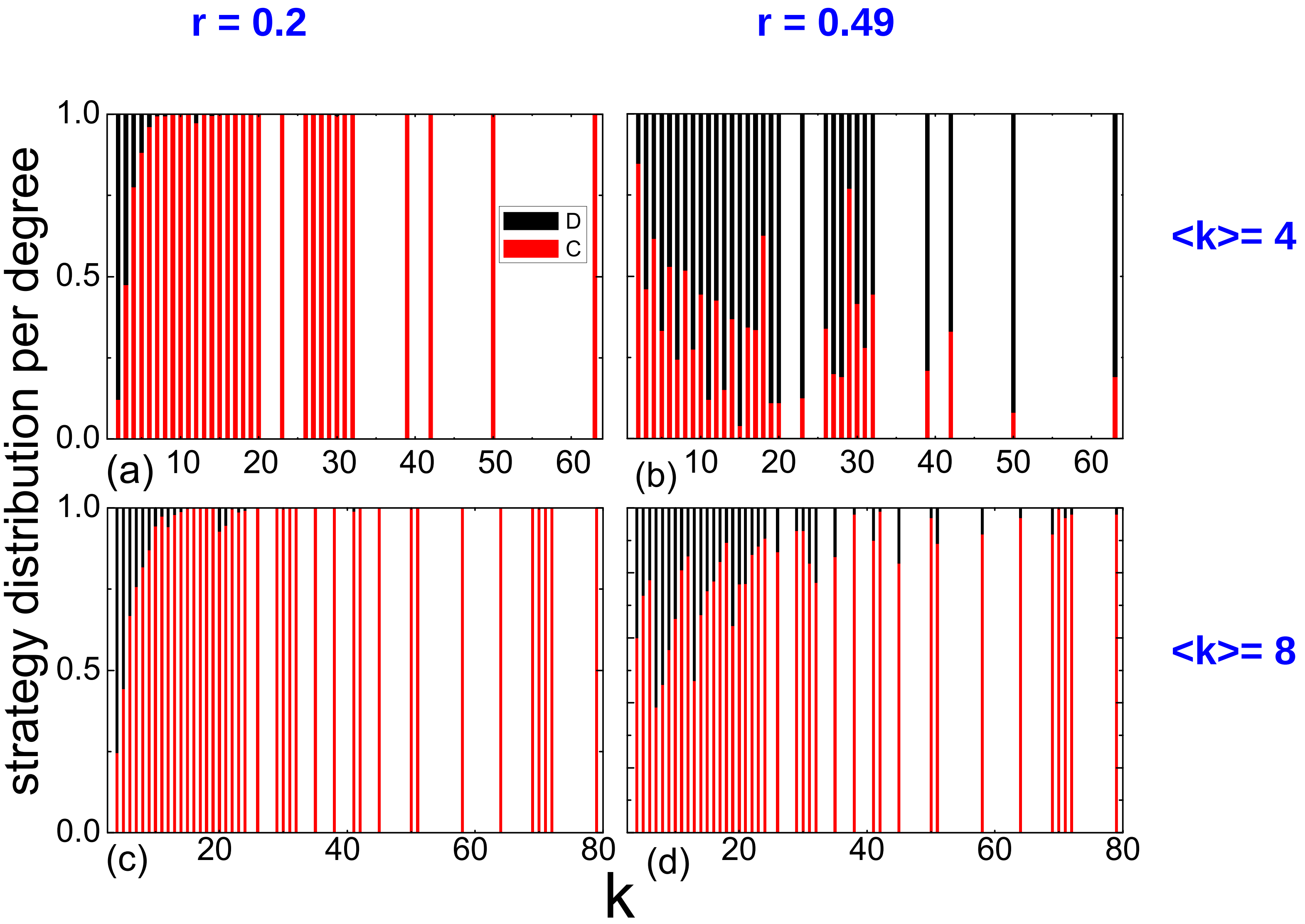


Cooperation on scale-free networks

Average degree is 4 **Average degree is 8**



Strategy occupation distribution for scale-free networks



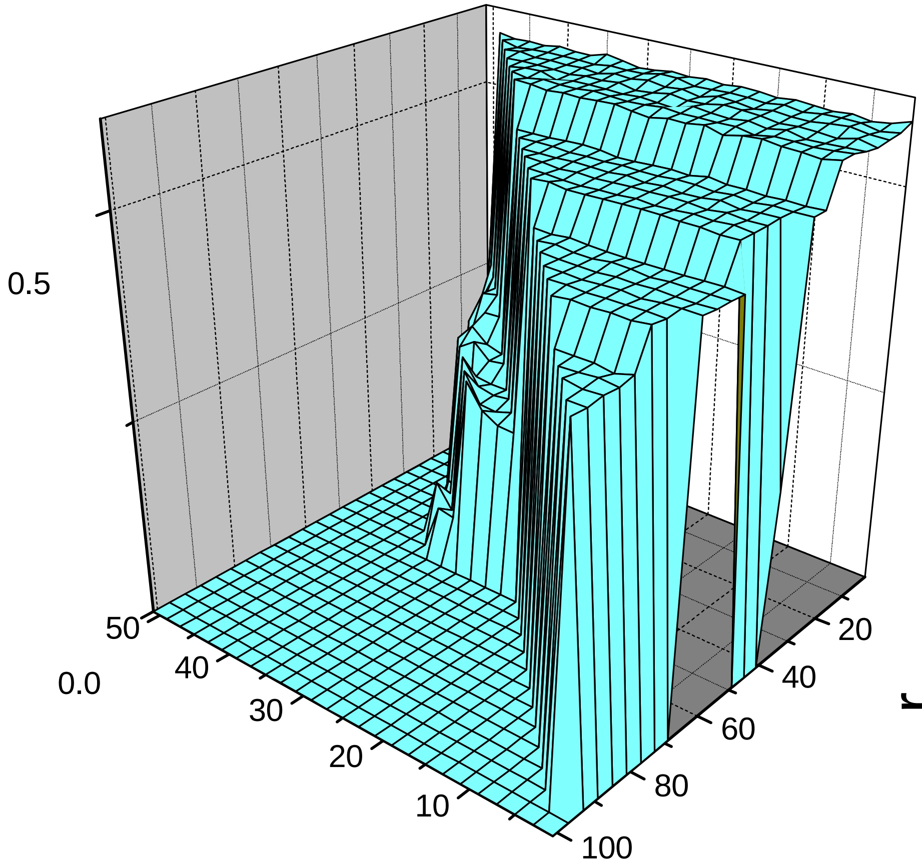
Generalize memory-based snowdrift game to public goods game

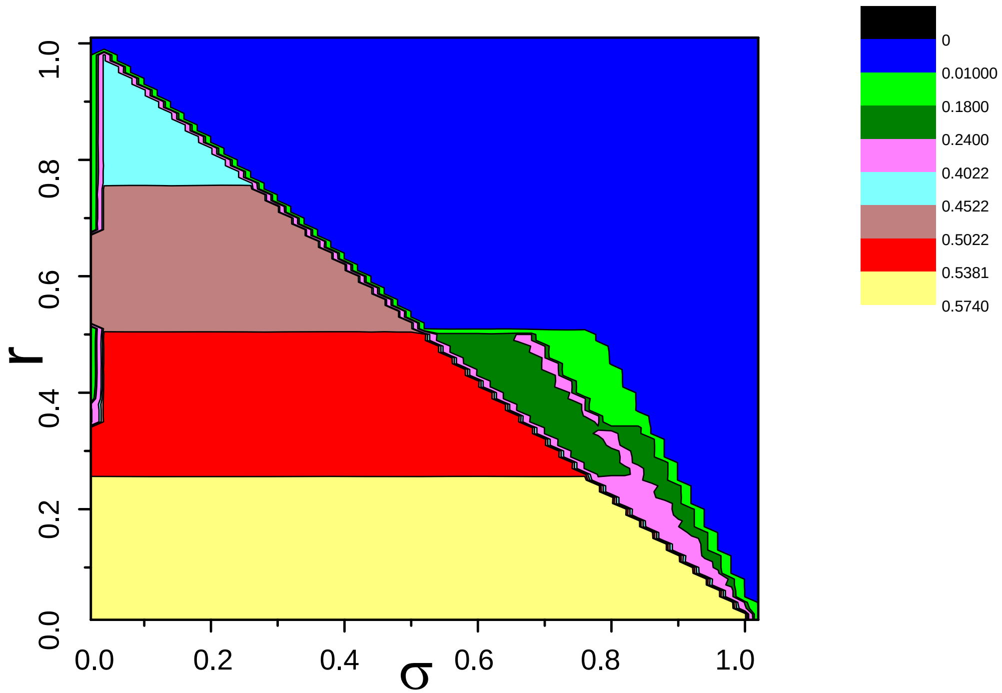
Payoff Matrix

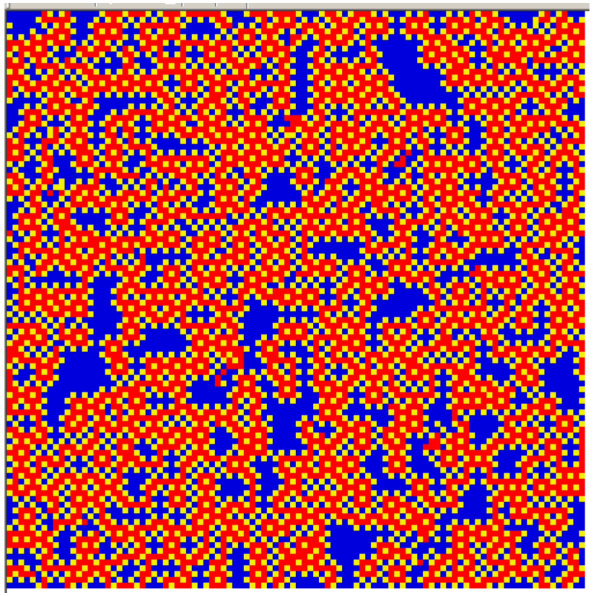
C: cooperator
D: defector
L: loner

$X \backslash Y$	D	C	L
D	$0 \backslash 0$	$b \backslash c$	$\sigma \backslash \sigma$
C	$c \backslash b$	$1 \backslash 1$	$\sigma \backslash \sigma$
L	$\sigma \backslash \sigma$	$\sigma \backslash \sigma$	$\sigma \backslash \sigma$

Phase diagram for the memory-based public goods game

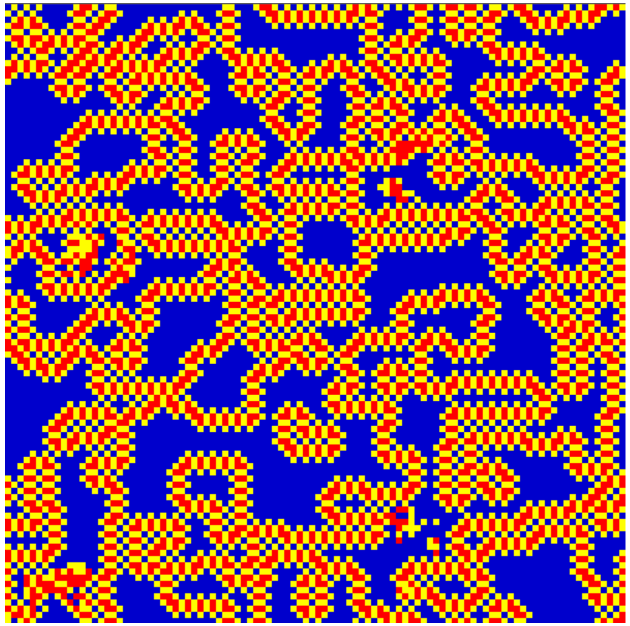


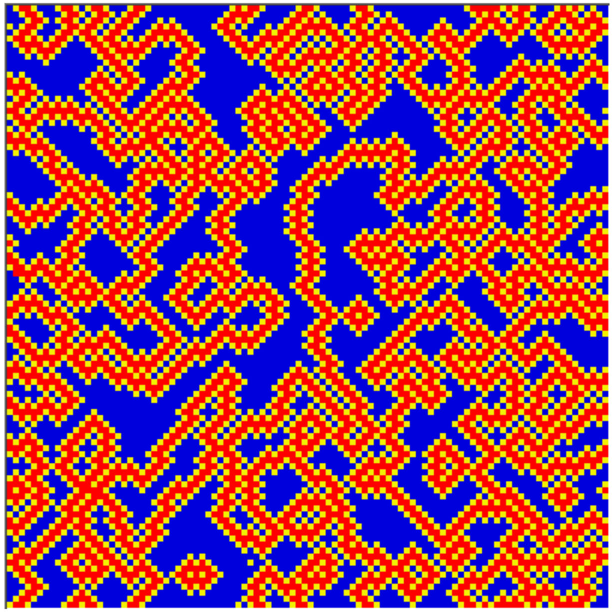




Sigma=0.9, r=0.15,M=7

Sigma=0.6, r=0.45,M=7
Sigma=0.7, r=035,M=5





Sigma=0.85, r=0.15,M=5
Sigma=0.9, r=0.1,M=11

PHYSICAL REVIEW E (In press)

Randomness enhances cooperation:
a resonance type phenomenon in evolutionary games

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[3] *Biotechnology and Bioengineering Center and Department of Physiology, Medical College of Wisconsin, Milwaukee, Wisconsin 53226, USA*
(Dated: January 19, 2007)

- Topological and dynamical randomness
- A resonance type phenomenon

Topological randomness *P*

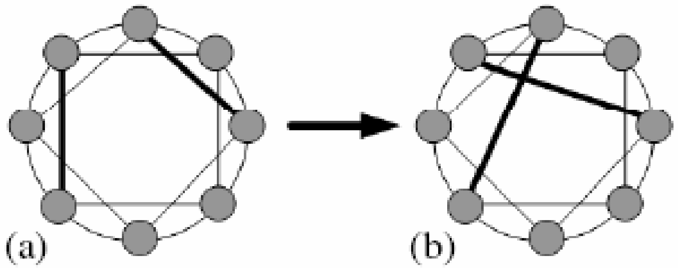


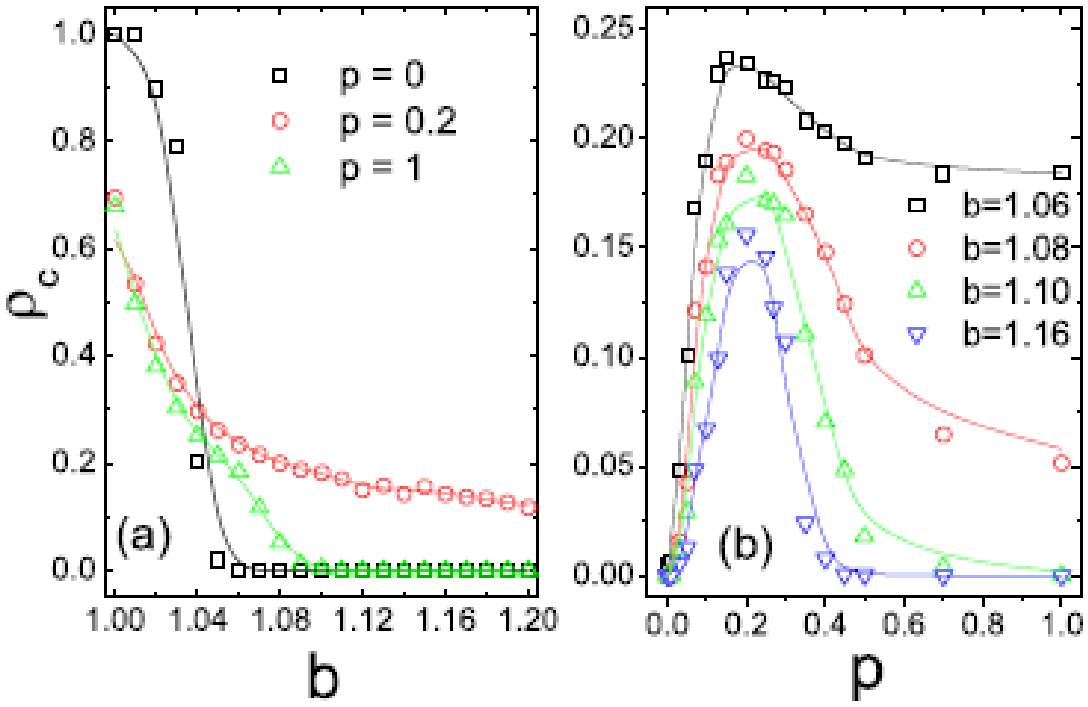
FIG. 1: (a) Illustration of a regular ring graph with connectivity $z = 4$. Two edges are chosen and marked by thick lines. (b) Swap the ends of the two chosen edges. The swapped edges are marked by thick lines.

Dynamical randomness *T*

$$W_{s_x \leftarrow s_y} = \frac{1}{1 + \exp[(M_x - M_y)/T]},$$

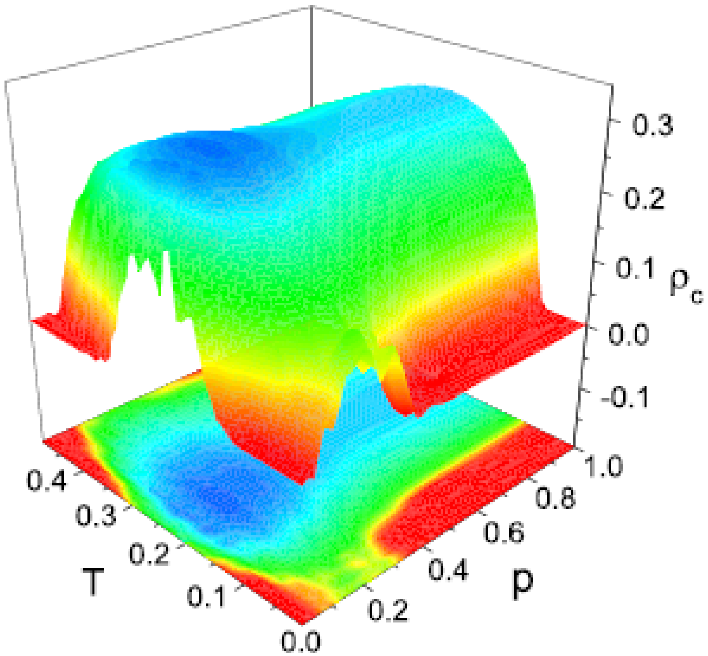
where *T* characterizes the stochastic uncertainties, including errors in decision, individual trials, *etc*. $T = 0$ denotes the complete rationality, where the individual always adopts the best strategy determinately.

Cooperation depends on parameters *b* and *P*



b: parameter in payoff matrix of the prisoner's dilemma game
P: the probability of swapping links

Cooperation depends on both topological and dynamical randomness



The existence of the optimal island resembles a resonance type phenomenon.

More information can be seen in
http://www.univie.ac.at/virtuallabs/

Future works

- The spreading pattern of the cooperative action
- Evolutionary games on weighted adaptive networks
- Apply game theory to other dynamics
- Study the strategy evolution by using series analysis

Recent publications

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Y.-B.Xie**, W.-X. Wang,** and B.-H.Wang, ***Phys. Rev. E* 75**, 026111 (2007).
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C.-Y. Yin, B.-H. Wang, **W.-X. Wang**, G. Yan, H.-j. Yang, ***Eur. Phys. J. B* 49**, 205 (2006).
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M.-B. Hu, **W.X.Wang**, R. Jiang, Q.-S. Wu, B.-H. Wang, and Y.-H. Wu, ***Eur. Phys. J. B* 53**, 273 (2006).

Thanks for your attention