Coordinative Control over Complex Networks

複雜網絡上的協調控制

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Topics for Today

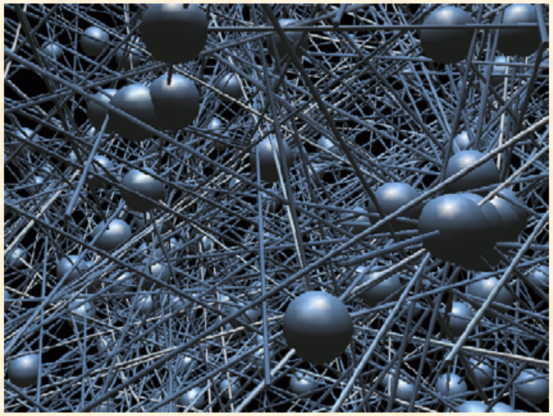
- **A Quick Review of Three Network Models:**

  Random-Graph, Small-World, and Scale-Free Networks

- **Consensus and Control over Complex Networks:**
  - **Pinning Control**
  - **Swaming, Consensus, Flocking**
  - **Coordinated Control**

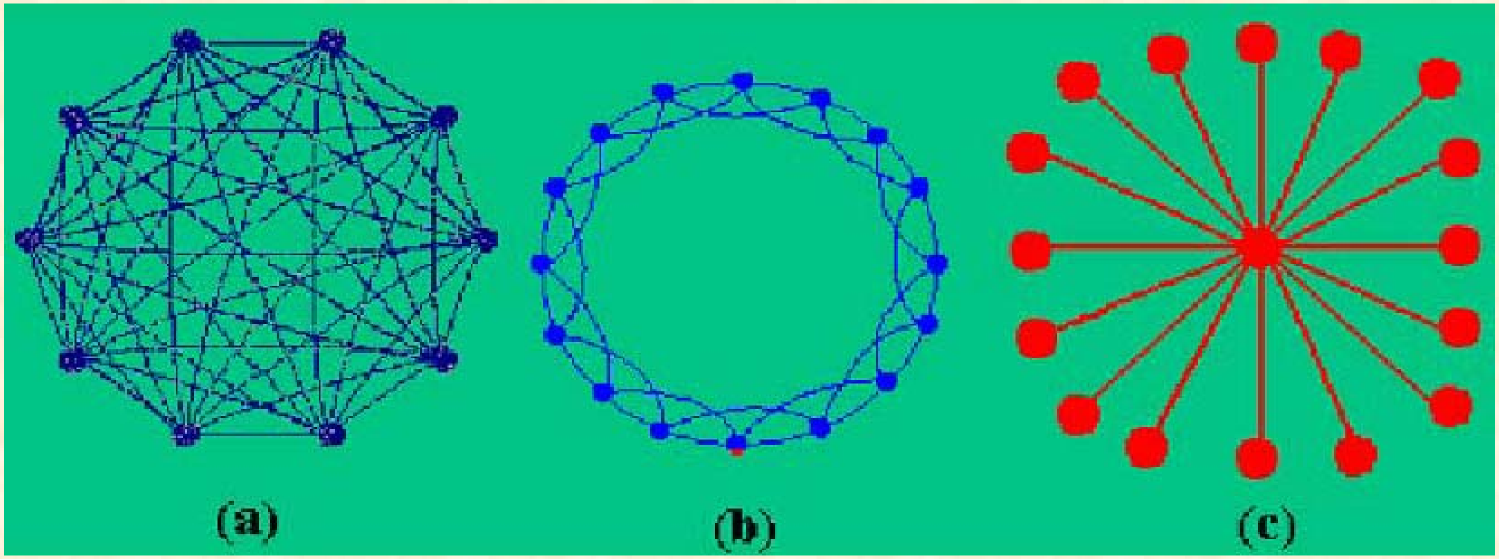
- **DEMO**

Network Topology



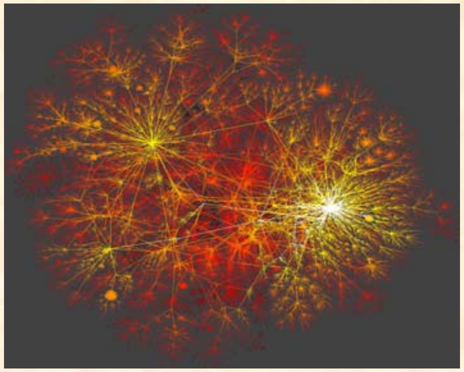
- A **network** is a set of **nodes** interconnected via **links**
  - **Internet:** **Nodes** – routers **Links** – wires
  - **Neural Network:** **Nodes** – cells **Links** – nerves
  - **Social Networks:** **Nodes** – individuals **Links** – relations
  - … …

Regular Networks



(a) Globally coupled network
(b) Ring-coupled network
(c) Star-coupled network

Basic Network Models



- <u>Random Graph</u> Theory (Erdös and Rényi, 1960)
- ER Random Graph model dominates for near 50 years

…… till today ……

- Availability of databases and supper-computing

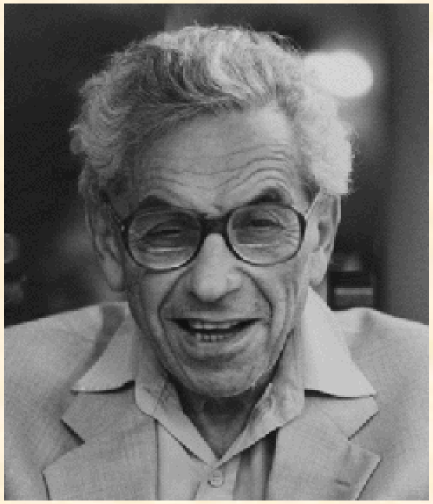
→ rethinking of approach

- Recent significant discoveries:
  - <u>Small-World</u> effect (Watts and Strogatz, Nature, 1998)
  - <u>Scale-Free</u> feature (Barabási and Albert, Science, 1999)

Random Graph Theory

-- A **revolution** in the 1960s

Paul Erdös



Alfred Rényi

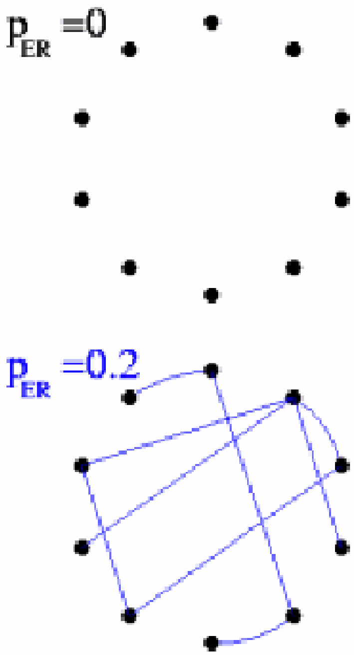
- Simplest model for most complex networks
- Rigorous mathematical theory

ER Random Graph Models

Erdős-Rényi

(Publ. Math. Inst. Hung. Acd. Sci. **5**, 17 (1960))

N nodes, each pair of node is connected with probability p



Features:

- **Connectivity**

  Poisson distribution

- **Homogeneity**

  All nodes have about the same number of links

- **Non-growing**

Random Graph and Poisson Degree Distribution

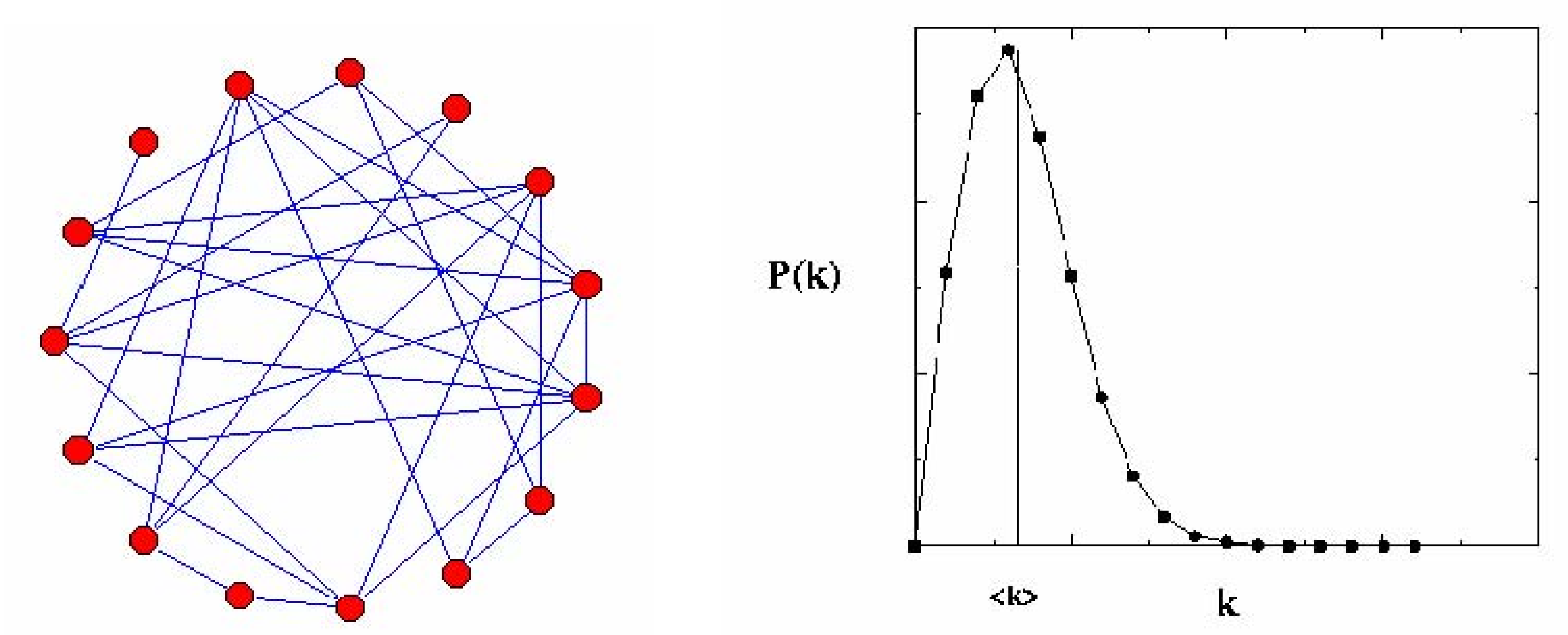


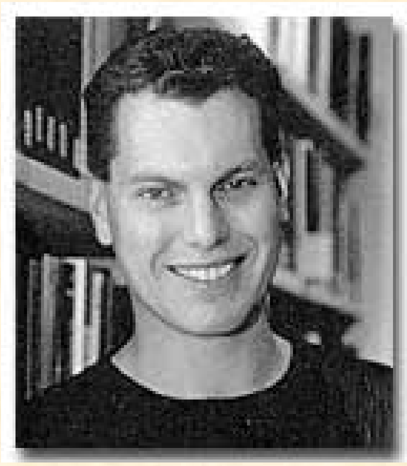
Illustration of Erdös-Rényi randon-graph network model

Small-World Networks

“Collective dynamics of
'small-world' networks”

--- Nature, 393: 440-442, 1998

D. J. Watts



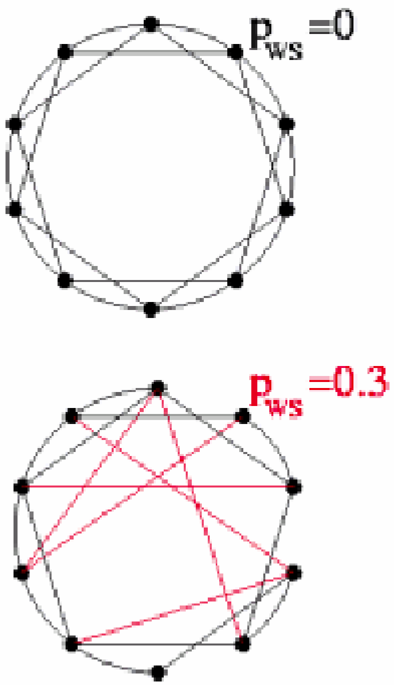
S. H. Strogatz

Cornell University

Small-World Networks

Watts-Strogatz

(Nature **393,** 440 (1998))



N nodes forms a regular lattice. With probability p, each edge is rewired randomly

Features:

(Similar to ER Random Graphs)

- **Connectivity**

  Poisson distribution

- **Homogeneity**

  All nodes have about the same number of links

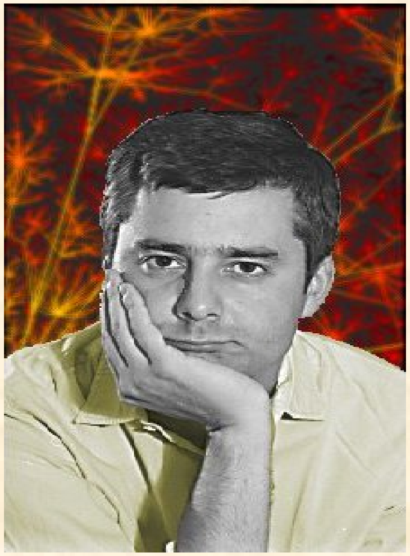
- **Non-growing**

Scale-Free Networks

“Emergence of scaling in random networks”

Science, 286: 509 (1999)

A.-L. Barabási



R. Albert

Norte Dame University

Scale-Free Networks

(**Barabasi-Albert**, **Science**, 1999)

Start with a network of size m_0 **(initialization)**

(i) **Add new nodes (incremental growth)**:

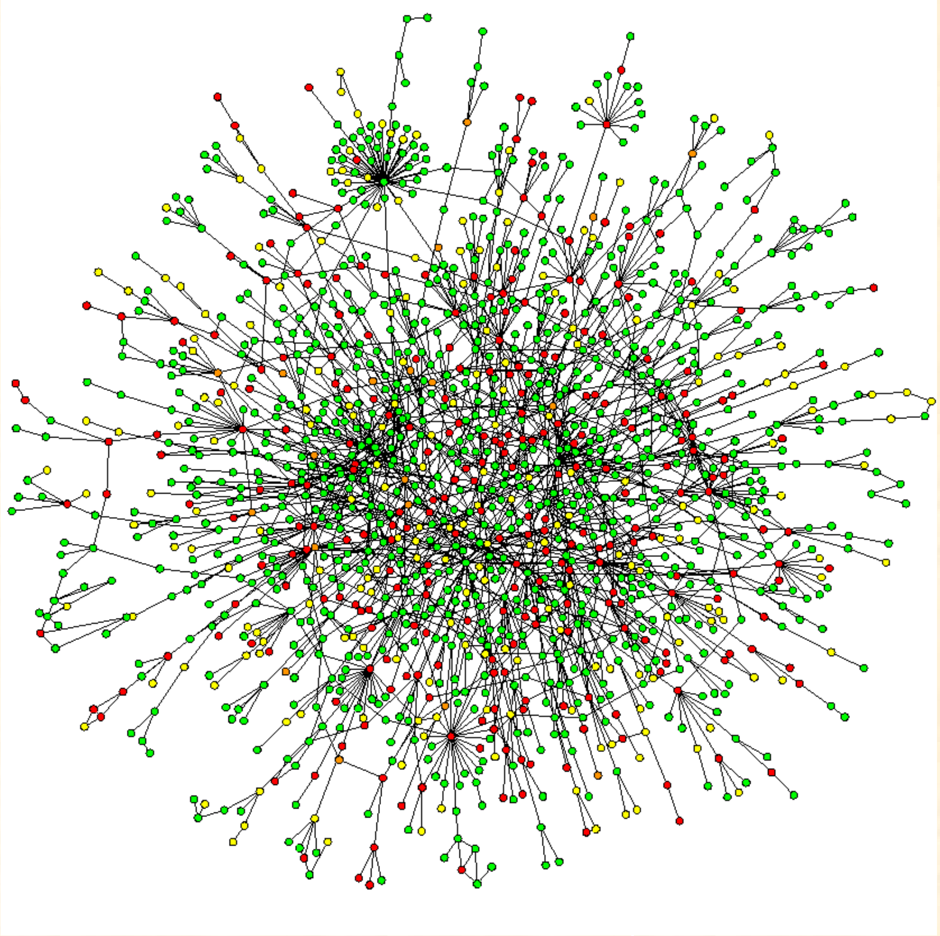
With probability p, a new node is added into the network

(ii) **Add new links (preferential attachment)**:

The new node has m ($m \le m_0$) new links to the already existing nodes in the network, with probability

$$\Pi(k_i) = \frac{k_i + 1}{\sum_l (k_l + 1)}$$

Scale-Free Networks



Features:

- **Connectivity:**

  in power-law form

  P(k) ~ k^{ - r }

- **Non-homogeneity:**

  Very few nodes have many links but most nodes have very few links

- **Growing**

(Hawoong Jeong)

Complex Networks and ICM

International Congress of Mathematics (ICM)

22-28 August 2006, Madrid, Spain

Jon M Kleinberg (Cornell Univ) received the Nevanlinna Prize for Applied Mathematics

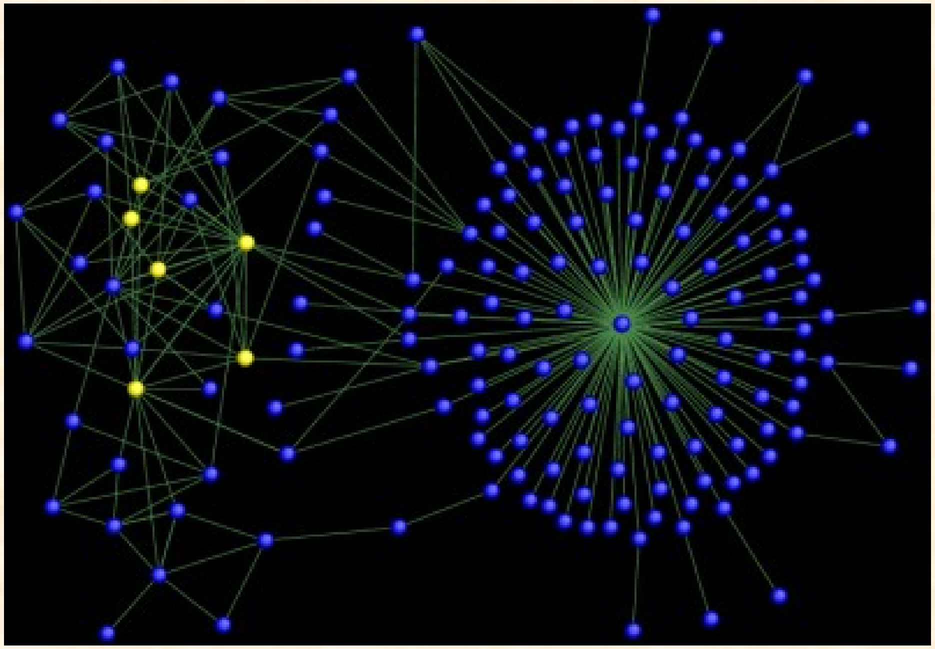
He gave a 45-minute talk –

"Complex Networks and Decentralized Search Algorithms"

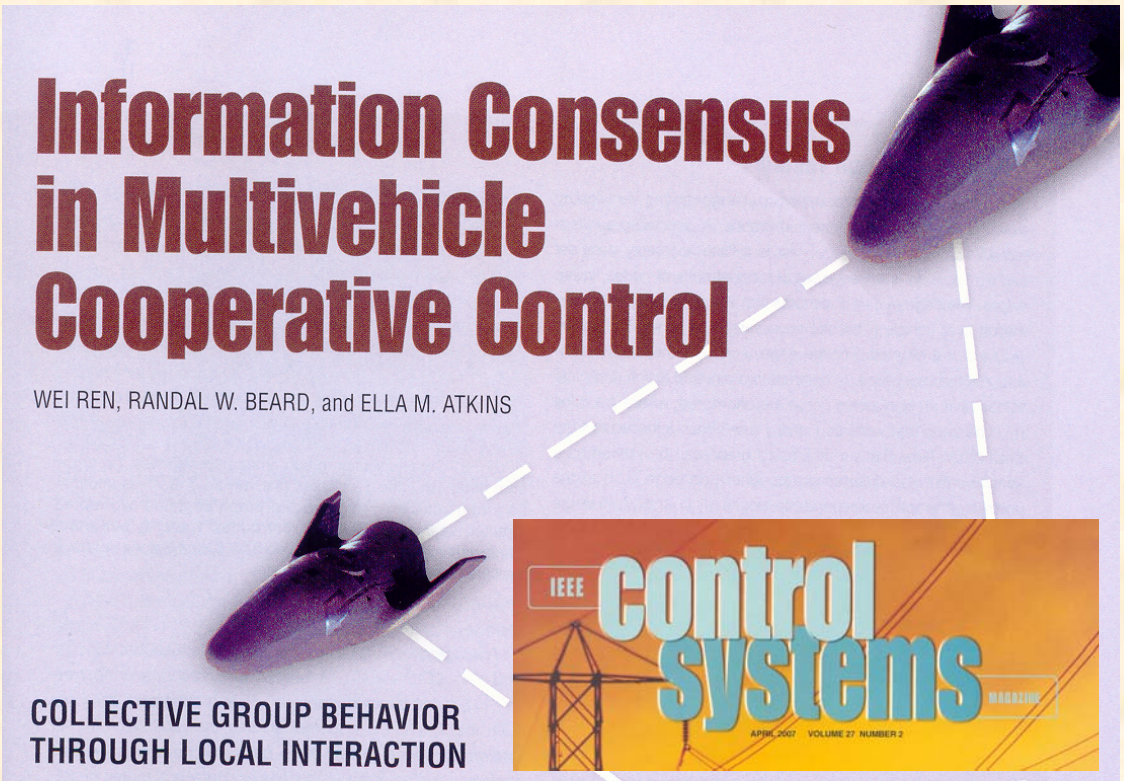
J M Kleinberg, "Navigation in a small world," Nature, 2000

Consensus and Control over Complex Networks

It has already started …



IEEE Control Systems Society



Pinning Control of
Complex Dynamical Networks

- **Pinning Control:**

  -- Only pin a small portion of nodes

- **Random / Specific Pinning:**

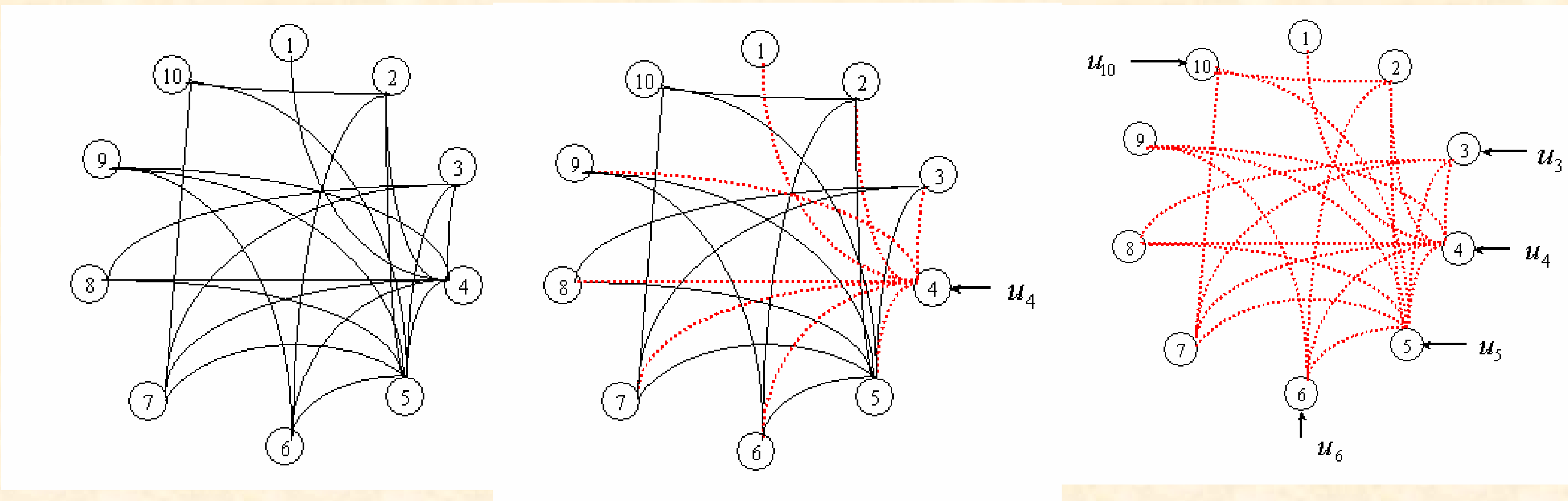
  R: Pin a portion of randomly selected nodes

  S: First pin the most important node

  Then select and pin the second-most important node

  Continue … till control goal is achieved

Pinning Control Example



A network with 10-nodes

generated by the B-A scale-free model (N=10, m=$m0$=3)

X. F. Wang, G. Chen, Physica A (2002)

X. Li, X. F. Wang, G. Chen, IEEE T-CAS (2003)

Control of Complex Networks

A network with N linearly coupled nodes:

$$\dot{x}_i = f(x_i) + c\sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ij}\Gamma(x_j - x_i), i = 1,2,\cdots,N$$

Here:

$x_i = (x_{i1}, x_{i2}, \cdots, x_{in}) \in R^n$ - state variables

$f(\cdot)$ - nonlinear continuously differentiable function

$\Gamma \in R^{n\times n}$ - constant 0-1 coupling matrix

Assume: $\Gamma = diag(r_1, \cdots, r_n)$ is diagonal with $r_i = 1$ for a particular i and $r_j = 0$ for $j \neq i$.

Control of Complex Networks

Control Objective: To stabilize the network onto a particular solution of the network:

$$x_1(t) = x_2(t) = \cdots = x_N(t) \to \bar{x}, \quad as \quad t \to \infty$$

Here, $\bar{x} \in R^n$ is an equilibrium point of an isolated node.

Selective Control:

Only a portion of nodes are selected for stabilization control

1. Specifically selective scheme
2. Randomly selective scheme

X. F. Wang, G. Chen, Physica A (2002)
X. Li, X. F. Wang, APWCCS (2003)
X. Li, X. F. Wang, G. Chen, IEEE T-CAS (2003,2004)
Z. P. Fan, G. Chen, Handbook (2004), CCDC (2005)

Control of Complex Networks

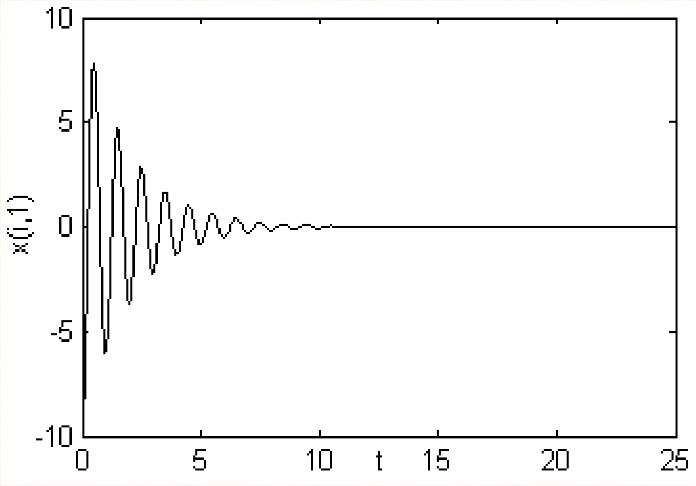
Example: Consider a coupled CNN:

$$\dot{x}_i = \begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \\ \dot{x}_{i4} \end{pmatrix} = \begin{pmatrix} -x_{i3} - x_{i4} + c\sum_{j=1}^{N} a_{ij} x_{j1} \\ 2x_{i2} + x_{i3} + c\sum_{j=1}^{N} a_{ij} x_{j2} \\ 14x_{i1} - 14x_{i2} + c\sum_{j=1}^{N} a_{ij} x_{j3} \\ 100x_{i1} - 100x_{i4} \\ \quad + 100(|x_{i4}+1| - |x_{i4}-1|) \\ \quad + c\sum_{j=1}^{N} a_{ij} x_{j4} \end{pmatrix} \quad (i = 1, 2, \cdots N)$$

Control of Complex Networks

Here, network size $N=60$, coupling strength $c=8.246$, and number of controlled nodes $l=15$

Specifically Selective Scheme: Only control the first 15 largest-degree nodes, with state-feedback control. Control gains are



The controlled state x1

$$k_i = 29.7603$$

Control of Complex Networks

Comparison:

Randomly Selective Scheme: Randomly select 15 notes to pin.

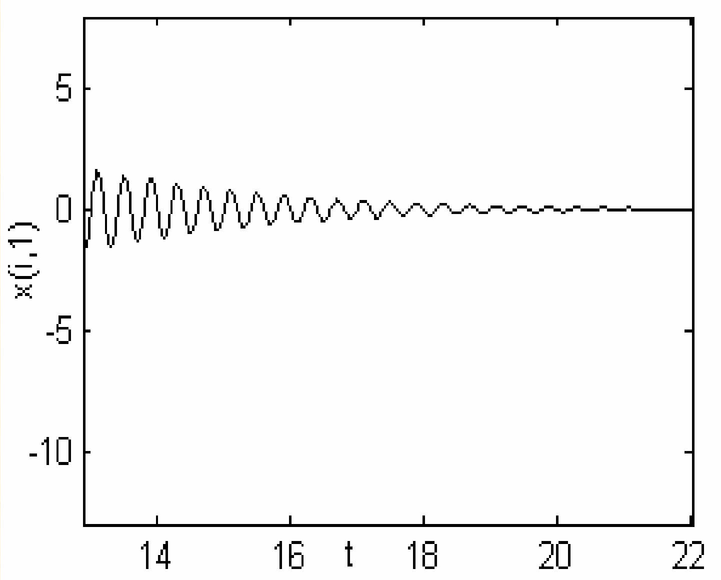
Control gains are much larger:

$$k_i = 513.3709$$

compared to the previous one:

$$k_i = 29.7603$$

And, it takes twice longer time to stabilize the network



The controlled state x1

Consensus and Control over Complex Networks

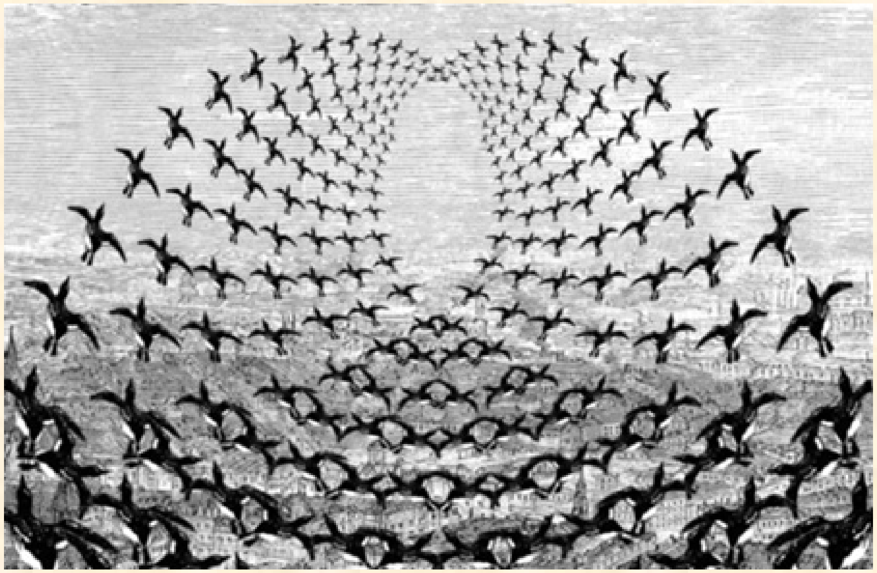
- Swarm Dynamics/Modeling
- Consensus Protocols/Analysis
- Flocking Algorithms/Control
- DEMO

Fish Swarming

- Swarming:

  to move or gather in group

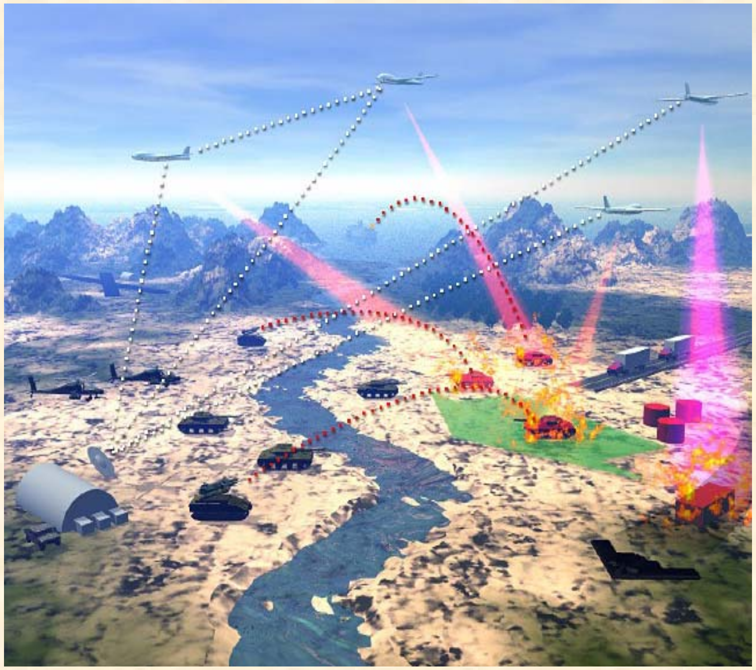
Birds Flocking

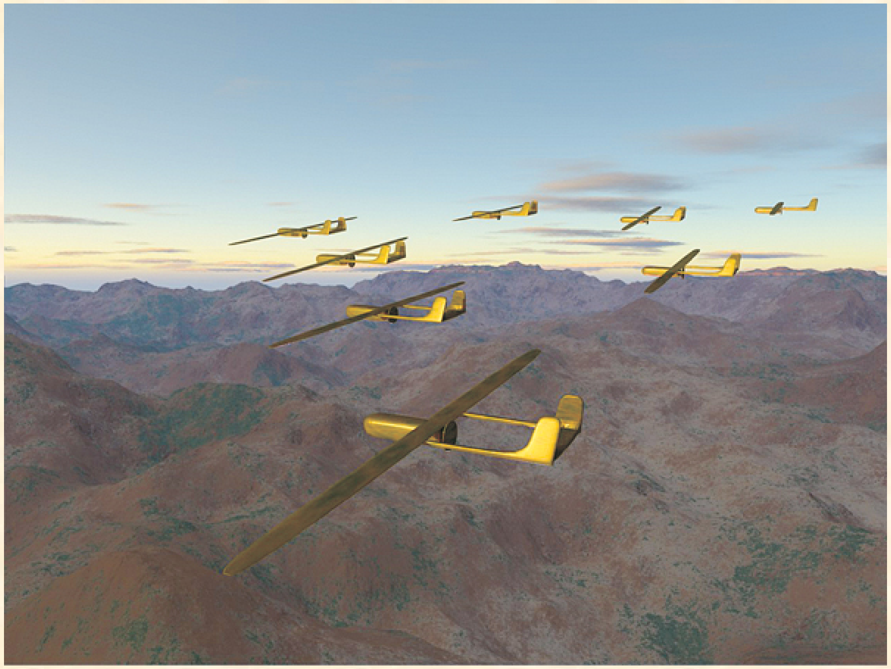


Flocking: to congregate or travel in flock

Consensus

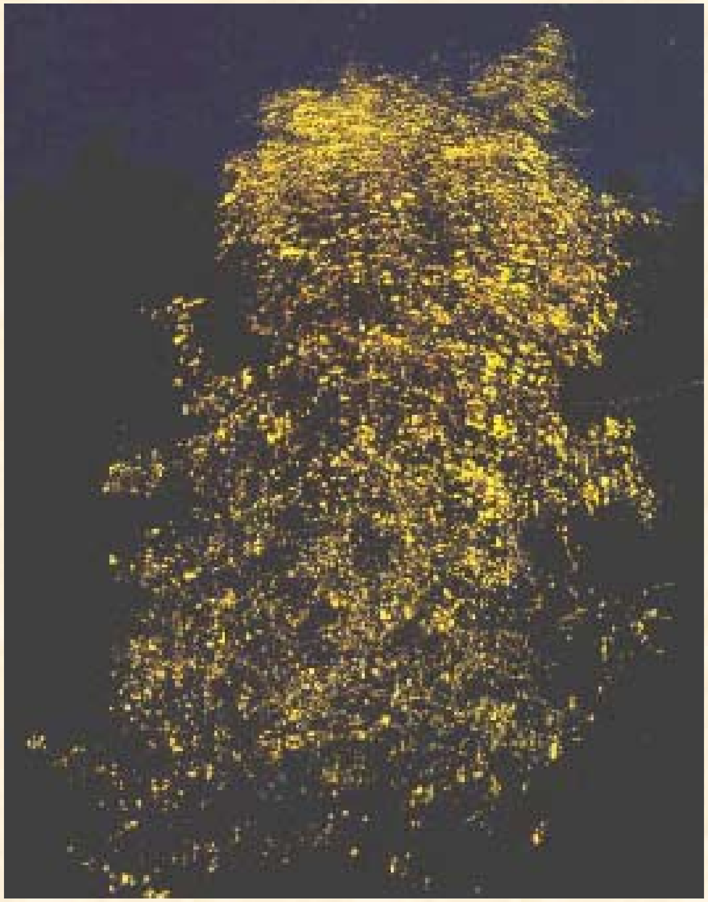
A position reached by a group as a whole



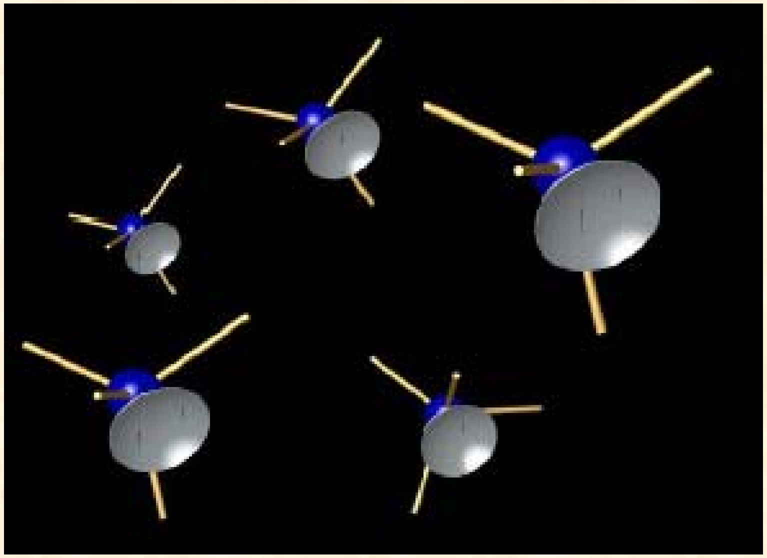


Battle space management scenario

Fireflies Synchronization



Attitude Alignment



The attitude of each spacecraft is synchronized with its two adjacent neighbors via a bi-directional communication channel

What are in common ?

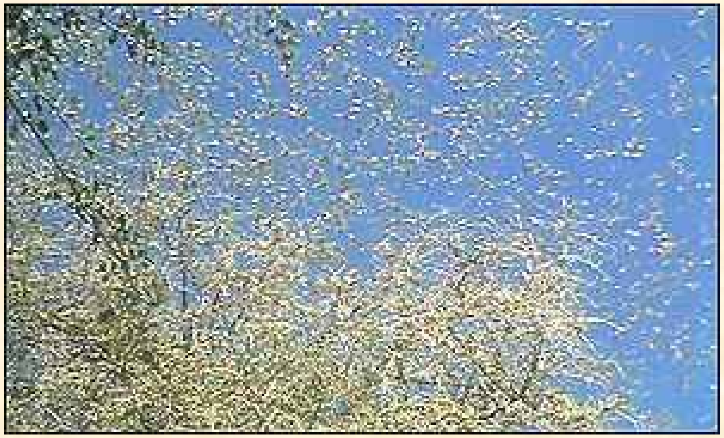
- Swarming
- Flocking
- Rendezvous
- Consensus
- Synchrony
- Cooperation
- ......

⇨

Distributed coordination of a network of agents:

- ✓ Agents
- ✓ Network
- ✓ Distributed local control
- ✓ Global consensus

Swarming



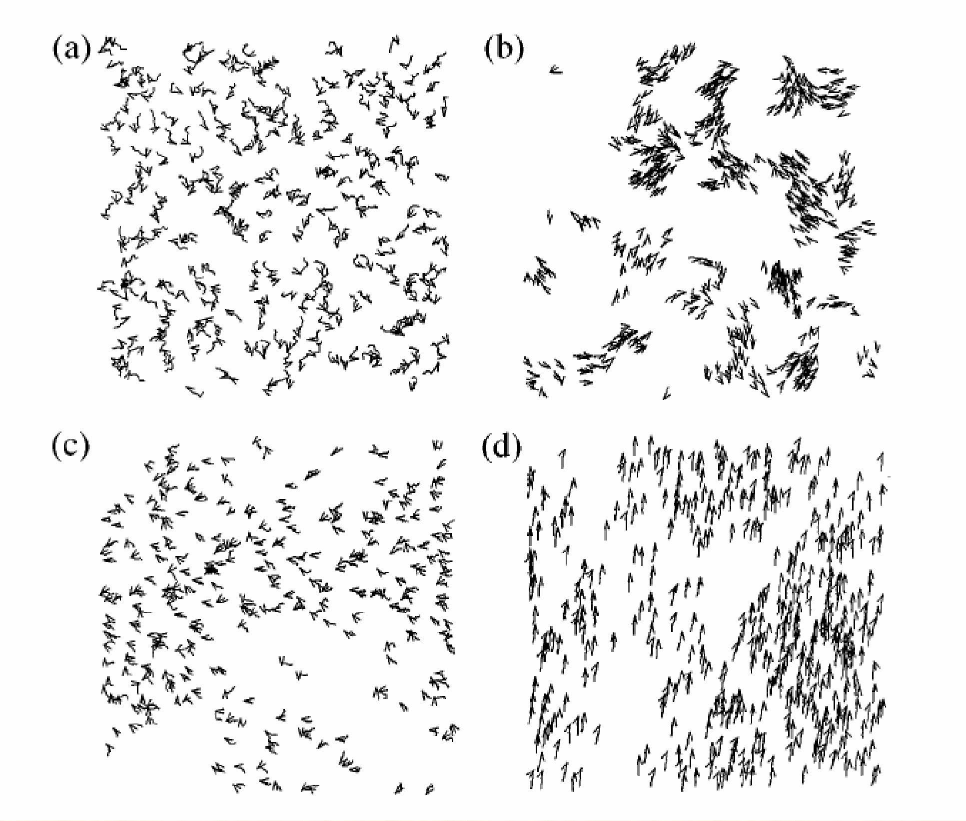
Vicsek Model

- Position:

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t$$

- Heading:

$$\theta(t+1) = <\theta(t)>_r + \Delta\theta$$



(a) Initial: Random positions/velocities

(b) Low density/noise: grouped

(c) High density/noise: correlated

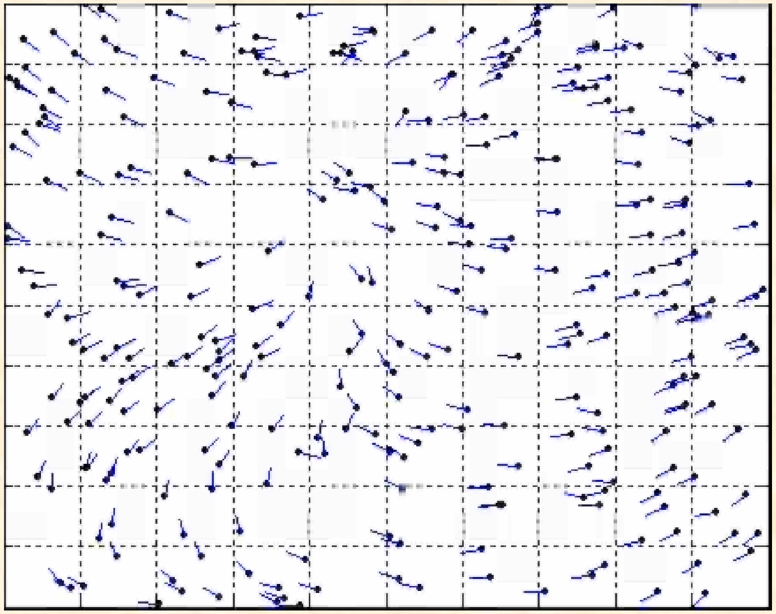
(d) High density / Low noise

→ coordinated motion

Vicsek, Czirok, Jacob, Cohen, Shochet, "Novel type of phase transition in a system of self-driven particles," Phys. Rev. Lett., 1995, 75 (6): 1226

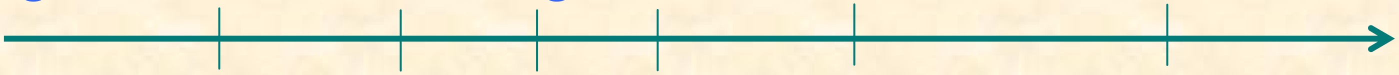
Convergence

$$\theta(t+1) = F_{G_p(t)}\theta(t)$$



- **Strong condition:**
  agents are always linked
- **Weak condition:**
  on some time intervals
  $[t_i, t_{i+1}), i = 0,1,2,\ldots\infty,\ t_0 = 0$

  agents are linked together



→ $$\lim_{t\to\infty} \theta(t) = \theta_{ss}1$$

Jadbabaie, Lin, Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," IEEE Trans. Auto. Control, 2003, 48(6): 988-1001

Moreau Model

Linear or linearized model:

$$x(t+1) = A(G(t))x(t)$$

with a stochastic matrix

$$(A(G))_{kl} = \begin{cases} \dfrac{w_{lk}}{1 + \sum\limits_{i \in Neighbors(k,A)} w_{ik}}, & if \;\; l \in Neighbors(k,A) \\ \dfrac{1}{1 + \sum\limits_{i \in Neighbors(k,A)} w_{ik}}, & if \;\; k = l \\ 0, & otherwise \end{cases}$$

Moreau, “Stability of multiagent systems with time-dependent communication links,” IEEE Trans. Auto. Control, 2005, 50(2): 169-182

Stability Analysis

$$x(t+1) = A(G(t))x(t)$$

It is uniformly globally attractive with respect to the collection of equilibrium solutions if and only if there exists a $T > 0$ such that for all t there is a node (the “leader”) connecting to all other nodes, directly or indirectly, over the time window $[t, t+T]$.

Moreau, “Stability of multiagent systems with time-dependent communication links,” IEEE Trans. Auto. Control, 2005, 52(2): 169-182

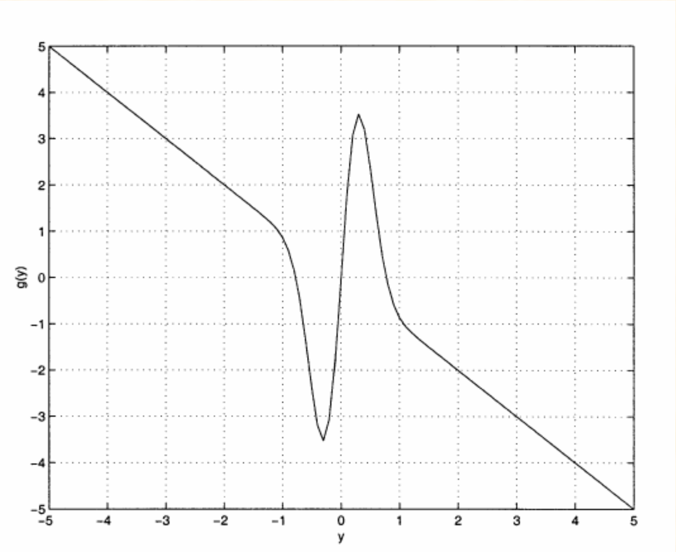
Stability Analysis of a Swarm Model

- A swarm of M globally nonlinearly coupled individuals:

$$\dot{x}^i = \sum_{j=1,\, j\neq i}^{M} g(x^i - x^j), \quad i = 1, \ldots\ldots M$$

Attraction/Repulsion function:

$$g(y) = -y\left(a - b\exp\left(-\frac{\| y \|^2}{c}\right)\right)$$



- All the agents will converge to a hyperball:

$$B_\varepsilon = \{x : \| x - \bar{x} \| \leq \varepsilon\} \qquad \bar{x} = 1/M \sum_{i=1}^{M} x^i \qquad \varepsilon = \frac{b}{a}\sqrt{c/2}\,\exp(-1/2)$$

Gazi, Passino, IEEE Trans. Auto. Control, 2003, 48: 692–697

Stability Analysis of Foraging Swarms

$$\dot{x}^i = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1, j\neq i}^{M} g(x^i - x^j), i = 1, \ldots, M$$

A/R function: $g(y) = -y[g_a(\| y \|) - g_r(\| y \|)]$

Linear attraction

$$g_a(\| x^i - x^j \|) = a,$$

bounded repulsion

$$g_r(\| x^i - x^j \|) \| x^i - x^j \| \leq b,$$

$\| \nabla_y \sigma(y) \| \leq \bar{\sigma}$ ➡ $x^i(t) \to B_\varepsilon(\bar{x}(t))$ $\varepsilon = \varepsilon_1 = \frac{(M-1)}{aM}[b + \frac{2\bar{\sigma}}{M}]$

$M \to \infty$ ➡ $\varepsilon = b / a$

Gazi, Passino, IEEE Trans. Sys. Man Cybern., B: 2004, 34(1): 539-557

Another Model

$$d_i(t) = \sum_{j \neq i} \frac{p_j(t) - p_i(t)}{|p_j(t) - p_i(t)|} + \sum_j \frac{\theta_j(t)}{|\theta_j(t)|}$$

$d_i(t)$ -- Desired direction of travel

$p_i(t)$ -- Position

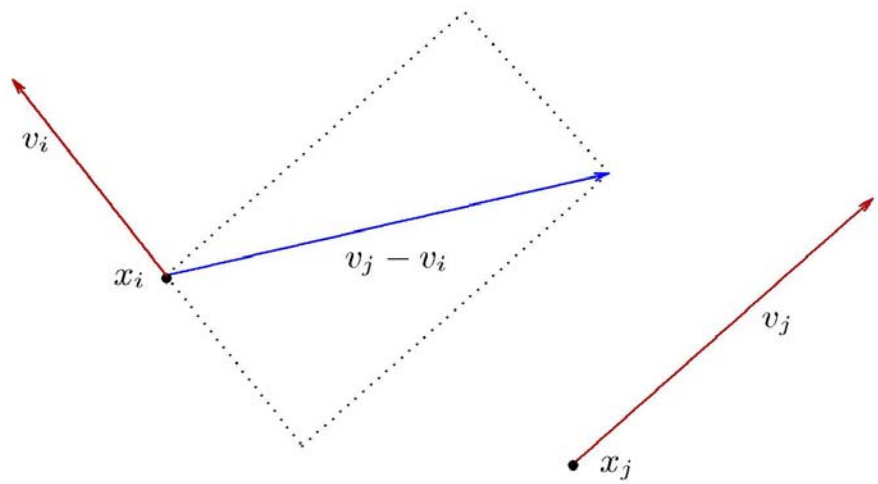
$\theta_i(t)$ -- Direction

Couzin, Krause, Franks, Levin,
"Effective leadership and decision-making in animal groups on the move,"
Nature, Feb. 3, 2005, 513-516

Yet Another Model
-- with more precise analysis

Model: every bird adjusts its velocity by adding to it a weighted average of the differences of its velocity with those of the other birds. That is, at time $t \in \mathbb{N}$, and for bird i,

$$v_i(t+1) - v_i(t) = \sum_{j=1}^{k} a_{ij}(v_j(t) - v_i(t)). \tag{1}$$



Cucker, Smale, "Emergent behavior on flocks," **IEEE Trans. Auto. Control,** 52: 852-862, 2007

We assume that this influence is a function of the distance between birds, namely

$$a_{ij} = \frac{K}{(\sigma^2 + \|x_i - x_j\|^2)^\beta}. \tag{2}$$

for some fixed $K, \sigma > 0$ and $\beta \geq 0$.

Convergence in continuous time

For $x, v \in \mathbb{E}^k$ we denote

$$\Gamma(x) = \frac{1}{2} \sum_{i \neq j} \|x_i - x_j\|^2$$

$$\Lambda(v) = \frac{1}{2} \sum_{i \neq j} \|v_i - v_j\|^2.$$

Theorem 1 *Assume that*

$$a_{ij} = \frac{K}{(\sigma^2 + \|x_i - x_j\|^2)^\beta}.$$

Assume also that one of the three following hypothesis hold:

(i) $\beta < 1/2$,

(ii) $\beta = 1/2$ *and* $\Lambda_0 < \frac{K^2}{18k^2}$,

(iii) $\beta > 1/2$ *and*

$$\left[\left(\frac{1}{2\beta}\right)^{\frac{1}{2\beta-1}} - \left(\frac{1}{2\beta}\right)^{\frac{2\beta}{2\beta-1}}\right]\left(\frac{K^2}{18k^2\Lambda_0}\right)^{\frac{1}{2\beta-1}} > 2\Gamma_0 + \sigma^2.$$

Then there exists a constant B_0 *such that* $\Gamma(t) \leq B_0$ *for all* $t \in \mathbb{R}_+$. *In addition, when* $t \to \infty$, $\Lambda(t) \to 0$ *and the vectors* $x_i - x_j$ *tend to a limit vector* $\widehat{x_{ij}}$, *for all* $i, j \leq k$.

Random Model

G. Tang and L. Guo, "Convergence of a class of multi-agent systems in probabilistic framework," J. of Sys. Sci. and Complexity, 20 (2007): 173-197.

Model:

$$\begin{cases} \theta(t) = P(t-1)\theta(t-1) \\ x(t) = x(t-1) + vs(\theta(t)) \end{cases}$$

where x is the position, $\theta \in (-\pi, \pi]$ is the angle, $s(\theta) = (\cos\theta, \sin\theta)$, $P(t-1)$ is the average angle of the whole network at time $t-1$, and v is a constant speed.

Neighborhood:

$$N_k(t) = \{ j : \| x_j(t) - x_k(t) \| < r \}, \quad 1 \le k \le n$$

where n is the number of nodes in the network and r is the radius of the ball (neighborhood).

Optimization:

$$\min_{j \in N_k(t-1)} \left(\theta - \theta_j(t-1) \right)^2$$

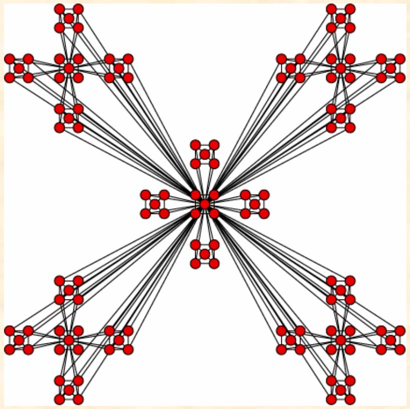
where θ is the target angle.

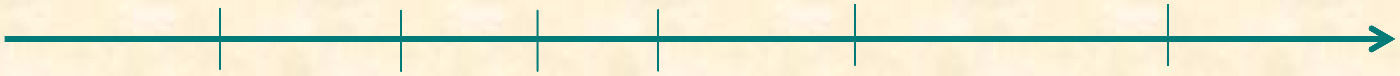
<u>**Theorem**</u>: For any given speed $v > 0$, any given radius $r > 0$, and any large enough size n, the network will synchronize with probability no less than $1 - O\left(n^{g_n}\right)$, where $g_n = n / \ln^6 n$.

Consensus Protocol

Design a network connection topology, or design local control law, so that $\boxed{\|x_i - x_j\| \to 0}$ (consensus = synchronization)

Consensus is reached asymptotically **if** there exists an infinite sequence of bounded intervals such that the union of the graphs over such intervals is totally connected.



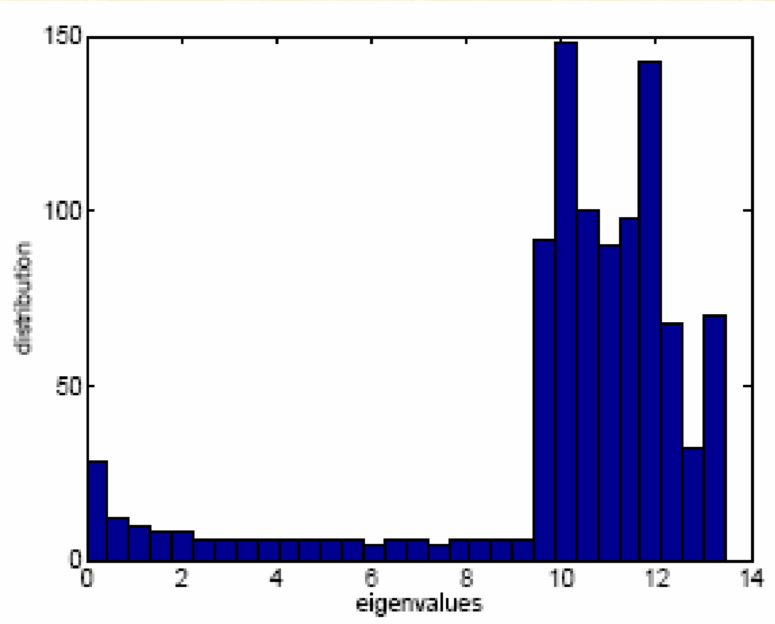


Olfati-Saber, Murray, "Consensus problems in networks of agents with switching topology and time-delays," IEEE Trans. Auto. Control, 2004, 49(9): 1520-1533

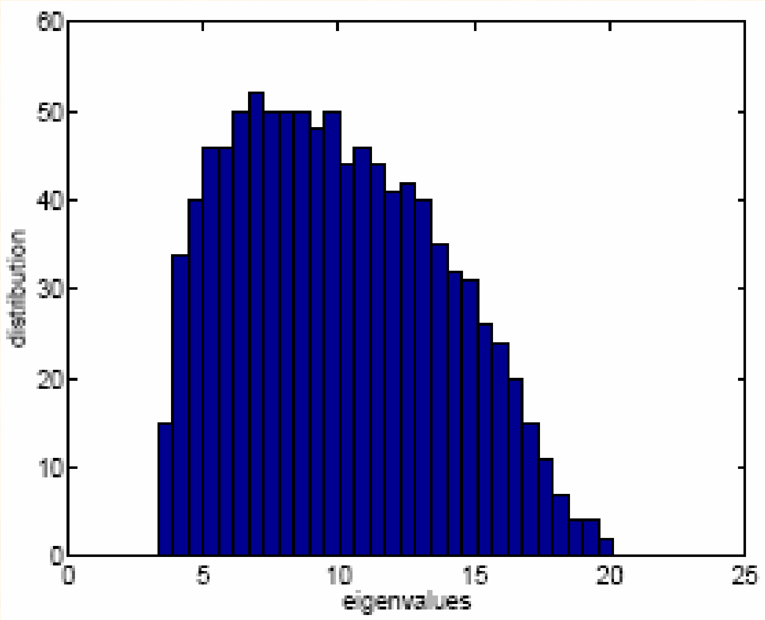
Small-World Networks
are better for Consensus

λ_n / λ_2 Condition number -- the smaller, the better

Regular networks



Small-world networks



1000 times average

Olfati-Saber, "Ultrafast consensus in small-world networks," Amer. Control Conf., 2005

Flocking

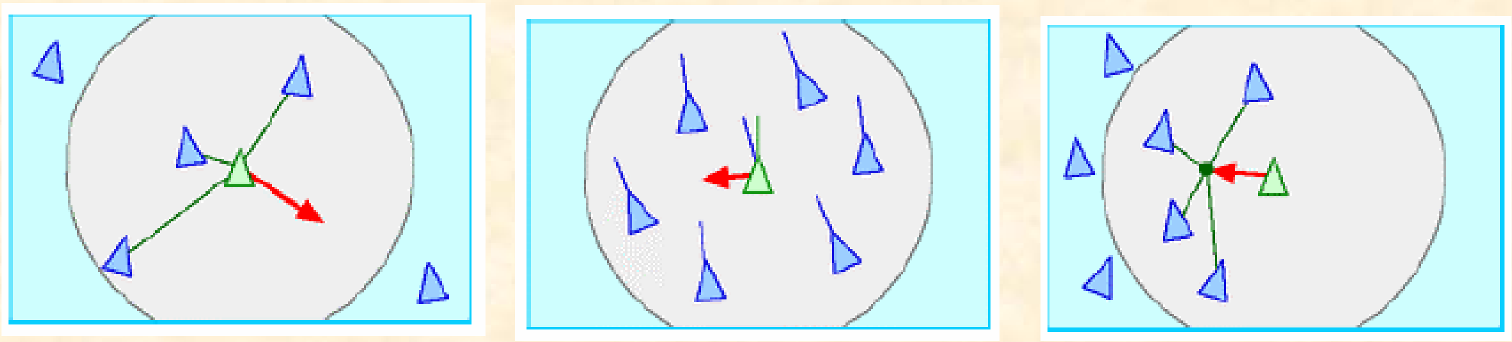
Boids Flocking Model

Three Rules:

Separation: Steer to avoid crowding local flockmates

Alignment: Steer to move toward the average heading of local flockmates

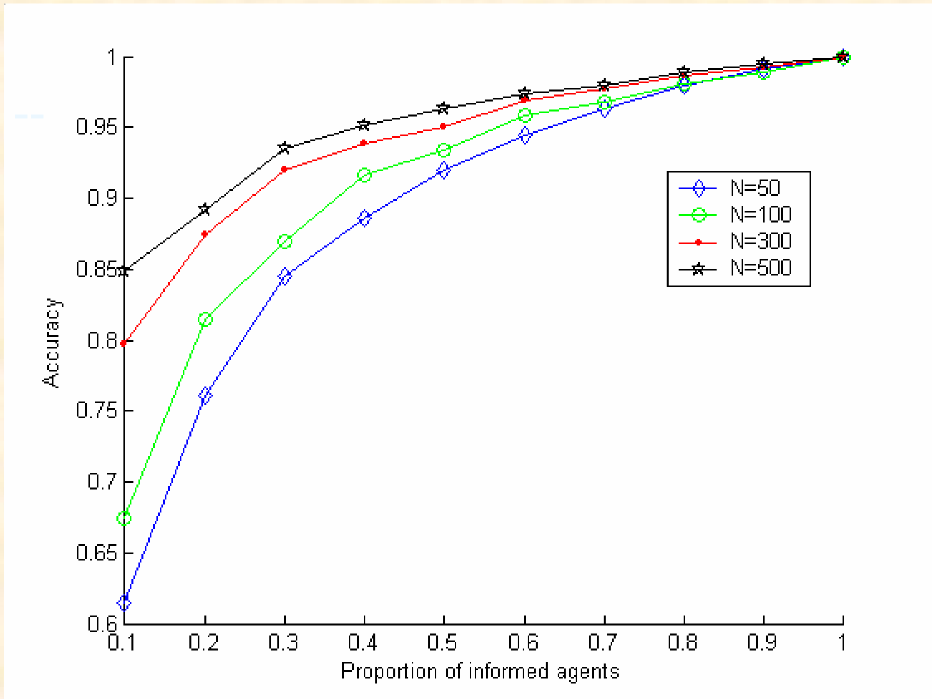
Cohesion: Steer to move toward the average position of local flockmates



Reynolds, "Flocks, herd, and schools: A distributed behavioral model," Computer Graphics, 1987, 21(4): 15-24 **http://www.red3d.com/cwr/boids/**

Flocking: Feedback Control

"The larger the group, the smaller the portion of agents who need navigational feedback" (H. S. Su, X. F. Wang, Phys Rev E, 2007)

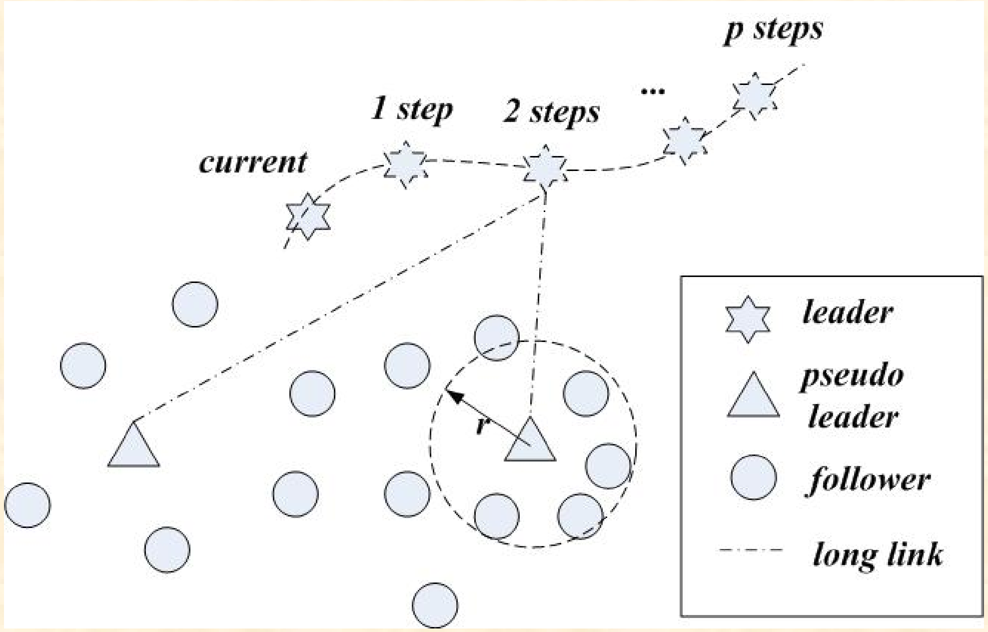


"We reveal that the larger the group the smaller the proportion of information individuals needed to guide the group"

I. D. Couzin, J. Krause, N. R. Franks, S. A. Levin, Nature, Feb. 3, 2005

Flocking: Model Predictive Control

Small-world communication generates “pseudo-leaders” who control their neighbors

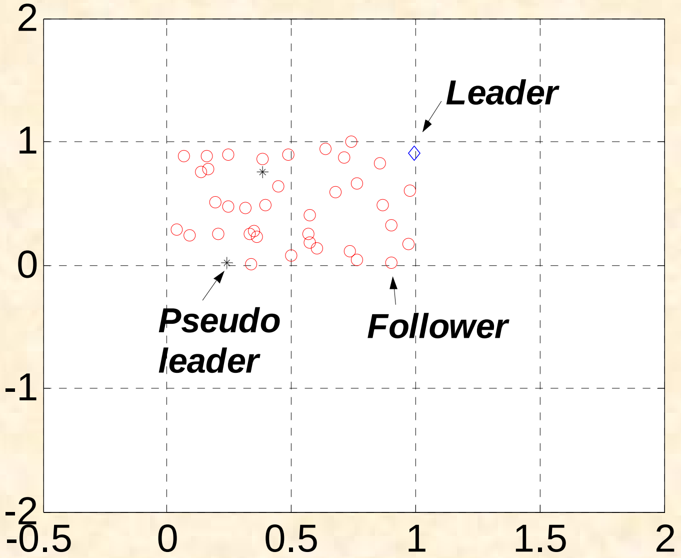


Minimizing:

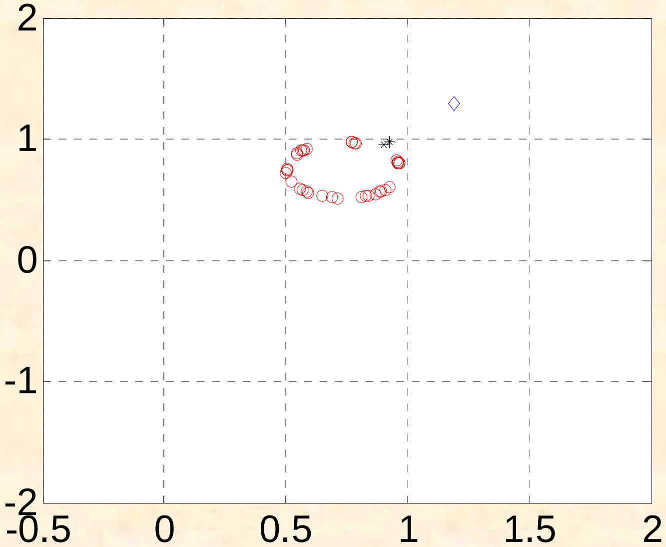
$$(1/N)\sum_{i=1}^{N}\left|v_i - v_{leader}\right|$$

Zhang, Chen, “Small-world network-based coordinate predictive control of flocks,” PhysCon, Germany, 2007

Flocking: Model Predictive Control



Initial position of the flock

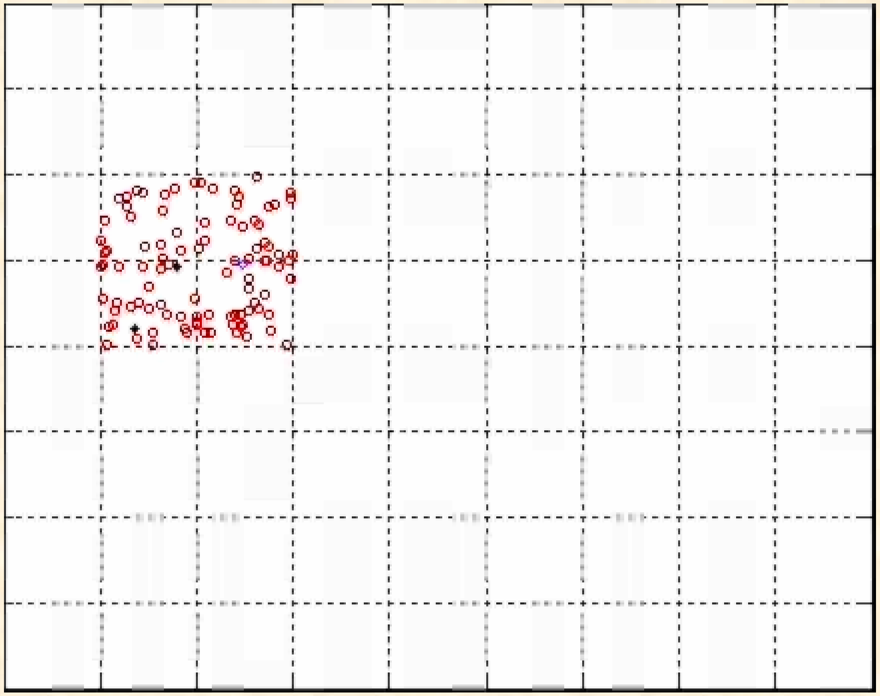


Flock position after 40 iterations under MPC

Zhang, Chen, "Small-world network-based coordinate predictive control of flocks," PhysCon, Germany, 2007

Flocking: MPC DEMO

Small-world MPC



Zhang, Chen, "Small-world network-based coordinate predictive control of flocks," PhysCon, Germany, 2007

Another Flocking Algorithm: DEMO

Lu *et al.* in progress (2007)

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- My personal technical papers:
- **http://www.ee.cityu.edu.hk/~gchen/Internet.htm**

Thank You !