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# Information traffic in scale-free networks with fluctuations in packet generation rate

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## ABSTRACT

We study the information traffic in scale-free networks where the information generation rate varies with time as a periodic function. We observe that when the fluctuation in packet generation rate increases, the average transit time increases and network performance degrades. In order to improve the transportation efficiency in this situation, we propose a new routing method called mixed routing. It operates in two modes: (1) when the packet generation rate is small, the shortest paths are used to deliver the packets to the destination; (2) when the packet generation rate is large, the traffic loads in central nodes are redistributed to other non-central nodes, using the so-called efficient routing method. We find that the time shifting between the two modes is very critical for the routing performance. Consequently, we provide an efficient method to determine the critical times to shift the routing modes for achieving good network performance.

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## 1. Introduction

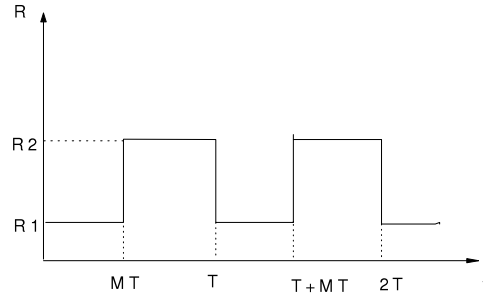
Since the seminal work on scale-free networks by Barabási and Albert (BA model) [1] and on the small-world phenomenon by Watts and Strogatz [2], the structures and dynamics of complex networks have attracted a tremendous amount of interest and attention from the physics and engineering communities [3–5]. It has been well proved that the topological features of the underlying networks have great impacts on the final outcomes of the dynamics taking place on them [3,4,6–8].

Among all the transport problems investigated in various kinds of complex networks [3,4,8–12] information transport in communication networks such as the Internet may be of practical importance. In the study of information traffic in complex networks, there have been many reports indicating traffic fluctuations on networks [13]. Therefore, it is of great interest to study the impact of fluctuations in traffic density. Here the traffic fluctuation means at different times we have different amount of traffic to handle. But the average packet generation rate is fixed. In this paper, we investigate in detail how the fluctuations of traffic density affect the data traffic behavior in complex heterogeneous networks.

Typically, the packet generation rates are assumed to be invariable [14]. During the simulation time, for every time step we have the same number of packets generated. However, due to the change of users, at different times we may have different number of packets to deliver. For example, in the telephone network, we have business data traffic during office hours [15]. In the Internet, we observe time-dependent traffic on Internet routers of Mid-Atlantic Crossroads network by MRTG (Multi-Router Traffic Grapher software) [13]. It is sensible to take the traffic changing behavior into account in the traffic study. In this paper, we consider packet generation rate varying with time, and study how this traffic behavior affects

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**Fig. 1.** The changing  $R$  against time.

network performance. In order to improve the transportation efficiency in this situation, we propose a routing method called mixed routing. It operates in two modes: (1) when the packet generation rate is small, the shortest paths are used to deliver the packets to the destination; (2) when the packet generation rate is large, the traffic loads in central nodes are redistributed to other non-central nodes, using the so-called efficient routing method [14]. We find that the time shifting between the two modes is very critical for the routing performance. Consequently, we provide an efficient method to determine the critical times to shift the routing modes for achieving good network performance.

## 2. The model

In this paper, we define all the nodes as both hosts and routers [16,17]. For simplicity, we use the well-known BA scale-free network model as the basic infrastructure on top of which a packet delivery process is taking place. The BA model contains two generic mechanisms: growth and preferential attachment [1,6], which can be constructed as follows. Starting from  $m_0$  nodes, one node with  $m$  links is added at each time step in such a way that the probability  $\prod_i$  of a new link being connected to an existing node  $i$  is proportional to the degree  $k_i$  of node  $i$ , i.e.,  $\prod_i = k_i / \sum_j k_j$ , where  $j$  runs over all existing nodes. In the present work, the total network size is fixed with  $N = 1000$  and the parameters are set to be  $m_0 = 4$ ,  $m = 2$ . Hence, the average connectivity of the network is  $\langle k \rangle = 4$  [6]. The degree distribution of the generated BA network,  $P(k)$ , which denotes the probability of a randomly selected node in the network having exactly degree  $k$ , follows a power law  $P(k) = k^{-\gamma}$  with the exponent  $\gamma = 3$ .

On the other hand, the packet transmission on the network, i.e., the packet generating process and the packet delivery process, is implemented by a discrete-time parallel update algorithm. At each time step  $t$ , there are  $R(t)$  packets generated from the system with randomly chosen sources and destinations, where  $R(t)$  is a periodic function of time step  $t$  with period  $T$ . In this paper, we select the function as follows:

$$R(t) = \begin{cases} R_1 & nT < t < (n+M)T \\ R_2 & (n+M)T \leq t \leq (n+1)T \end{cases}$$

where  $n = 0, 1, 2, \dots$  is a non-negative integer,  $R_1 \leq R_2$ , and  $M$  ( $0 \leq M \leq 1$ ) is the proportion of the small packet generation rate in the system. Fig. 1 illustrates the function of  $R(t)$ .

For example, for the time-dependent traffic on Internet routers of the Mid-Atlantic Crossroads network [13] (Please see Fig. 1(a) of Ref. [13]), the period  $T$  is almost 20 h. The traffic throughput changes almost every 10 h, which means  $M$  is almost 0.5.

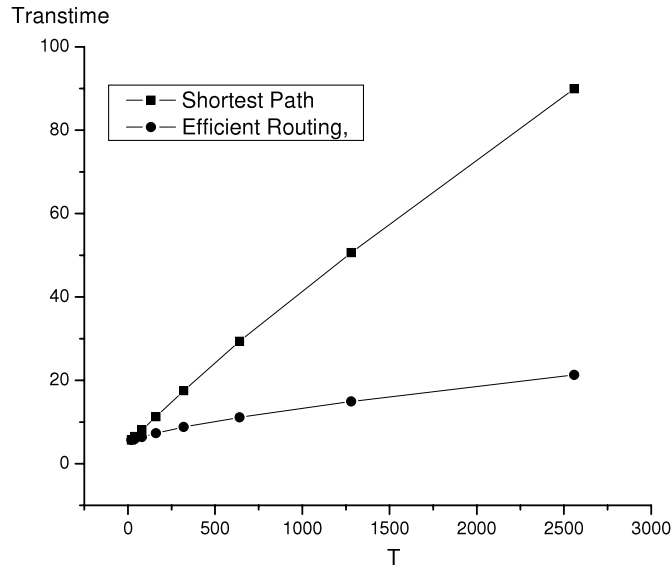
All of our results in the following parts are presented for average packet generation rate  $\bar{R} = 5$  so that we have the same throughput. In addition, we assume that all the routers have the same capabilities in delivering and handling information packets. That is, at each time step all the nodes can deliver at most  $C$  packets one step toward their destinations according to the routing table. For simplicity, we set  $C = 1$  and assume every node has infinite buffer. A packet, upon reaching its destination, will be removed from the system.

## 3. The basic mixed routing strategy, results and discussion

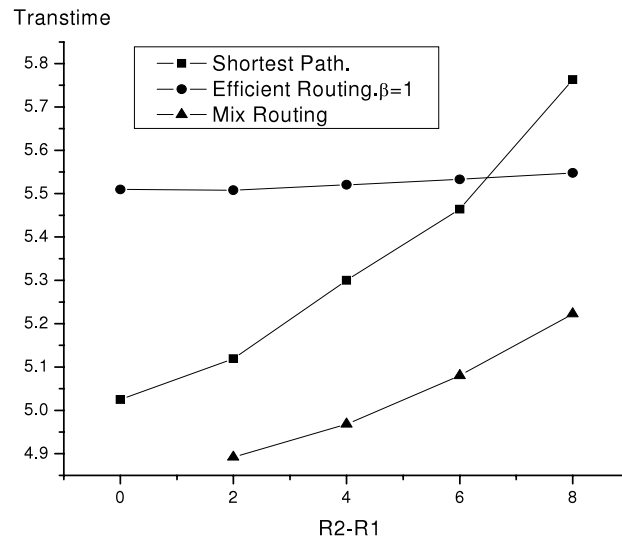
The mixed routing algorithm works in two modes:

- (1) Light traffic mode:  
When the packet generation rate is small, we use the shortest path routing.
- (2) Heavy traffic mode:  
When the packet generation rate is large, we redistribute the traffic load from the central nodes to other non-central nodes, using the so-called efficient routing proposed in Ref. [14]. For any path between nodes  $i$  and  $j$ :  $P(i \rightarrow j) := i \equiv x_0, x_1, \dots, x_{n-1}, x_n \equiv j$ ,  
Denote:  $L(P(i \rightarrow j)) = \sum_{i=0}^{n-1} k(x_i)$ .

The efficient path between  $i$  and  $j$  is corresponding to the route that makes the sum  $L(P(i \rightarrow j))$  minimum, which is the optimal routing method to get the largest critical packet generation rate [14]. We use a parameter  $R_c$  to determine how to



**Fig. 2.** Relationship between the parameter  $T$  and the average transit time. For the shortest path method:  $R_1 = 1, R_2 = 9, M = 0.5$ . For the efficient routing:  $R_1 = 1, R_2 = 81, M = 0.95$ .



**Fig. 3.** Relationship between the fluctuation of  $R$  and average transit time for  $M = 0.5, T = 20$ . (From left to right: the values of  $(R_1, R_2)$  are  $(5, 5), (4, 6), (3, 7), (2, 8), (1, 9)$ .)

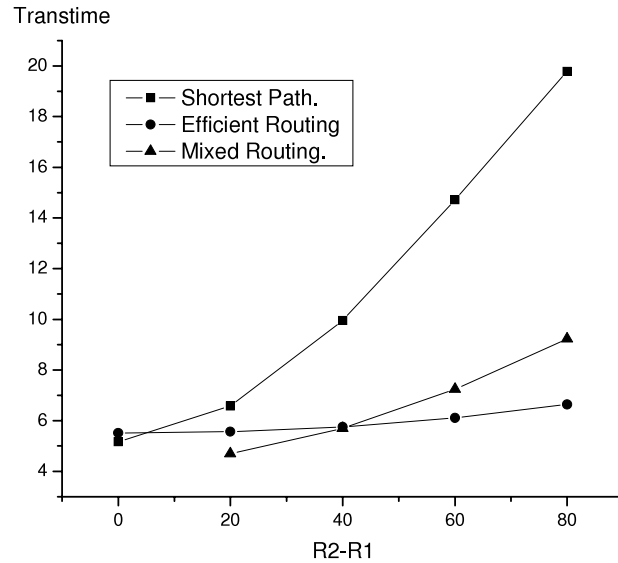
shift between these two routing modes. In particular, when  $R < R_s$ , we use the shortest path routing to deliver the packet to the destination; when  $R \geq R_s$ , we use the efficient routing to redistribute the traffic load. In simulation, we consider  $R_s = \bar{R} = 5$ .

We are interested in studying the relation between the parameter  $T$  and the average transit time when  $R(t)$  is varied. Also, we are interested in knowing how the difference between  $R_1$  and  $R_2$  affects the average transit time.

Based on the above setting of the mixed routing, we run the simulation and obtain some results as follows:

In Fig. 2, we show the relationship between the parameter  $T$  and the average transit time. Because different routing methods correspond to different critical generation rates  $R_c$  [14], by simulations we get that  $R_{c1} \approx 5.5$  for the shortest path method and  $R_{c2} \approx 43$  for the efficient routing. Although we have the same average packet generation rate, we obtain different critical generating rates, as can be seen from this figure the curve with the shortest path ascends more rapidly.

In Fig. 3, we show the relationship between the fluctuation of  $R$  and the average transit time with fixed period  $T$  and proportion  $M$ . The three curves correspond to different routing methods. We can see that the mixed routing has the best performance. On the other hand, we see that the efficient routing is the steadiest routing method, which keeps almost unchanged with the increase of  $R_2 - R_1$ . But we can also see the shortcoming of the efficient routing: when the information



**Fig. 4.** Relationship between the fluctuation of  $R$  and the average transit time for  $M = 0.95$ ,  $T = 100$ . (From left to right: the values of  $(R_1, R_2)$  are (5, 5), (4, 24), (3, 43), (2, 62), (1, 81).)

flow is steady and the traffic density is small, it spends more time to transport a packet to the destination as compared with other routing methods.

#### 4. The mixed routing with congestion avoidance, results and discussion

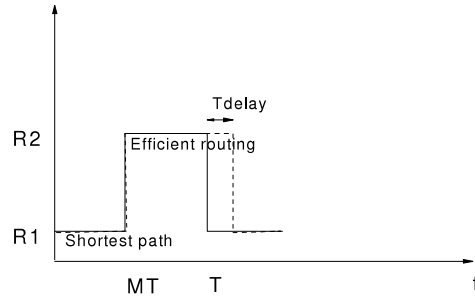
Next, we show another figure of the relationship between the fluctuation of  $R$  and the average transit time using the basic mixed routing strategy. With fixed period  $T$  and proportion  $M$ , the three curves correspond to different routing methods. We can see that the mixed routing has the best performance at the start, but when the fluctuation becomes larger, the efficient routing has the best performance. We will explain the reason in the next paragraph.

According to the mixed routing rule, when the information generation rate is large, we redistribute the traffic load; when the packet generation rate is small, we use the shortest path to deliver the packet to destination. In Fig. 4, when  $R_2 - R_1 > 40$ , the result of the mixed routing becomes not as good as that of the efficient routing. It is because when  $R_2 - R_1 > 40$ ,  $R_2$  is large so the network becomes congested when packet generation rate is  $R_2$ . However, when packet generation rate becomes small, according to the mixed routing method, the routing mode will be shifted to the shortest path routing immediately. Those packets queued in the network due to previous heavy traffic could have high possibility to be still around the nodes with large degrees at the time the generation rate becomes small. We can imagine that shifting to the shortest path routing immediately is not good for this situation. This explains why the mixed routing does not perform well when  $R_2 - R_1 > 40$  in Fig. 4. In order to improve the routing strategy, we introduce a modified version of the mixed routing, called the mixed routing with congestion avoidance. It works as follows. When  $R$  changes from a large value to a small value, we delay the packet for a period of time  $T_{\text{delay}}$  (which is a design parameter and is given below) to shift the routing mode from the efficient routing to the shortest path routing. On the other hand, when  $R$  changes from a small value to a large value, we shift the routing method from shortest path routing to efficient routing immediately as in the basic mix routing. The rationale of introducing the parameter  $T_{\text{delay}}$  is when the network packet generation rate changes from high to low, there are still a large number of packets being transmitted in the network and it takes time to clear them up from the network. However, if we switch the routing algorithm right away to the shortest path routing, all the traffic will be switched to the central nodes and could lead to network congestion there.

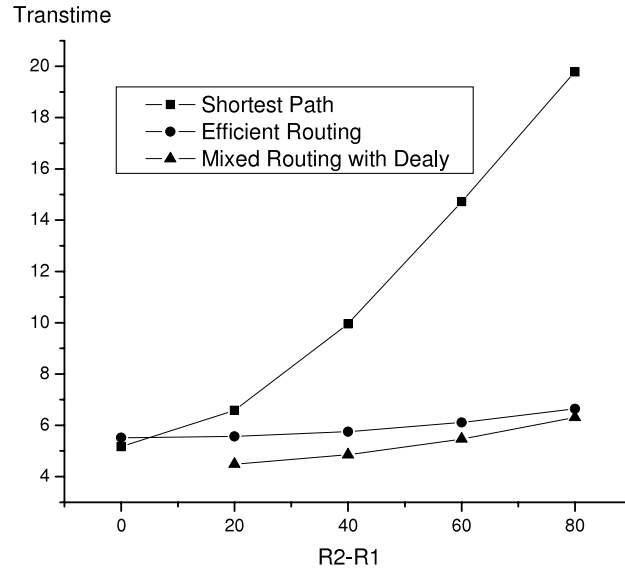
Here, we try to determine a proper value of  $T_{\text{delay}}$  given a set of system parameters (i.e.,  $R_2 - R_1$ ,  $T$  and  $M$ ) such that a good network performance can be obtained for the mixed routing with congestion avoidance. From the above analysis, we can see that the larger the period  $T$ , the worse the network performance, and the larger the fluctuation, the larger the average transient time. With the consideration of effects on the traffic period, the traffic fluctuation and the total throughput, we choose  $T_{\text{delay}}$  as follows:

$$T_{\text{delay}} = c(R_2 - R_1) * T/M$$

where  $c$  is a constant,  $T$  is the period of packet generation,  $R_2 - R_1$  is the fluctuation of  $R$  and  $M$  is the proportion of the small packet generation rate in the system which reflects the whole network throughput. The above formula for  $T_{\text{delay}}$  reflects that the fact that the larger the values of  $T$  and  $R_2 - R_1$ , the more packets queued in the network and the more time (i.e., the larger  $T_{\text{delay}}$  value) needed to clear up those packets queued in the network. On the other hand, the larger the value of  $M$ , the less packets queued in the network and the less time (i.e., the smaller  $T_{\text{delay}}$  value) needed to clear up those queued packets.



**Fig. 5.** The introduction of time delay  $T_{\text{delay}}$  for switching efficient routing to shortest path routing in order to avoid congestion.



**Fig. 6.** Relationship between  $R_2 - R_1$  and average transit time with delay to switch routing modes for  $M = 0.95$ ,  $T = 100$ .  $T_{\text{delay}} = 0.0005(R_2 - R_1) * T/M$ . (From left to right: the values of  $(R_1, R_2)$  are (5, 5), (4, 24), (3, 43), (2, 62), (1, 81).)

Fig. 6 shows that using congestion avoidance scheme with  $c = 0.0005$ , the mixed routing gets a much better performance as compared with the case shown in Fig. 5 where the congestion avoidance scheme is not used, i.e.,  $T_{\text{delay}}$  is zero.

Using the Mid-Atlantic Crossroads network [13] as an example again, with  $T$  is almost 20 h, we get  $T_{\text{delay}} = c(R_2 - R_1) * T/M$ , or

$$T_{\text{delay}} = 0.0005 * 20 * 60 * (R_2 - R_1) / 0.5 \text{ min} = 1.2(R_2 - R_1) \text{ min}.$$

For a scale-free network with size  $N = 1000$ , the critical value of packet generation rate  $R_{c1} \approx 5.5$  for shortest path routing. With such a value of  $R_{c1}$ , a reasonable engineering choice for  $R_1$  and  $R_2$  is  $R_1 = 2$  and  $R_2 = 10$  where 10 is a very large packet generation rate for this network size. For such a choice, the average rate, i.e.,  $(R_1 + R_2)/2$ , is 6 which is larger than  $R_{c1} \approx 5.5$ , meaning that the network is congested. This justifies our choice. From the above formula, we can get the effective delay  $T_{\text{delay}}$  is around 10 min for normal fluctuation of packet generation rate with the period of 24 h.

We have also performed simulations for other values of  $R_1$ ,  $R_2$ ,  $M$  and  $T$ . The qualitative behaviors of the relationship curves shown in Fig. 6 remain the same.

## 5. Conclusion

We have studied information traffic in BA scale-free heterogeneous networks. The nodes are assumed to have infinite buffers and same capacity of processing packets in each time step. We considered a practical situation: packet generation rate is varied with time. To be consistent with some findings in the real life [13], we used a periodic function to describe the dynamic behavior of the packet generation rate. We found that with the increase of the fluctuation in packet generation rate, the average transit time increases and the network performance degrades. According to the dynamic characteristics of the packet generation rate, we have proposed a routing method called the mixed routing which is a switching scheme between the shortest path routing and the efficient routing. Numerical results have shown that with proper design on the shifting instances between the two routing modes according to the traffic dynamic behavior, the mixed routing has better network

performance than the shortest path routing and the efficient routing. We find that the time shifting between the two modes is very critical for the routing performance. Consequently, we provide an efficient method to determine the critical times to shift the routing modes for achieving good network performance.

Although we use packet transmission for simulation, we believe that the conclusion and insights obtained in this paper can also apply to other complex systems having periodic traffic behavior. Although the average transit time improvement is not very marked, we believe the packets generated in the light traffic mode can be delivered to the destinations much more rapidly. This has practical significance when we focus on the unit in the complex network.

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