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Integrating network structure and dynamic information for better routing strategy on scale-free networks

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ABSTRACT

We study information packet routing processes on scale-free networks by mimicking the Internet traffic delivery strategies. We incorporate both the global network structure information and local queuing information in the dynamic processes. We propose several new routing strategies to guide the packet routing. The performance of the routing strategies is measured by the average transit time of the packets as well as their dependence on the traffic amount. We find that the routing strategies which integrate both global network structure information and local dynamic information perform much better than the traditional shortest-path routing protocol which takes into account only the global topological information. Moreover, from comparative studies of these routing strategies, we observe that some of our proposed methods can decrease the average transit time of packets but the performance is closely dependent on the total amount of traffic while some other proposed methods can have good performance independent of the total amount of traffic with hyper-excellent average transit time of packets. Also, numerical results show that our proposed methods integrating network structure information and local dynamic information can work much better than the methods recently proposed in [S. Sreenivasan, R. Cohen, E. López, Z. Toroczkai, H.E. Stanley, Phys. Rev. E 75 (2007) 036105, Zhi-Xi Wu, Gang Peng, Eric W.M. Wong, Kai-Hau Yeung, J. Stat. Mech. (2008) P11002.], which only considered network structure information.

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1. Introduction

In recent years, tremendous interest has been devoted to studying statistical and dynamical properties of largescale networks with complex structures. After the seminal works of [3,4], extensive research has been carried out, for example, on the following topics: network growth and self-organization, degree and betweenness distributions, complex network resilience and cascading breakdown, epidemiological processes, community structures, and network stability and synchronization (see Refs. [5–7] and references therein). Complex networks thus have become an active field in nonlinear science.

Network transport is a problem encountered in a variety of systems, including biological, social, and a multitude of natural and human-made transport and communication systems. The quantities to be transported can be information transported on the Internet or other networks and systems such as cars in transport system [3,6,8–14,1,15–27]. Routing protocol of network transport is well known to be an important problem in many fields of science and technology today [8–14,1,15–32]. A great body of work on this subject has been carried out. In Refs. [13,28], Kevin et al. proposed an optimal

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routing method and applied it to three different networks, Erdős–Rényi, [33] Barabási–Albert, [4] and uncorrelated scalefree networks generated using the configuration model, [34]. With this method, the three networks can sustain significantly higher traffic without jamming than in the case of shortest path routing. In Refs. [29,30], Wang et al. proposed some local routing strategies, in these two papers, both the average transit time and critical packet generation rate are discussed. According to Refs. [29,30], we know that integrating the network dynamic information can make the routing strategy work much better. In Refs. [14,1,2] the authors designed some global routing strategies based on network structure. In Refs. [17, 18,31] the traffic awareness routing strategy is designed by the authors using network structure and dynamic information.

In this paper, we study the impact of traffic routing protocols on the performance of communication systems by incorporating traffic awareness. More specifically, we are exploring how the average network performance depends on the ability of the routing protocol to divert traffic across paths other than the shortest ones. To this end, we numerically explore some new routing protocols in which tunable parameters account for the degree of traffic awareness incorporated in packet delivery. We find that the routing strategies which integrate both global network structure information and local dynamic information perform much better than the traditional shortest-path routing protocol, which takes into account only the global topological information, as well as the methods recently proposed in Refs. [1,2], which only considered network structure information.

2. The model

It was argued that the Internet at the autonomous system level shows a scale-free degree distribution [35]. We thus focus our attention on information packet traffic in scale-free networks. First, we adopt the Barabási–Albert algorithm to build a scale-free network on top of which a packet delivery process is taking place [4]: Starting from two connected nodes, we add new nodes with two links to the existing network one by one; each link of the new node is attached to an existing node *i* in the network with a probability proportional to its degree:

$$\Pi_i = k_i / \Sigma_j k$$

where *j* runs over all existing nodes; during the growth process of the network, duplicate links between nodes are forbidden, and the growth process stops at the time of network size attaining N = 5000 in our simulations. According to Refs. [4], the average connectivity and the degree distribution of the generated network are, respectively, $\langle k \rangle = 4$ and $P(k) \sim k^{-\gamma}$ with the exponent $\gamma = 3$ in the large degree limit.

With the underlying infrastructure at hand, we let each node act as a host and a router at the same time. We also allow each node to have an infinite queue length, i.e., it can store as many packets as necessary. In order to have a realistic framework of communication, we limit the ability of nodes to deliver packets and assume that, at every time step, each node *i* can deliver at most $1 + k_i^{\theta}$ packets one step toward their destinations, where $\theta \ge 0$ characterizes the extent of heterogeneity of their packet-handling ability. We call θ the capacity parameter. If $\theta = 0$, all nodes have the same ability irrespective of their link degrees, while $\theta > 0$ means larger degree possess stronger capacity. At the beginning of the delivery process, each node creates m_0 information packets with destinations being randomly selected among the remaining N - 1nodes. Thus, the total number of information packets is $N_p = Nm_0$. In subsequent time steps, the packet transmission on the network is implemented by a parallel update algorithm. Each node *i* processes at most $1 + k_i^{\theta}$ packets in its queue, based on first-in-first-out rule, and selects the next routing node for the packets to go according to the routing strategy given below.

3. The routing protocol

In Ref. [2], Wu et al. proposed a routing strategy based on local degree information and global network structure information, whose performance is found to be better than the shortest path (SP) routing and robust against the traffic amount. Motivated by the fact that integrating network dynamic information can make the routing strategy works much better [29,30], we propose the following routing methods: at each time step, all the packets move from their current position, *i*, to the next node in the path, *j*, with a probability Q_{ij} defined by either one of the following four formulas:

$$Q_{ij} = \frac{c_j^{\alpha} \exp[-\beta(D_{id} - D_{jd} - 1)]}{\sum_{l \in \Omega_i} c_l^{\alpha} \exp[-\beta(D_{id} - D_{ld} - 1)]}$$
(1)

$$Q_{ij} = \frac{(c_j/k_j)^{\alpha} \exp[-\beta(D_{id} - D_{jd} - 1)]}{\sum_{l \in \Omega_i} (c_l/k_l)^{\alpha} \exp[-\beta(D_{id} - D_{ld} - 1)]}$$
(2)

$$Q_{ij} = \frac{(c_j/E_j)^{\alpha} \exp[-\beta(D_{id} - D_{jd} - 1)]}{\sum_{l \in \mathcal{Q}_i} (c_l/E_l)^{\alpha} \exp[-\beta(D_{id} - D_{ld} - 1)]}$$
(3)

where $E_j = 1 + k_j [1 - \exp(-\overline{ac_j^t})]$ with $\overline{c_j^t} = \frac{1}{4}(c_j^{t-3} + c_j^{t-2} + c_j^{t-1} + c_j^t)$, *a* is a tunable parameter, a > 0, in this simulation, we set a = 4, and *t* is the simulation time step, or

$$Q_{ij} = \frac{(c_j/E_j)^{\alpha} \exp[-\beta(D_{id} - D_{jd} - 1)]}{\sum_{l \in \Omega_i} (c_l/E_l)^{\alpha} \exp[-\beta(D_{id} - D_{ld} - 1)]}$$
(4)

with $E_i = 1 + k_i^{\theta}$.

Also, we compare our routing protocols with the routing protocol discussed in Ref. [2]:

$$Q_{ij} = \frac{k_j^{\alpha} \exp[-\beta(D_{id} - D_{jd} - 1)]}{\sum_{l \in \Omega_i} k_l^{\alpha} \exp[-\beta(D_{id} - D_{ld} - 1)]}.$$
(5)

As was suggested, in heterogeneous networks, the SP routing strategy is easy to cause overloading of the hub nodes because so many paths go through them [36]. However, the SP routing provides a benchmark for newly proposed routing protocols. Only those protocols superior to SP routing are of theoretical and practical importance. To allow a direct comparison with SP routing, we discuss the relative difference between the average transit times of our strategies with that of SP routing in this paper.

In all the above formulas Eqs. (1)–(5), Ω_i is the set of neighboring nodes of *i*, c_j is the number of packets queued in node *j* queue length, k_j is the link degree of node *j*, $1 + k_j^{\beta}$ is the processing ability of node *j*, and D_{id} is the minimum number of hops which one starting from *i* has to pass by in order to reach the destination *d*, i.e., the shortest path between *i* and *d*. The parameter α and β are tunable parameters with $\alpha \in (-\infty, 0]$ and $\beta \in [0, \infty)$. For $\alpha = 0.0$ and $\beta \rightarrow \infty$, we recover the SP routing strategy. With the decrease of β to zero, the global topological information involved in the routing protocol is reduced. It is worth noting that when $\beta = 0$, we implement exactly a local routing protocol for the packet traffic.

The probability Q_{ij} can be used to quantify communication between agents *i* and *j* [36]. The essential idea behind Eqs. (1)–(5) is that we prefer to send packets to a transmitter with one step closer to their destinations, but when there is more than one alternatives, we would like to select a proper one according to the available local topological information (the degrees of the candidate nodes) and local dynamical information (the number of packets queued in the node). A finite value of β means that the selection of a transmitter with an equal or even larger distance to the destination (as compared to that of the present sender) is also possible. The rationale behind the form of Q_{ij} comes from the following considerations. The heterogeneous abilities of the nodes to deliver information result in heterogeneous numbers of packets waiting in their queues. In order to improve the efficiency, we prefer to send the packets to those nodes with as small a waiting time as possible before they are handled. Assuming that we have two alternative nodes to receive a packet along its shortest path, one is a hub node (with a stronger ability to handle the task in each time step) and the other is non-hub. If the packet waits in the queue of the hub node for too long to compensate the time, it could stay in the queue of the non-hub node. We of course would like to select the non-hub one as the next sender of the packet in this case. In practice, this can be achieved by adjusting the parameter α to help us make an appropriate decision.

In (1), we consider the queue length as an influence factor when selecting the next hop. With $\alpha \in (-\infty, 0]$, we prefer to select the node with a smaller queue length, which indicates that the packet may wait for less time to be handled. For $\theta > 0$, larger degree nodes have stronger capacities to process packets, so we make a revision in (1) which we just consider for the queue length in the first part of the equation. In (2), we set c_j/k_j (the queue length divided by the node degree) as an influence factor for selecting the next hop. With $\alpha \in (-\infty, 0]$, we prefer to select the node with a smaller queue length and a larger node degree. The meaning of $\frac{c_j}{k_j}$ is, at this time step, the average number of packets the node should send out to its every neighbor in order to process the collection in its buffer. In (3), we try to combine the strong points of methods (1) and (2). We will explain (3) in more detail in the next section. In (4), we set $E_j = 1 + k_j^{\theta}$, which is the processing capacity of node *j*. We replace c_j by c_j/E_j as an influence factor. The meaning of the first part in (4) is clear: at this time step, with every processing capacity, the number of packets should be sent out in order to empty the node buffer. With $\alpha \in (-\infty, 0]$, we prefer to select the node with a smaller queue length and a larger processing capacity. In (5), the authors in Ref. [2] just considered the local degree information and the topological information, ignoring the dynamic information of the network. In Eqs. (1)–(5), the parameters α and β matter greatly in determining the performance of the proposed method, hence we denote the routing strategy by (α , β).

4. Discussion of results

To determine how the above routing strategies influence the efficiency of the information traffic, we implement different realizations of the dynamics for several values of N_p and θ by smoothly varying the values of α and β , and monitoring the relevant quantities $\langle T \rangle$, which is the average time it takes for all N_p packets to travel from their sources to their destinations. To allow a direct comparison with the efficiency of SP routing, we summarize our simulation results in Figs. 1–5, where the relative difference between the average transit time of our strategies with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, is shown by varying the values of α and β for two combinations of θ and N_p . Note that, for SP routing, the average transit time of the packets is determined by the values of N_p and θ [2]. As can be seen from Figs. 1–5, there exist optimal combinations of α and β for the routing algorithms to achieve their best performance. The darker the region, the more efficient the routing strategy.



Fig. 1. The relative difference between the average transit time of our method (1) with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, on the $\alpha - \beta$ parameter space for two initial numbers of packets N_p and capacity parameter values of θ . From left to right, the parameters N_p and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively.



Fig. 2. The relative difference between the average transit time of our method (2) with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, on the $\alpha - \beta$ parameter space for two initial numbers of packets N_p and capacity parameter values of θ . From left to right, the parameters N_p and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively.

We can see that when N_p is small; the routing strategy (1) has excellent performance. The average transit time is much smaller than that of SP routing in most areas. In the left panel of Fig. 1, for a fixed α , we find an optimal range for β . Since β is a finite value, we can say that a proper detour is beneficial to the network performance. But, when N_p becomes larger, in the right panel of Fig. 1, the usable area superior to SP routing shrinks considerably. For example, when $N_p = 10^4$, $\alpha = -2.5$, $\beta = 2.5$, the new routing algorithm is useful since its average transit time is much smaller than that of SP routing; but when $N_p = 10^5$ and $\alpha = -2.5$, $\beta = 2.5$, the new routing algorithm is totally useless. So we can say that this new algorithm is useful if the total network traffic is low. With the simulation results, we can find the optimal $\alpha - \beta$ at the point the network will get much better performance than SP routing. But, if the network has a heavy amount of traffic, this new routing strategy is not that helpful; because the optimal $\alpha - \beta$ changes with the total traffic amount.

In (2), we use c_j/k_j to replace c_j , considering the queue length divided by the node degree instead of the queue length itself. Comparing these two graphs, we can see that this new method is much steadier in spite of the total traffic fluctuations. The usable area superior to SP routing is almost unchanged with the total packet N_p increasing, although the best performance is not as good as the routing algorithm (1). For example, with $N_p = 10^4$, $\alpha = -2.5$, $\beta = 2.5$, the new routing algorithm is useful since its average transit time is much smaller than that of SP routing; with an increase of the total traffic amount N_p , when $N_p = 10^5$ and $\alpha = -2.5$, $\beta = 2.5$, the new routing algorithm is still better than SP routing. Also, we can see that with a fixed α , we can get an optimal range for β and, if we fix β , we also get an optimal range for α .



Fig. 3. The relative difference between the average transit time of our method (3) with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, on the $\alpha - \beta$ parameter space for two initial numbers of packets N_p and capacity parameter values of θ . From left to right, the parameters N_p and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively.



Fig. 4. The relative difference between the average transit time of our method (4) with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, on the $\alpha - \beta$ parameter space for two initial numbers of packets N_p and capacity parameter values of θ . From left to right, the parameters N_p and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively.

Comparing the results based on the first two routing algorithms, i.e. (1) and (2), we can see that the first routing algorithm can have an excellent average transit time. With the simulation, we observe that the optimal $\alpha -\beta$ is related to the total traffic amount which is an unknown parameter in practical networking applications. However, in the second routing algorithm, the performance is independent of the traffic amount. So, we wonder if we can combine the strong points of the two algorithms. As we know, the more packets that need to be delivered, the more packets are waiting in the queue. So, we consider the routing algorithm (3) instead.

In this method, we use a relatively complicated formula. If every node in the network has few packets waiting (c_j is small), then E_j is approximately equal to 1 and the algorithm reduces to the routing algorithm (1). If every node in the network has many packets waiting in the queue, then E_j is approximately equal to $1 + k_j$ and the routing algorithm reduces, nearly, to (2). With the simulation, we get the above results from which we can see that the algorithm essentially combines the strong points of the two algorithms. When the traffic is light, we get a very large usable area superior to SP routing and the average transit time is very short. When the traffic is heavy, we still have a large usable area and its average transit time is much shorter than that of SP routing.

In (4), we consider the processing ability rather than the degree information, and we show the simulation results for the routing strategy (4) in Fig. 4. Comparing this method (4) with the first method (1), we can see that the presence of the node processing ability does not affect the routing performance by too much. Based on this observation, when designing routing protocols, we can simplify the design complexity by ignoring the node processing ability.



Fig. 5. The relative difference between the average transit time of method (5) [2] with that of SP routing, $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$, on the $\alpha - \beta$ parameter space for two initial numbers of packets N_n and capacity parameter values of θ . From left to right, the parameters N_n and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively.

The authors of Ref. [2] only considered the global shortest-paths information and local degree information when proposing their routing algorithm. There exist optimal combinations of α and β for the routing algorithm (5) to achieve the best possible performance. It seems that a large value of β benefit the improvement of the efficiency of the routing strategy irrespective of other parameters, and it strengthens the important role of the SP information in packet traffic. However, it should be noted that a traditional SP routing (with $\alpha = 0$ and large value limit of β) performs worse than a routing strategy where the global SP information of the nodes and the local degree information are appropriately combined, for example ($\alpha = -2.5$ and $\beta = 5.5$).

In order to illustrate the importance of the local dynamical information, we compare our methods with this method [2]. From the above figure we can see that the performance of the routing method is steady in spite of the varying total traffic. But the performance is worse compared to our methods (1)–(4) which additionally takes into account the local dynamical information. As we can see from Figs. 1–5, the best performance of [2] is $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle} \ge -0.5$, the average transit time $\langle T \rangle$ reduces to half of $\langle T_{sp} \rangle$, whereas our method can make $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sp} \rangle}$ up to -0.7, which means $\langle T \rangle = 0.3 \langle T_{sp} \rangle$. In addition, we can see the gray area where $\langle T \rangle < \langle T_{sp} \rangle$ is much smaller than our methods.

Also we compare our models to the routing algorithm proposed in Ref. [1], where the authors proposed a routing strategy based on big nodes which have more neighbors compared with others. In Ref. [1], the information packets try to keep away from the hub nodes when delivered to the destination. In practice, we:

(I) Remove a number *nH* of the highest degree nodes. The network could now consist of several disconnected clusters. $nH = \frac{\text{The number of removed nodes}}{\text{network size}}$.

(II) In every disconnected cluster, assign a routing path for every pair of nodes using Shortest Path protocol.

(III) Replace the removed nodes with their edges.

(IV) For every pair of nodes which have not been assigned a routing path in step (II), assign one using the Shortest Path protocol.

We show the simulation results in Fig. 6. It can be seen that with the removal of highest degree nodes, a minimal $\langle T \rangle$ at a certain value of $nH = nH^*(N)$ despite the traffic amount rising. Moreover, the advantage of the algorithm [1] decreases as the total traffic increases. Compare the improvement of this algorithm on y-axis with our methods; we can clearly see that our methods of integrating dynamic information and topological information can get much better performance with a proper selection of (α , β).

We have also compared the method in Ref. [1] with other 3 methods we proposed; the qualitative result is the same as Fig. 6.

Now we investigate other parameters which may have an influence on the average transit time. For example, when (α, β, θ) are fixed, N_p changes linearly, the improvement in the average transit time is given in Fig. 7.

As we can see from Fig. 7, when the traffic is very low, the relative improvement of all the 5 methods is not so obvious. This is because when traffic is low, the shortest path can already work very well. As the whole traffic increases, the improvement of method (2) and (3) becomes more evident. But with methods (1) and (4), there exists an optimal value of K_0 for the improvement. If the total traffic keeps increasing behind that optimal point, the performance becomes worse. Also we can see from the graph that the best performance of methods (1) and (4) is better than methods (2) and (3). In addition, the improvement for method (5) is almost unchanged with an increase of the total traffic amount. This is because method (5) has no consideration of the network dynamic information.



Fig. 6. The relative difference between the average transit time of method [1] and method (2) with that of SP routing, $\frac{(T)-(T_{Sp})}{(T_{Sp})}$, with different percentage of removed nodes for two initial numbers of packets N_p and capacity parameter values of θ . The parameters N_p and θ are (10⁴, 0.0) and (10⁵, 0.4), respectively. Left: Method [1], Right: Method (2).



Fig. 7. The relative difference of the average transit time $\frac{(T)-(T_{sp})}{(T_{sp})}$ against K_0 , where $N_p = K_0 \times N$ and N is the network size.

Now, we consider the case where (α, β, N_p) is fixed and θ changes linearly and see the improvement of the average transit time changes.

From Fig. 8, we can see that the improvement of the routing protocols becomes worse as θ increases. This is because all our methods try to divert the traffic flow from hub nodes to non-hub nodes to avoid the congestion in hubs with the cost that the packet may need to pass through more nodes. However, with the increase of θ , the hub nodes get a stronger processing ability and hence the congestion can be handled and avoided effectively by these hub nodes and it no longer needs to divert the traffic to non-hub nodes. So our routing becomes less efficient as θ increases.

5. Conclusions

In this paper, we have studied some alternative strategies for traffic delivery on scale-free heterogeneous networks. The proposed methods integrate both the global topological information and the local dynamical information of the network in the packet routing process. The performance of the routing protocols is weighted by the average transit time of the packets. Through numerical simulations, we have shown that with an appropriate selection of the tunable parameters the proposed routing algorithms are superior to the traditional shortest-path routing protocol which takes into account only the global topological information in designing routing protocols, the protocol performance is independent of the total traffic, but by also considering local dynamic information in designing routing protocols, the protocol design, the traffic amount can affect the protocol performance. In order to lighten the total traffic influence, we have developed several routing algorithms.



Fig. 8. The relative difference of the average transit time $\frac{\langle T \rangle - \langle T_{sp} \rangle}{\langle T_{sn} \rangle}$ against θ .

have shown that the new routing algorithms (2) and (3) essentially achieve the goal. We hope that our work provides some insight into the design of routing protocols for complex-network communication systems.

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