

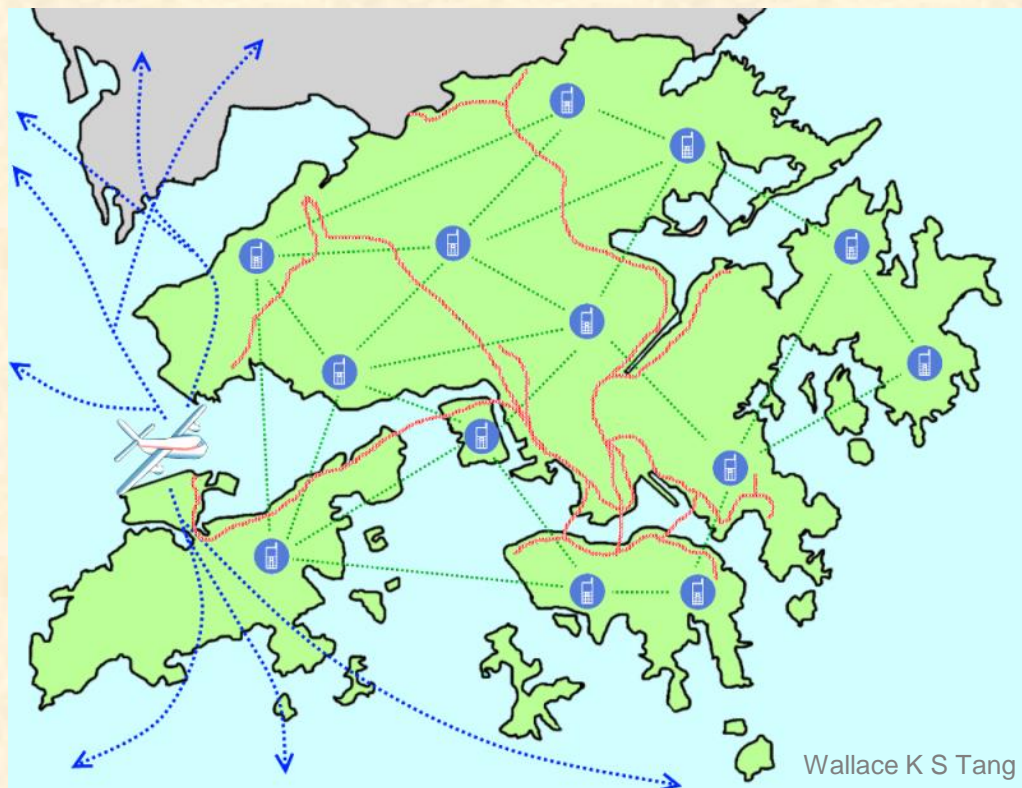
日常生活中的复杂网络

Complex Networks in Daily Life

陈关荣

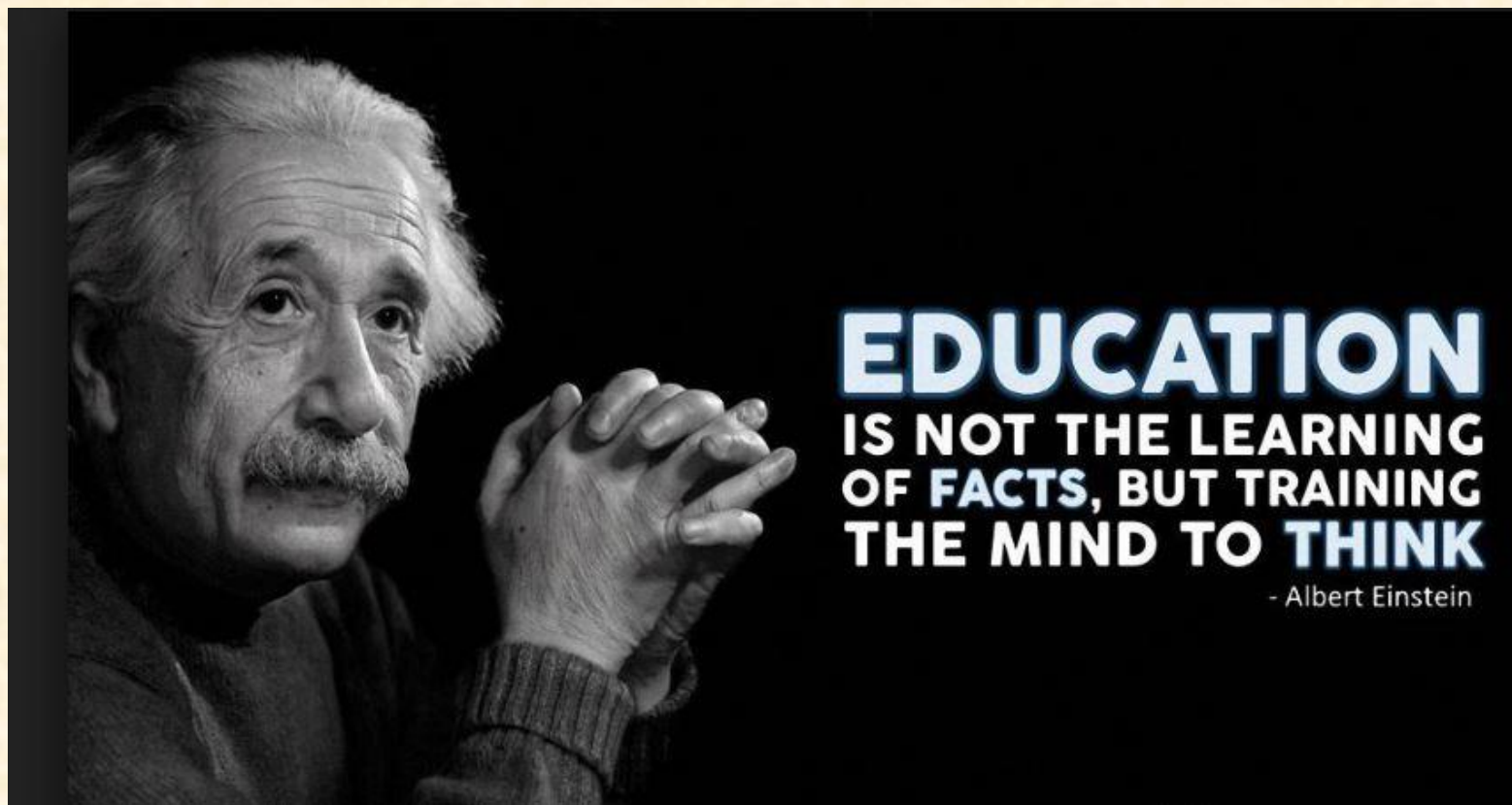


EEGChen@cityu.edu.hk



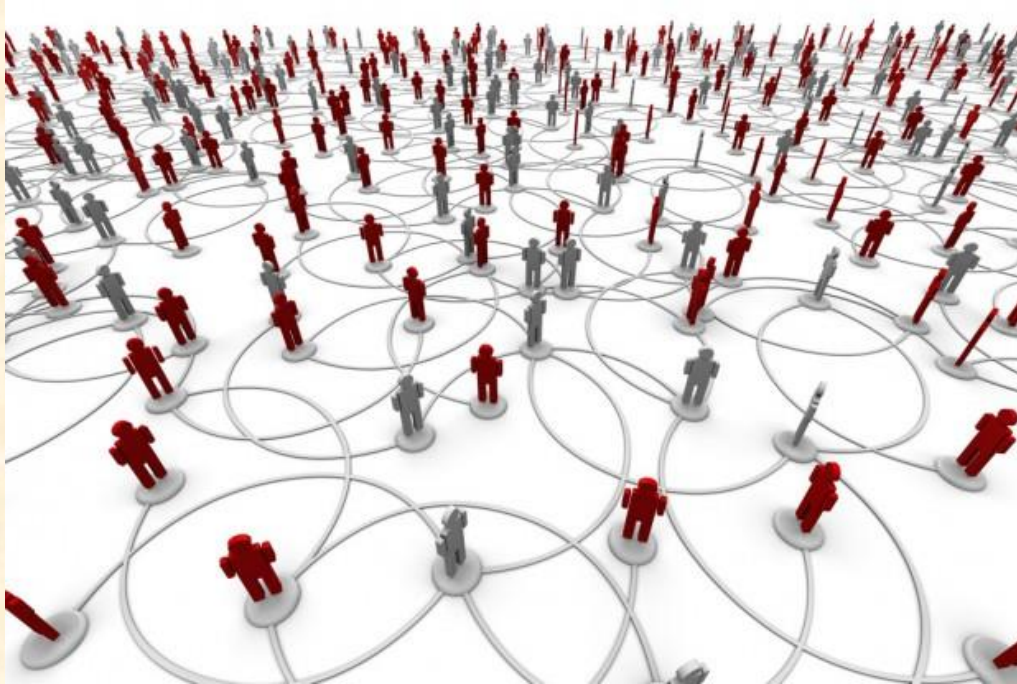
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谨向原作者致谢



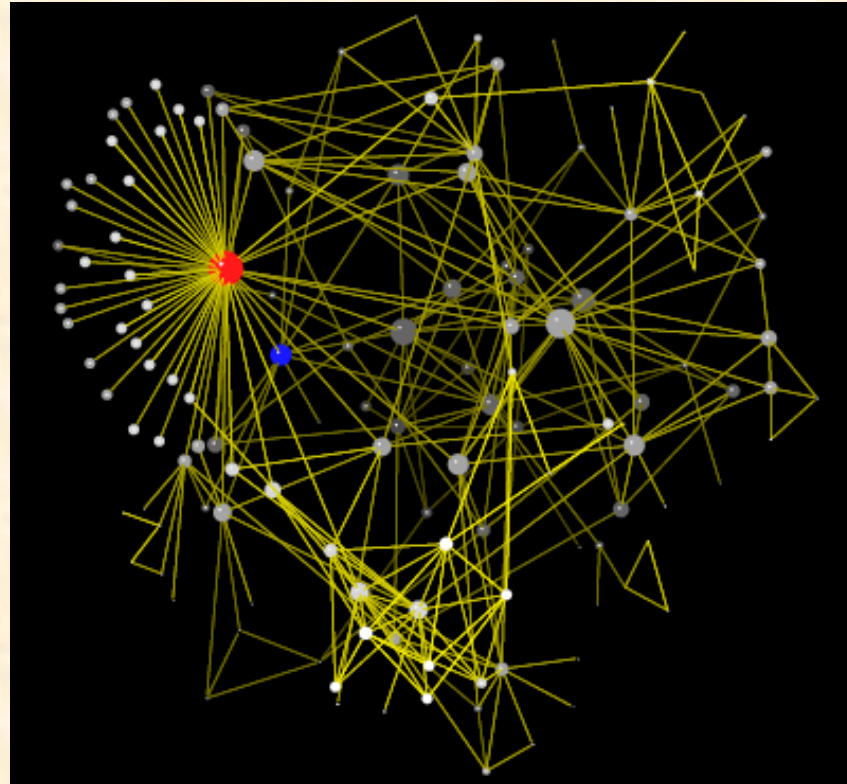
“教育的价值不在于学知很多事实，而在于训练大脑学会思考。”

我们生活在网络世界之中

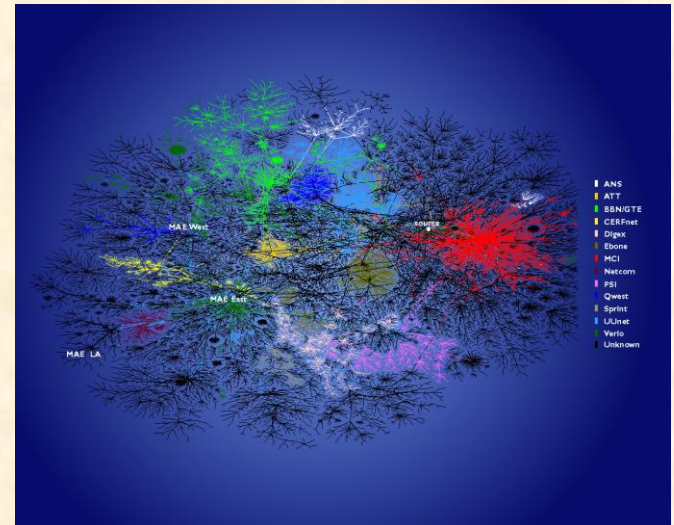
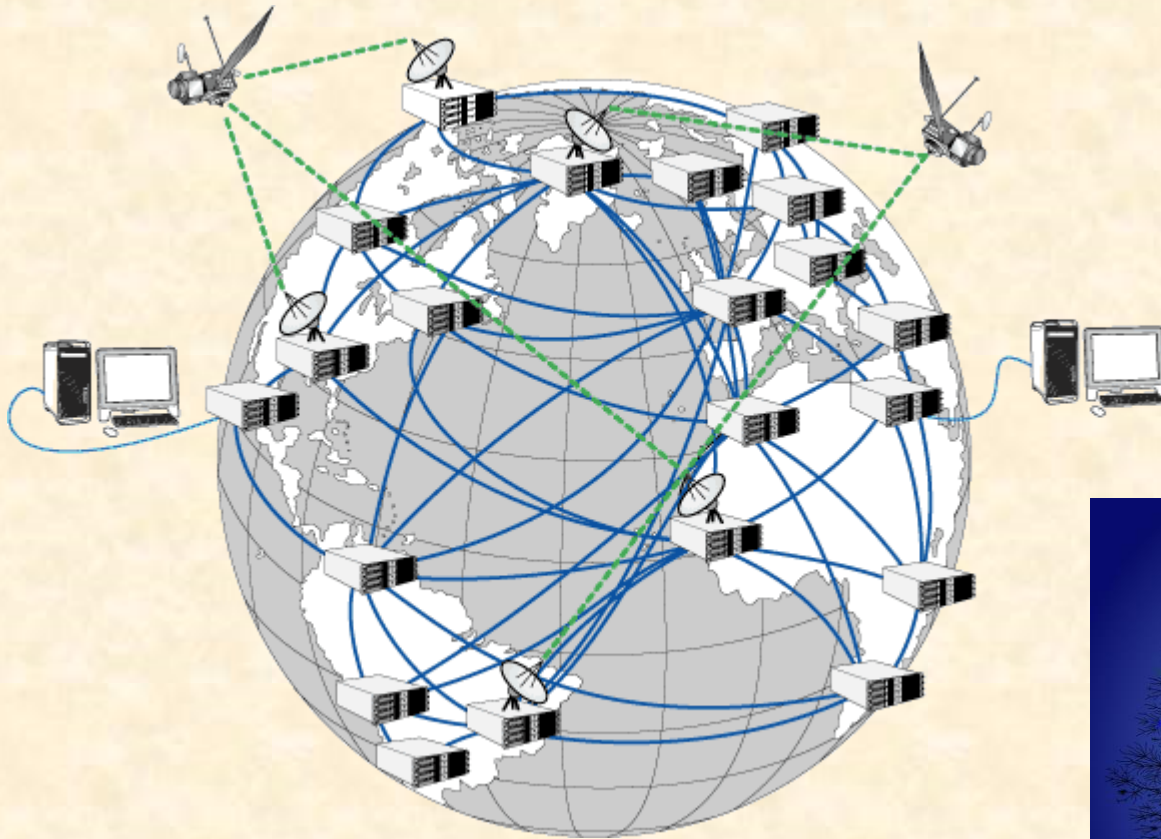


如果随意地在世界上某个角落
找出一个人，他（她）会通过
多少中间朋友和你连接在一起？

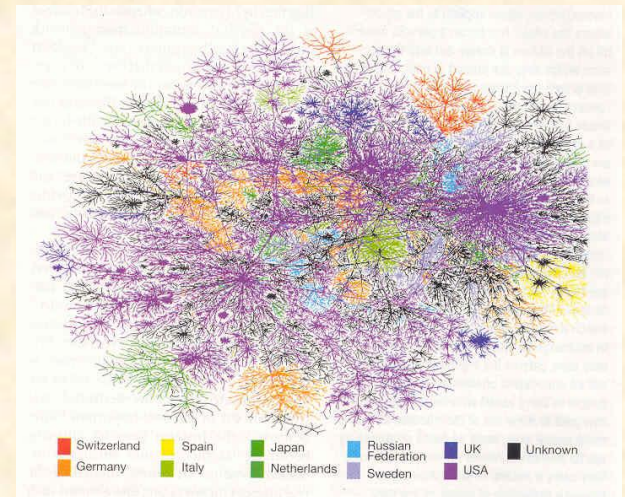
复杂网络简介



互联网: Internet



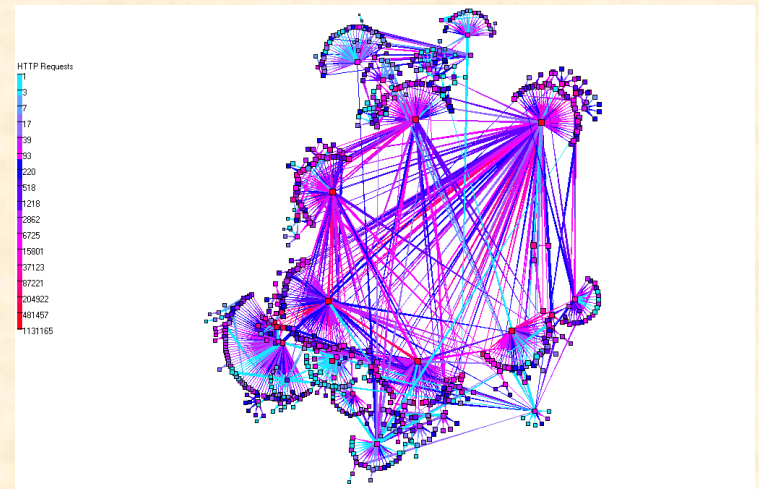
万维网: WWW



超文本传输协议: HTTP



Hyper Text Transfer Protocol

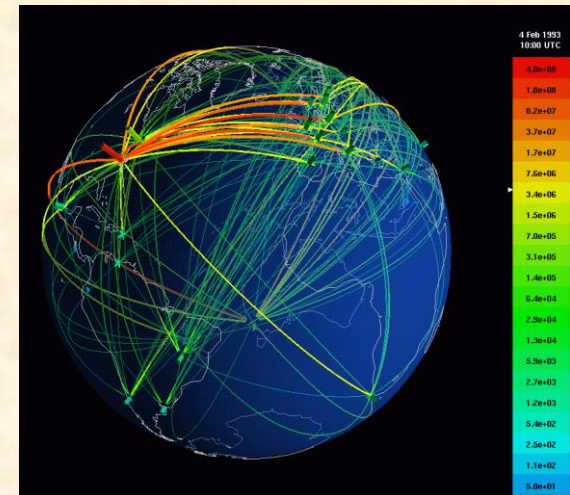
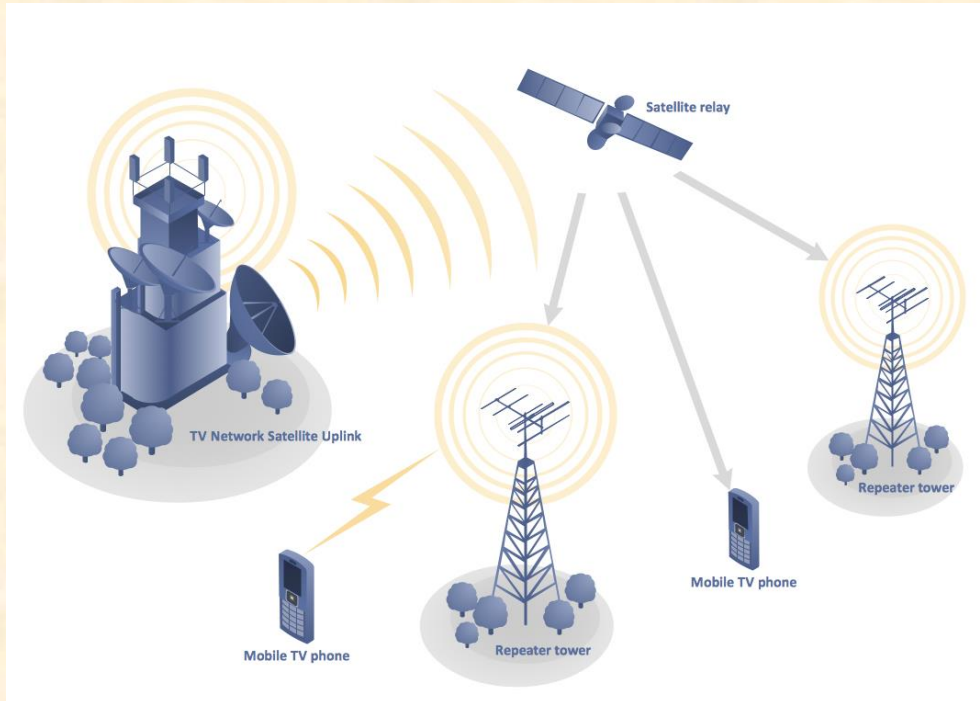


脸书： Facebook

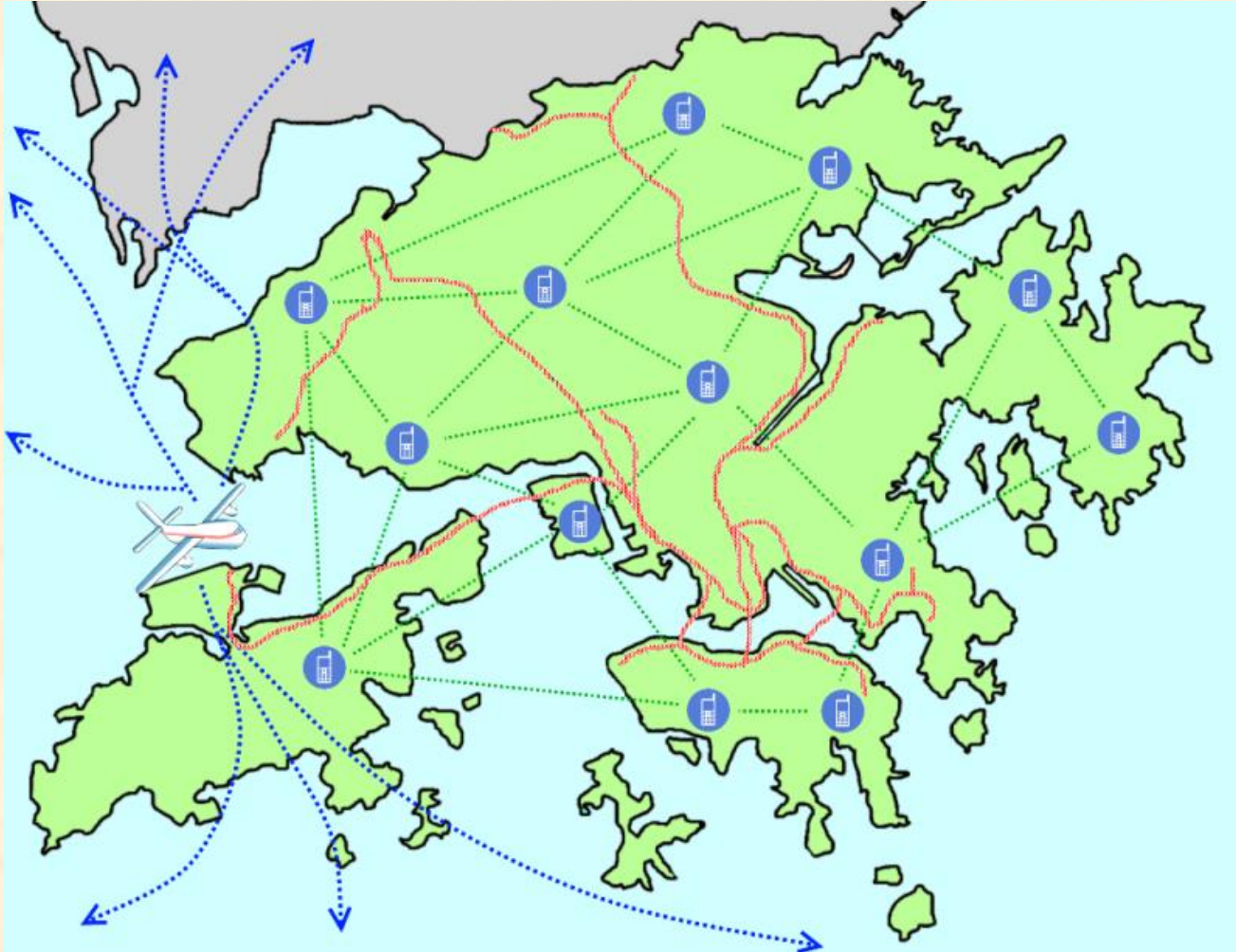


www.bbc.co.uk (24 Dec 2010)

无线通信网：Telecomm Networks



通信和交通网络



(Wallace K S Tang)

股票网络： Hang Seng Index



飞机航线图：Airlines Route Maps





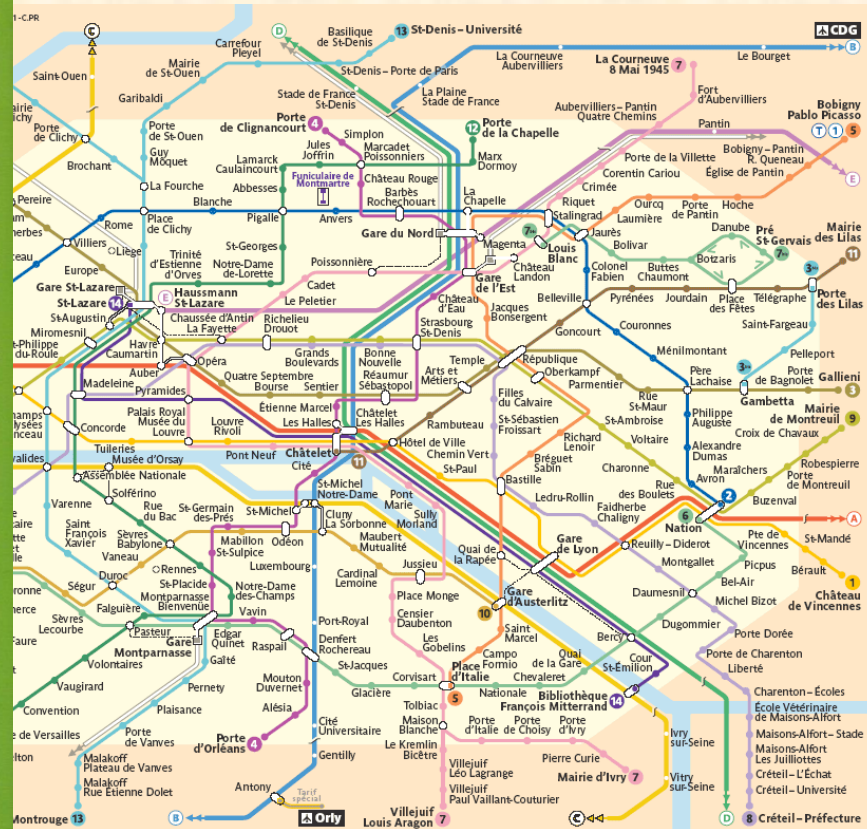
中国国航
航线图

铁路、公路、地铁路线图

欧洲铁路



均匀网络



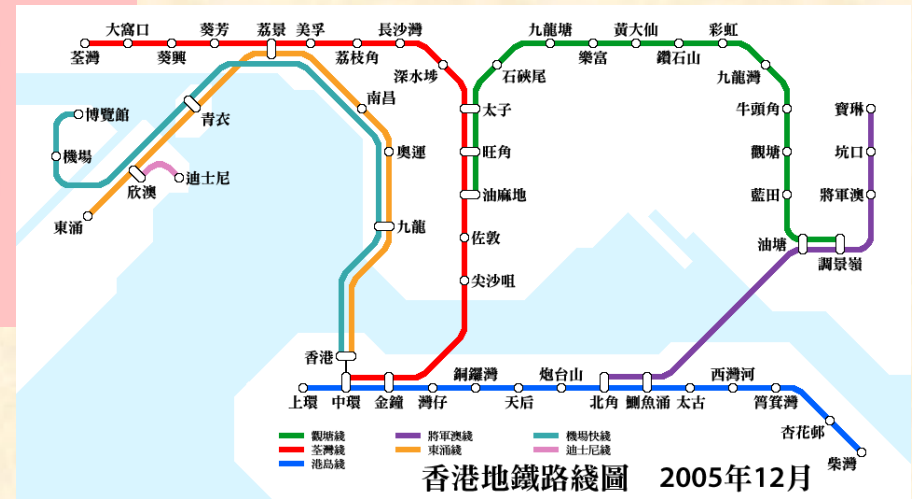
东京地铁

铁路、公路、地铁路线图

中国铁路

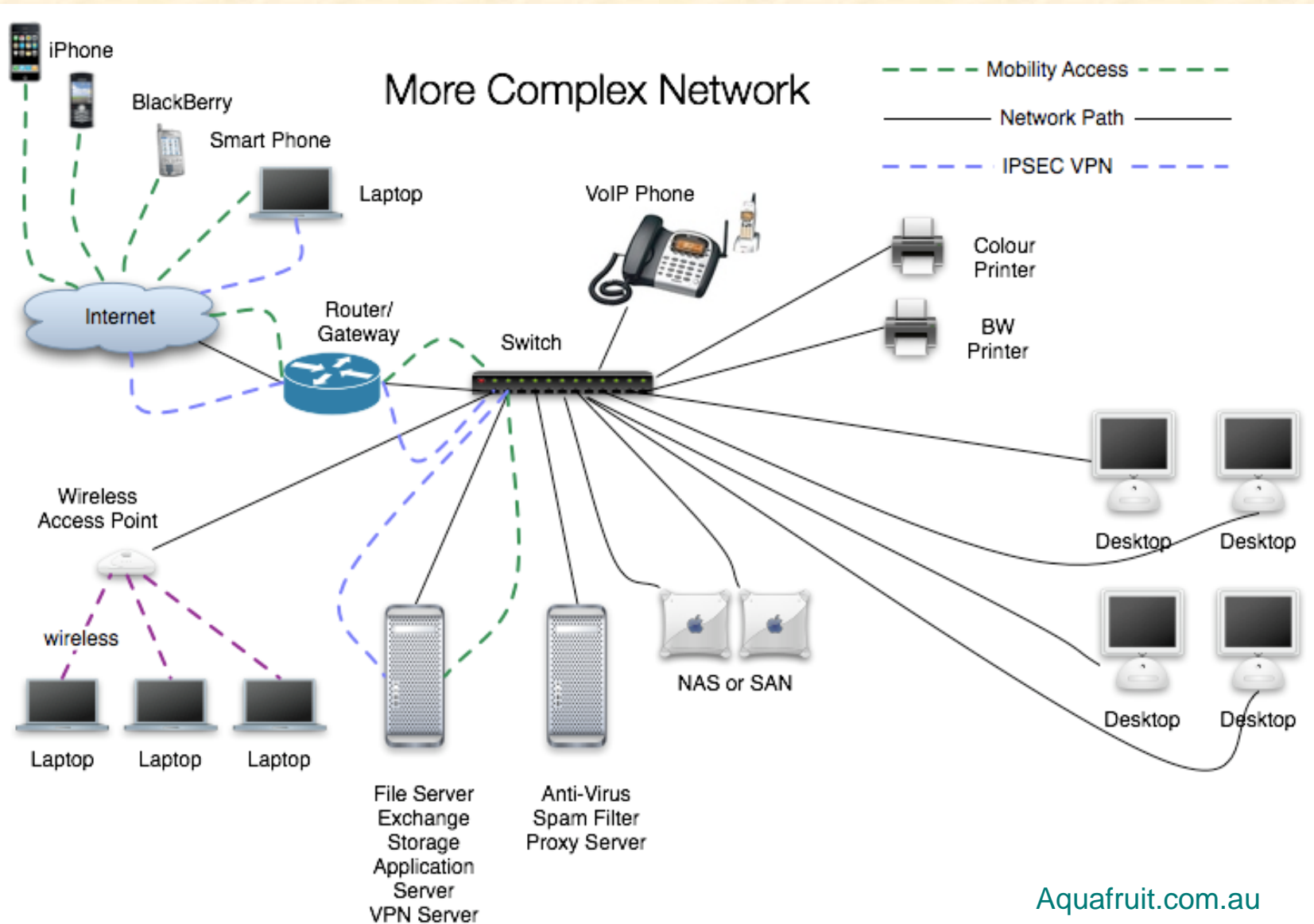


均匀网络

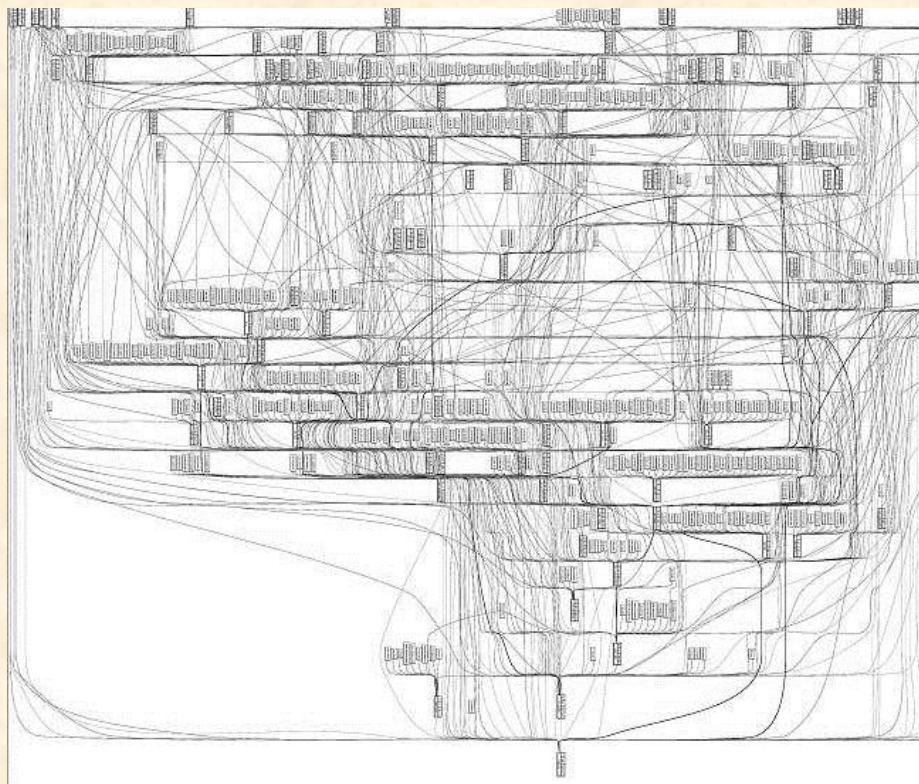
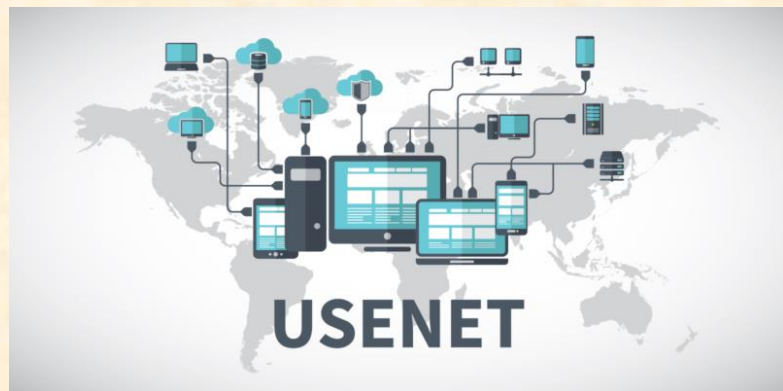


香港地铁

信息网络

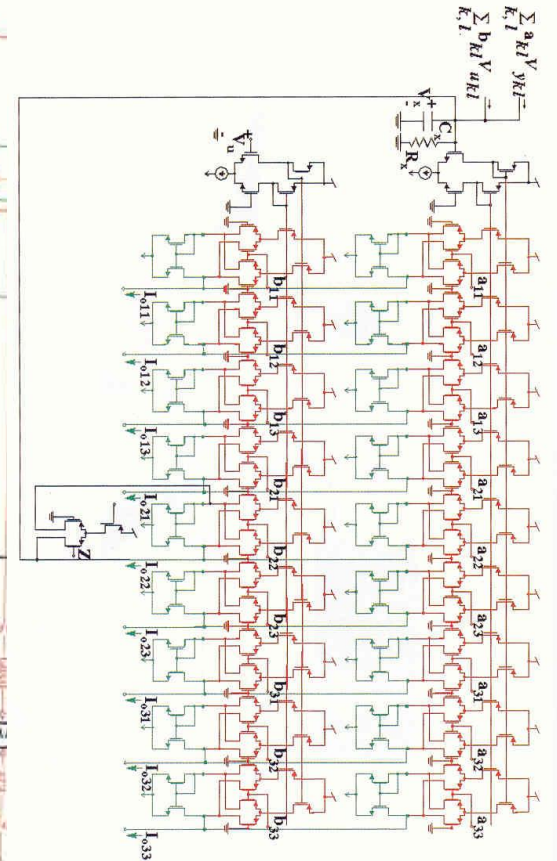
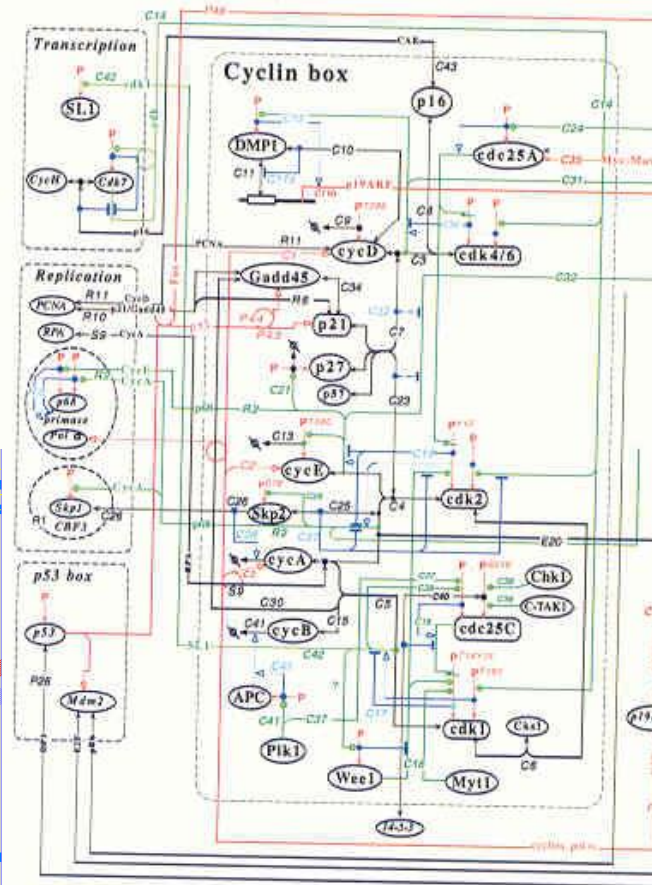
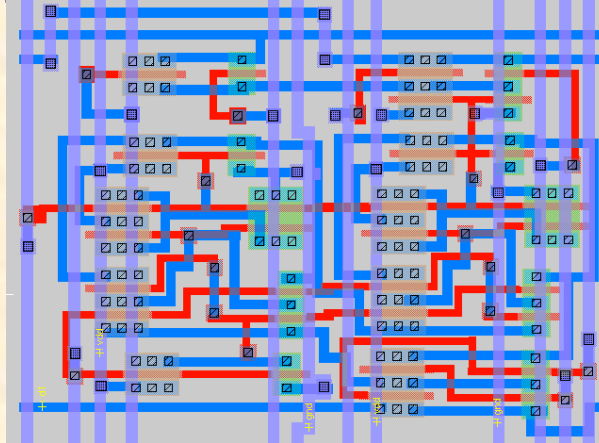
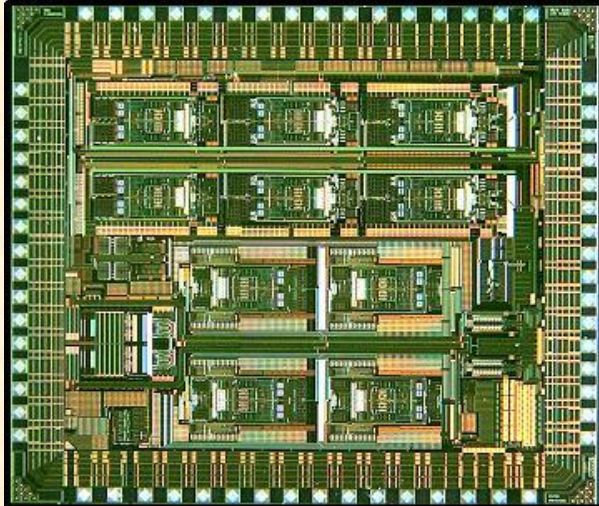


新闻组网络 Usenet



(Naveen Jamal)

大规模集成电路

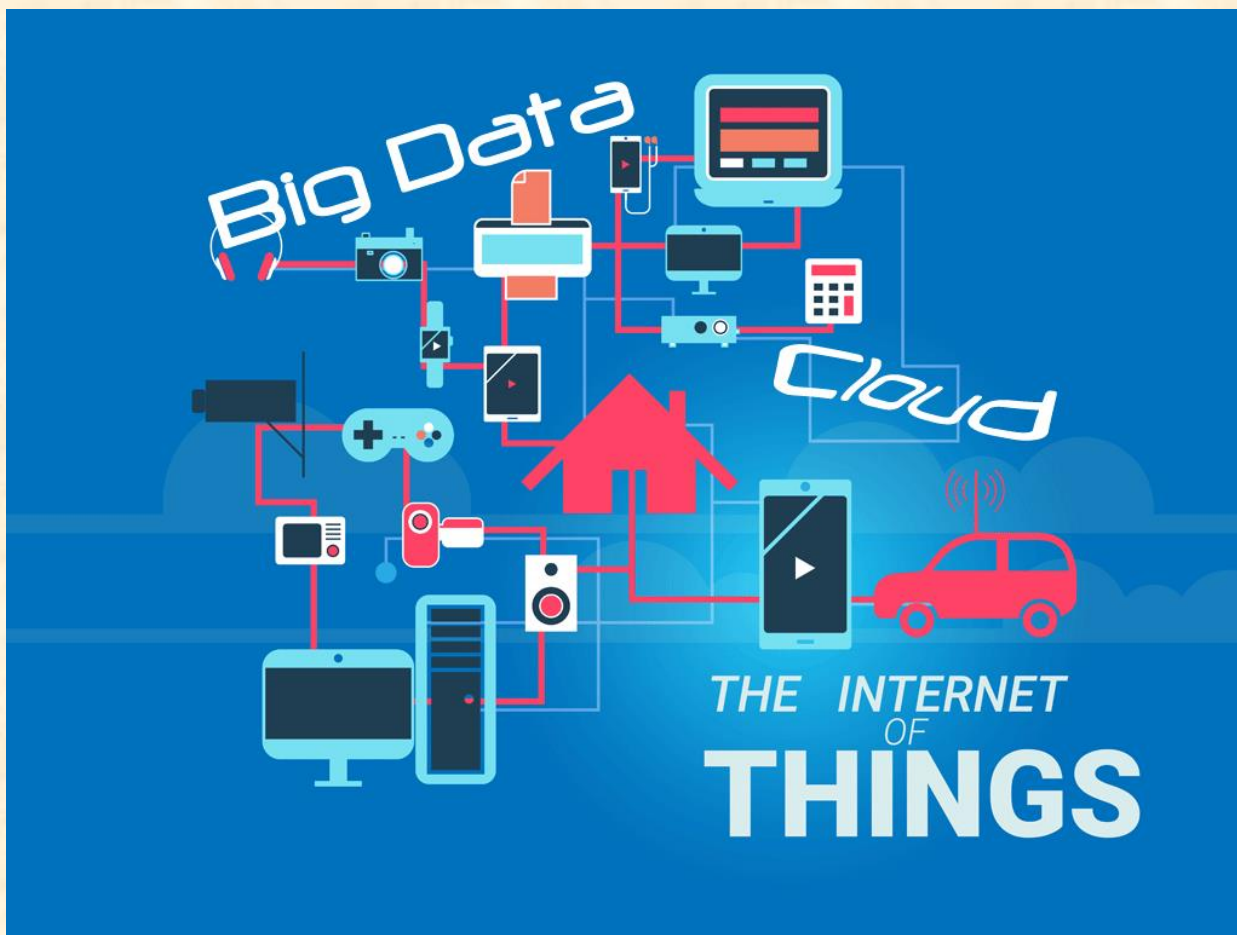


机械结构网络

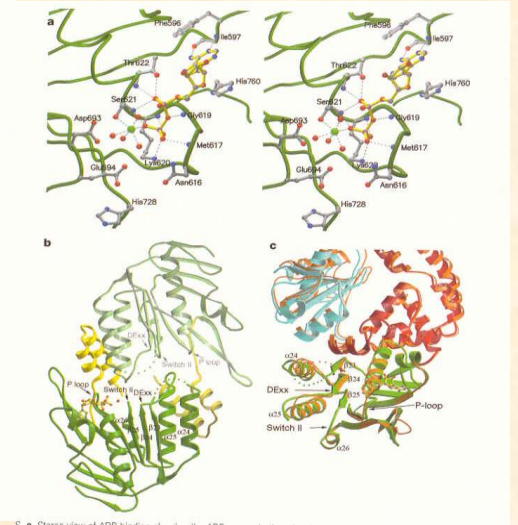
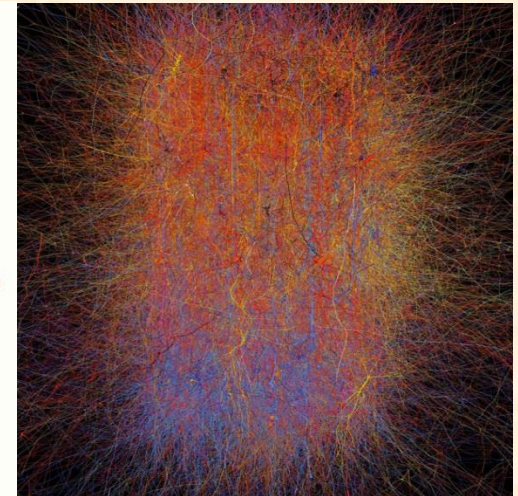
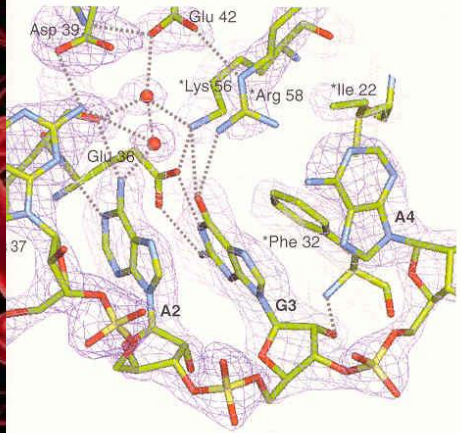
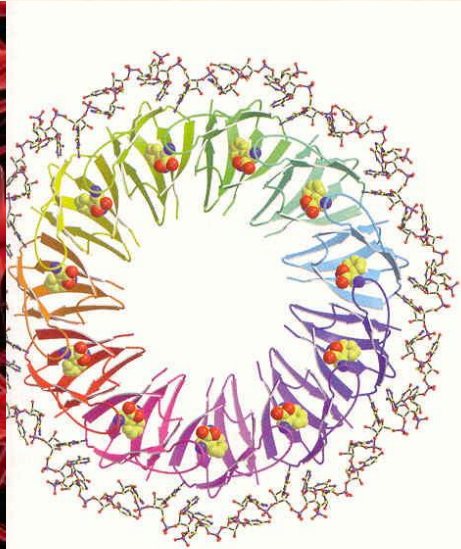
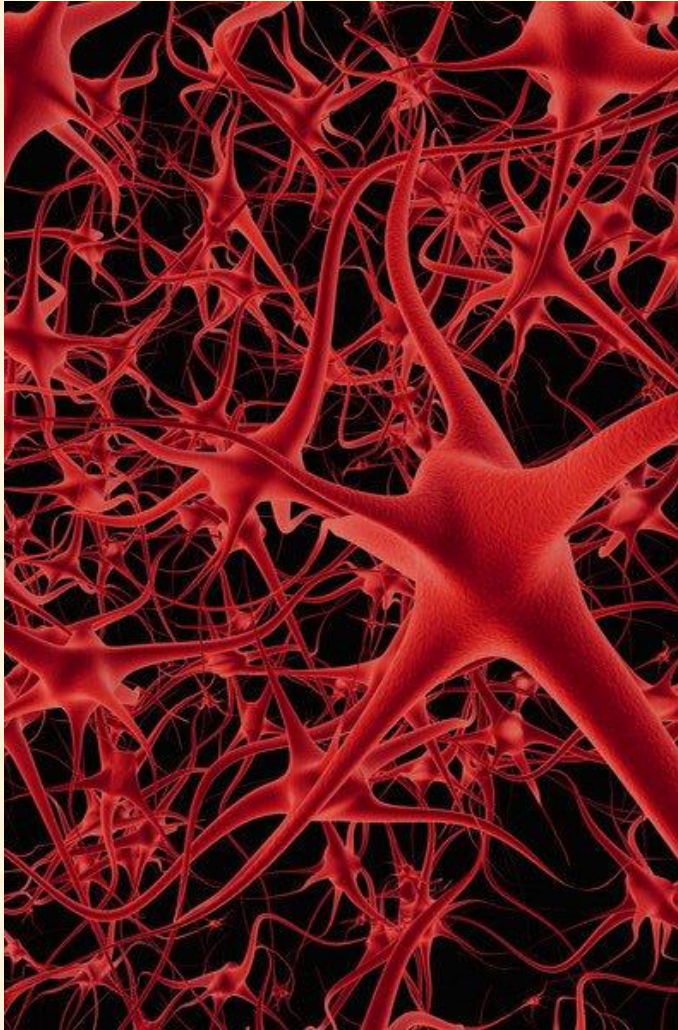


New Faculty building of University of Ljubljana, Slovenia
Courtesy of Ljubljana university incubator technology transfer - Certus

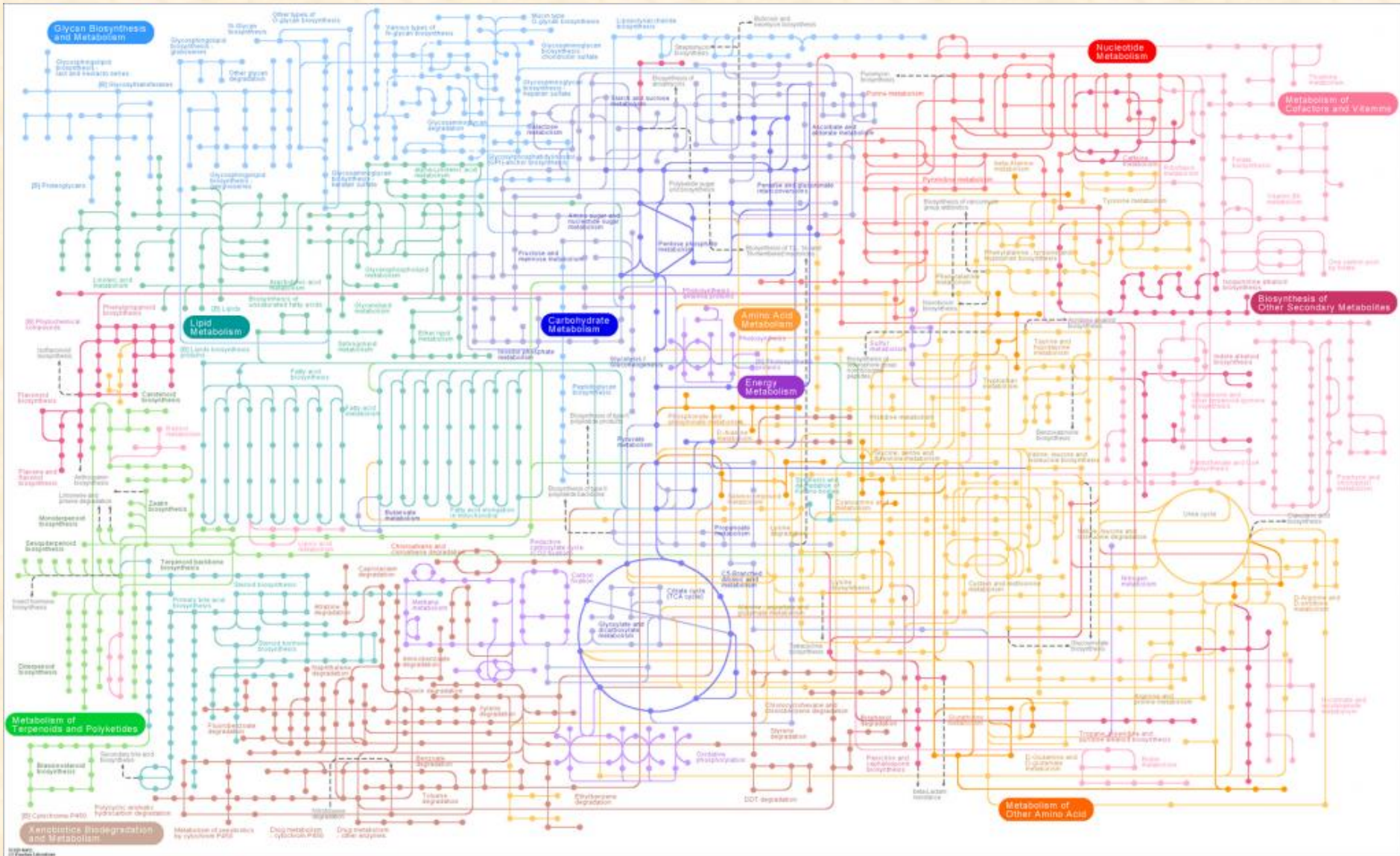
物联网



生物网络



新陈代谢系统：Metabolism Pathway



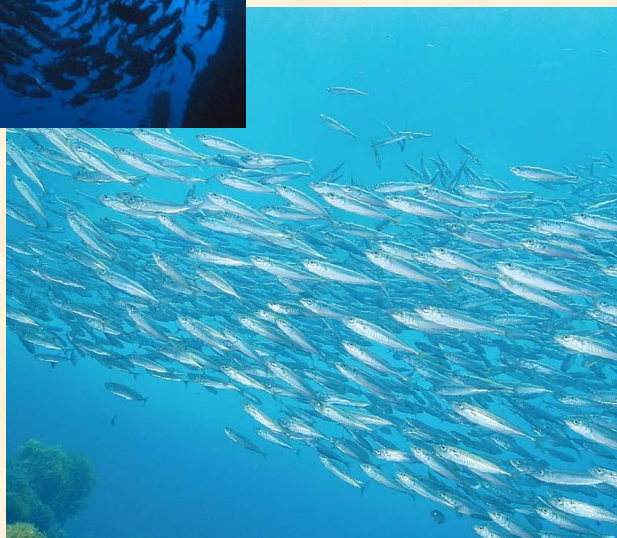
社会网络



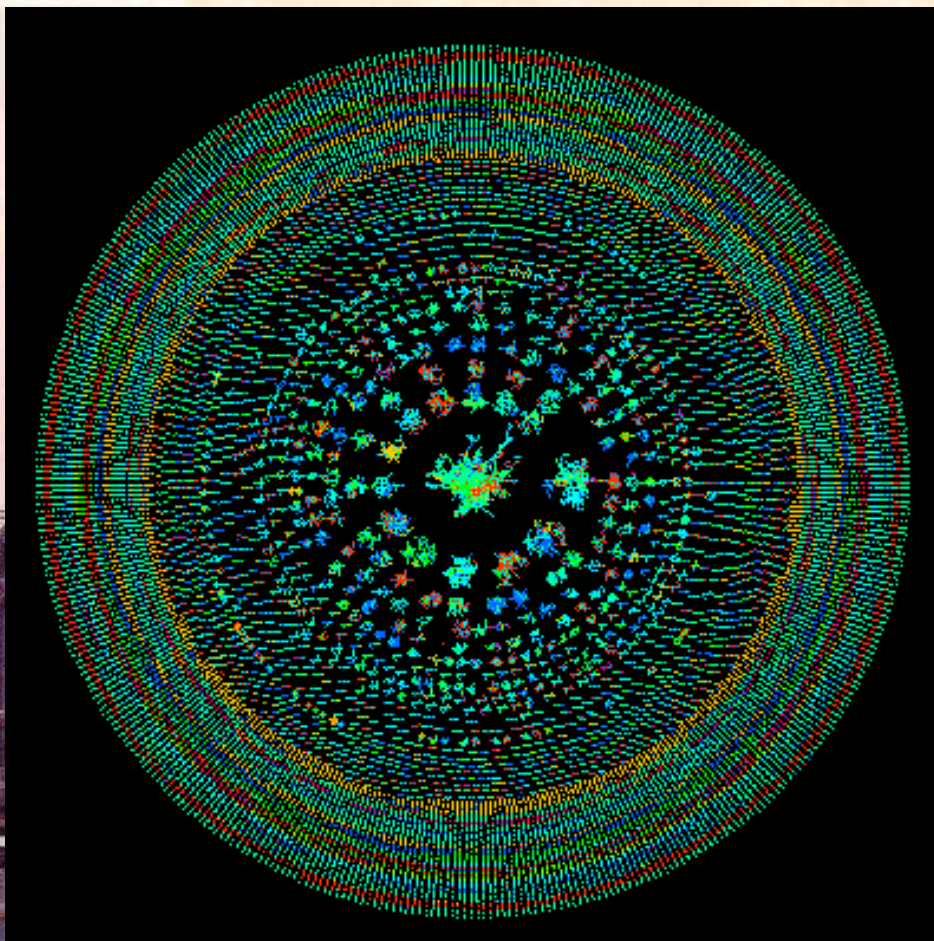
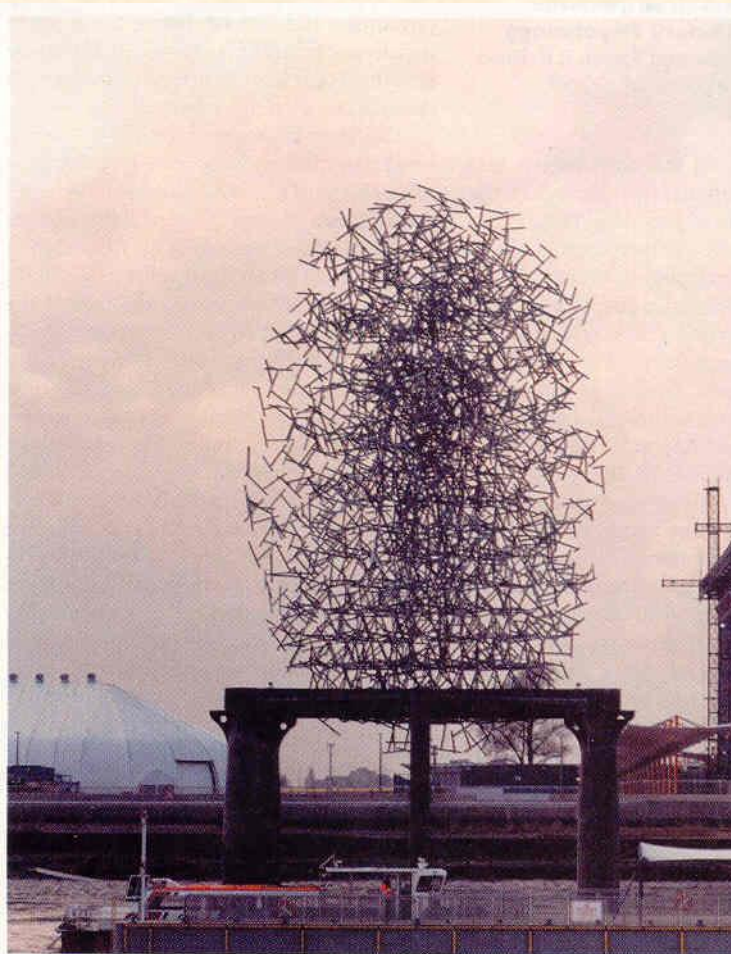
社会网络



动物世界



艺术网络: Arts



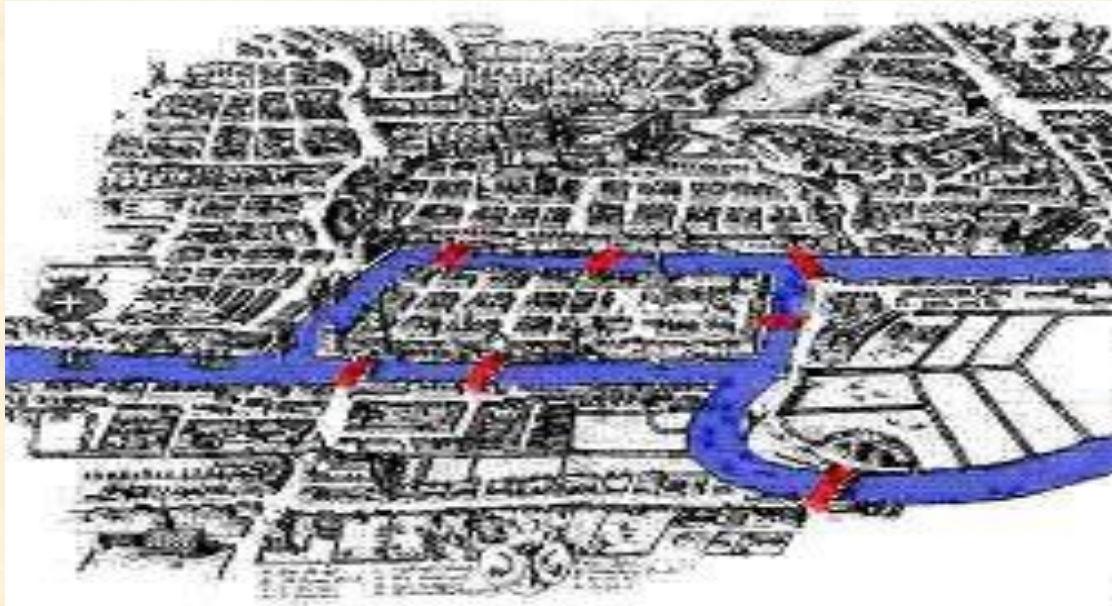
内容

- 图论 **Graph Theory**
- 网络模型
- 随机网络 **Random-Graph Networks**
- 小世界网络 **Small-World Networks**
- 无标度网络 **Scale-Free Networks**



- 现实世界例子
- 技术网络
- 生物网络
- 社会网络

Königsberg 七桥问题

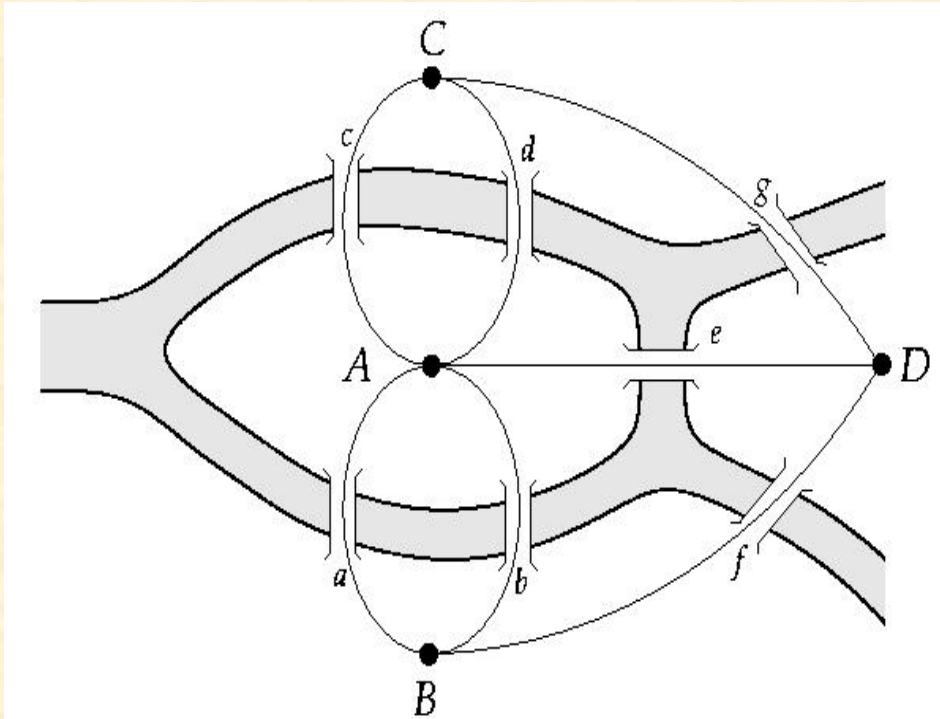


Königsburg 市区地图 (1736年)

问题：

能否从某个地方开始，走过每一道桥，
並且只走过一次，最后回到起点？

欧拉 Euler



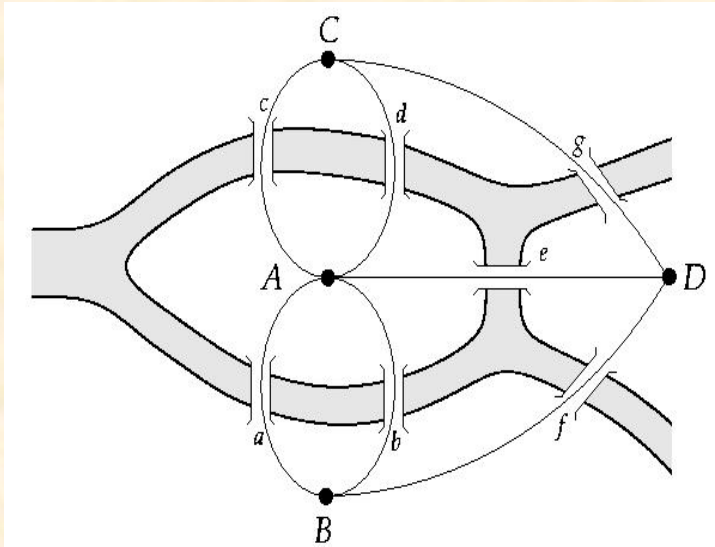
欧拉从此创立了 数学图论



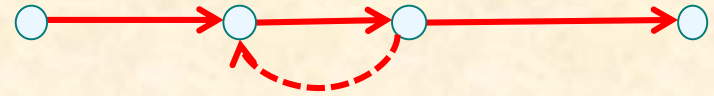
(1707-1783)

他把问题描述为 4 个
节点和 7 条连边的图

然后证明了问题无解



欧拉的证明

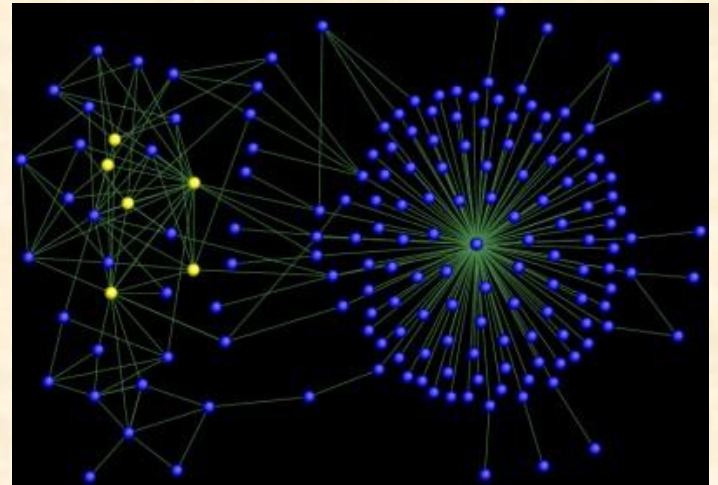


- 每一条路径只有一个起点和一个终点
- 除了起点和终点之外，其他所有节点都有偶数条连边
- 七桥问题要有解的话，所有节点必须有偶数条连边
- 七桥地图并非这样 → 问题没有解！

(Euler, 1735)

网络拓扑（结构）

Network Topology (Structure)

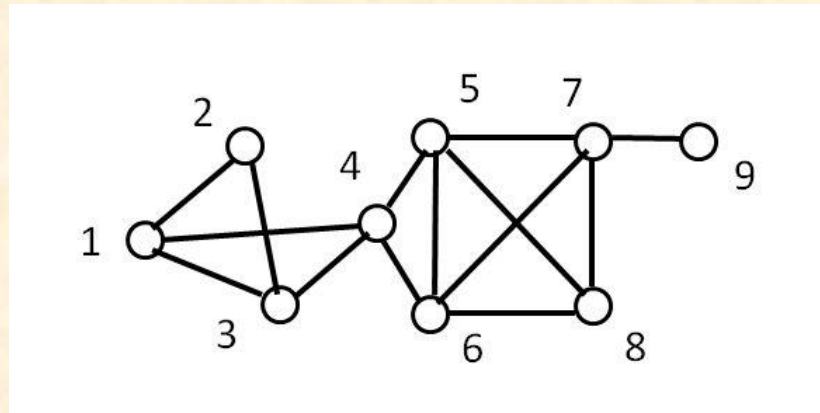


网络基本元素： 节点 (node, vertex)
连边 (edge, link)

- ❖ 计算机网络: 节点 - 电脑 连边 - 光纤
- ❖ 神经网络: 节点 - 神经元 连边 - 神经线
- ❖ 社会网络: 节点 - 个人 连边 - 关系
- ❖

两个基本概念

- ◆ 节点度 (degree) 和 度分布 (degree distribution)
- ◆ 距离 和 平均路径长度 (average path length)
- ◆

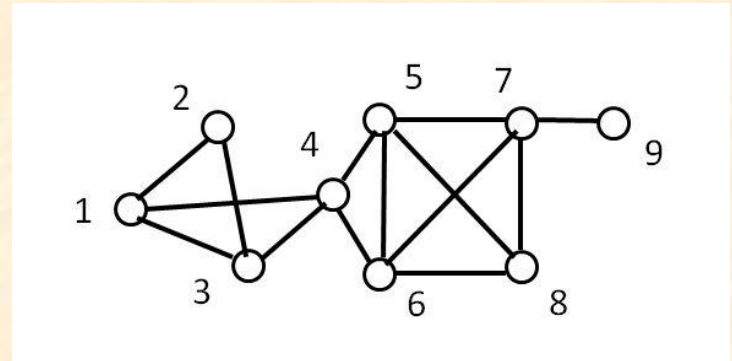


度和度分布

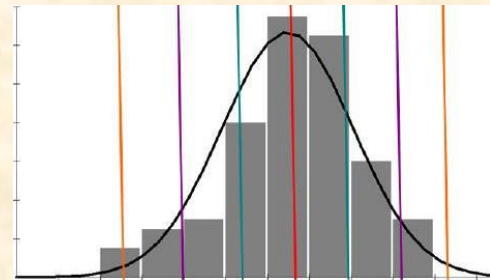
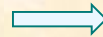
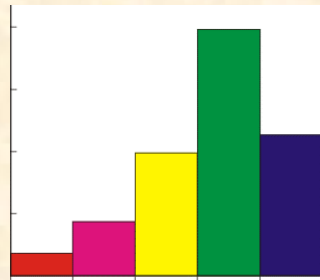
❖ 节点 i 的度 $k_i =$ 连边数目

▪ 网络平均度 $\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i$

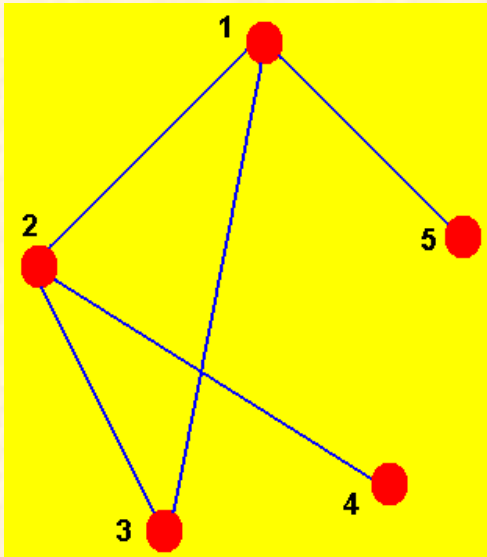
▪ 度分布



histogram



例:



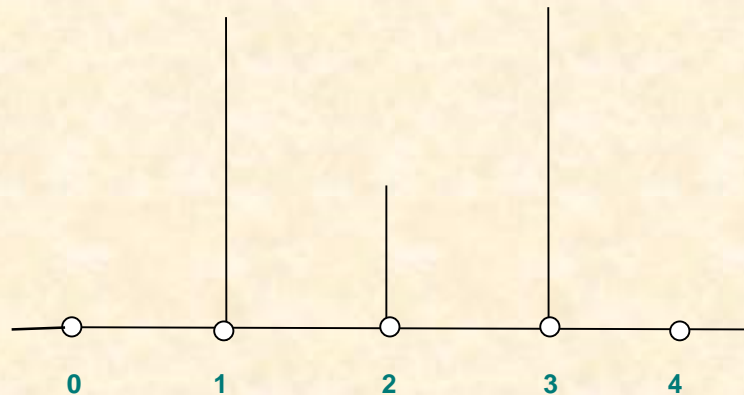
➤ 度:

node1 = 3, node2 = 3, node3 = 2,
node4 = 1, node5 = 1

➤ 平均度:

$$\langle k \rangle = (3 + 3 + 2 + 1 + 1) / 5 = 2$$

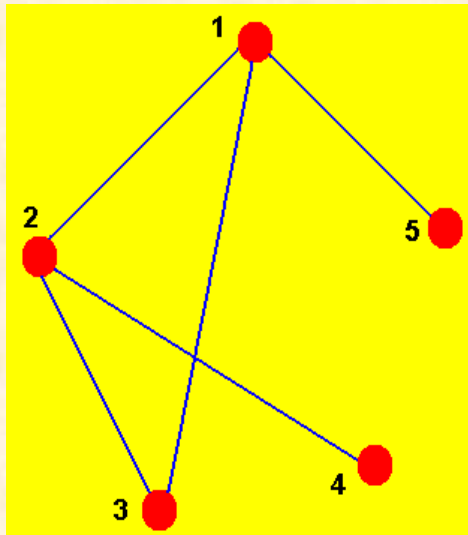
➤ 度分布:



(概率密度函数)

距离和平均路径长度

例:



$$d_{12} = 1$$

$$d_{13} = 1$$

$$d_{14} = 2$$

$$d_{15} = 1$$

$$d_{23} = 1$$

$$d_{24} = 1$$

$$d_{25} = 2$$

$$d_{34} = 2$$

$$d_{35} = 2$$

$$d_{45} = 3$$

$$\text{Total} = 16$$

平均路径长度:

$$L = 16 / 10 = 1.6$$

基本网络模型

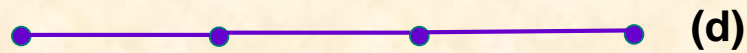
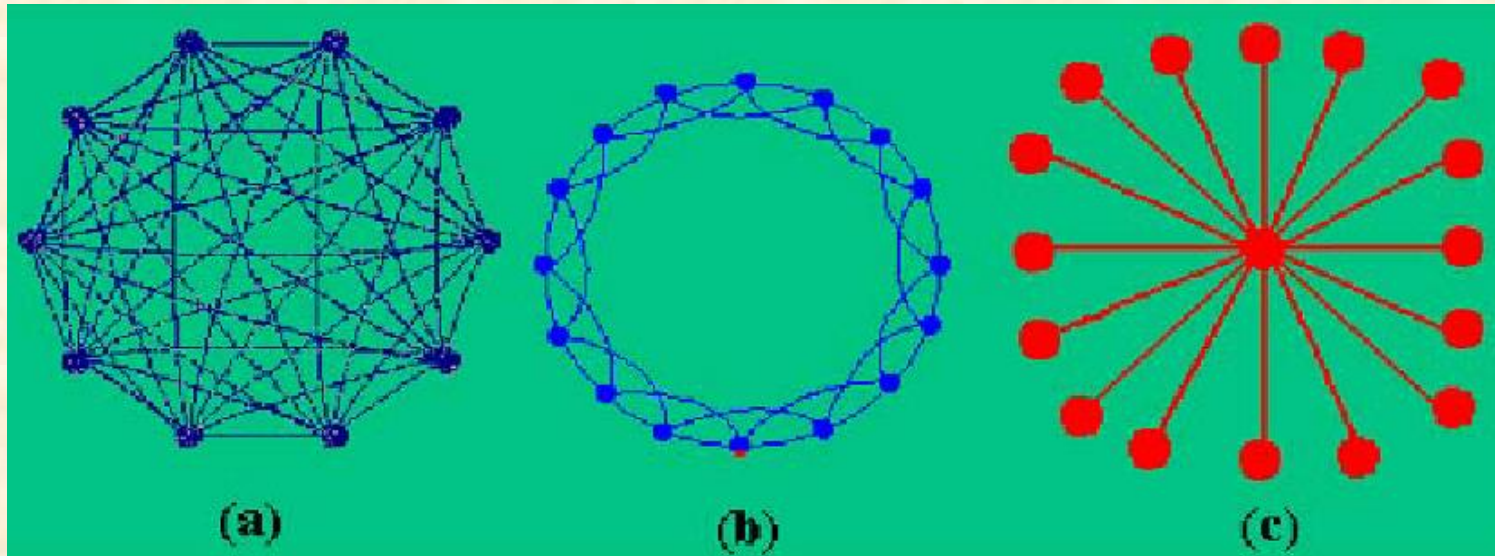
❖ 经典:

- 欧拉图论 (1735)
- 随机图模型 - Erdős - Rényi (1959)

❖ 新近:

- 小世界模型 - Watts - Strogatz (1998)
- 无标度模型 - Barabási - Albert (1999)

规则网络

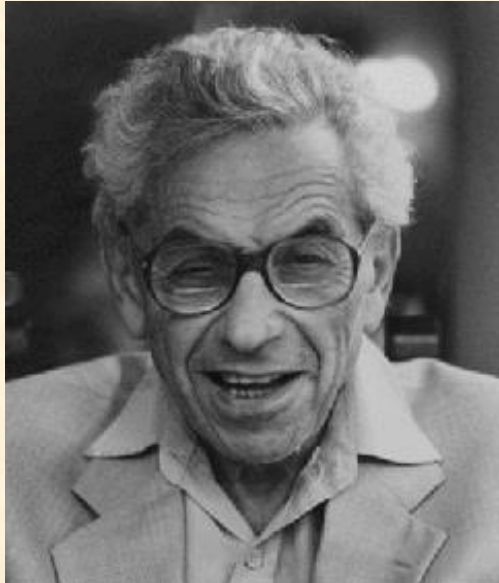


- (a) 全连接网络 fully-connected network
- (b) 环形网络 ring-shaped network
- (c) 星形网络 star-shaped network
- (d) 链形网络 chain-shaped network

随机图理论

Random Graph Theory

Paul Erdős



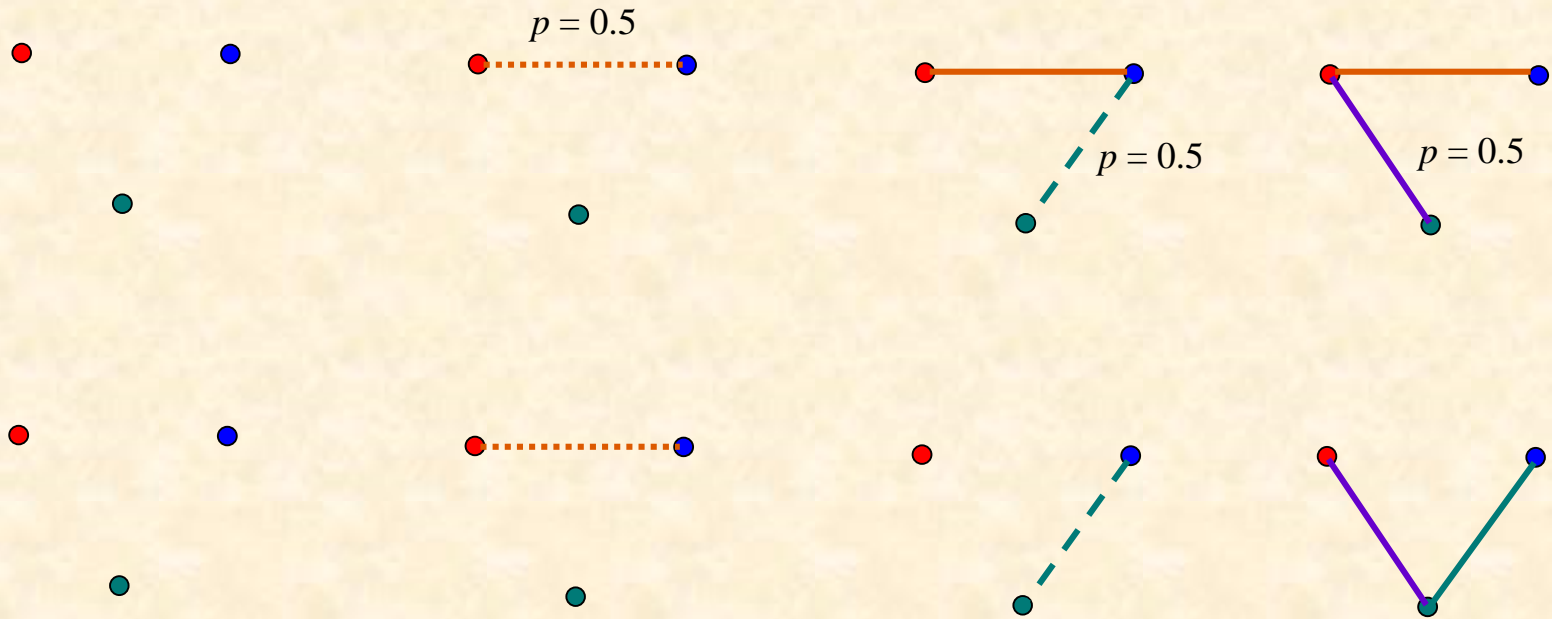
Alfred Rényi



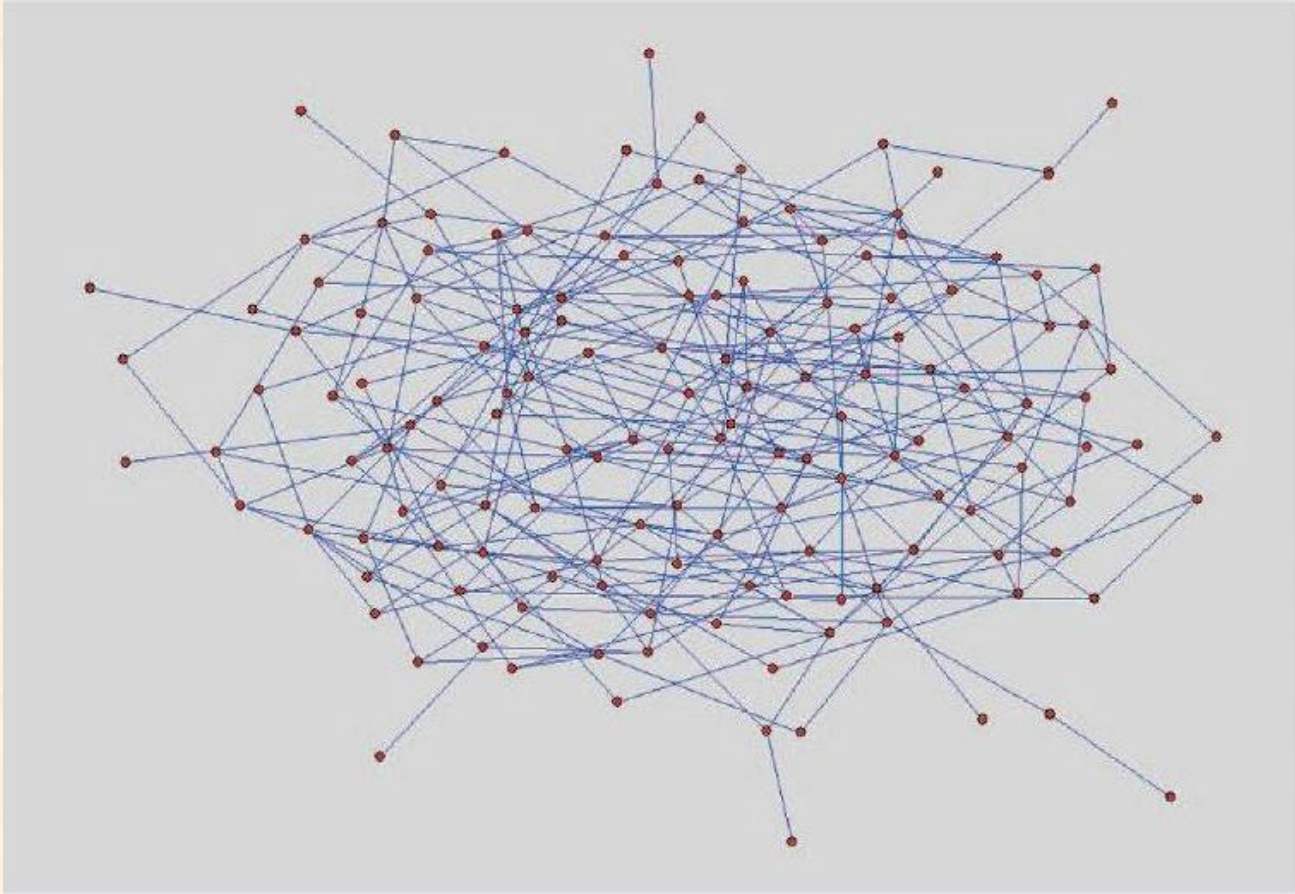
1960年代 数学图论的突破性发展

ER 随机图模型

例:



ER 随机图模型



均匀性：所有节点都有大致相同的度

随机图 和 泊桑(Poisson)分布

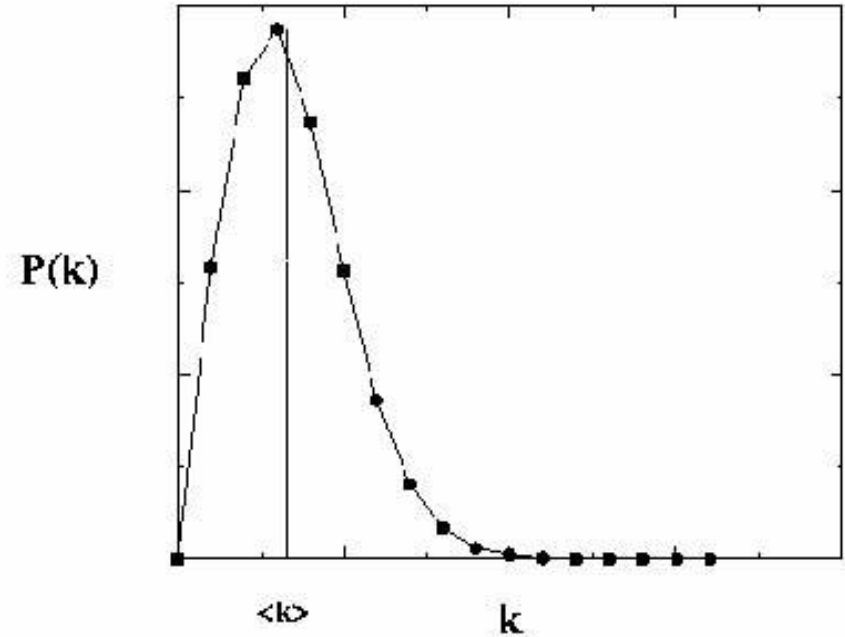
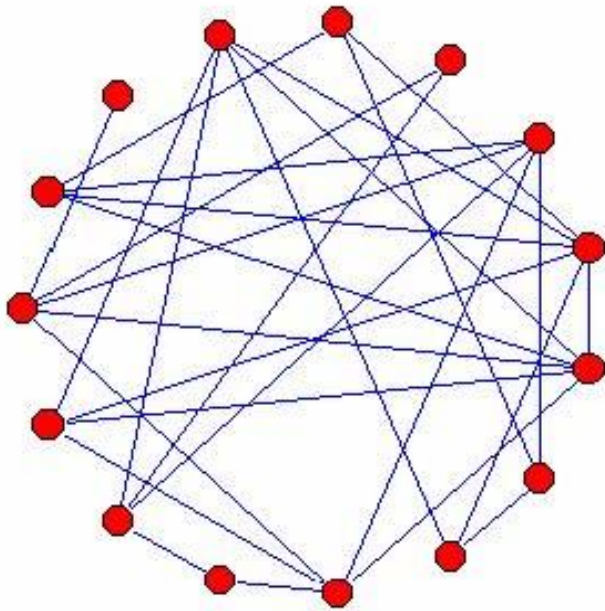


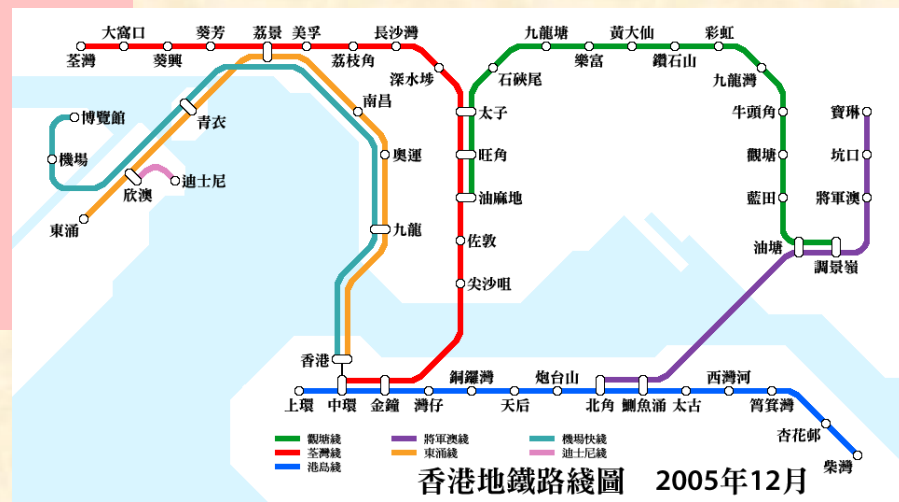
Illustration of Erdős-Rényi random-graph network model

铁路、公路、地铁路线图

中国铁路



非随机
均匀网络



香港地铁

小世界网络

Small-World Networks

海内存知己，天涯若比邻。

(唐) 王勃

小小世界

CHAIN-LINKS

by

Frigyes Karinthy

(1929, **Everything is Different**)

Planet Earth has never been as **tiny** as it is now.

... anyone on Earth, at my or anyone's will, can now learn in just a few minutes what I think or do.

We should select any person from the 1.5 billion inhabitants of the Earth – anyone, anywhere at all. He bet us that, using no more than **five** individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the **network** of personal acquaintances.

小世界网络

Small-World Networks

--- *Nature*, 393: 440-442, 1998

D. J. Watts

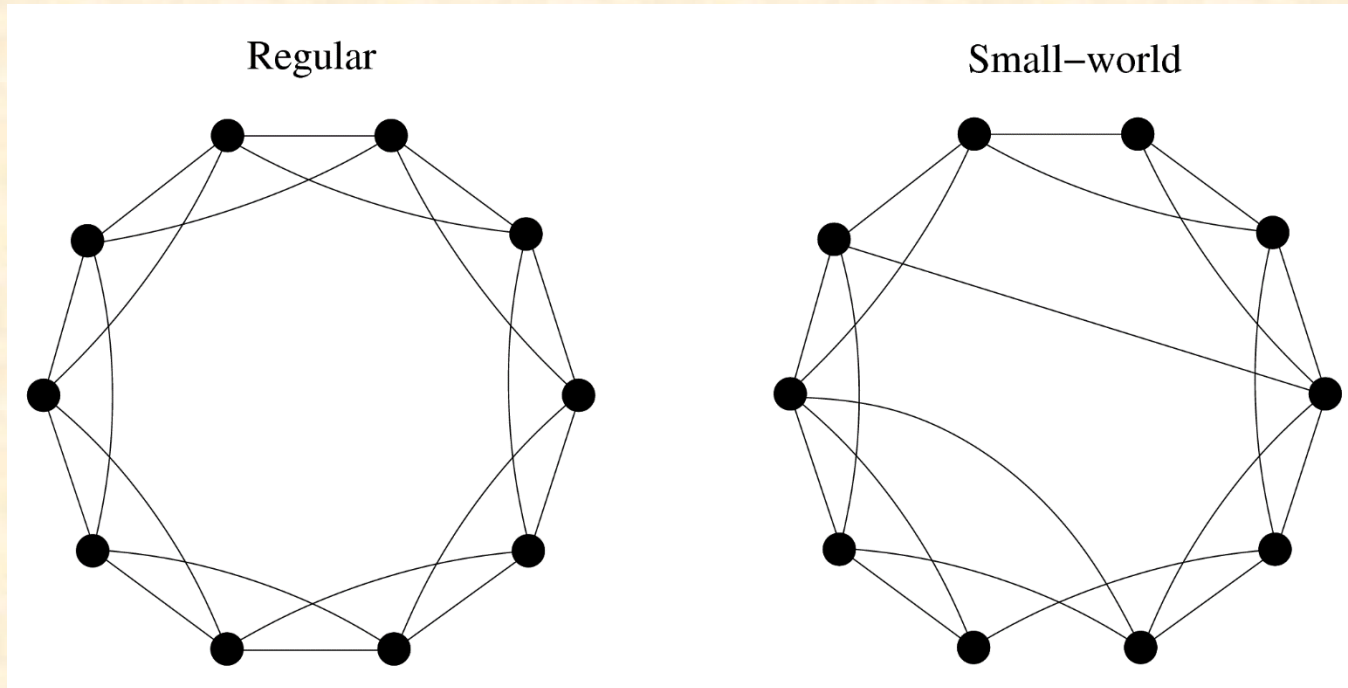


S. H. Strogatz



康奈尔大学 Cornell University

小世界网络模型

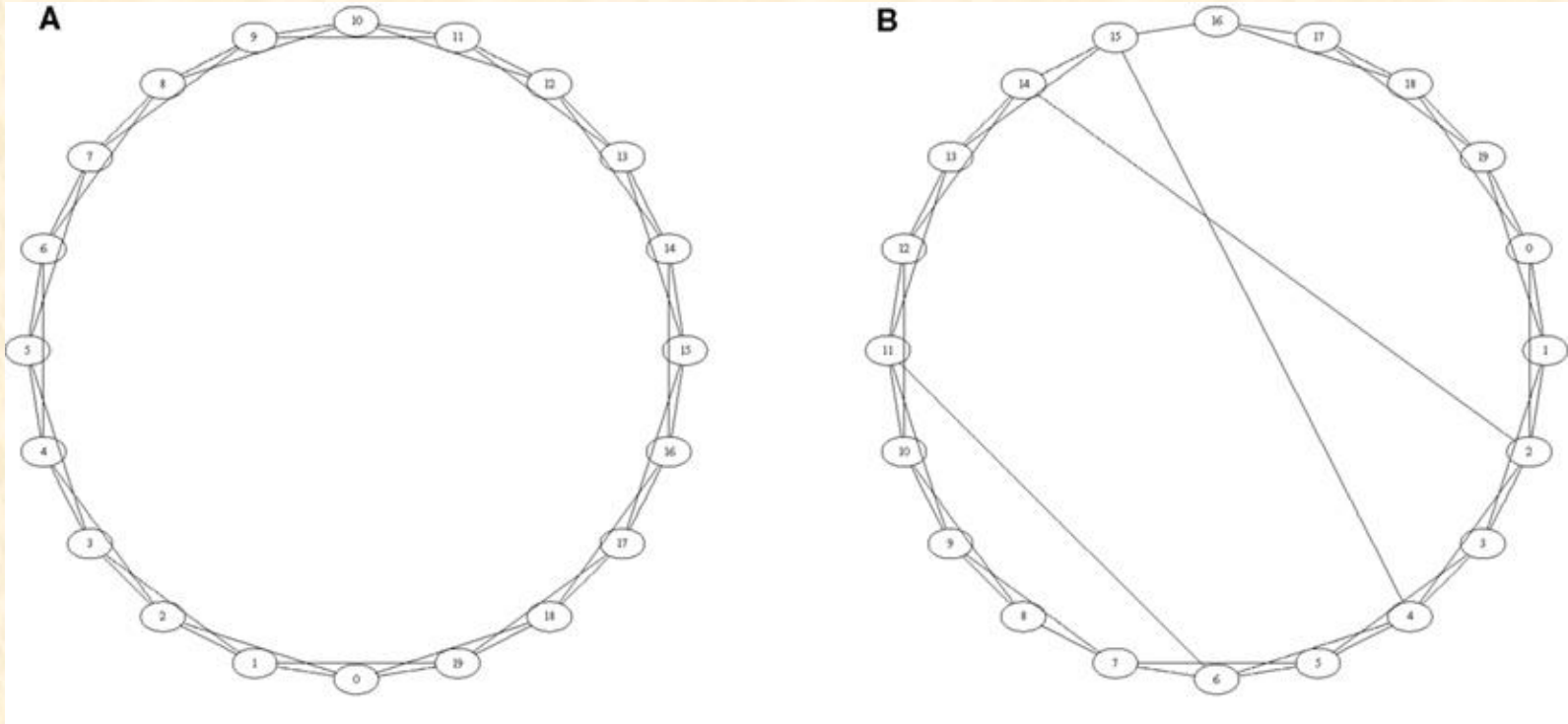


生成：（1）从一个规则环形网络开始

（2）每个节点对进行如下一次操作（不重复不遗漏）：

以一定概率断开原有的连边，把该边随机地连接到另一个节点

WS 小世界网络



观察发现: 小世界特性!

任意两点之间的“距离”都很短

无标度网络

Scale-Free Networks

Science, 149(3683) : 510-515, July 30, 1965

Networks of Scientific Papers

The pattern of bibliographic references indicates the nature of the scientific research front.

Derek J. de Solla Price

This article is an attempt to describe in the broadest outline the nature of the total world network of scientific papers. We shall try to picture the network which is obtained by linking each published paper to the other papers directly associated with it. To do this, let us consider that special relationship which is given by the citation of one paper by another in its footnotes or bibliography. I should make it clear, however, that this broad picture tells

chinese-handled citation studies, of large and representative portions of literature, which are much more tractable for such analysis than any topical indexing known to me. It is from such studies, by Garfield (1, 2), Kessler (3), Tukey (4), Osgood (5), and others, that I have taken the source data of this study.

Incidence of References

percent of the papers contain no references at all; this notwithstanding, 50 percent of the references come from the 85 percent of the papers that are of the "normal" research type and contain 25 or fewer references apiece. The distribution here is fairly flat; indeed about 5 percent of the papers fall in each of the categories of 3, 4, 5, 6, 7, 8, 9, and 10 references each. At the other end of the scale, there are review-type papers with many references each. About 25 percent of all references come from the 5 percent (of all papers) that contain 45 or more references each and average 75 to a paper, while 12 percent of the references come from the "fattest" category—the 1 percent (of all papers) that have 84 or more references each and average about 170 to a paper. It is interesting to note that the number of papers with n references falls off in this "fattest" category as $1/n^2$, up to many hundreds per paper.

These references, of course, cover the entire previous body of literature. We can calculate roughly that, since the body of world literature has been growing exponentially for a few centuries (6) and probably will continue

无标度网络

Scale-Free Networks

--- *Science*, 286: 509 (1999)

A.-L. Barabási



R. Albert



圣母大学 Norte Dame University

无标度网络

(Derek J de Solla Price, *Science*, 1965)
(Barabasi-Albert, *Science*, 1999)

生成：

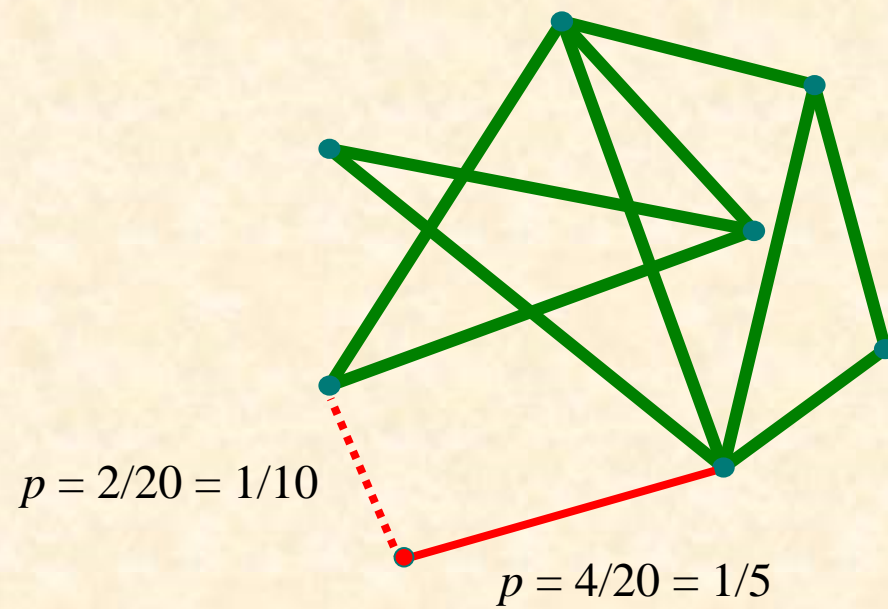
(0) 从一个小的连通网络开始

每一步：

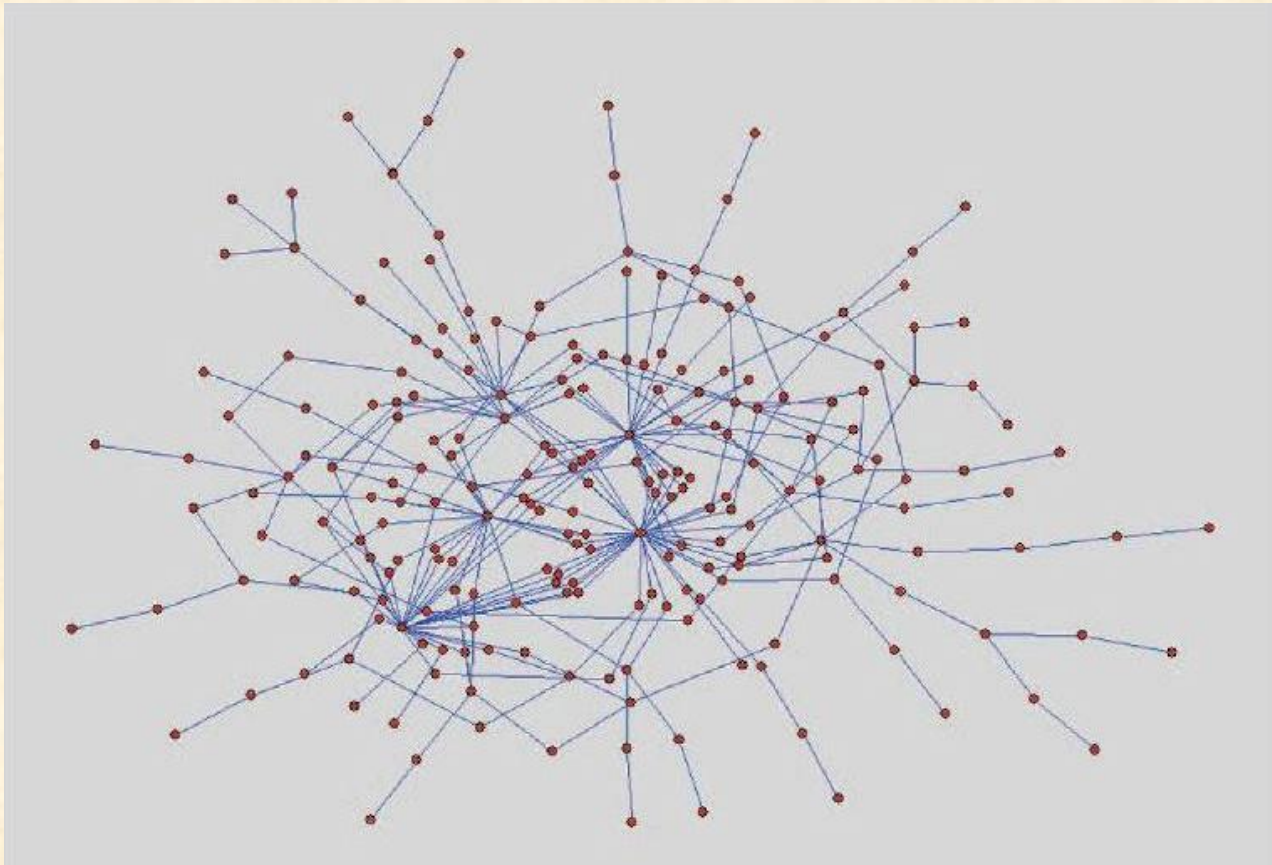
(1) 增加一个新的节点

(2) 新节点和老节点作有偏好性的连接
(其偏好性正比于老节点的度)

例：

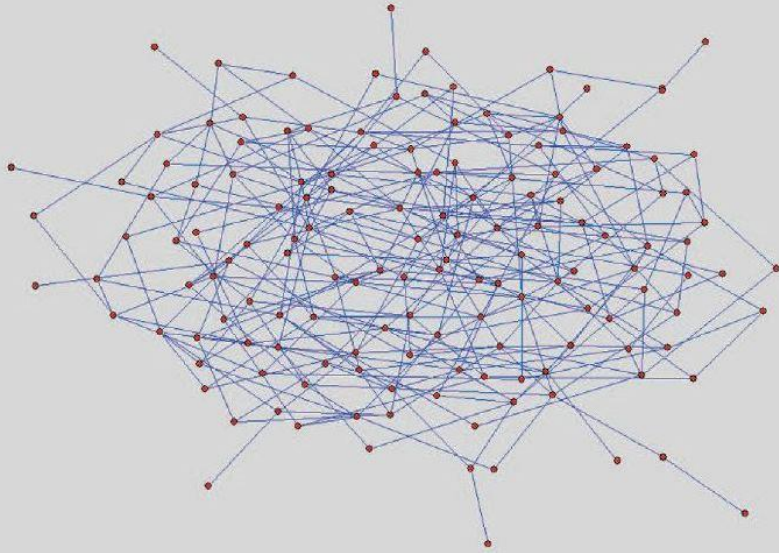


无标度网络

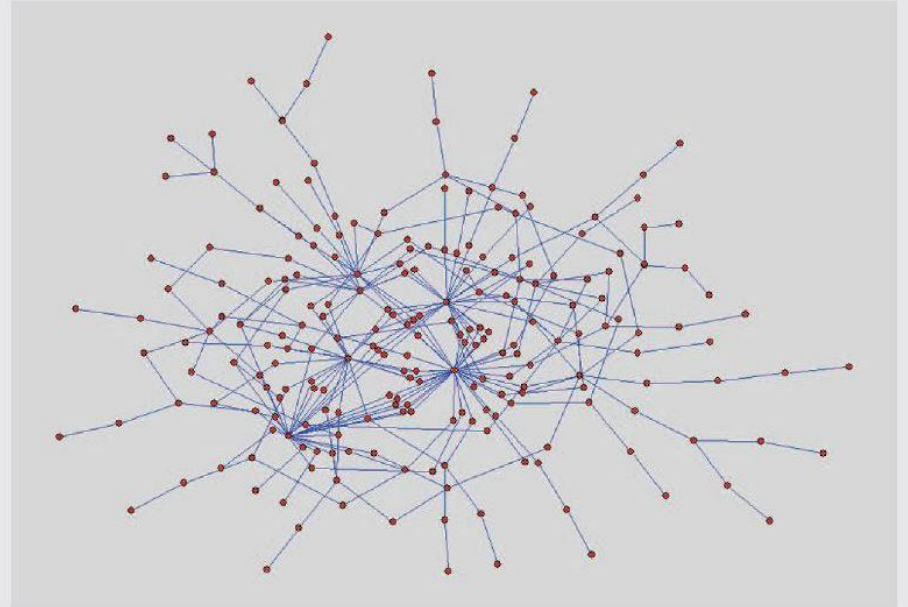


非均匀性：少部分节点有很多连线
大部分节点有很少连线

对比 Comparison



(a)



(b)

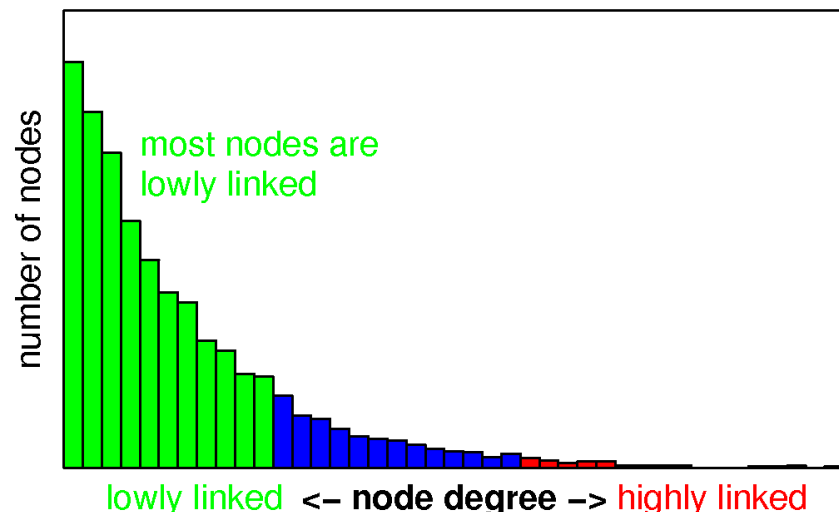
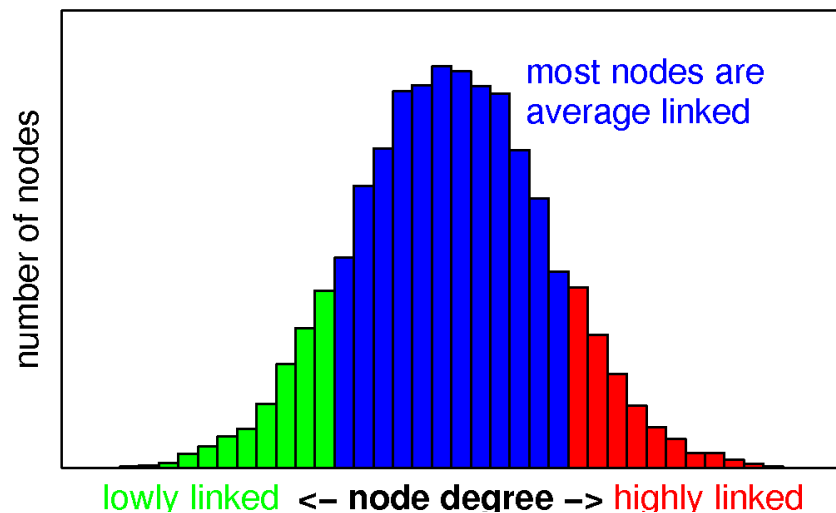
Random Network versus Scale-Free Network

(200 nodes)

随机网络 versus 无标度网络

对比 Comparison

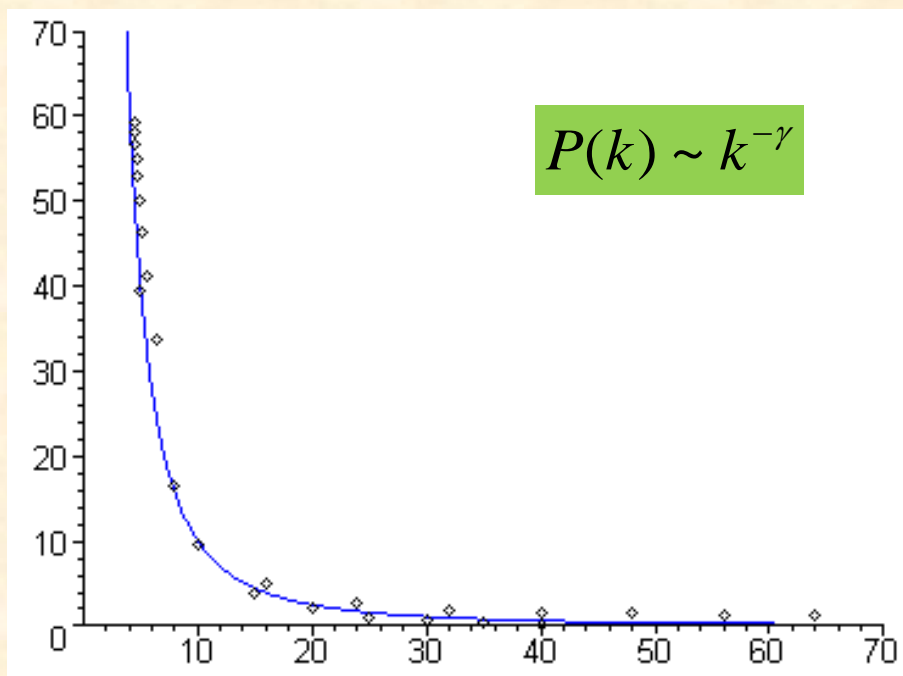
节点度分布图



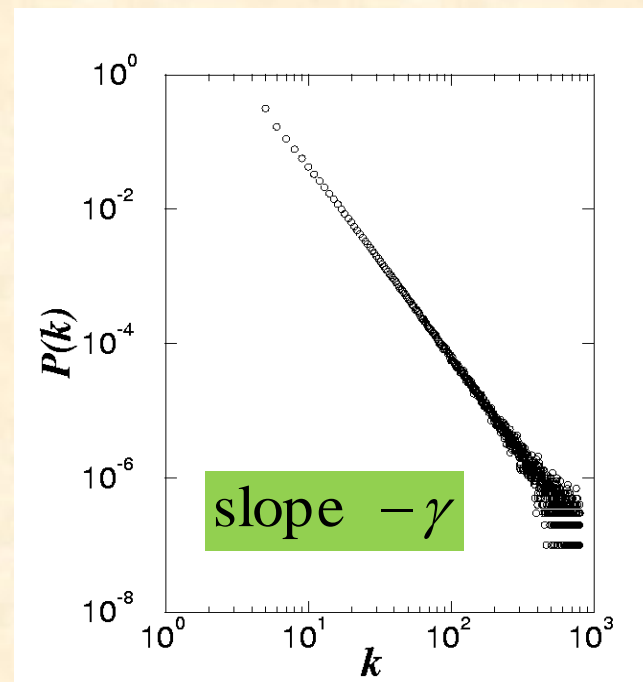
Random Network versus Scale-Free Network

随机网络 versus 无标度网络

无标度网络的度分布



幂率 (Power-Law)



Log-Log Plot

无标度：与网络的规模无关

飞机航线图：Airlines Route Maps



现实世界复杂网络例子

❖ 技术网络:

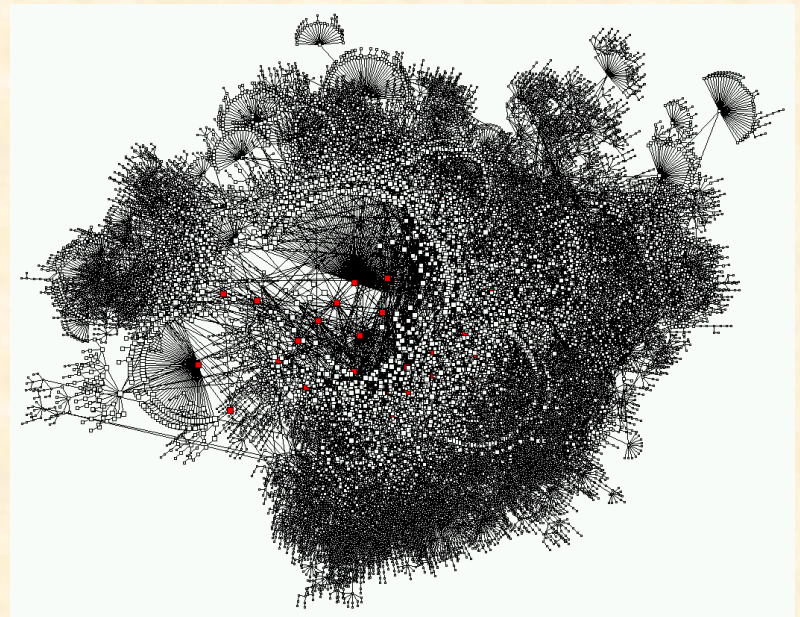
- 互联网 Internet
- 万维网 WWW

❖ 生物网络:

- 基因、蛋白
- 线虫
- 大脑

❖ 社会网络:

- 6度分离现象
- 电影明星网络
- 科研合作网络

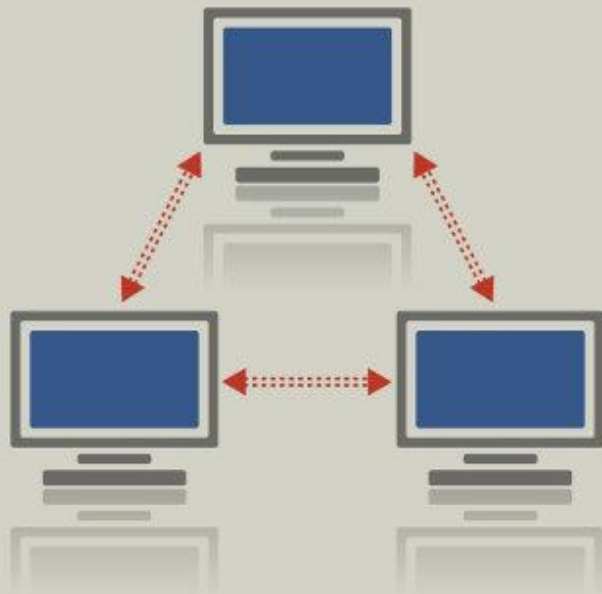


(Bradley Huffaker)

技术网络

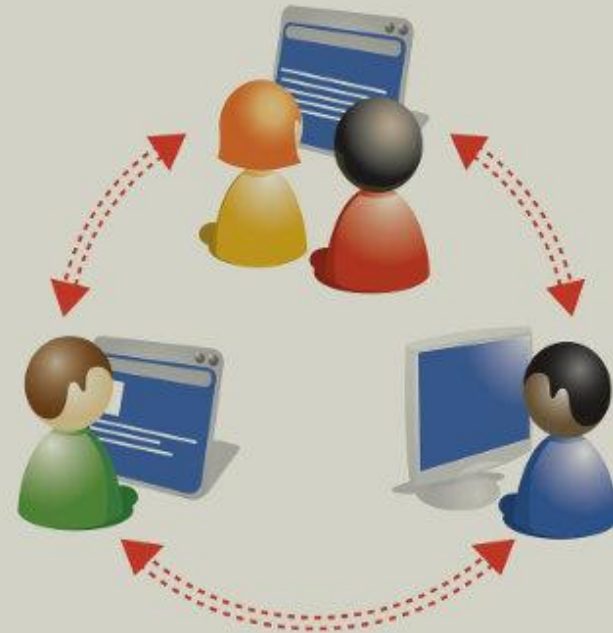


互联网 Internet vs 万维网 WWW



The Internet: Connecting Computers

硬件



The Web: Connecting People

软件

互联网把机联网

万维网把人联网

互联网 Internet

Data: 1995-1999, at both domain level and router level

❖ 平均路径长度 Average distance

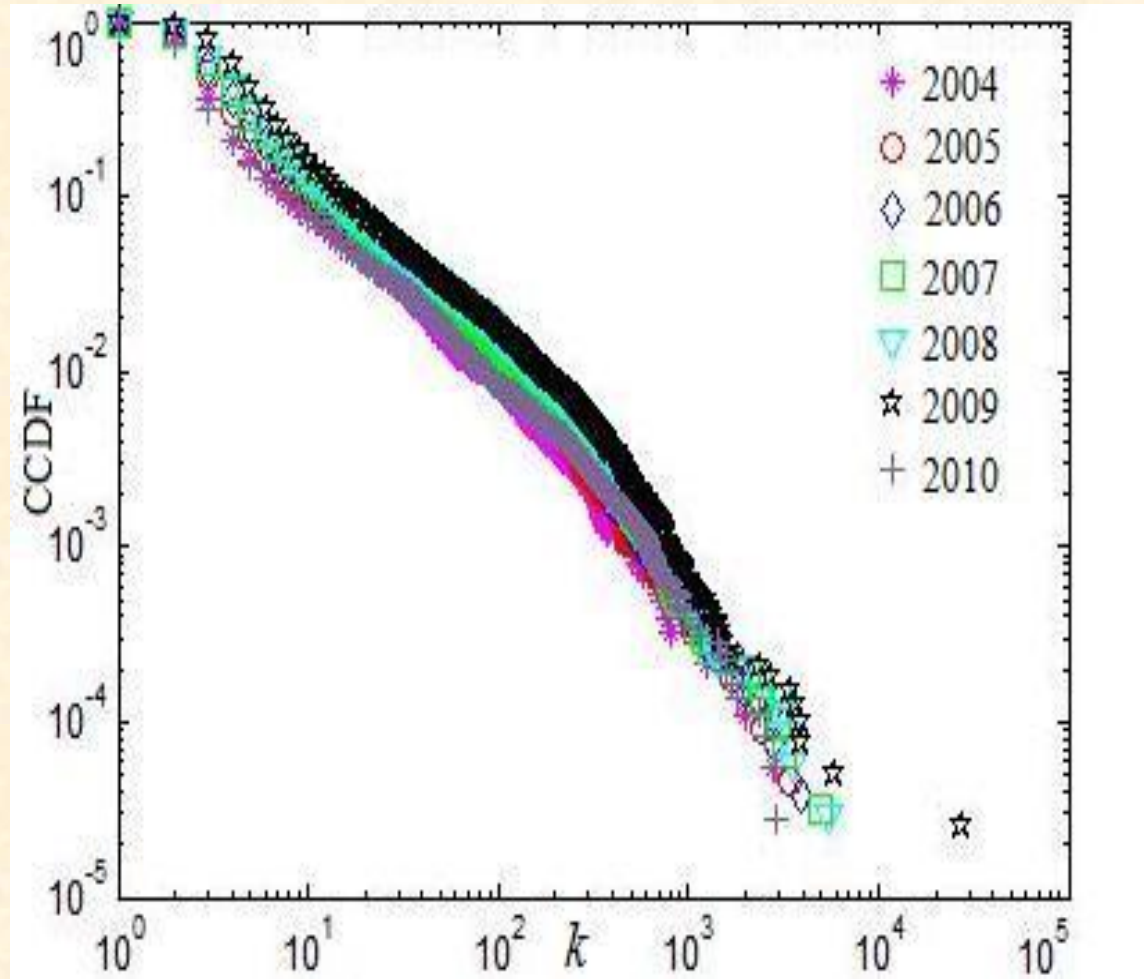
- $L = 4.0$ (small)
- → 互联网是小世界网络

❖ 度分布 Degree distribution

- 幂率 power law: $P(k) \propto k^{-\gamma}$, $\gamma = 2.2$
- → 互联网是无标度网络

许多现实网络同时具有两种形态（分类的重合部分）

真实的互联网 Internet (AS)



$$P(k) \propto k^{-\gamma} \quad \gamma = 2.14 \sim 2.21$$

Z. P. Fan and
G. R. Chen (2011)

万维网 WWW

S. Lawrence (1999): 800 million documents (8 亿个节点)

❖ 平均距离 Average distance → 小世界网络

➤ $L = 14$

➤ 直径 $D = 19$

❖ 度分布 Degree distribution → 无标度网络

➤ 出度 Outgoing edges: $P(k) \sim k^{-\gamma}$ ($r = 2.38 \sim 2.72$)

➤ 入度 Incoming edges: $P(k) \sim k^{-\gamma}$ ($r = 2.1$)

许多现实网络同时具有两种形态 (分类的重合部分)

万维网 World Wide Web

互联网 Internet

大数据表明:

WWW 是小世界网络也是无标度网络



Internet

大数据表明:

Internet 是小世界网络也是无标度网络



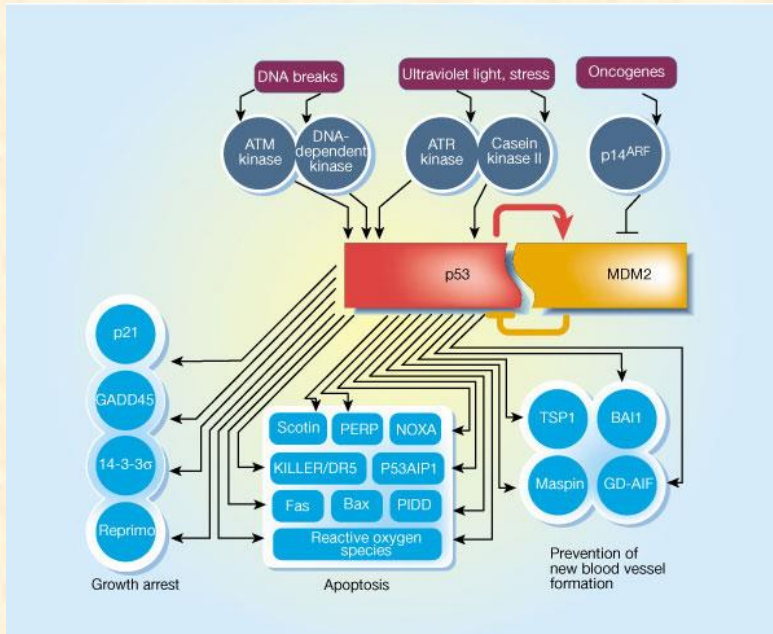
生物网络



Surfing the p53 network

Bert Vogelstein, David Lane and Arnold J. Levine

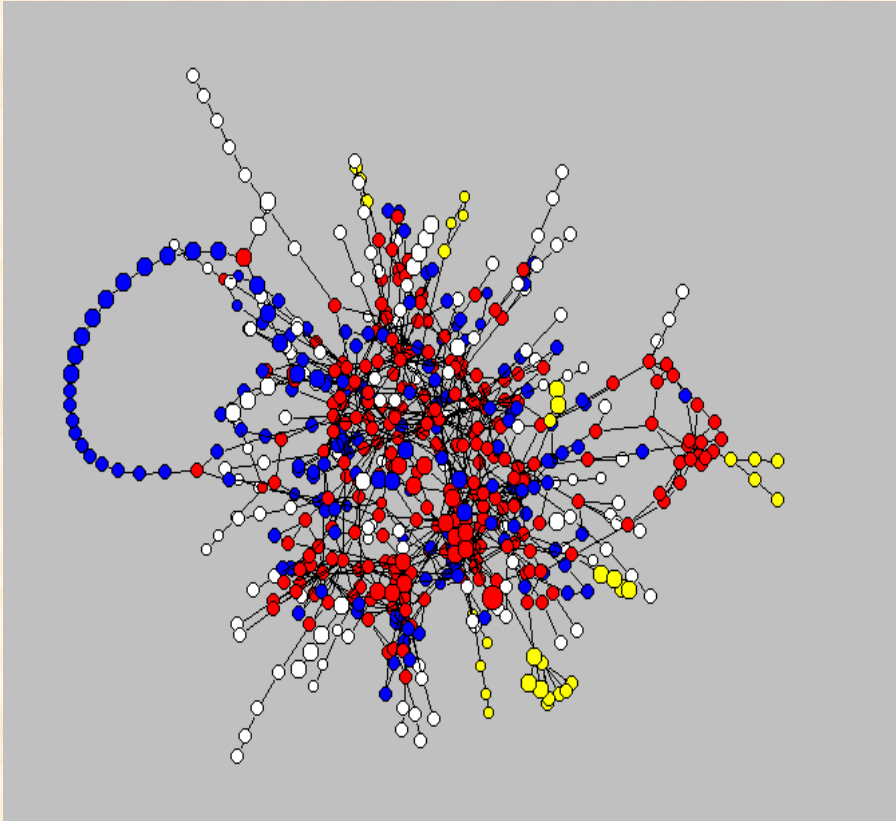
The p53 tumour-suppressor gene integrates numerous signals that control cell life and death. As when a highly connected node in the Internet breaks down, the disruption of p53 has severe consequences.



p53 基因控制癌细胞的生与死。它的发现被认为是癌症研究中最重要成果之一。

--- “了解 p53 网络的方法是把它与互联网做直接对比。这基因和互联网一样，是个无标度网络”

新陈代谢网络

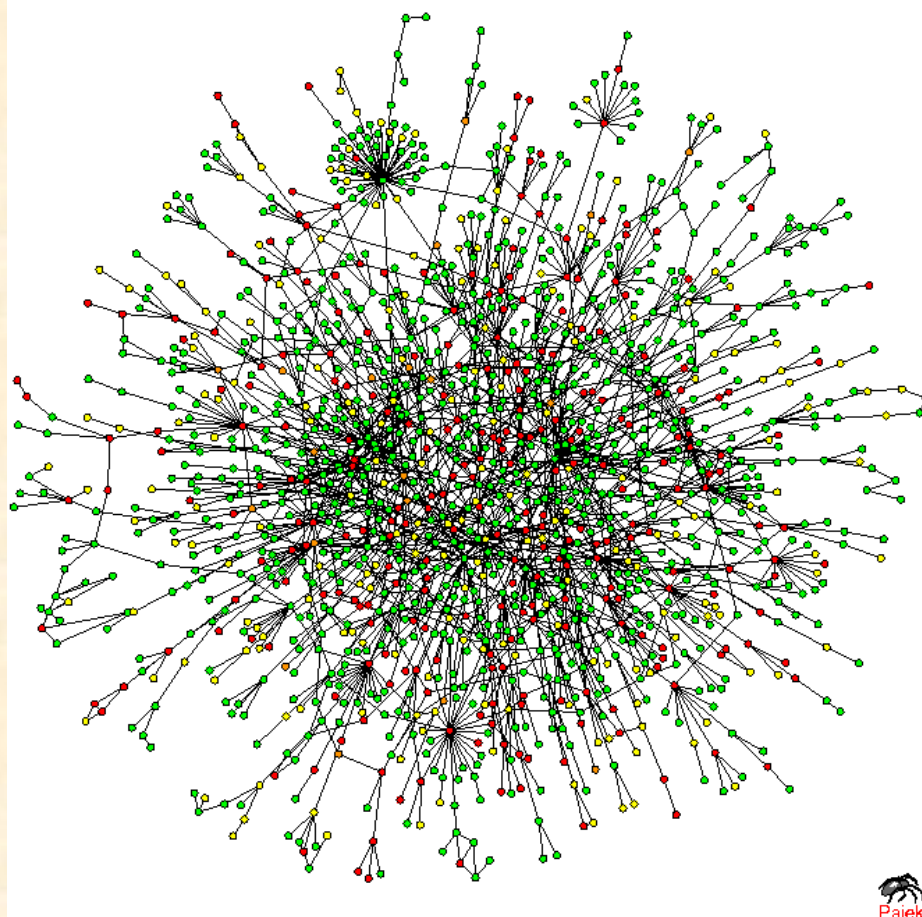


人体具有43个器官的
新陈代谢 (化学反应)
网络是无标度网络和小
世界网络

H. Jeong *et al.*,

Nature, 407: 651-654, 2000

酵母蛋白作用网络



无标度网络

节点：蛋白
连边：相互作用

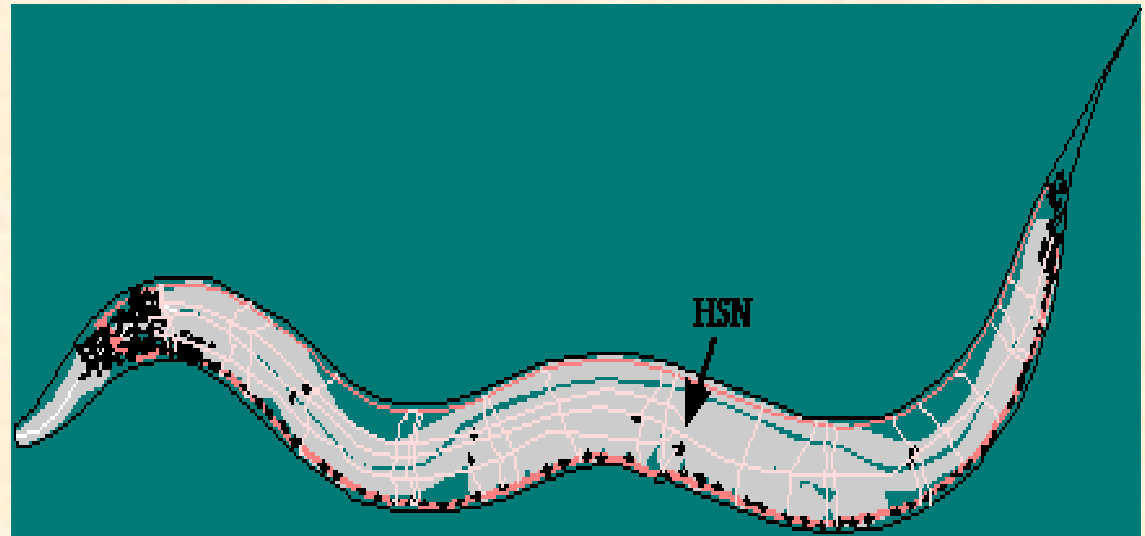
Red: lethal
Green: non-lethal
Orange: slow growth
Yellow: unknown

H. Jeong *et al.*, *Nature*, 411: 41-42, 2001

线虫 *C. Elegans* 神经网络

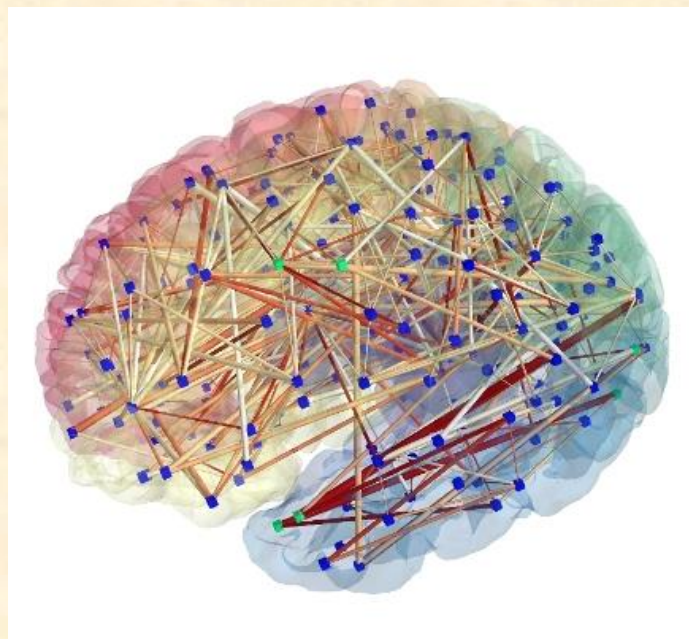
小世界网络

L. A. N. Amaral et al., *PNAS* (2000)



节点: ~300 神经元 连边: ~7000 突触

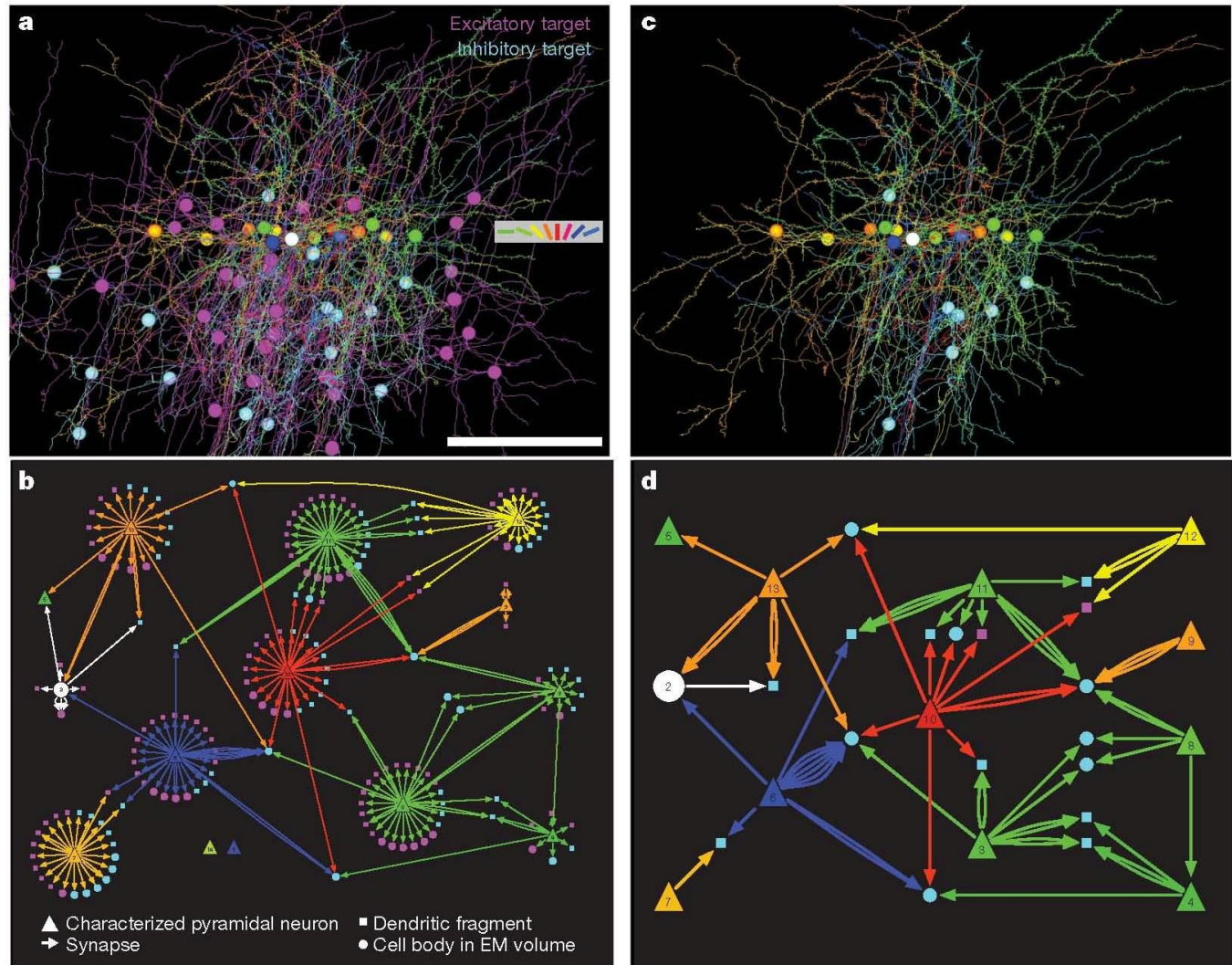
大脑



大脑神经网络

小世界拓扑

社团结构



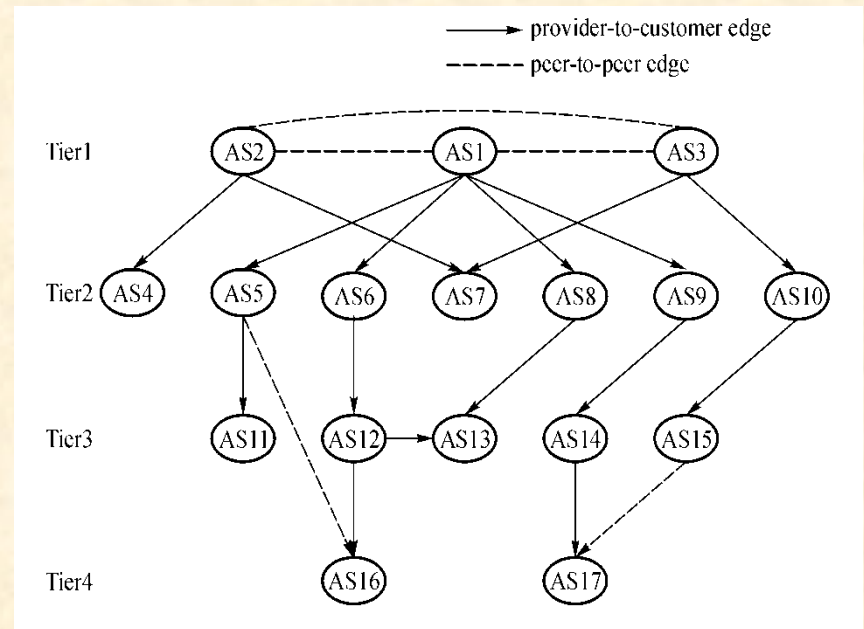
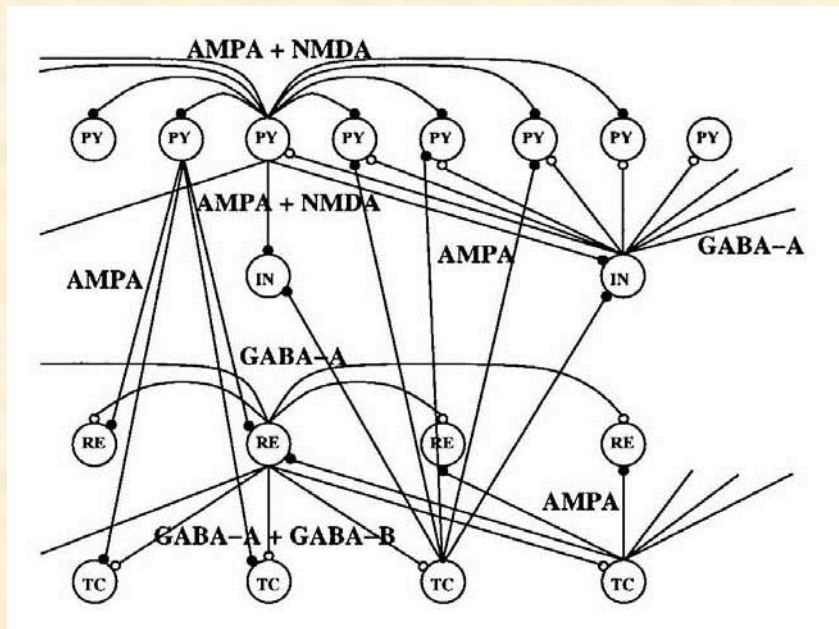
D. D. Bock et al., *Nature*, 2011

大脑结构

大脑 Brain



互联网 Internet



M. Bazhenov *et al.*, *J. Neuroscience*, 2002

S. I. Cai *et al.*, *Proc. IEEE CDC*, 2004

HUMAN BRAIN CELLS, OUR UNIVERSE AND THE INTERNET HAVE VERY SIMILAR STRUCTURES!!

December 13, 2013 by marcanthony213 in Uncategorized and tagged brain cells, cellular structures, energy, frequencies, internet, structures, universe, vibrations

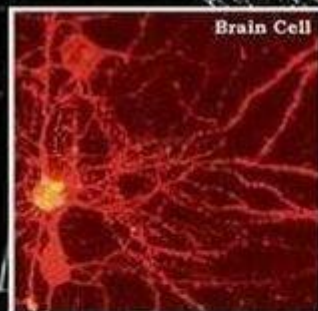
D. Krioukov et al., *Scientific Reports*, 2012

www.unbelievable-facts.tumblr.com

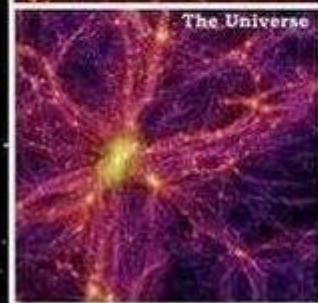
都是小世界网络

都是小世界網絡

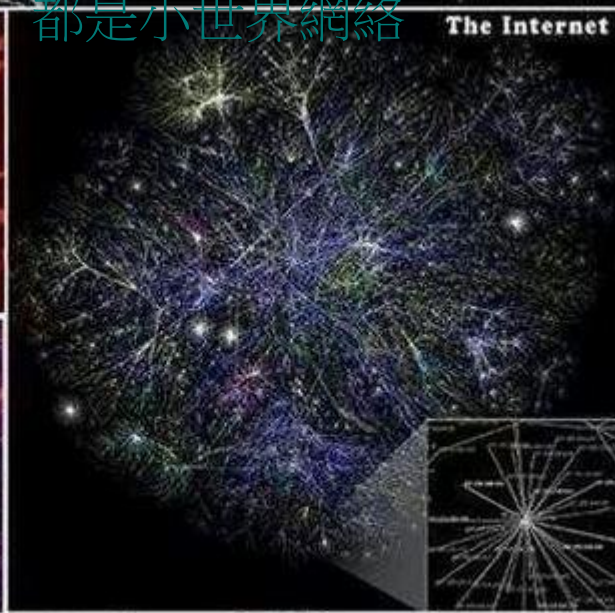
大脑



Brain Cell



The Universe

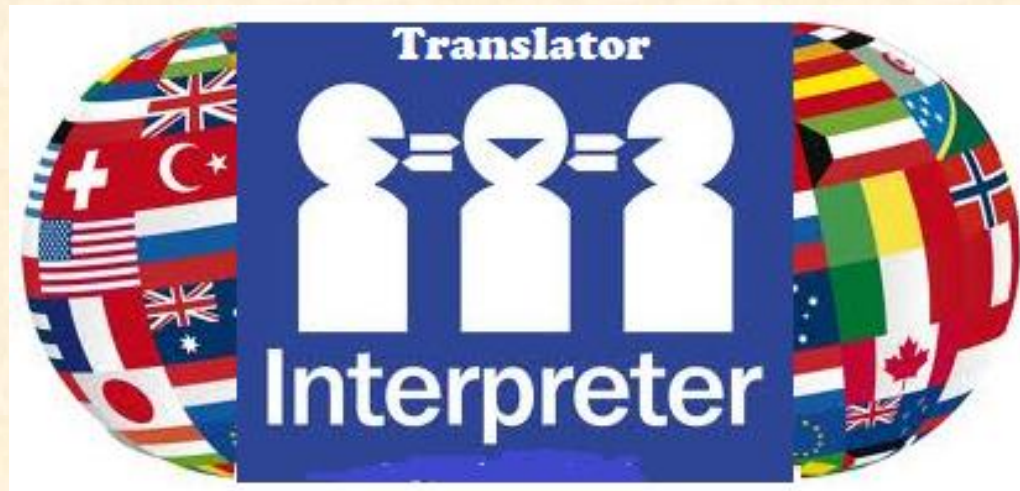


The Internet

宇宙

互联网

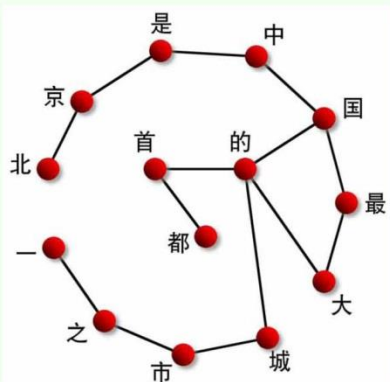
语言网络



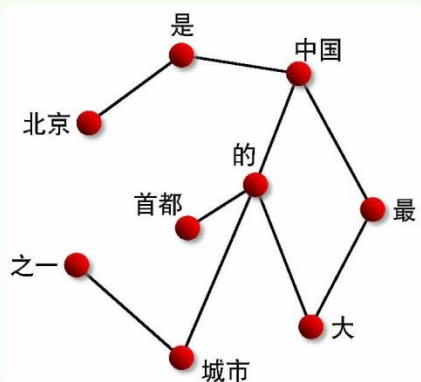
中文网络: 无标度和小世界网络

北京是中国的首都，是中国最大的城市之一。

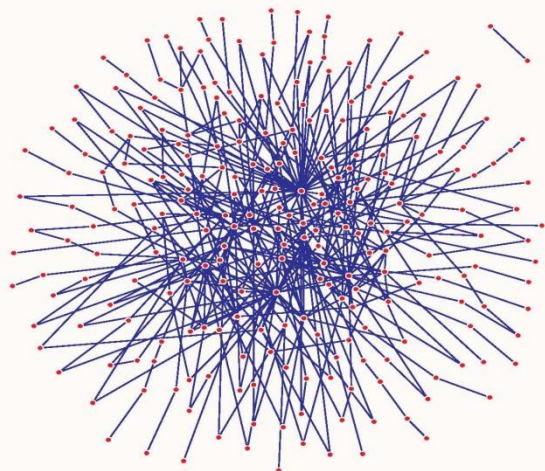
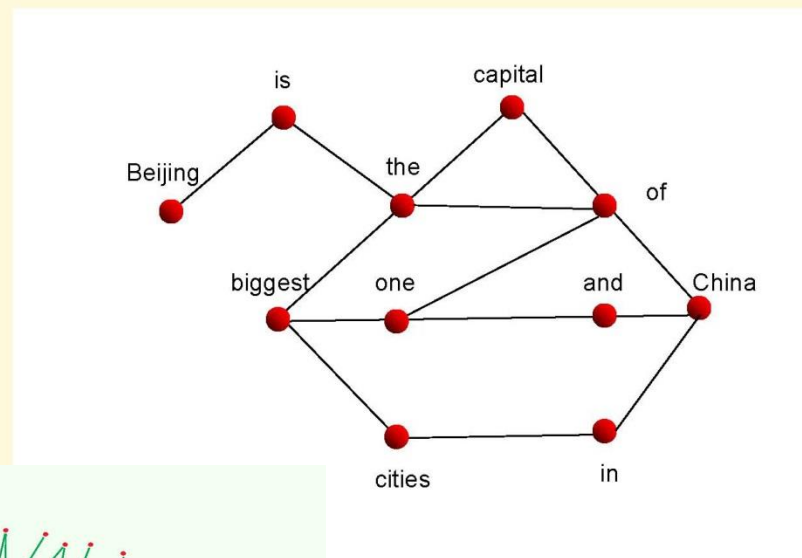
Beijing is the capital of China and one of the biggest cities in China.



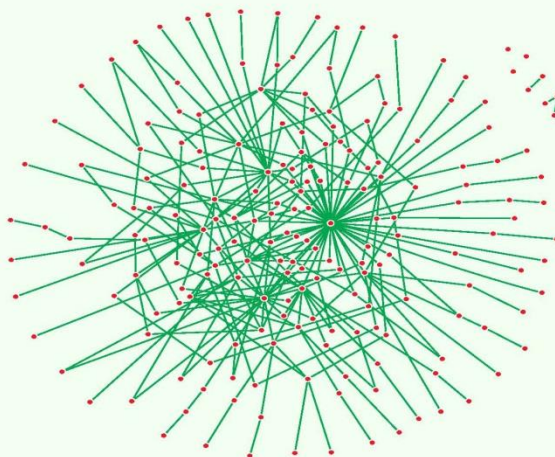
(a) 字同现网络



(b) 词同现网络



散文文字网络: $N = 253, E = 408$.

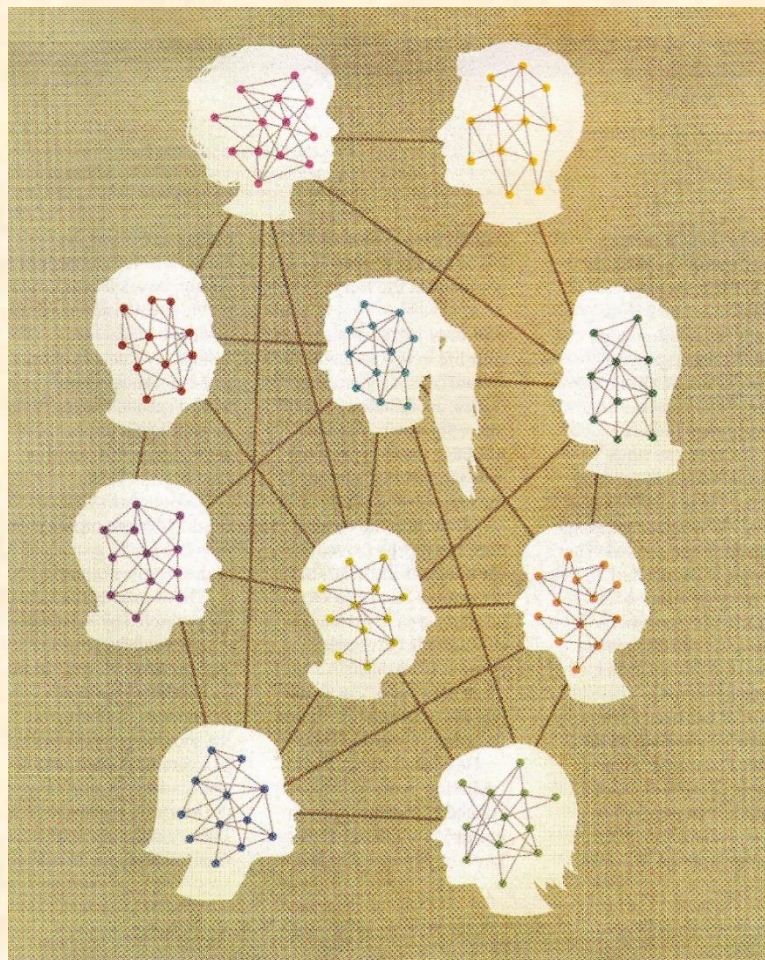


散文词网络: $N = 198, E = 279$.

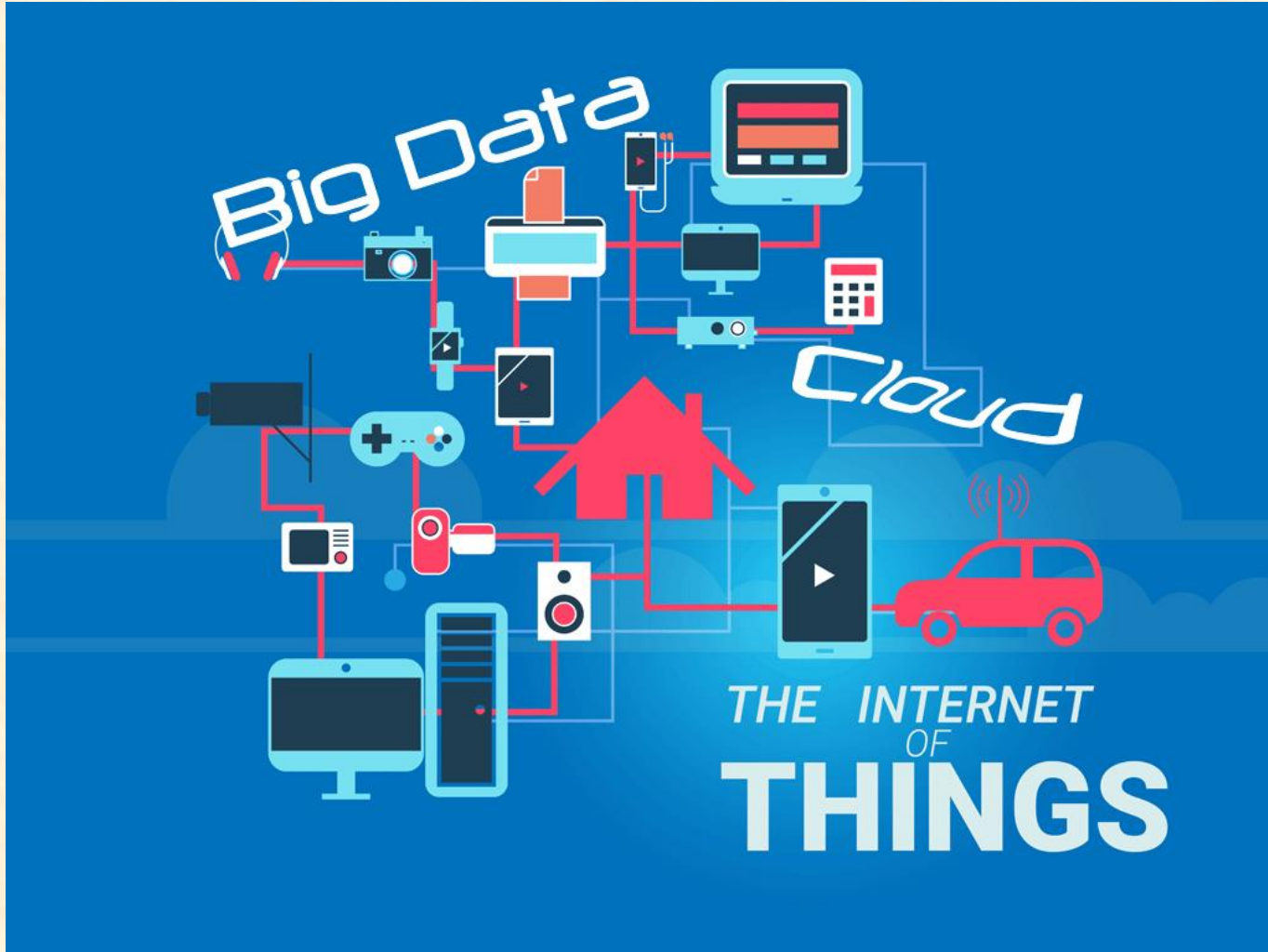
汉字语句网络

汉语拼音网络

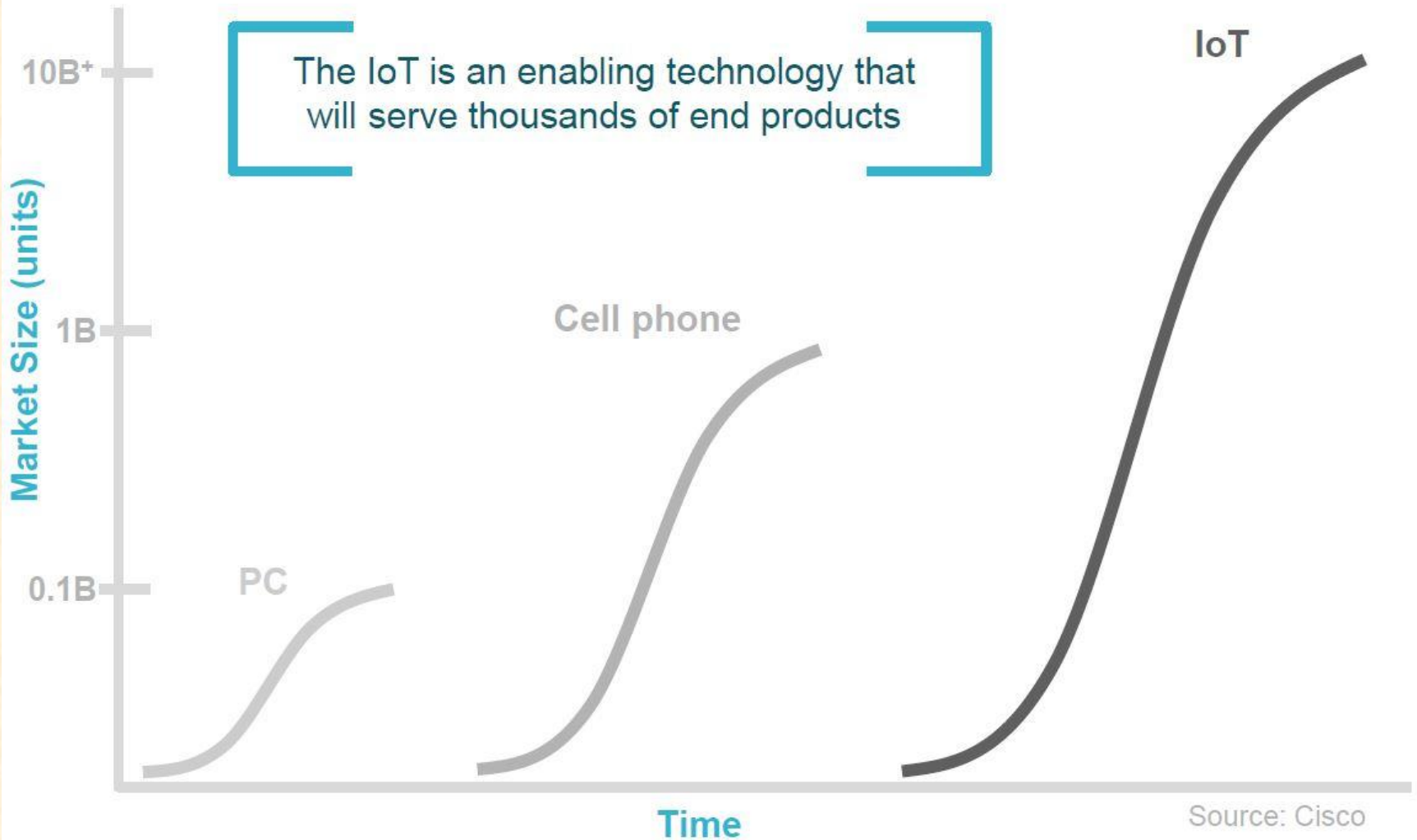
网络的网络



物联网 Internet of Things



Why the IoT is attractive



万物皆联网

Everything is Networked Together

Internet of Things

Total number of things

Things

Connected Things

Things on the Internet

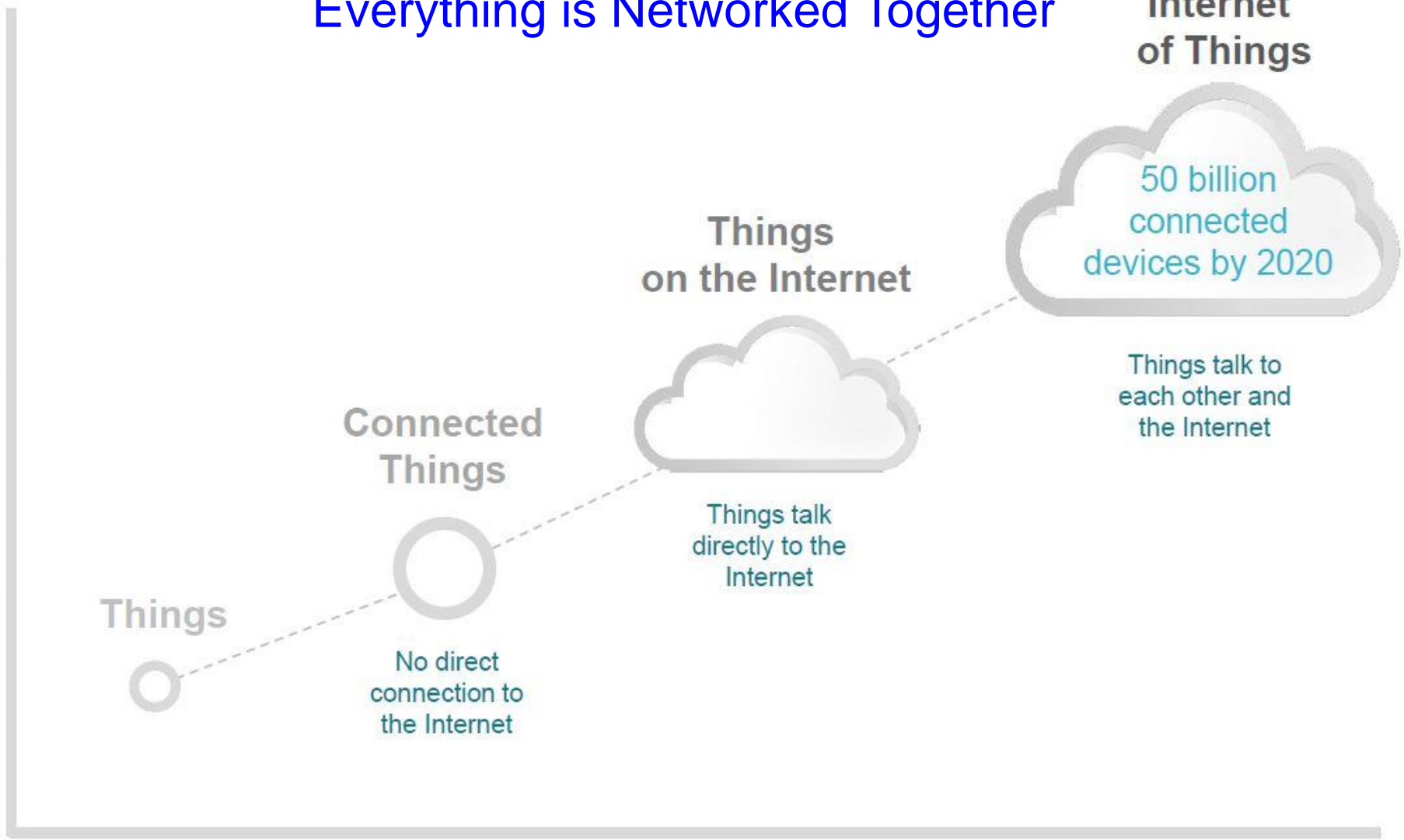
50 billion connected devices by 2020

No direct connection to the Internet

Things talk directly to the Internet

Things talk to each other and the Internet

Time



社会网络

有朋自远方来，不亦乐乎？

—— 孔子



小世界实验 (1967)

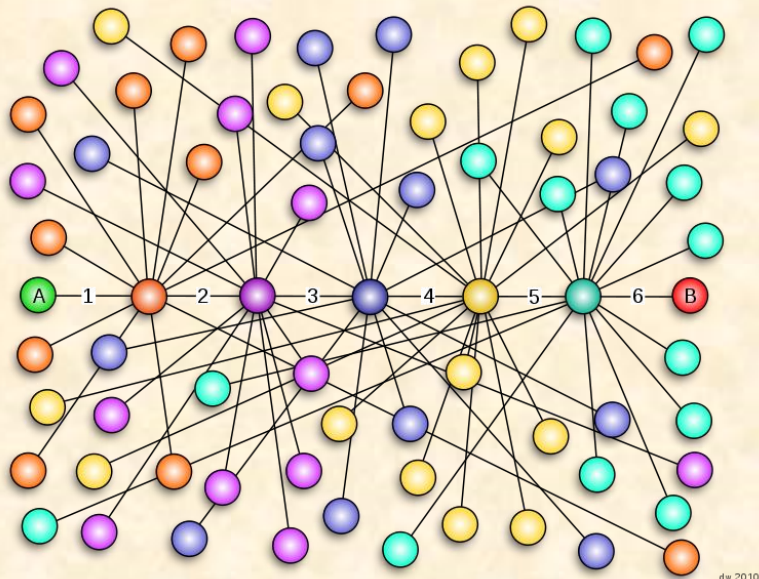
Stanley Milgram, 哈佛大学 Harvard University



問題：在美国随机地找两个人，他們会通过多少个中間朋友连接在一起？

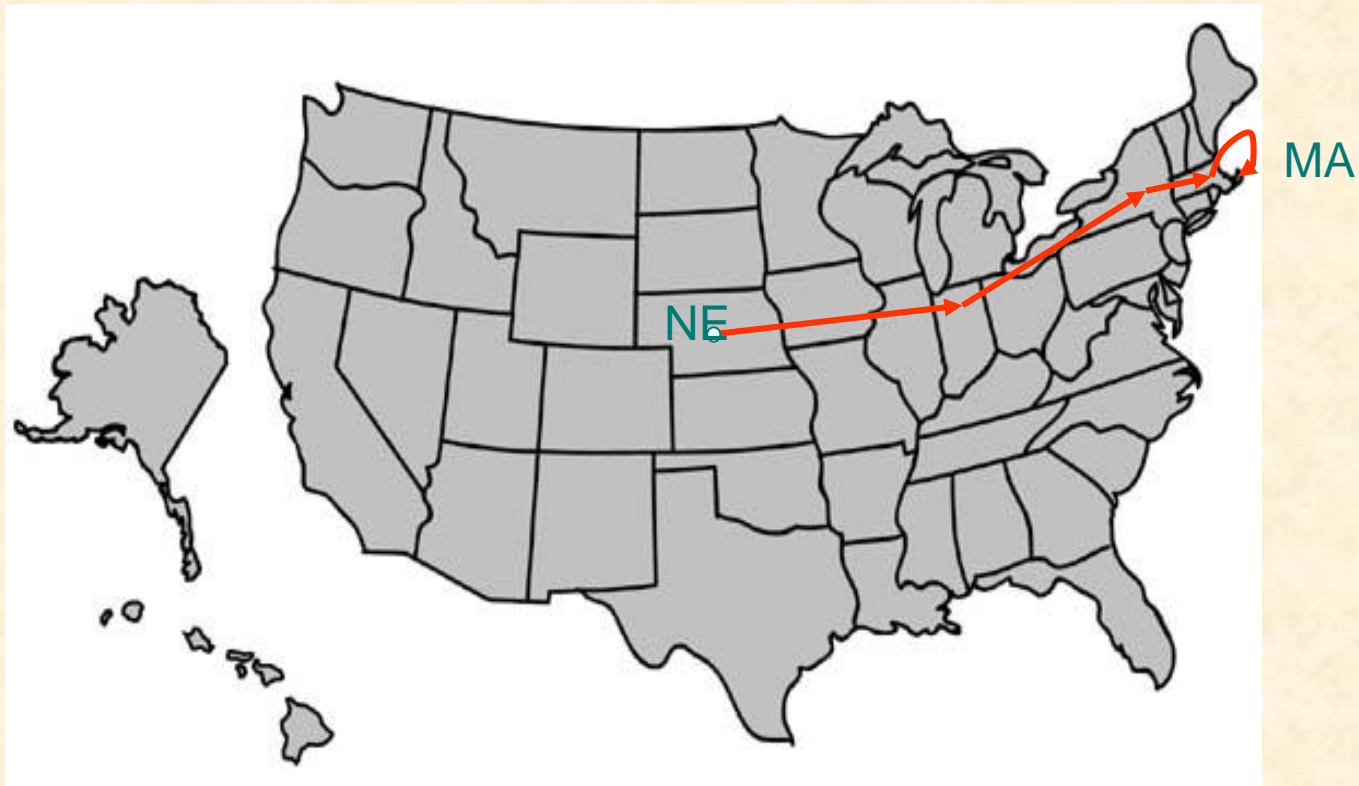
答案：6

Alice → B → C → D → E → F → George



S. Milgram, The small world problem,
Psychology Today, May 1967, 60-67.

小世界实验



- 設定一个“收信人”（在 Boston）
- 随机地邀请 300 个“发信人”（在 Omaha, Nebraska）
- “发信人”把信件发给可能在 Boston 有联络的朋友
- 结果：平均 6 人

新近大规模的小世界实验

例 1: Duncan Watts 在 *Science* (August 2003) 报告说：他们在历时一年多的社会实验中，联络了 60,000 个志愿者（来自 166 国家），给 18 个“收信人”（分散在 13 个国家里）发电子邮件，最后成功到达的电子邮件都经过了 5 ~ 7 个用户的传递。

例 2: Facebook Data Team 在 2011 年 11 月 21 日宣布，他们调查了 7.21 亿个 Facebook 活跃用户（占全球人口的 10%），通过用户 690 亿条联络，发现两个用户之间的联络平均只有 4.74 个朋友的间隔。（2016 年 Facebook 报告：3.57）

→ 世界将变得越来越小

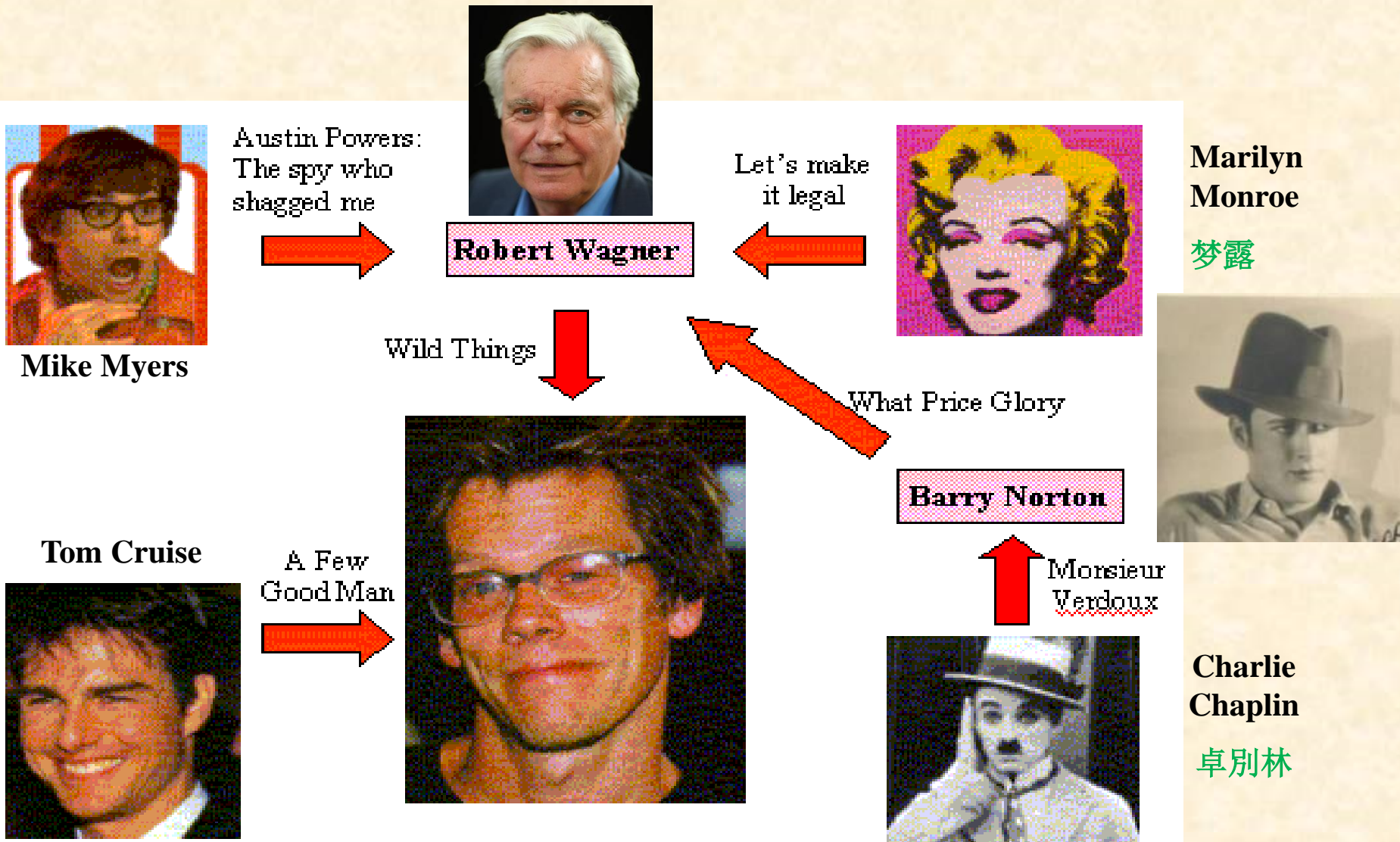
Kevin Bacon 网络

美国电影明星 Kevin Bacon 和其他影星的“距离” = ?



- ▶ 如果两个演员在同一部电影里演出，就认为他们之间有一条连边（“距离” = 1）
- ▶ 1994年, CNN TV 的 Stewart Show 节目讨论了这样一个问题：从荷里活中随机地挑出一个演员，问他（她）和 Bacon 之间的“距离”是多少（即，有多少个其他演员把他们联系起来）？
- ▶ 结果: 平均来说，荷里活中任何一个演员和 Bacon 的“距离”为 ≈ 3

Kevin Bacon 小世界网络




Bacon Number (as of 2020-1-1)

<u>Kevin Bacon</u> Number	# of People
0	1
1	855
2	62266
3	238646
4	120565
5	9585
6	799
7	137
8	37
9	16

Total number of linkable actors: 432907

Weighted total of linkable actors: 1377703

Average Kevin Bacon number: 3.182 

<https://oracleofbacon.org/onecenter.php?who=Kevin+Bacon>

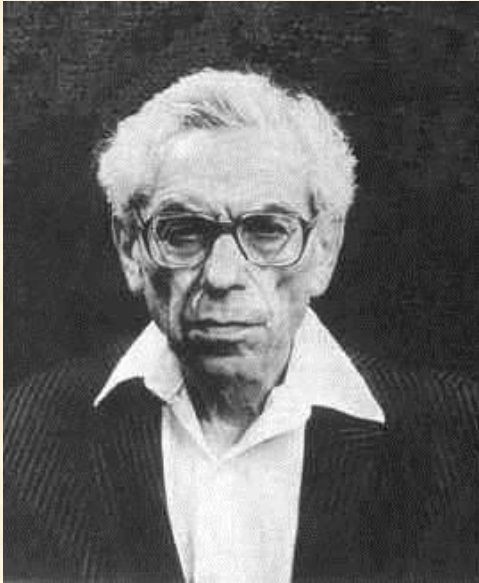
科研合作网络



节点：科学家

连边：合作写文章

Erdős Number 网络



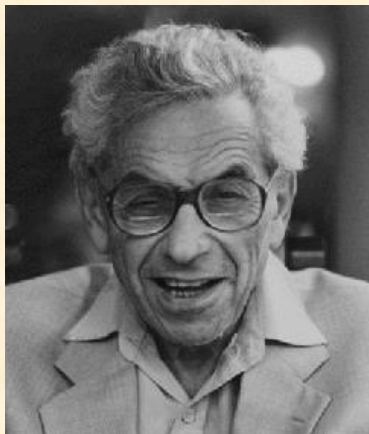
Pál Erdős
(1913-1996)

- Erdős 一生发表了 1600 多篇论文
有 500 多个合作者
- 從20岁起到83岁止，平均每个月发表 2 篇论文
- Erdős number 网络是小世界和无标度网络

请你估計一下：

Erdős 与 我 之间的 “距离” 是多少？

我的 Erdős Number 是 2



P. Erdős



C. K. Chui



G. R. Chen

小小世界！

Borosh, I., Chui, C. K. and Erdos, P., On changes of signs in infinite series,
Anal. Math. 4(1), 3-12, 1978.

Chui, C. K. and Chen, G., Kalman Filtering with Real-Time Applications,
Springer-Verlag (1st ed., 1987; ... ; 4th ed., 2009)

欧拉与我？



欧拉 Euler
(1707-1783)

博士导师和学生的师徒关系



欧拉与我？

欧拉 Euler
(1707-1783)

Academic Genealogy

(A → B means that A was PhD adviser of B)

博士生导师和学生的师徒关系表

欧拉 L. Euler → J.-L. Lagrange → J.-B. J. Fourier → G. P. L. Dirichlet → L. Kronecker

→ M. Lerch → M. Plancherel → A. Edrei → S. Hellerstein → C. K. Chui → G. R. Chen 陳關榮



高斯 Gauss
(1777-1855)

高斯与我？

博士生导师和学生的师徒关系



高斯 Gauss
(1777-1855)

高斯与我？

Academic Genealogy

(A → B means that A was PhD adviser of B)

博士生导师和学生的师徒关系表

高斯 C. F. Gauss → F. W. Bessel → H. F. Scherk → C. C. Kummer →

J. von Neumann

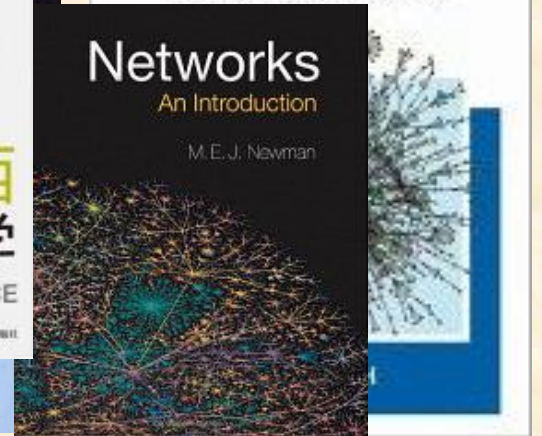
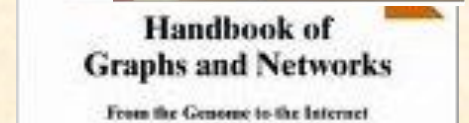
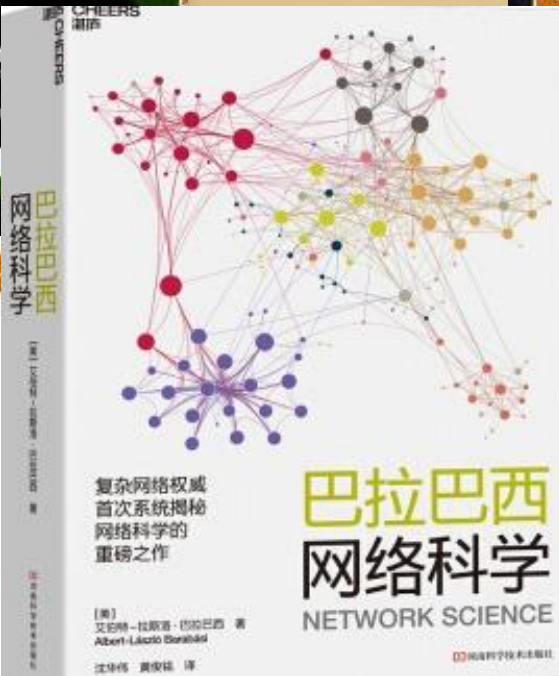
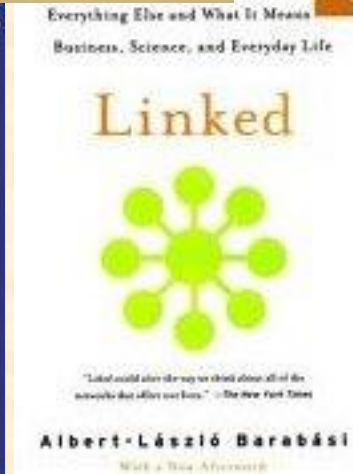
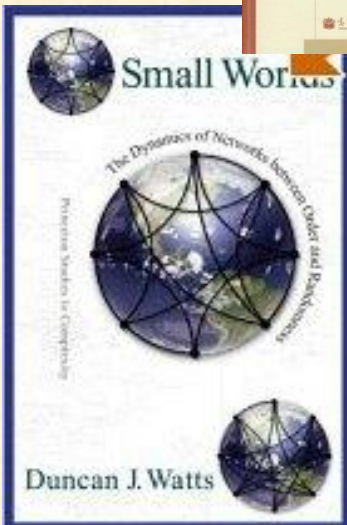
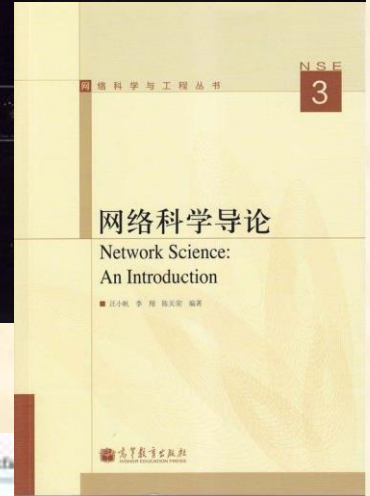
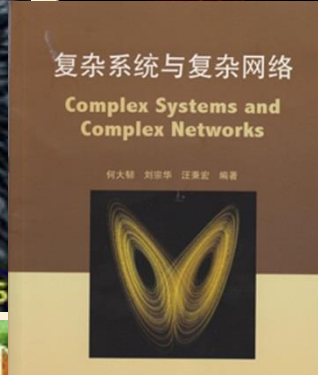
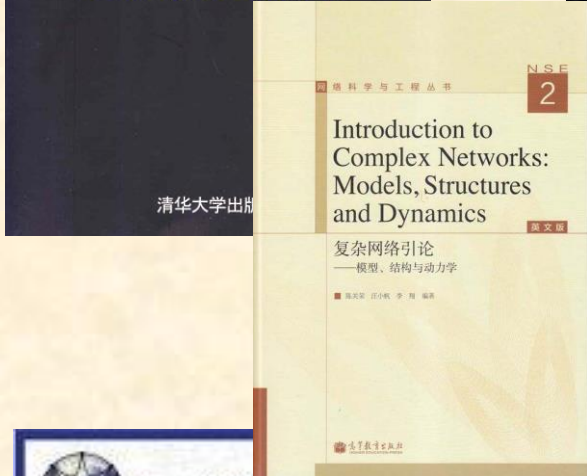
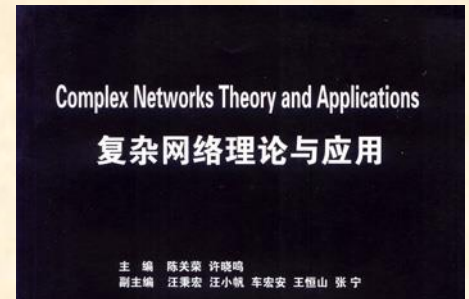
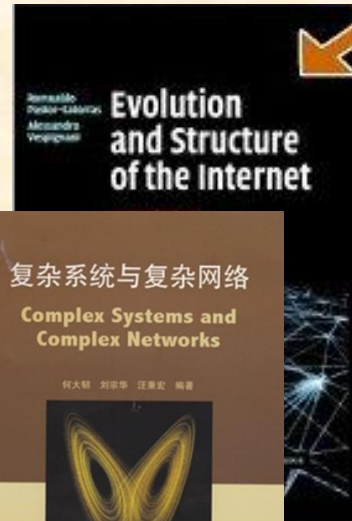
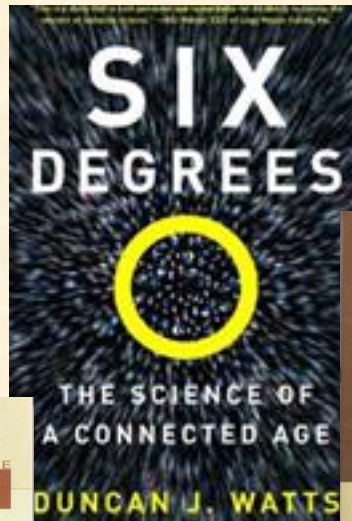
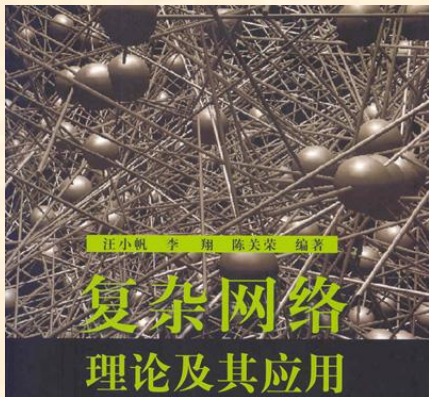
→ H. A. Schwarz → L. Fejer → G. Polya → A. Edrei → S. Hellerstein →
P. Erdős

→ C. K. Chui → G. R. Chen 陳關榮

It is a small world after all



小小世界！



many more . . .

Sean Meyn

Thanks

