



In Memoriam: Gennady Alekseevich Leonov



Gennady Alekseevich Leonov
(February 2, 1947–April 23, 2018)

N. V. Kuznetsov

*Faculty of Mathematics and Mechanics, St. Petersburg State University,
Peterhof, St. Petersburg, Russia*

*Department of Mathematical Information Technology,
University of Jyväskylä, Jyväskylä, Finland*

S. Abramovich

State University of New York at Potsdam, USA

A. L. Fradkov

*Faculty of Mathematics and Mechanics, St. Petersburg State University,
Peterhof, St. Petersburg, Russia*

Institute of Problems of Mechanical Engineering RAS, Russia

G. Chen

City University of Hong Kong, Hong Kong SAR, P. R. China

Gennady Alekseevich Leonov passed away on April 23, 2018 after a short battle with grievous illness.

The prolific life of Gennady Leonov as a scholar and educator ended tragically at the zenithal time of his indisputable scientific creativity and academic leadership, bestowing on us the legacy of more than 500 publications, 20 monographs, ten patents, and the diligent supervision of five Doctors of Science (a habilitation degree), 16 Doctors of Philosophy (PhD), and 37 Candidates of Science.

Gennady Leonov earned numerous awards and honors over the course of his career. He was a corresponding member of the Russian Academy of Sciences, a Foreign Member of the

Finnish Academy of Science and Letters, a Council Member of the International Federation of Automatic Control (2011–2017), a Highly Cited Researcher in the Russian Federation — the list of his honors and awards is indeed too long¹ to be fully presented in this brief obituary.

Gennady Leonov was born in St. Petersburg (then Leningrad) on February 2, 1947 into the family of Antonina Andreevna Leonova, an accountant, and Aleksey Michailovich Leonov, a blue-collar worker. Growing up, just like many ambitious youngsters of his generation, Gennady was working daily in a manufactory while studying at night in school and completed his high school education on a fast track (two versus three years). In 1964, he passed the highly competitive entrance exams to become a mathematics major student at the Mathematics and Mechanics Faculty of Leningrad State University, the very place he spent the rest of his life, serving as Dean of the Faculty for 30 years (1988–2018). In 1969, Gennady started his post-graduate studies (*aspirantura* in Russian) under the supervision of Arkadii Kh. Gelig, and defended his thesis in 1971, ahead of schedule. In his dissertation for Candidate of Science, his answer was negative to the famous conjecture of Aizerman [1949] in control theory of the most general form. He joined the newly established Department of Theoretical Cybernetics, headed by Vladimir A. Yakubovich [Abramovich *et al.*, 2015; Fradkov, 2017], as Assistant Professor in 1971, and was quickly promoted to the position of Associate Professor. In 1983, Gennady defended his second dissertation (for Doctor of Science), titled “*Stability in the Whole*”, which endowed him the Full Professorship.

By the early 1980s, under the supervision of Gennady Leonov, ten dissertations for Candidate of Science had been defended, where important problems in differential equations and control theory were investigated. One of the directions of Gennady’s research work at that time was the development of a mathematical theory for phase-locked loop systems [Gelig *et al.*, 1978], when only heuristic engineering methods existed for investigations. His deep mind and insightful view of mathematics for investigating and solving pivotal real-life problems resulted in a major professional achievement: “*the development of the theory of phase synchronization in radio engineering and communication*”, with which a team of researchers, Gennady Leonov included, was awarded the 1986 State Prize of USSR, the second in societal prestige and academic standing that existed in the Soviet Union.

The significance of his contributions to such highly recognized scientific work was undoubtably corroborated during the last decade of the 20th century, when systems of phase synchronization became useful tools of vital importance for ubiquitous integration into various new information systems. In particular, frequency synthesis devices in every modern computer are based on such systems of phase synchronization. Phase-locked loops have also been used in the Global Navigation Satellite System (GLONASS) and the Global Positioning System (GPS). It is fair to say that the effectiveness and precise functioning of such systems would not be possible without utilizing the rigorous mathematical methods developed by Gennady Leonov [Leonov, 2006; Leonov *et al.*, 2012; Leonov & Kuznetsov, 2014; Leonov *et al.*, 2015c, 2015d; Best *et al.*, 2016; Kuznetsov *et al.*, 2017].

In 1988, in the spirit of *perestroika*, Gennady Leonov was elected through an open search process to become Dean of the Mathematics and Mechanics Faculty, a position he held until the very last day of his life. In the 1990s, during the difficult years for Russian science, Gennady was instrumental in preserving the rich traditions of scientific achievements of mathematicians, mechanical engineers, and astronomers of the Faculty. With dedication and determination, he demonstrated an extraordinary ability for preserving the celebrated traditions of the St. Petersburg University in mathematics, while advancing his own research.

¹Personal webpage: http://www.math.spbu.ru/user/leonov/index_en.html.

Google scholar profile: https://scholar.google.ru/citations?hl=en&user=_zv2pFwAAAAJ.

Mathematics genealogy project: <http://www.genealogy.ams.org/id.php?id=105152&fChrono=1>.

For one, Gennady proposed to integrate Lyapunov functions into the attractors dimension theory [Leonov, 1991; Leonov & Boichenko, 1992; Boichenko *et al.*, 2005; Kuznetsov, 2016], something that enabled him to obtain exact analytic formulas for the Lyapunov dimension of attractors for a number of well-known dynamical systems (see, e.g. some recent works [Leonov *et al.*, 2015a; Leonov *et al.*, 2016; Leonov, 2017; Kuznetsov *et al.*, 2018]). For another, he solved a problem in the most general form of stabilization of linear differential equations [Leonov, 2001; Leonov & Shumafov, 2010] formulated by Brockett [1999] as an open problem in control theory.

By continuing and expanding the work of Vladimir A. Yakubovich and Viktor A. Pliss, his major academic forefathers, Gennady Leonov was able to establish his own scientific school of control theory, qualitative theory of dynamical systems and their applications in science and engineering. One such application is in the study of chaotic dynamics and hidden attractors. Over the past 40 years, the mathematical theory of chaos grew into one of the most vigorous domains of scholarly research. The significance of this theory is due to the complex phenomena of turbulence in hydrodynamics, the challenging problem of weather prediction, the intrinsic ocean current flows, and the wide appearance of chaotic oscillations in electric circuits and systems. More recently, homoclinic and heteroclinic orbits were found to play an important role in the dynamics of transition to chaos. To this end, Gennady proposed a general methodology (called *Fishing principle*) for investigating such orbits [Leonov, 2012, 2013, 2014; Leonov *et al.*, 2015b]. Consequently, the Fishing principle has provided mathematicians with a universal tool for analytically proving the existence of homoclinic orbits in a number of well-known dynamical systems. Last, but not least, the notion of *hidden attractors* and the development of new mathematical methods for investigating such attractors in classic dynamical systems [Leonov & Kuznetsov, 2013; Kuznetsov & Leonov, 2014; Leonov *et al.*, 2015b; Kuznetsov *et al.*, 2018] have attracted a great deal of interest from researchers in various fields, especially mathematicians, engineers, and physicists [Bragin *et al.*, 2011; Sharma *et al.*, 2015a, 2015b; Dudkowski *et al.*, 2016; Chen *et al.*, 2017; Stankevich *et al.*, 2017]. Largely due to the discovery of hidden attractors in dynamical systems, Gennady Leonov and his former doctoral student Nikolay V. Kuznetsov were proclaimed as the Highly Cited Researchers in the Russian Federation twice over two consecutive years (2016–2017) by Clarivate Analytics.

When compiling the achievements of Gennady Leonov, it is impossible to ignore his administrative talents and contributions. In 2007, in close collaboration with Nikolay V. Kuznetsov, he established a Department of Applied Cybernetics. The goal was to connect fundamental mathematics education at the Faculty of Mathematics and Mechanics with the demands of international collaboration in information technologies that graduates of the Faculty have to fulfill. With this goal, Gennady drew on the experience from the Faculty of Information Technology, University of Jyväskylä, Finland, and the affiliated Technopark established under the auspices of Nokia. Over the last decade, the department has accepted annually about 20 third-year university students; the most successful ones being invited for post-graduate studies and the best of the best were selected for participation in the joint Russia-Finland program of PhD studies, which was organized in cooperation with the Dean of the Faculty of Information Technology, Pekka Neittaanmäki (University of Jyväskylä). Productive collaboration between St. Petersburg State University and University of Jyväskylä has been in place for the last ten years. As a recognition, in 2017, the Finnish Academy of Science and Letters elected Gennady Leonov to be a foreign member.

There is too much to remember about the great mathematician, scientist and administrator Gennady A. Leonov. It is with great respect, immense admiration and profound grief that we celebrate the prolific life of our never-to-be-forgotten friend, colleague and mentor by acknowledging his intellect, talents, kindness, wisdom and acumen. He was a great role model for many, and will continually be so for many more to come.

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