DYNAMICAL SYSTEMS

PATHS OF TWELVE MATHEMATICIANS AND PHYSICISTS TO CHAOS AND ITS APPLICATIONS

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4. My academic life through a winding path by Guanrong Chen

Ever since my childhood, I have always been and still am a forward-looking person, with little interest in looking back to the winding path of my life and academic career. But when beckoned to reflect, I concede that I have accumulated many small stories that are perhaps worth telling.

I was born in 1948 and raised in a rather poor family in Guangzhou, a city in southern China. My parents grew up in a small village in the Zhongshan county of the Guangdong Province, near the home of Sun Yat-sen (1866–1925), who was the founding president of the Republic of China. Not being able to finish their elementary school education, my parents moved to Guangzhou to work for survival in the 1930s. Obviously, I did not receive any kind of intellectual assistance in education from them. Nevertheless, I did well in both elementary and middle schools on just about all subjects, particularly in mathematics and Chinese literature. That, in some way, nurtured my self-studying and self-learning abilities with intrinsic motivation. In middle school, driven by curiosity and interest, I started to self-learn mathematics beyond the course curriculum from some interesting books written for middle and high school students by, for example, the famous Chinese mathematician Loo-Keng Hua[4] (1910–1985), who published a series of elementary mathematics treatises for the youths that stimulated and guided me to pursue further studies of mathematics. Hua and Shiing-Shen Chern[5] (1911–2004) were considered the greatest mathematicians of modern China in their time. While Hua lived in China, Chern was Professor at the University of California at Berkeley in the USA.

I entered high school in 1964 and had a chance to present several rather complicated mathematical formulas in trigonometric functions and combinatorics that I derived by myself to my mathematics teacher, whose significantly positive appraisal of my "impressive re-discoveries" greatly encouraged me to look forward to studying higher mathematics, hopefully in college.

Two years later, however, the so-called Chinese Cultural Revolution took place throughout the whole country that lasted for an entire decade, during which all universities in China were closed. Of course, my dream of going to college was shattered, as were those of the rest of a whole generation of youth. Together with several middle school and high school students, I was sent to a mountain village in the inland of Hainan Island in the South China Sea to reclaim wastelands. There, during the following seven years, we lived in grass huts, engaged in hard manual labor, and experienced constant hunger.

Unlike the other young fellows in the village who completely gave up their hopes for the future and passively spent their time in playing poker and chess games after work

4 Hua's main contributions to number theory include the Goldbach-Waring problem (on formulating every integer as the sum of powers of primes). A number of important mathematics results, such as Brauer-Cartan-Hua theorem, Hua's lemma, Hua's inequality and Hua's identity in Jordan algebra, are named after him.

5 Chern, commonly regarded as one of the greatest mathematicians of the twentieth century, is the "father of modern differential geometry".

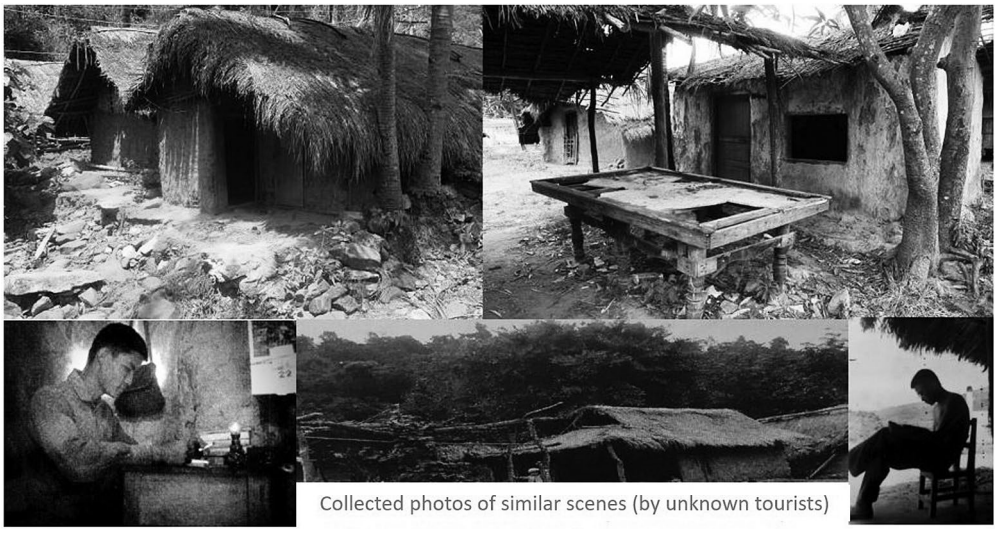


Figure 20. I was sent to a mountain village in the inland of the Hainan Island to work for seven years (1968–1975), where I lived in a hut and self-studied mathematics at night (recent photos taken by some unknown tourists).

every day, I did not want to waste my life without learning new scientific knowledge. Since there were absolutely no conditions for anyone to learn physics or engineering that require experimentation, I realized that the only thing I could do was to read books. Fortunately, we were allowed to visit our parents once a year for ten days each time, which gave me opportunities to bring some books in mathematics and literature back to the village from Guangzhou, since I knew that reading books in these two subjects is most time-consuming. Motivated by my strong interests in mathematics and Chinese classic poetry, I brought back from every home visit some mathematics books discarded by previous university professors and students, along with some collections of classic poems. I told myself that if I could solve one mathematics exercise problem and read one poem every night, then I might be able to finish a mathematics course and write a number of poems in a year and if so I would not be bored from the world-isolated, rural mountain life.

Driven by my determination to accomplish that goal, I lit up a small kerosene lamp every night in the hut to study mathematics. When I felt tired of mathematics, I read and wrote poems.[6] I studied alone because no one was interested in joining me to learn "useless mathematics" that had already been abandoned by the revolution. However, I strongly felt that learning mathematics was an enjoyable way to enrich my tiring and boring life in the mountain village. Figure 20 resembles a scenario of my living environment, with recent photos taken by some unknown tourists.

I started learning Calculus from a college textbook with exercises and solutions and made a clear plan of reading the textbook page by page and trying to solve one exercise problem each day. Moving forward steadily, I spent more than a year to basically complete the self-study of Calculus, of course leaving some difficult exercise problems unsolved.

6 See concluding remark.

I then moved on to learn Linear Algebra. In this way, through the seven years in the mountain village, I essentially completed teaching myself the first courses of Calculus, Linear Algebra, Ordinary Differential Equations, and Theoretical Mechanics. Meanwhile, I also studied English with some help from an enthusiastic elementary school teacher who was sent from the South China Normal University in Guangzhou to work in the same village. I started from the English alphabet, following a couple of discarded middle school textbooks. But listening and speaking English were out of the question. Yet by the time when I left the island, I could read simple English articles with a dictionary.

In 1975, near the end of the Cultural Revolution, all of us young fellows found our ways to return home. At that time, my father filed an excuse of being seriously sick and requested from the authority for me to replace him as a porter in a railroad station in Guangzhou, at my return from the island. In Guangzhou, while I worked in the railroad station during the weekdays, I was able to continue learning mathematics by myself in the evenings and the weekends.

For my self-study, I was fortunate to be introduced by a family friend to two mathematics professors, Mingjun Chen (1934–2008) and Yuesheng Li (1930–2024), of the Sun Yat-sen University in Guangzhou (see Figure 21). During those days, they were my highly respected mentors, who had helped me to become a self-taught "amateur mathematician". In particular, my beloved mentor, Mingjun Chen, hosted my frequent visits to his home and often treated me to lunches with his family after giving me free tutorials. I also paid occasional visits to Yuesheng Li, who was a pioneer and leader in computational mathematics in China and specialized in spline function theory and numerical analysis. In fact, I visited them almost every weekend, and they were happy to provide me with free tutorials, explaining some basic concepts and solving some difficult exercise problems. This was timely in my favor, as the university was still closed and they had no teaching duties during that period. Within three years, I had partially completed learning the basics of Probability Theory, Statistics, Partial Differential Equations, and Functional Analysis.

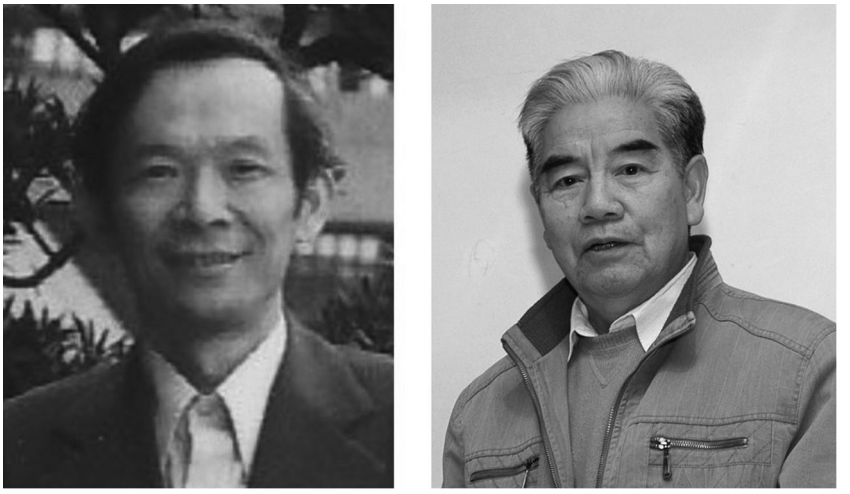


Figure 21. Mingjun Chen (1934–2008) and Yuesheng Li (1930–2024).

In 1976, the Cultural Revolution officially ended and all universities were re-opened by the end of 1977. I took and passed a stringent nationwide entrance exam on several high school subjects and thereby was admitted to the undergraduate program of the South China University of Technology in Guangzhou. Three months later, in early 1978, I took and passed a higher level entrance exam in several advanced mathematical subjects and was consequently admitted into the postgraduate program of mathematics at Sun Yat-sen University. There, I studied computational mathematics, with an emphasis on spline function theory and numerical analysis under the supervision of Yuesheng Li. In the first two years, I took some courses on Advanced Linear Algebra, Real Analysis, and Partial Differential Equations.

Three years later, I gained my Master's degree. Then, through a rather tedious process I moved on to the USA to study for my Ph.D. degree in applied mathematics at Texas A&M University. There, I studied mathematical approximation theory and computational methods for complex systems modeling and control, under the supervision of Charles K. Chui (see Figure 22), who was a world-leading expert in these fields and, since 1990, one of the pioneers of wavelets and a leading authority in computational and applied harmonic analysis. Continuously driven by my innate desire of making up for the loss of ten years' time, I studied tirelessly on several advanced subjects in applied mathematics as well as systems engineering. Meanwhile, I also started to do research on complex dynamical systems modeling, control, analysis, and computation. There was no hurry for me to graduate, though, partially because I might have to return to China to work thereafter. Admittedly, instead of submitting my thesis earlier, I started to write, together with my Ph.D. advisor, several papers and books, resulting in a textbook on optimal estimation, entitled *Kalman Filtering with Real-Time Applications* [16] and a research monograph on optimal systems reduction, entitled *Discrete H^∞ Optimization* [17].

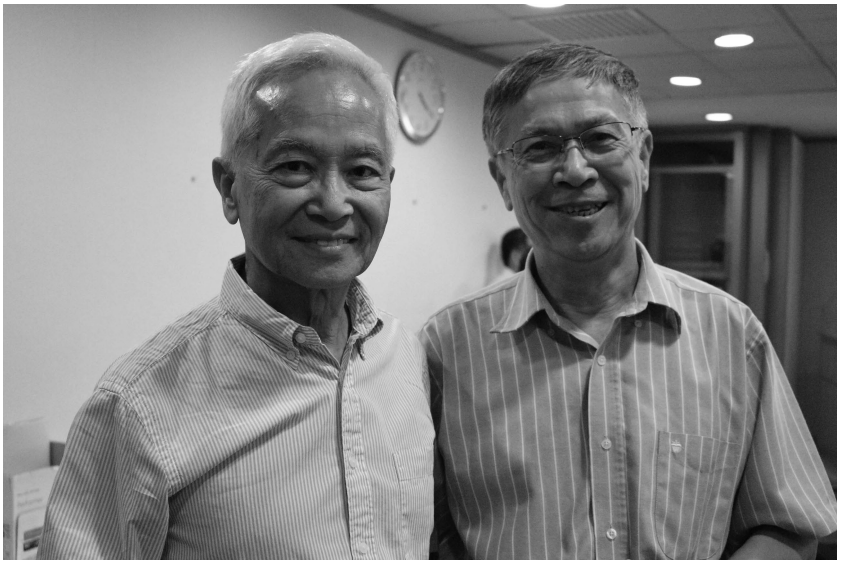


Figure 22. With Charles K. Chui (photo taken in 2016).

It happened one day in the spring of 1987, just before I defended my Ph.D. thesis, I received a phone call from Rui J. P. de Figueiredo (1929–2013) at Rice University. Although we did not know each other, he asked if I would be interested in working with him as a postdoc fellow, based on his reading of a couple of my papers. I consequently became Visiting Assistant Professor at Rice for three years, where I started to work with Rui on nonlinear control systems. Rui is an Indian born in Goa, a Portugal colony, therefore inheriting a Portuguese name. He had great leadership in the IEEE Circuits and Systems Society and was a leading expert in the fields of nonlinear systems, signal processing, and circuits theory (see Figure 23). At Rice, together with Rui, we completed and later published a research monograph entitled *Nonlinear Feedback Control Systems—An Operator Theory Approach* [20].

After my postdoctoral research at Rice, my focus turned to nonlinear science and engineering, encompassing a wide spectrum of research topics in nonlinear control systems and complex dynamical analysis. My interest was attracted to chaos theory and bifurcation analysis since the late 1980s, starting from my first encounter with Leon O. Chua from the University of California at Berkeley, who visited Rui at Rice in 1987. I immediately recognized that Chua's circuit is an excellent platform for me to work on regarding chaos theory.

With control theory and systems engineering background, I first worked on chaos control by means of stabilizing chaotic dynamics when they are harmful, using various feedback control techniques. Those mathematical and engineering methodologies differ from the parameter-variation approach that was popular in the physics community, typically the Ott-Grebogi-Yorke perturbation method for "controlling chaos" [38]. After several years of pursuit, I summarized the research progress of the field in a large-sized book,

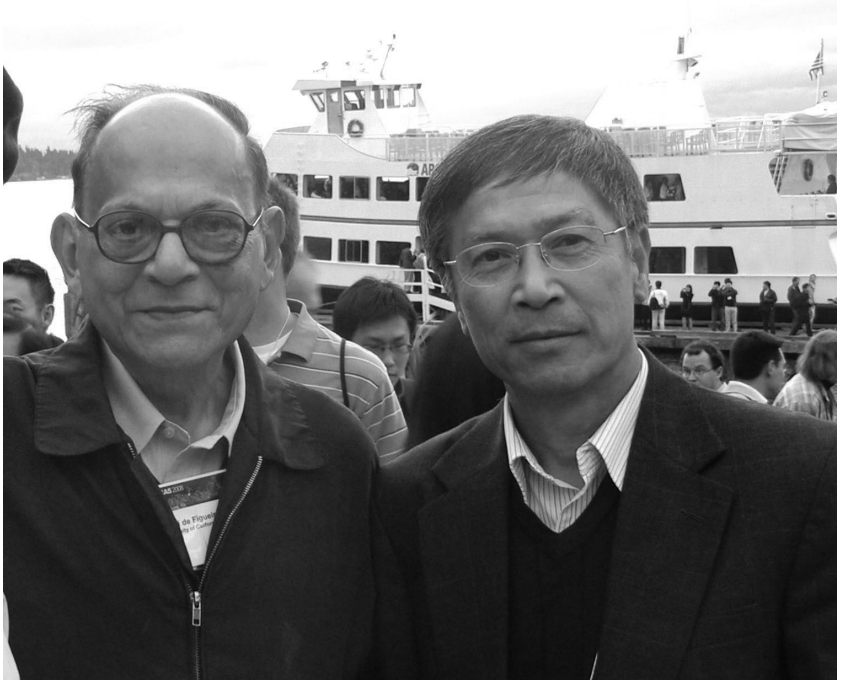


Figure 23. With Rui J. P. de Figueiredo (photo taken during the IEEE ISCAS in 2008).

written with my Ph.D. student Xiaoning Dong, entitled *From Chaos to Order: Methodologies, Perspectives and Applications* [13].

In the early 1990s, I also worked on engineering applications of chaos, specifically using chaos to benefit secure communication, data encryption, and liquid mixing. For those applications, we expect to generate or sustain chaos by means of control, which I called anti-control of chaos or chaotification. One day in 1998, I looked at the classic Lorenz system again,

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz, \end{cases} \tag{7}$$

where a, b, and c are real parameters. As is well known, when $a = 10$, $b = 8/3$, and $c = 28$, the system is chaotic, with an attractor, as shown in Figure 24 (a). With the parameters a, b, and c not in the range of chaos, I applied my anti-control technique to add a linear controller

$$u = \alpha x + \beta y + \gamma z \tag{8}$$

to the right-hand side of the second equation of the Lorenz system (7), where α, β, γ are real constants to be determined. Then, under the Shilnikov condition for having chaos, I determined these three coefficients such that the resultant controlled system is chaotic. It turned out that one simple choice is $\alpha = a$, $\beta = c + 1$, $\gamma = 0$, yielding the so-called "Chen system", named by others afterward, described by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - xz + cy, \\ \dot{z} = xy - bz, \end{cases} \tag{9}$$

where a, b, and c are the Lorenz parameters. When $a = 35$, $b = 3$, and $c = 28$, the system is chaotic, with an attractor, as shown in Figure 24 (b), which was immediately verified and visualized by my postdoctoral fellow Tetsushi Ueta.

The Chen system appears to be similar in formulation, yet nevertheless is not mathematically equivalent to the Lorenz system. In fact, in the sense of algebraic structures it is a dual system to the Lorenz system and it has much more complex dynamics. Soon after, with my Ph.D. student Jinhu Lu, we coined another chaotic system bridging the two dual chaotic systems. Later, together with my colleague Sergej Celikovsky, based on the three closely related chaotic systems we constructed a large family of generalized Lorenz systems [11].

Meanwhile, with my colleague Simin Yu and also Jinhu Lu, we developed an effective methodology to generate all sorts of multi-scroll and multi-wing chaotic attractors from electronic circuits, using multiple saturated functions, as demonstrated by the example shown in Figure 25 [33], for which we can control the number, location, and orientation of the scrolls and wings of the chaotic attractor.

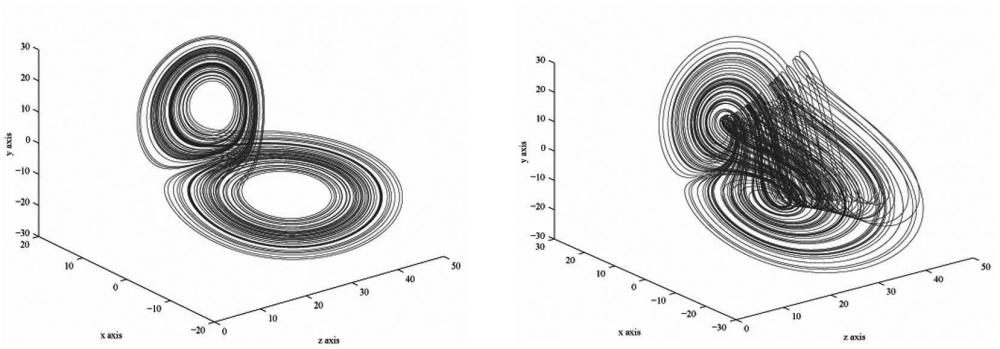


Figure 24. Phase portraits of the two chaotic systems: (a) Lorenz attractor and (b) Chen attractor.

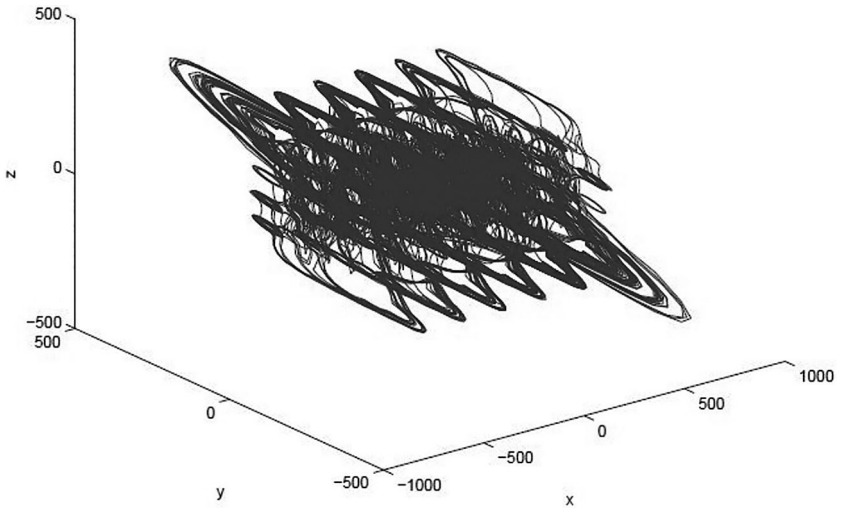


Figure 25. A chaotic attractor with multi-scrolls in three directions.

For linear maps, on the other hand, in collaboration with colleagues Dejian Lai and especially Yuming Shi, my anti-control technique provides a unified way to rigorously generate chaos in the sense of Li-Yorke or in the sense of Devaney via simple and implementable feedback controls [14].

Recently, my Ph.D. student Xiong Wang and I, along with several pivotal collaborators, found chaos from some first-order three-dimensional quadratic polynomial systems that have no equilibria [51] (see Equation (10) and Figure 26 (a)), or have only stable equilibria [50] (see Equation (11) and 26 (b)), or have infinitely many equilibria that form a curve or a surface or even a subspace, summarized in our edited book (with Nikolay V. Kuznetsov) entitled *Chaotic Systems with Multistability and Hidden Attractors* [52]. One example is

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = -y + 3y^2 - x^2 - xz - a, \end{cases} \tag{10}$$

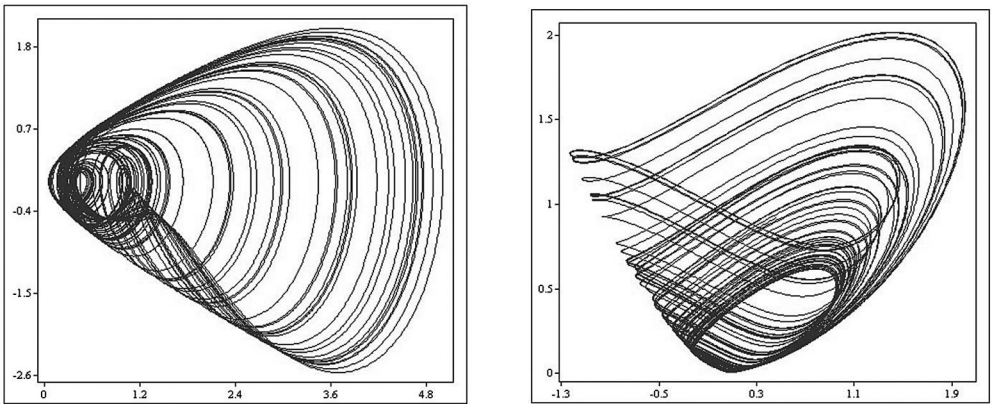


Figure 26. Chaotic attractors: (a) System (10) and (b) System (11).

which, for any real parameter $a > 0$, has no equilibrium point, but interestingly it has a chaotic attractor, as shown in Figure 26 (a). Another example is

$$\begin{cases} \dot{x} = yz - a, \\ \dot{y} = x^2 - y, \\ \dot{z} = 1 - 4x, \end{cases} \tag{11}$$

which, with $a = 0.005$ or $a = -0.05$, has only one stable equilibrium point, but surprisingly it has a chaotic attractor, as shown in Figure 26 (b). Both systems (10) and (11) can be verified to be chaotic by all measures such as positive Lyapunov exponents and fractional dimensions. This great collaborative research effort widens up a research direction on non-hyperbolic chaotic systems.

I moved to the City University of Hong Kong from the University of Houston in year 2000, to work as Chair Professor and now Hong Kong Shun Hing Education and Charity Fund Chair Professor in Engineering, where I founded the Research Centre for Complexity and Complex Networks. In my endeavour to extend the study of synchronization from two coupled chaotic oscillators to a network of chaotic oscillators, I started to look for efficient complex network topologies. I was one of the very first in China to notice and apply the small-world network model [53] and the scale-free network model [8]. I consequently led great efforts in the Chinese scientific communities to pursue a research development in complex networks and organized the first national conference in 2004, which has then become an annual conference series on complex networks in China (see Figure 27).

With my colleagues Xiaofan Wang, Xiang Li, and Zhisheng Duan, we initiated the pinning control approach to achieving complex network synchronization, resulting in a unified pinning control methodology, and established the first criterion for network synchronization, both published in 2002. We put all these earlier advancements together into a textbook type of research monograph entitled *Introduction to Complex Networks—Models, Structures and Dynamics* [15]. Recently, I also published a textbook entitled *Nonlinear Systems: Stability, Dynamics and Control* [12]. Figure 28 shows the two book covers.

Figure 27. Chinese forums on complex networks (from the first in 2004 to the twentieth in 2024).

Throughout my academic career, I received a number of encouraging awards and recognitions: I was elected IEEE Fellow in 1997 and am now Life Fellow. I was awarded the 2011 Euler Gold Medal from Russia and conferred Honorary Doctor Degrees by the Saint Petersburg State University, Russia, in 2011 and by the University of Le Havre Normandy, France, in 2014. I received the China National Natural Science Awards for three times in 2008, 2012, and 2016. Moreover, I became a member of Academia Europaea in 2014 and a fellow of The World Academy of Sciences in 2015. Since 2010, I have succeeded my mentor Leon O. Chua (see Figure 29) to serve as the Editor-in-Chief for the *International Journal of Bifurcation and Chaos*. These are honors both bestowed and earned through sheer diligence, dedication, and dogged persistence of academic endeavors, under a long-term unconditional strong support from my wife Helen and our two lovely daughters, Julie and Leslie.

In retrospect, I did not wish to inherit a legacy from the ten-year deprivation of my youth marked by helplessness or lack of control over my life. Instead, by relentlessly pursuing my goals, with unwavering belief in myself, and drawing on my strong discipline for self-study of new subjects and self-exploration of new research directions, I have recovered a decade of my life that was lost to the Chinese Cultural Revolution. I have since turned that loss into a personal abundance that has launched a fulfilling career and enriched an entire academic life.

Finally, as a closing remark, I have officially retired and am currently on a short-term extension of work at the same City University of Hong Kong. When having breaks during teaching and research work, I maintain my habit of reading Chinese literature books. But I have not been writing Chinese classic poems for years, leaving a collection of my old

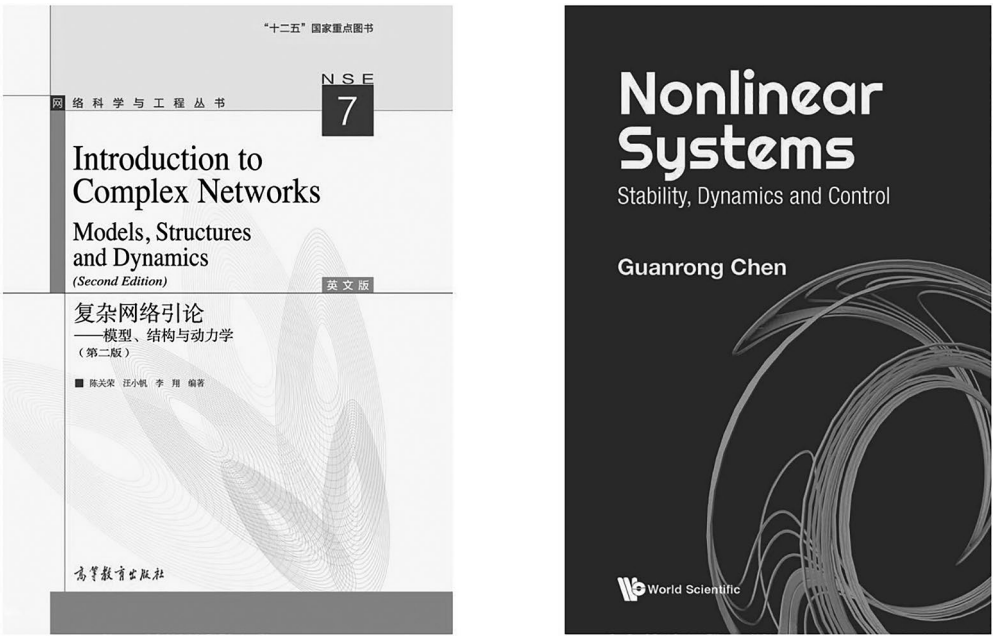
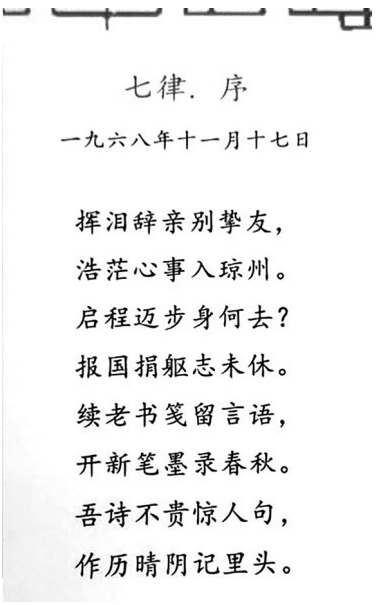


Figure 28. Two recent books: (a) A textbook on complex networks (2012; 2nd ed. 2015) [15] and (b) A textbook on nonlinear systems (2023) [12].

Figure 29. With Leon O. Chua (photo taken in 2012).

Figure 30. A self-printed collection of my Chinese classic poems. On the left is the cover page, in the middle and on the right is the table of contents


七律. 序

一九六八年十一月十七日

挥泪辞亲别挚友，
浩茫心事入琼州。
启程迈步身何去？
报国捐躯志未休。
续老书笺留言语，
开新笔墨录春秋。
吾诗不贵惊人句，
作历晴阴记里头。

Figure 31. My very first poem (see translation in foot note).

poems in the bookshelf (see Figures 30, 31).[7] Perhaps as I look forward to tomorrow, I will rededicate myself to writing new chapters of my life!

7 In Figure 30, on the left is the cover page, in the middle and on the right is the table of contents. In Figure 31, is my very first poem:
A New Start (November 17, 1968)
Saying goodbye to my family and friends with tears, my heart is filled with worries.
Where will I be going?
I am not afraid of sacrificing my life for my country.
I will continue to write my life history on the old papers, but to write it with a new pen.
I will not value my poems with surprising sentences; they record my life in the sunny and cloudy days.

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