



Chen System as a Controlled Weather Model — Physical Principle, Engineering Design and Real Applications

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This paper presents the Chen system as a controlled weather model. Mathematically, the Chen system is dual to the Lorenz system via time reversal. Physically, the Chen system can be viewed as a controlled weather model from the anti-control perspective. This paper illustrates the physical principle of this controlled weather model, and develops an engineering design of the model for real indoor climate (temperature-humidity) regulation, with a perspective on outdoor weather control application.

Keywords: Chaos; Chen system; Lorenz system; temperature-humidity control; climate control.

1. Introduction

The Chen system [Chen & Ueta, 1999] is described by

$$\begin{aligned}\dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x - xz + cy, \\ \dot{z} &= xy - bz,\end{aligned}\quad (1)$$

where, to generate chaos, typical parameters are $c = 28$, $a = 35$, and $b = 3$. It was originally derived from anti-control of the classical Lorenz system [Lorenz, 1963]

$$\begin{aligned}\dot{x} &= a(y - x), \quad \dot{y} = cx - xz - y, \\ \dot{z} &= xy - bz,\end{aligned}\quad (2)$$

where typical parameters are $c = 28$, $a = 10$, and $b = 8/3$. The so-called “anti-control of chaos” was meant to introduce a control input to make an originally nonchaotic system become chaotic [Chen, 1997]. Here, a simple linear anti-controller of the form $u = \alpha x + \beta y + \gamma z$ was applied to the right-hand side of the second equation of the Lorenz equation (2), which had the three parameters originally not in the chaotic region, to try to generate (new) chaotic dynamical behaviors. Based on the well-known criterion of Shilnikov [1965], it was found that the anti-controller could be chosen as

$$u = -ax + (1 + c)y + 0z \quad (3)$$

thereby yielding the Chen system (1).

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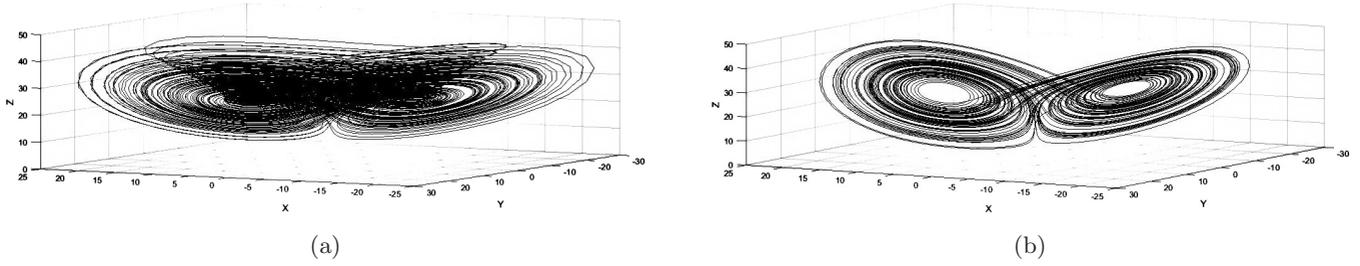


Fig. 1. Comparison of two chaotic attractors. (a) Chaotic Chen attractor and (b) chaotic Lorenz attractor.

As a side note, by similarly choosing $u = -cx + (1 + c)y + 0z$, one obtains the Lu system [Lu & Chen, 2002]

$$\begin{aligned} \dot{x} &= a(y - x), & \dot{y} &= -xz + cy, \\ \dot{z} &= xy - bz, \end{aligned} \quad (4)$$

where typical parameters are $c = 22.2$, $a = 30$, and $b = 44/15$.

By comparing the chaotic attractors of the Chen system to the Lorenz system, generated with the same initial condition $x_0 = -3$, $y_0 = 2$, $z_0 = 20$ as shown by Fig. 1, the new attractor moves upward much more fiercely, prominently driven by the internal force governed by the new input (3) to the system equations. This paper illustrates the physical principle and mechanism that create this difference in their chaotic dynamics, and explains why some seemingly simple modifications of the Lorenz system provide drastically different dynamics. It will be further shown that the new dynamical mechanism embedded in the Chen system, which by nature is a controlled Lorenz system, could be useful as an engineering design for some real applications, with two examples presented on indoor climate (temperature-humidity) regulation and a perspective on outdoor weather control application.

The rest of the paper is organized as follows. Section 2 illustrates the physical principle of the Chen system developed from the Lorenz system. Section 3 develops an engineering design utilizing the new characteristics of the Chen system. Section 4 shows two examples of indoor and outdoor climate controls. Section 5 concludes the paper with an outlook on potential future applications.

2. Physical Interpretation

Similarly to the Lorenz system, which is a simplified weather dynamics model, the parameters in

the Chen system (1) can be interpreted as follows [Rohsenow *et al.*, 1985]:

- x is a variable of the spatial average of hydrodynamic velocity, which is proportional to the intensity of the fluid convective motion;
- y is a variable of temperature, which is the temperature difference between the ascending and descending currents;
- z is a variable of the temperature gradient or deviation, which is proportional to the distortion of the vertical temperature profile from linearity;
- dimensionless positive constant a is the Prandtl number;
- dimensionless positive constant c is the Rayleigh number, which is the product of the Grashof number and the Prandtl number, therefore a and c are mutually dependent;
- dimensionless positive constant b is related to the ratio of the spatial dimensions involved.

Recall that the Prandtl number (Pr) is the ratio of momentum diffusivity to thermal diffusivity. A smaller Pr value implies a more rapid heat diffusion compared to the momentum, whereas a larger Pr value means that the momentum becomes more diffusivity dominant on the dynamics. Recall also that the Grashof number is defined as the ratio of the buoyancy force to the viscous force acting on the fluid, which often arises in the study of free convection or natural convection. And, the Rayleigh number can be described by the product of these two ratios. In the atmospheric convection model, the Rayleigh number is proportional to the difference in the temperatures from the warm base to the cool top in a convective cell. Unlike the Prandtl number, both the Grashof number and the Rayleigh number depend on the differences between the surface temperature and the bulk temperature, as well as on a characteristic length in the model.

Although the variables and parameters in Eqs. (1)–(3) are dimensionless after reasonable

mathematical simplifications, their corresponding physical properties are preserved, which are further discussed below. Specifically, in the following, variable x is considered as the air flow, y as the temperature, and z as the relative humidity. The z variable represents a cross-coupling effect and is inherently a fluctuated variable of the temperature. Usually, temperature affects humidity, namely the water vapor or humidity is limited by the temperature in a parcel of air. In fact, humidity responds to temperature in such a way that with higher temperature the air holds more water vapor. Note, however, that humidity has a rather different temperature profile in the vertical direction in space.

Now, consider the Lorenz system (2), with control input (3) being added to the right-hand side of its second equation, leading to the Chen system (1).

Physically, the change of the temperature, namely the term \dot{y} in the Lorenz system (2), is being modified by the control input (3):

- the first term of the control input actually adds a negative driven flow;
- the second term of the control input adds more heat flux into the second equation of the system.

As a result, the free convective motion in the original Lorenz system becomes a forced convection process. As pointed out by Barboza [2018], the second equation of the Chen system (1) is regenerative, characterized by the positive-feedback term $+cy$ in

the controller (3), which is a fundamental property of the Chen system distinguishing itself from the Lorenz system.

More precisely, keeping in mind the magnitudes of the parameters and signs of the variables in the control input (3), the first term $-ax$ can be interpreted as reversing the direction of the air flow, which reduces the rate of temperature change along the x -axis scaled by a factor of a . The second term $(1+c)y$ can be interpreted as modifying the temperature by adding more heat flux to the system, scaled by a factor of $(1+c)$. The schematic diagram shown in Fig. 2 illustrates the underlying physical process.

It is remarked that, as pointed out by Leonov and Kuznetsov [2015], for physical realization of the control usually it is necessary to have bounded control, i.e. dissipativity in the sense of Levinson (having an absorbing set); while it is trivial for the Lorenz system, which has an absorbing set for any parameter used, it is nontrivial for the Chen system (1) and the Lu system (4).

Numerically, the magnitudes of the system parameters (a, b, c) of the Chen system are (35, 3, 28) and that of the Lorenz system are (10, 8/3, 28). Thus, the Prandtl number of the Chen system is higher than that of the Lorenz system. More importantly, in the Chen system the direction of the increasing temperature, $+cy$, is in the opposite direction with a greater magnitude of the Rayleigh number ($c \gg 1$) compared to that in the Lorenz

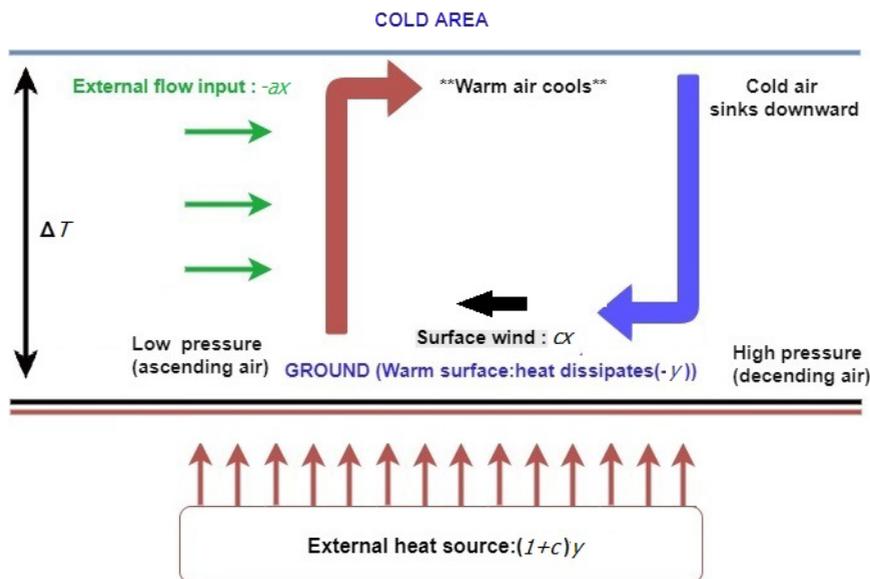


Fig. 2. Physical process of the controlled air convection.

system, $-y$, which was already pointed out by Barboza [2018], as mentioned above.

By nature, the Chen system describes a state of severe conditions of the weather system, where the temperature and its fluctuation and therefore the air flow are all higher than those of the Lorenz system, verified also by its larger maximum Lyapunov exponent [Ueta & Chen, 2000].

Now, imagine that a storm is forming from the Lorenz system, and an external control force (e.g. heated air) is being injected into the air from the ground. Effectively, the external air-flow and heat-flux input will force the original air flow to move upward much more fiercely, resulting in a hotter and more humid air convection system as a whole, with a much stronger momentum of flow motion. Besides, the air flow is contaminated with various types of particles besides water vapors ($a = Pr$ of water ranges only within the scale of 1–10). All these together generates the drastically rising hat in the center of the chaotic Chen attractor from the originally flat butterfly-shaped Lorenz attractor (see Fig. 1 again for a visual comparison).

Note that the above control effects can be realized physically. Indeed, it has been known [Landahl & Mollo-Christensen, 1992] that applying gradient pressure to disturb a flow can be done by ejecting a rapid outflow of a low-speed fluid from the surface region into the inflow of a high-speed fluid. This can create a shear flow that eventually yields turbulence, as illustrated by Fig. 3(a). And, applying a heat flux of a high Rayleigh number to a flow by heating the horizontal surface can likewise create a turbulent flow of the fluid, as illustrated by Fig. 3(b). Now, it can be easily imagined that, under the combined effect of the above two actions, the regular cellular air-flow structure in the air will disappear and a fully turbulent flow will take place. As a matter of fact, this phenomenon has been repeatedly observed, for example in The Burning Man art [fasly007, 2003; Jukin Media, 2016] and even in tornado formations.

As shown in the Appendix, the above controlled Lorenz system, namely the Chen system, is both linearly controllable and observable, and it possesses a minimum realization, namely, x , y and z

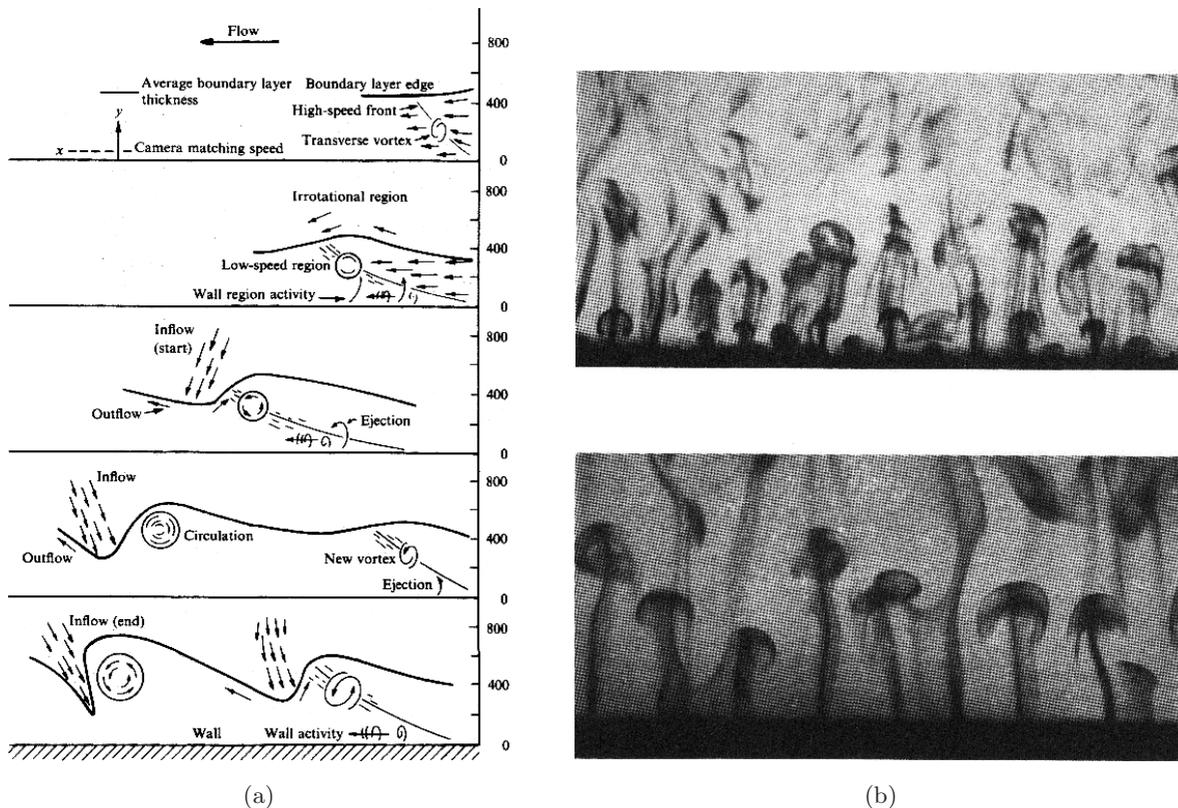


Fig. 3. Turbulence forming processes: (a) Conceptual illustration of progression of flow formation of span-wise vortices (Fig. 8.5 on p. 111 in [Landahl & Mollo-Christensen, 1992]) and (b) thermals rising from a heated horizontal surface (Fig. 11.2 on p. 146 in [Landahl & Mollo-Christensen, 1992]).

are minimal to describe the controlled weather system. These nice properties of the Chen system make its engineering design possible, towards a controlled weather system for some real and potential applications. This will be further discussed next.

3. Engineering Design for Indoor Climate Control

Consider a familiar HVAC (Heating, Ventilation, and Air Conditioning) system. The objective here is to design, based on the typical HVAC system, an indoor climate control system comprising a humidifier unit controlled by an ultrasonic circuit to automatically adjust the relative humidity (RH) to be beneficial for human health and comfort.

It has been known for quite a long time [ANSI/ASHRAE, 2012] that, in spaces with air speed less than 0.2 m/s, the RH within the range of $40 < \text{RH} < 60$ and the temperature within the range of 22–27°C together provide a comfort zone for humans and, meanwhile, this condition can eliminate pathogens (e.g. viruses, bacteria, fungi) and allergic substances as well as reducing or even eradicating house dust mites [Arlian *et al.*, 2001; Mounghong *et al.*, 2014; Sookchaiya *et al.*, 2010].

To design a good sanitizing room, such as a surgery room that requires precise climate control, a conceptual control system model is needed. First, observe from Eq. (1) or (2) that the flow, the temperature and the RH are described by the nonlinear cross-coupling terms. A typical control design strategy is to decouple these variables for easier analysis and computation. To do so, one could first set the fan speed to constant; for example, fix the wind speed to be at the desired value of 0.2 m/s, which will continuously provide uniform ventilation to a concerned area in the room (or the entire room if it is relatively small). As a result, the constant wind speed gradually leads the temperature and RH to constant in the concerned area, so that the two wind-speed-dependent dimensionless variables $x \approx y$ in the vector field of the Chen system (1), yield

$$\dot{x} = 0, \quad \dot{y} = -ay - yz, \quad \dot{z} = y^2 - bz. \quad (5)$$

Although both temperature and RH in the dimension-reduced equation (5) are only approximated, this reduced-order system provides a good model for designing an effective control strategy for

the original system. In fact, this kind of complexity-reduced controller design methods are familiar to engineers as a practical approach.

Now, the third equation in (5) suggests that changing the temperature is more sensitive than changing the RH of the system, as the input variable on the right-hand side of the equation is in a parabolic form. This verifies the psychrometric chart shown in Fig. 4, the ANSI/ASHAE Standard 55-2010, to which Eq. (5) is applicable. As the temperature increases by around 3°C, the RH is correspondingly changed by roughly 9%, for the region within the comfort zone.

It is remarked that although more accurate but also more complex models are available for analysis [Bosen, 1958; Romps, 2014], which depend even on the height above the Earth's surface, geographic region, air pressures and other related factors, for the objective of climate control to track a desired temperature, T_d , and a desired RH, RH_d , Eq. (5) is adequate and good enough, as will be seen from the real application of an indoor climate control system to be further discussed below. A parallel control architecture for the above decoupling model is presented in Fig. 5.

It is also remarked that, through the indoor climate control process, once the desired temperature T_d is attained, which can be automatically accomplished by using an inverter control unit, one can turn on the heater and the humidifier so as to control the RH to reach the set point RH_d . This can be accomplished by setting an interval of tolerance, Δ , around T_d . If, during the process, the value T falls into the interval of $T_d \pm \Delta$, the RH controller is activated within its operational range. If T is not within the designed tolerance region, the inverter control unit is activated to enforce it to move to the set point. Once the temperature is settling into the set-point region, the cycle of RH control is activated again. Mathematically, from Eq. (5), the desired temperature T_d is reached when $\dot{y} = 0$. Consequently, it is easy to solve the equation for z , for $T \in [T_d - \Delta, T_d + \Delta]$.

Next, to design and implement a real indoor climate control system, two benchmark models are reviewed to reveal some basic ideas. In [Mounghong *et al.*, 2014], a 42 m³ room is used to install an indoor climate control system. The set-point values are $T_d = 25^\circ\text{C}$, $\Delta = 0.5$, and $\text{RH}_d = 55\%$. The room is fumed with a large amount of dangerous fungus called *Aspergillus flavus* (*A. flavus*) and

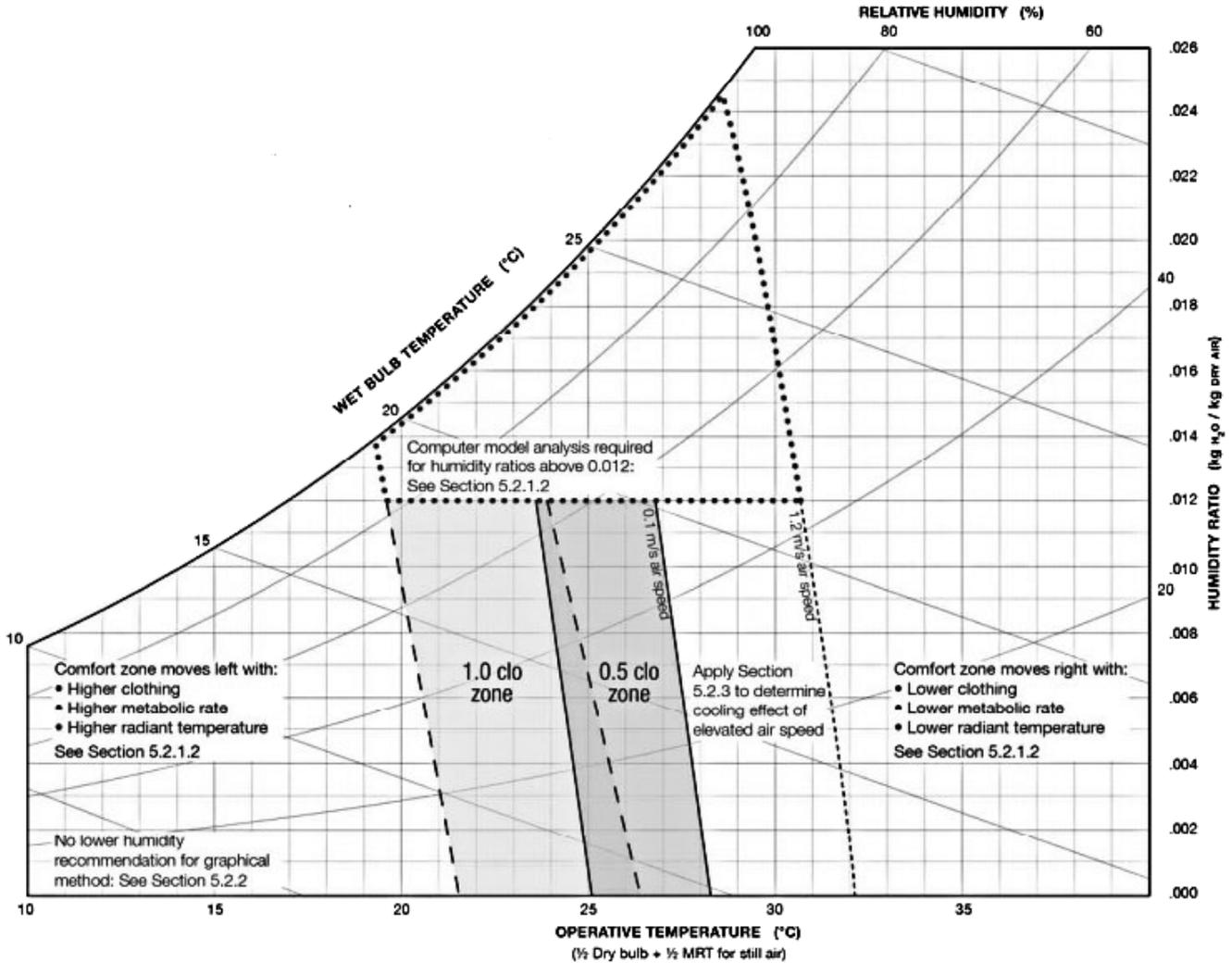


Fig. 4. A comfort zone defined by ANSI/ASHAE Standard 55-2010 [ANSI/ASHRAE, 2012].

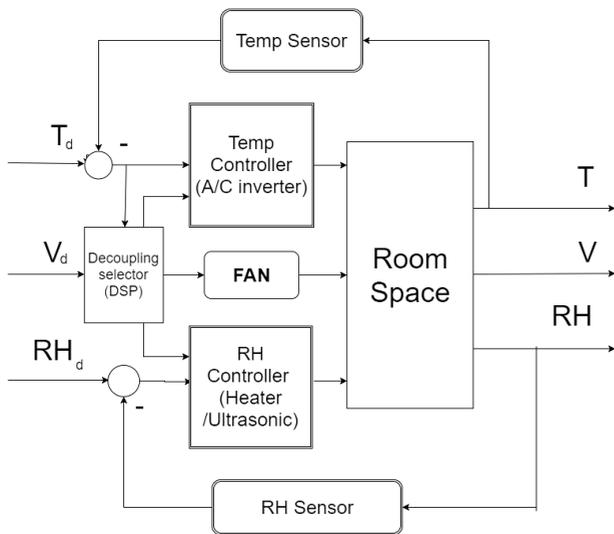


Fig. 5. The conceptual decoupled system model for indoor climate control.

the room is closed for 20 days. The climate control results demonstrated that, for the first three days, there are nine *A. flavus* colonies found in the room. However, after six days, nine days, 12 days and 15 days, the air samples contain neither fungus nor bacteria. In [Sookchaiya *et al.*, 2010], two 20 m² rooms are used for testing. One is a classroom and the other is a bedroom. The desired RH value for both cases is set to be within the interval [55%, 60%], with $T_d = 25^\circ\text{C}$ and tolerance $\Delta = \pm 0.2^\circ\text{C}$ for the classroom and $\pm 0.35^\circ\text{C}$ for the bedroom. Satisfactory results were reported for both cases.

Now, an engineering design and practical implementation of a real indoor climate control system based on the mathematical principles of the controlled Lorenz system (2), namely the Chen system (1), is illustrated.

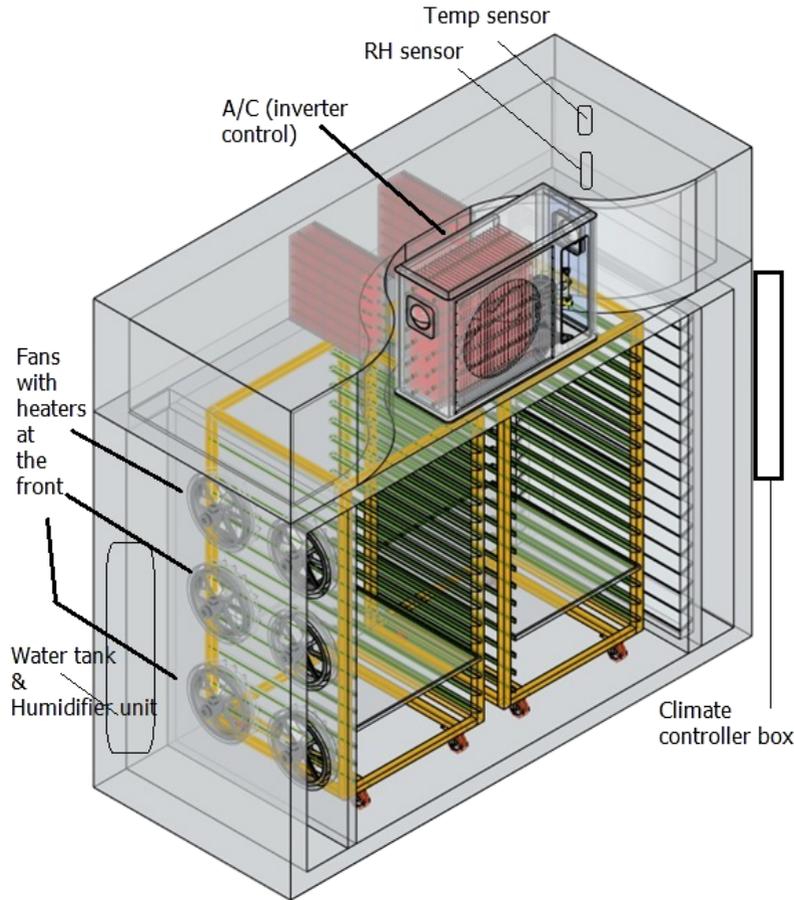


Fig. 6. The implemented climate control system for drying agricultural products.

Consider climate control system used for drying agricultural products, as shown by Fig. 6, with technical data for measuring instrument and component specification as given in Table 1. This system was built in a laboratory located in the King Mongkut's Institute of Technology, Ladkrabang, Bangkok, Thailand. The control system was implemented based on the structure shown in Fig. 5, which was designed following the Chen system model as discussed in detail above.

Table 1. Control system instrument/components and measurement specification.

Instrument/Component	Measurement/Specification	Accuracy
Anemometer	Air velocity	$\pm 1\%$
Temperature sensor	RTD (Pt100 X)	$\pm 0.05^\circ\text{C}$
RH sensor	STH15	$\pm 0.1\%$
A/C (inverter control)	12000 BTU	—
Heater	2kW	—
Humidifier (ultrasonic)	200 ml/h	—

In the chamber, coffee beans from the fermentation process are filled into each tray in a 1 m width \times 1.2 m length \times 1.8 m height compartment, to be dried to be of 11–12% moisture content. The control strategy discussed above is applied, as illustrated by Fig. 7, which presents both temperature and RH inside the chamber during the process. The desired RH, RH_d , is within [50%, 60%], $T_d = 25^\circ\text{C}$, and $\Delta = \pm 0.5^\circ\text{C}$. The desired wind speed, V_d , is fixed at 0.5 m/s.

It can be observed from Fig. 7 that there are peaks of the %RH output appearing periodically. This is because the decoupling strategy allows the temperature controller to turn on the control input whenever the temperature exceeds the allowable tolerance limit. The RH control loop is activated again once the temperature is within the desired range.

Experiments have confirmed that qualitatively the dried coffee beans preserve aroma, chlorogenic acid, color and other temperature-sensitive substances. Statistical comparisons have also confirmed

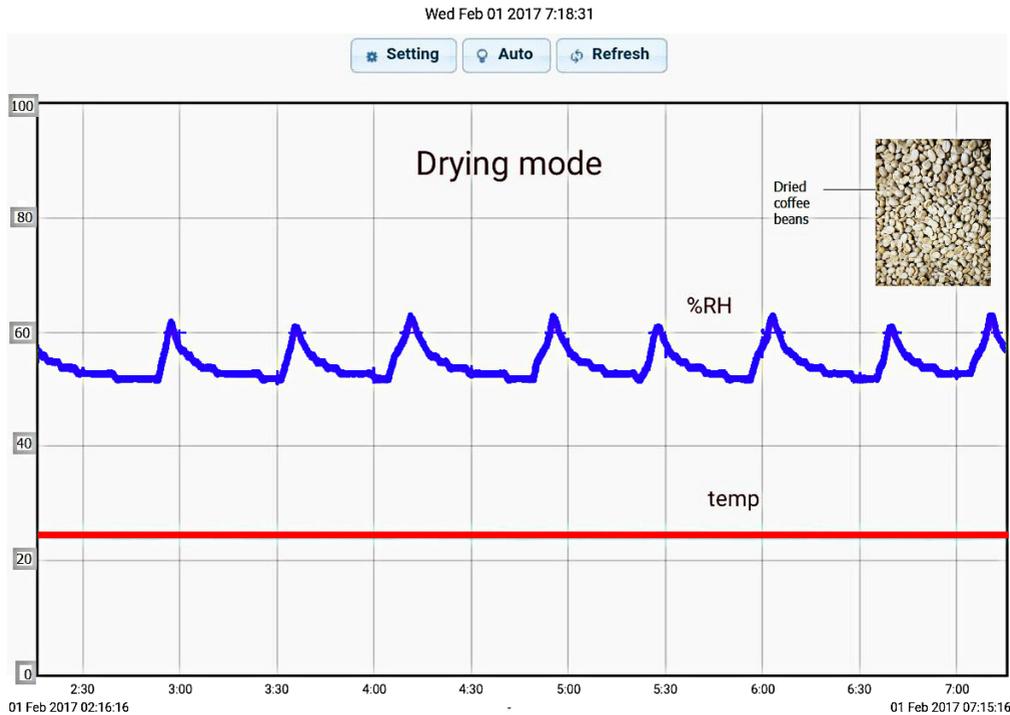


Fig. 7. Temperature and RH of the control system during the process (time unit is hour:minute).

that using the designed climate control system provides better results than that using the conventional means of sun- or wind-drying treatments of agricultural products like coffee beans.

4. Future Application in Outdoor Weather Control

This section illustrates how an application of the controlled Lorenz system, namely the Chen system, to outdoor weather control is possible. From a control-theoretic perspective, this section provides an anti-control conceptual framework for outdoor weather modification.

Historically, an experiment was conducted by Schaefer [1946] via spraying dry ice into a freezer containing supercooled droplets, which was cloud droplets being cooled well below 0°C. As a result, ice crystals were formed, which could be considered as weather modification. Later, it was also experimented by Langmuir [1948] via spraying silver iodide onto cold cloud at temperatures below the water vapor’s freezing point, which made the ice crystals grow bigger. The droplets then were solidified faster and the size was large enough to fall out of the cloud. As the temperature increased, the ice melted and became rain. More reports about

weather modification techniques can be found in the book by Cotton and Pielke [2007].

A simple, effective, and easy-to-understand demonstration of how the Lorenz system can be transformed to be the Chen system is perhaps the rainmaking technique suggested by the King of Thailand, His Majesty the King Bhumibol Adulyadej [2005], which was invented in 1969 [iamtube002, 2009].

Specifically, with great concern for the drought problem due to delayed seasonal rains once in Thailand, an artificial rainmaking technique was proposed by the Thailand King as illustrated by Fig. 8. For a brief introduction, the procedure of rainmaking is summarized into three operational steps, i.e. agitating, ripening and disturbing, respectively, as follows.

Step 1. Agitating

This step helps in forming rain clouds. The goal is to create humidity by a mass of air rising. Hygroscopic chemicals such as NaCl are used for stimulating cloud forming (Condition #1 in Fig. 8). From a scientific point of view, this step changes the values of Pr (i.e. the parameter a) and the Ra number c in the weather system within the targeted area, so as to increase the amount of potential rainfall.

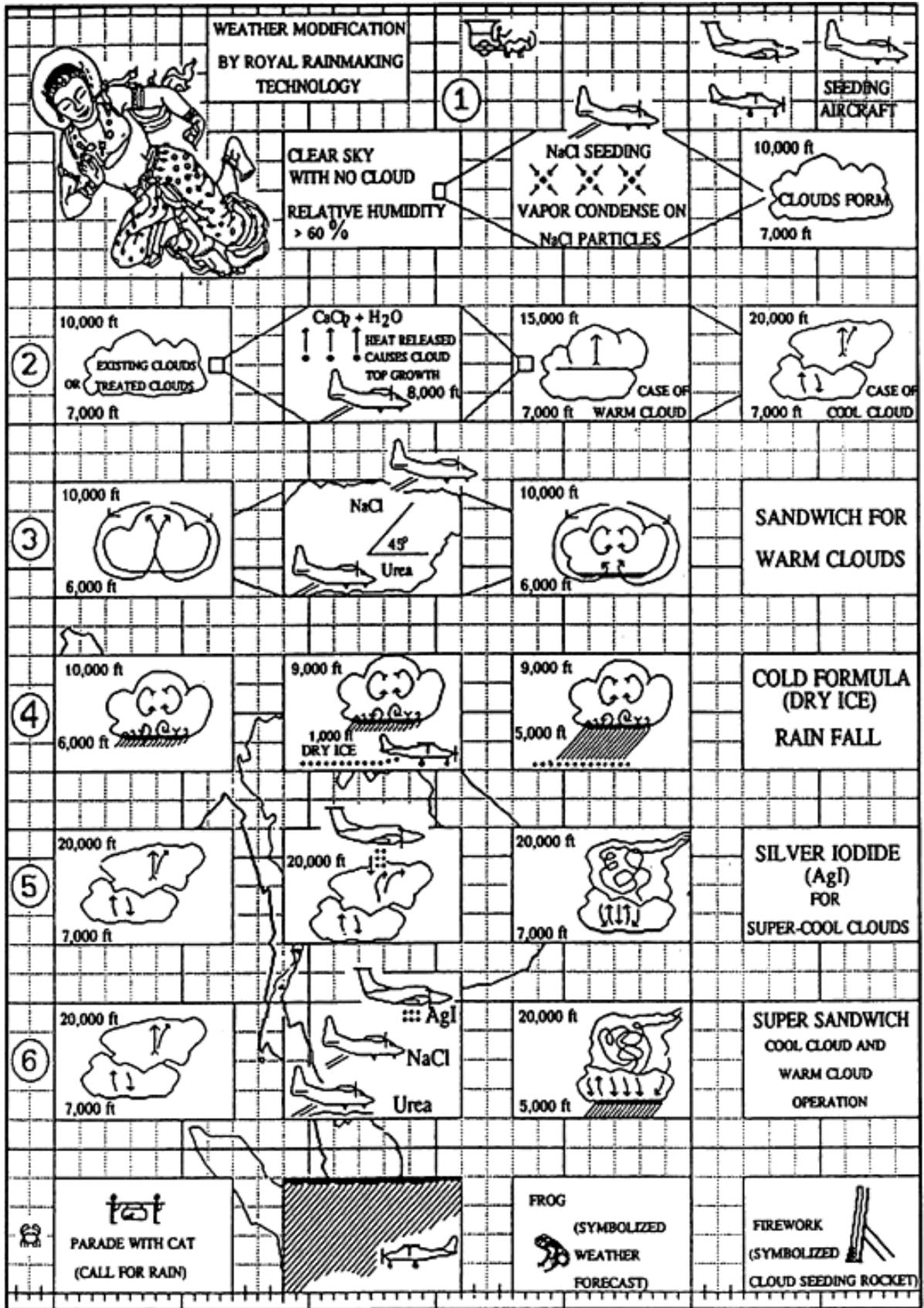


Fig. 8. Procedure of rainmaking for weather modification [His Majesty the King Bhumibol Adulyadej, 2005].

Step 2. Ripening

This step makes the droplets of water to be more condensed by scattering exothermic hygroscopic chemicals such as CaCl and water in Condition #2, NaCl and Urea in Condition #3, dry ice in Condition #4, silver iodide in Condition #5, and AgI + NaCl + Urea in Condition #6, respectively. All conditions are formulated to use hygroscopic chemicals for heating and cooling, which cause a significant temperature difference between the upper layer and the lower layer of the cloud, resulting in the change of the Ra number c . Mathematically, it is the action that the term $(1 + c)y$ in the control input (3) is introduced into the original Lorenz system, thereby converting it to the Chen system. This is in accordance with the description in [Langmuir, 1948] that the latent heat released as ice crystals grown by vapor deposition would warm the seeded part of the cloud, causing upward air-flow motion leading to turbulence. In addition, many pieces of cloud are buoyancy-driven. When an air parcel gets heated, this volume of air becomes warmer and so expands. The warmed air will be buoyed up in the upward direction, affecting the change of Ra and Pr in reality model. The expansion then changes the geometric shape of the cell, yielding a change of the constant b in Eq. (2), reminiscent of the description in [Siddheshwar & Titus, 2013].

Step 3. Disturbing

This step helps expedite the process of rain-falling by flying one or more airplanes through the heavy clouds, which is equivalent to creating a shear flow, the term $-ax$ in the control input (3), and injecting it to the original Lorenz system.

More details on modifying weather conditions for rainmaking can be found in the three-volume project report [Silverman *et al.*, 1994].

5. Conclusion

This article interprets the chaotic Chen system as a controlled Lorenz system with significant physical meanings of the control input terms, which can clearly explain some commonly observed natural phenomena and lead to an engineering design with practical implementation of a real indoor climate control system, showing a foreseeable potential in future applications to outdoor weather modification at various scales. Looking forward, the Chen system as a controlled weather system model may

even suggest military applications such as in heating electron clouds in the ionosphere to create thunderbolts and powerful pulse waves, which can provide tremendous amounts of heat and wind shears [Bailey & Worthington, 1997]. Such conditions could potentially lead to man-made hurricanes or tornadoes on the Earth, for better or for worse, which should be pursued or be prohibited by the scientific and engineering communities for good humanitarian reasons.

Acknowledgments

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Appendix

Controllability Matrix and Observability Matrix of the Linearized Chen System

Assume that the concerned weather system is evaluated in an infinitesimal region on an infinitesimal time scale, such that the incremental changes in the system dynamics are nearly linear. This leads to the system matrices and vectors equations (A.1)–(A.3), with variables x, y , and z representing the flow, temperature, and relative humidity as discussed above in the text. The input u in Eq. (A.1) is a feedback control term specified as in (A.3), which applies external flow and temperature to the original Lorenz system, leading to the Chen system as a controlled Lorenz system.

$$\frac{d}{dt}X = \begin{bmatrix} -a & a & 0 \\ c - z & -1 & 0 \\ y & 0 & -b \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(x, y, z), \quad (\text{A.1})$$

$$Y = X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (\text{A.2})$$

$$u = [-a \quad 1 + c \quad 0] X = -ax + (1 + c)y. \quad (\text{A.3})$$

Recall [Chui & Chen, 1989] that the Controllability Matrix (CM) of the above linearized system is defined by $\text{CM} = [B \quad AB \quad A^2B]$, which is

$$\text{CM} = \begin{bmatrix} 0 & a & -a(a + 1) \\ 1 & -1 & a(c - z) + 1 \\ 0 & 0 & ay \end{bmatrix}. \quad (\text{A.4})$$

Since the matrix CM has a full-row rank, the system is controllable.

Similarly [Chui & Chen, 1989], the Observability Matrix (OM) of the system is given by

$$\text{OM} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & a & 0 \\ c-z & -1 & 0 \\ y & 0 & -b \\ a(a+c-z) & -a(a+1) & 0 \\ -(a+1)(c-z) & a(c-z)+1 & 0 \\ -(a+b)y & ay & b^2 \end{bmatrix}. \quad (\text{A.5})$$

Since the matrix OM has a full-column rank, the system is observable. Hence, the weather system is both controllable and observable; therefore, the system is a minimum realization.