## 

## A Magic Number 142857

142857 is a cyclic number. If you multiply it by 2, 3, 4, 5 or 6, you always obtain a cyclic permutation of this number itself –

 $142857 \times 1 = 142857$   $142857 \times 2 = 285714$   $142857 \times 3 = 428571$   $142857 \times 4 = 571428$   $142857 \times 5 = 714285$  $142857 \times 6 = 857142$ 

If you multiply it by larger numbers, then you will get some variations of 142857 or 999999. For example:

142857 × 7 = 999999 142857 × 8 = 1142856 (adding the first digit to the last digit yields 142857) ..... 142857 × 42 = 5999994 (adding the first digit to the last digit yields 999999)

If you consider division instead, you will find that the digits of this number 142857 play around together as repeating decimals:

$$\frac{1}{7} = 0.\overline{142857}$$
$$\frac{2}{7} = 0.\overline{285714}$$
$$\frac{3}{7} = 0.\overline{428571}$$
$$\frac{4}{7} = 0.\overline{571428}$$
$$\frac{5}{7} = 0.\overline{714285}$$
$$\frac{6}{7} = 0.\overline{857142}$$

If you divide 142857 by 2 or 5, you will see that the digits of this number won't go away:

 $142857 \div 2 = 71428.5$  $142857 \div 5 = 28571.4$  If you square its last three digits and then subtract the square of its first three digits, you also get back to a cyclic permutation of the number itself –

 $857^2 - 142^2 = 714285$ 

Furthermore, you can easily verify that 142857 is divisible by the sum of its digits.

Last but not least, its square can be split into two parts that add up to the original number again:

 $142857 \times 142857 = 20408122449$ 

Namely, adding the first five digits to the last six yields 142857 again:

20408 + 122449 = 142857

GRC 🙂