Lecture 3. Diversity

- Detection in Fading Channel
- Diversity
  - ✓ Time Diversity
  - ✓ Space Diversity
  - ✓ Frequency Diversity

Detection in Fading Channel

- Consider a slow flat Rayleigh fading channel.

$$y[m] = h[m]s[m] + z[m] \qquad h[m] \sim \mathcal{CN}(0,1)$$

✓ How to detect?

✓ What is the error performance?

- Suppose the receiver has the information of $h[m]$ (i.e., CSIR).
  - Step 1: $\frac{h^*}{|h|} y = \frac{h^*}{|h|} hs + \frac{h^*}{|h|} z \quad \Longrightarrow \quad \tilde{y} = |h| s + \frac{h^*}{|h|} z$
  - Step 2: pass $\tilde{y}$ through the optimal detector for AWGN channel.

Coherent Detection

BER of Coherent Detection in Fading Channel

- With BPSK:

$$P_e \mid h = Q\left(\sqrt{2|h|^2 \, SNR}\right)$$

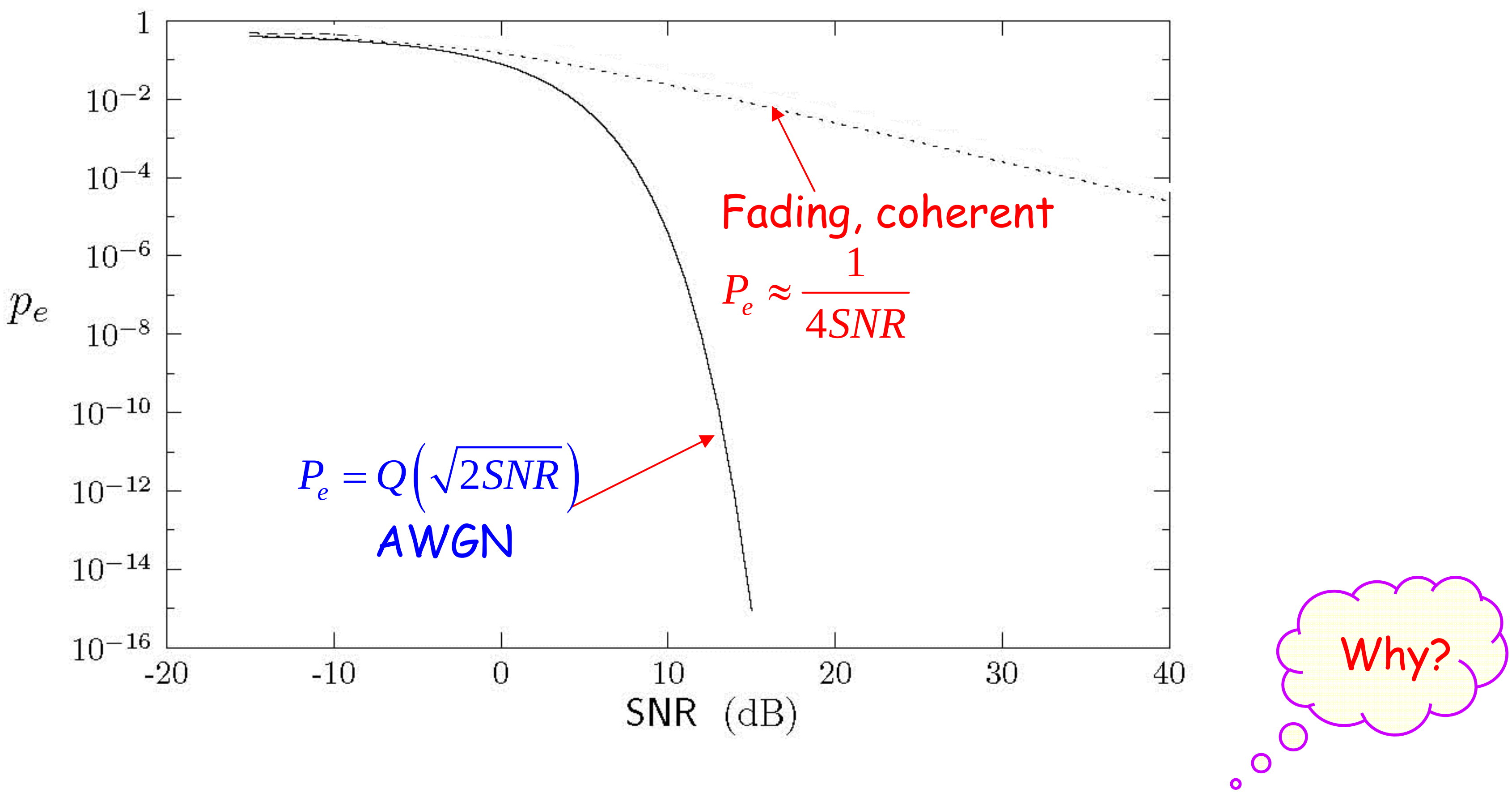
- The channel gain h varies with time. With $h \sim \mathcal{CN}(0,1)$,

$$P_e = \mathrm{E}_h\left[Q\left(\sqrt{2|h|^2 \, SNR}\right)\right] = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{1+SNR}}\right)$$

$$\Downarrow \qquad \sqrt{\frac{SNR}{1+SNR}} = 1 - \frac{1}{2SNR} + O\left(\frac{1}{SNR^2}\right)$$

$$P_e \approx \frac{1}{4SNR}$$ at high SNR

BER Comparison



The BER performance in fading channel is much worse than that in AWGN channel!

Deep Fade

$$P_e \mid h = Q\left(\sqrt{2 \mid h \mid^2 SNR}\right)$$

$P_e|h$ is small if $|h|^2 SNR$>>1; otherwise it is significant.

$$\Pr\{\underbrace{|h|^2 SNR < 1}_{\text{Deep fade event}}\} = \int_0^{1/SNR} e^{-x} dx = \frac{1}{SNR} + O\left(\frac{1}{SNR^2}\right)$$

- Detection in fading channels has poor performance because the channel gain is a random variable, and there is a significant probability that the channel is "bad". Deep fade

- How to overcome deep fade?

Diversity

Pass the information symbols through multiple signal paths, each of which fades independently, so that reliable communication is possible as long as one of the paths is strong.

Time Diversity

- Repetition Coding + Interleaving

Time Diversity

- Consider a slow flat Rayleigh fading channel.

$$y[m] = h[m]s[m] + z[m]$$

✓ Repetition Code: $s[i]=x,\ i=1,\ldots,L.$ → the number of diversity branches

(repeat the information symbol x by L times)

✓ Received signal: $\mathbf{y} = \mathbf{h}x + \mathbf{z}$

$$\mathbf{y} = [y[1], y[2], \ldots, y[L]]'$$
$$\mathbf{h} = [h[1], h[2], \ldots, h[L]]'$$
$$\mathbf{z} = [z[1], z[2], \ldots, z[L]]'$$

$$\Rightarrow \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{y} = \|\mathbf{h}\| x + \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{z}$$

Maximal ratio combiner: it weighs the received signal in each branch in proportion to the signal strength and also aligns the phases of the signals in the summation to maximize the output SNR.

Error Performance with Time Diversity

- With BPSK: $$P_e \mid \mathbf{h} = Q\left(\sqrt{2\|\mathbf{h}\|^2 SNR}\right)$$

  $\|\mathbf{h}\|^2 = \sum_{l=1}^{L} |h[l]|^2$ is Chi-square distributed with $2L$ degrees of freedom,

  if $h[l]$, $l=1,\ldots,L$, are i.i.d complex Gaussian random variables.

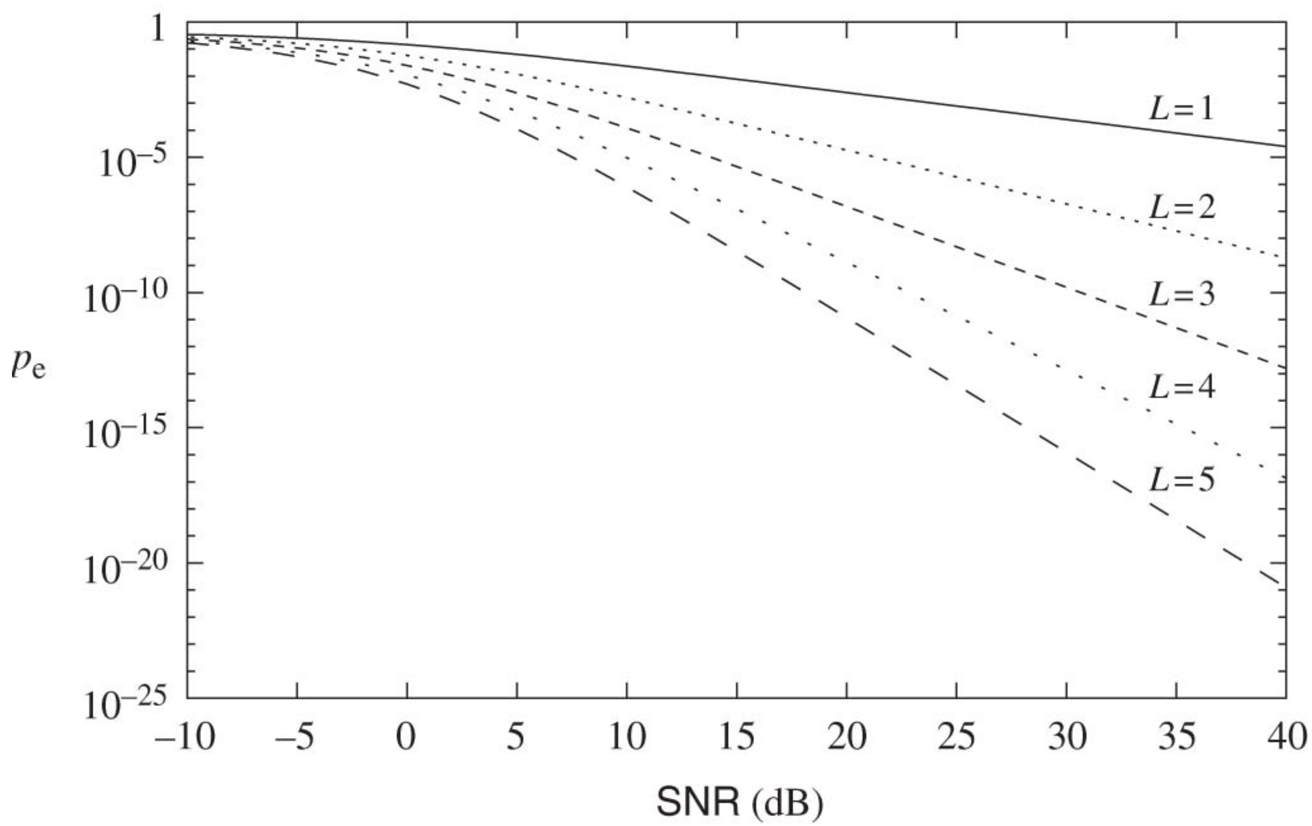
- At high SNR: $$P_e \approx \binom{2L-1}{L} \frac{1}{(4SNR)^L}$$

  diversity order

  Full diversity gain: The diversity order is equal to the number of diversity branches.

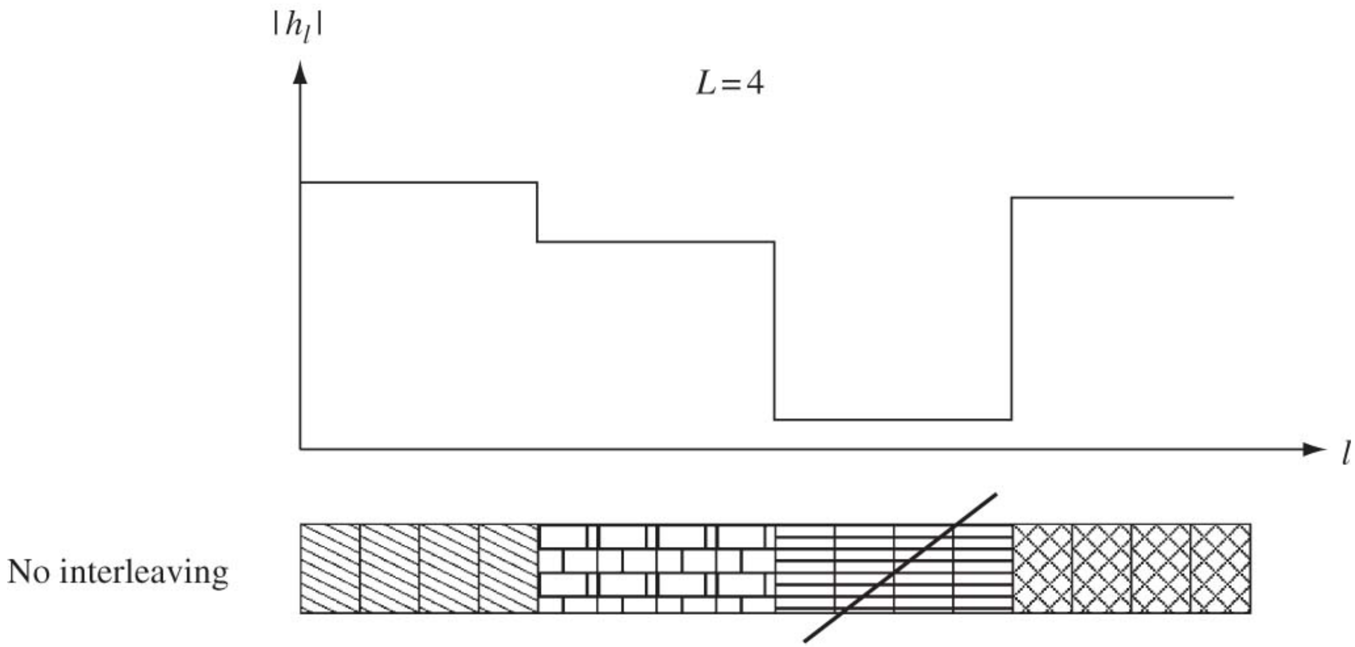
Can we obtain full diversity gain by using repetition codes alone?

Error Performance with Time Diversity

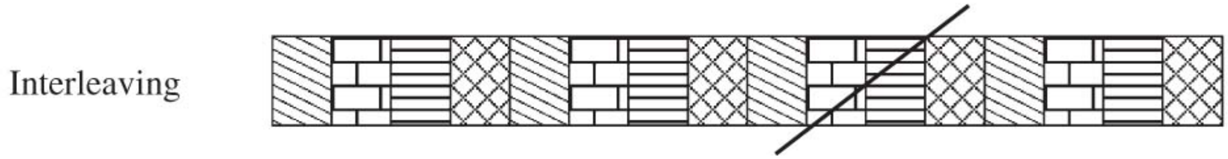


At high SNR: $$P_e \approx \binom{2L-1}{L} \frac{1}{(4SNR)^L}$$

Interleaving



- Interleaving of codewords is required to guarantee that each branch experiences independent fading.

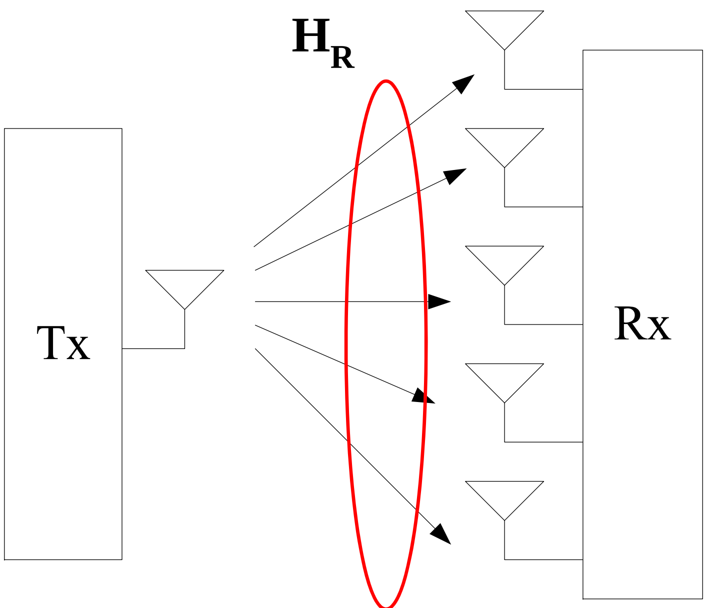


Space Diversity

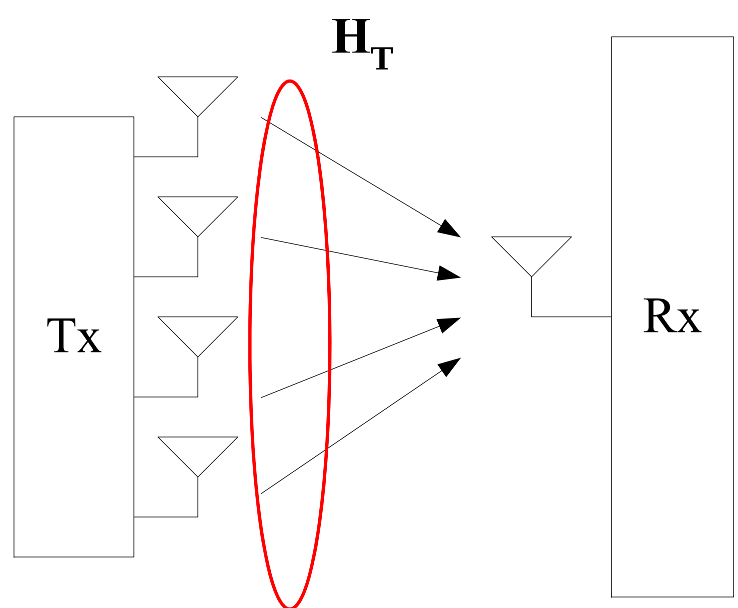
- Receive Diversity
- Transmit Diversity
- MIMO

Space Diversity

Receive Diversity
(SIMO)



Transmit Diversity
(MISO)



How to achieve full diversity gain? (with CSIR)

Receive Diversity

- Consider a slow flat Rayleigh fading channel.
  - With L receive antennas, the received signal $\mathbf{y}=[y_1, y_2, \ldots, y_L]'$

$$\mathbf{y}=\mathbf{h}x+\mathbf{z} \quad \Rightarrow \quad \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{y}=\|\mathbf{h}\|x+\frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{z}$$

$$\mathbf{h}=[h_1, h_2, \ldots, h_L]'$$

Maximal ratio combiner

- if h_l, l=1,…, L, are i.i.d complex Gaussian random variables:

With BPSK: $$P_e\,|\,\mathbf{h}=Q\left(\sqrt{2\|\mathbf{h}\|^2\,SNR}\right)$$

$$P_e \approx \binom{2L-1}{L}\frac{1}{(4SNR)^L} \longrightarrow \text{Full diversity gain}$$

Can we achieve the same performance by using transmit diversity?

Transmit Diversity I: with CSIT

- If the transmitter has full Channel State Information (CSIT):

Transmit Beamforming: $y = \mathbf{h}\frac{\mathbf{h}^*}{\|\mathbf{h}\|}x + z$ $\mathbf{h} = [h_1, h_2, ..., h_L]$

transmitted signal

$$= \|\mathbf{h}\| x + z$$

With BPSK: $P_e \mid \mathbf{h} = Q\left(\sqrt{2\|\mathbf{h}\|^2 \, SNR}\right)$

$$P_e \approx \binom{2L-1}{L}\frac{1}{(4SNR)^L}$$

Full diversity gain

What if the transmitter has no information about the channel?

Transmit Diversity II: No CSIT

- Scheme 1: All L antennas transmit the same symbol simultaneously

$$\begin{array}{c} \xrightarrow{\quad T \quad} \\ \mathbf{X} = \begin{bmatrix} s_1 & s_2 & \cdots \\ s_1 & s_2 & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix} \\ \downarrow L \end{array}$$

At any time slot t, $y = \mathbf{h}\mathbf{x}_t + z = s_t \sum_{i=1}^{L} h_i + z$

Diversity order =1

- No diversity gain
- Full rate: $r=k/T$=1

Transmit Diversity II: No CSIT

- Scheme 2: Turn on one antenna at each time slot

$$\mathbf{X} = \begin{array}{cc} & \xrightarrow{\qquad\qquad T \qquad\qquad} \\ L\Big\downarrow & \begin{bmatrix} s_1 & & & & s_2 & & & & \cdots \\ & s_1 & & & & s_2 & & & \cdots \\ & & \ddots & & & & \ddots & & \cdots \\ & & & s_1 & & & & s_2 & \cdots \end{bmatrix} \end{array}$$

Every symbol is repeated L times!

- Full diversity gain
- Low rate: $r=1/L$

Transmit Diversity II: No CSIT

- Scheme 3: Alamouti scheme (2 transmit antennas)

$$\mathbf{X} = \begin{array}{cc} & \xrightarrow{\quad T \quad} \\ \downarrow L & \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \end{array}$$

At the receiver side, we need to collect signals over 2 consecutive time slots for decoding.

$$[y_1 \quad y_2] = [h_1 \quad h_2] \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + [z_1 \quad z_2]$$

$$\Downarrow$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{\mathcal{H}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2^* \end{bmatrix}$$

$$\mathcal{H}^* \mathcal{H} = (|h_1|^2 + |h_2|^2)\mathbf{I}_2 = \|\mathbf{h}\|^2 \mathbf{I}_2$$

$$\mathbf{h} = [h_1, h_2]$$

Transmit Diversity II: No CSIT

- Scheme 3: Alamouti scheme (2 transmit antennas)

$$\mathcal{H}^* \times \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \underbrace{\mathcal{H}^* \times \mathcal{H}}_{\begin{bmatrix} \|\mathbf{h}\|^2 & 0 \\ 0 & \|\mathbf{h}\|^2 \end{bmatrix}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathcal{H}^* \times \begin{bmatrix} z_1 \\ z_2^* \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \|\mathbf{h}\|^2 \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix}$$

2-symbol joint decoding is decoupled into 2 separate one-symbol decoding.

- Full diversity gain
- Full rate: r=1
- Low complexity at the receiver

- 3dB gain loss compared to full CSIT case

What if we have L transmit antennas?

More about Alamouti Scheme

$\mathbf{s} \xrightarrow{f} \mathbf{X}$: Orthogonal Space-time Block Codes (STBC)

$$\underbrace{\mathbf{y}}_{1\times T} = \underbrace{\mathbf{h}}_{1\times L}\underbrace{\mathbf{X}}_{L\times T} + \underbrace{\mathbf{z}}_{1\times T} \Rightarrow \underbrace{f^{-1}(\mathbf{y})}_{T\times 1} = \underbrace{\mathcal{H}}_{T\times L}\underbrace{\mathbf{s}}_{L\times 1} + \underbrace{\tilde{\mathbf{z}}}_{T\times 1}$$

$\mathcal{H}$: $\mathcal{H}^*\mathcal{H} = \|\mathbf{h}\|^2 \mathbf{I}_L$

$$\tilde{y}_l = \|\mathbf{h}\|^2 s_l + \tilde{z}_l$$

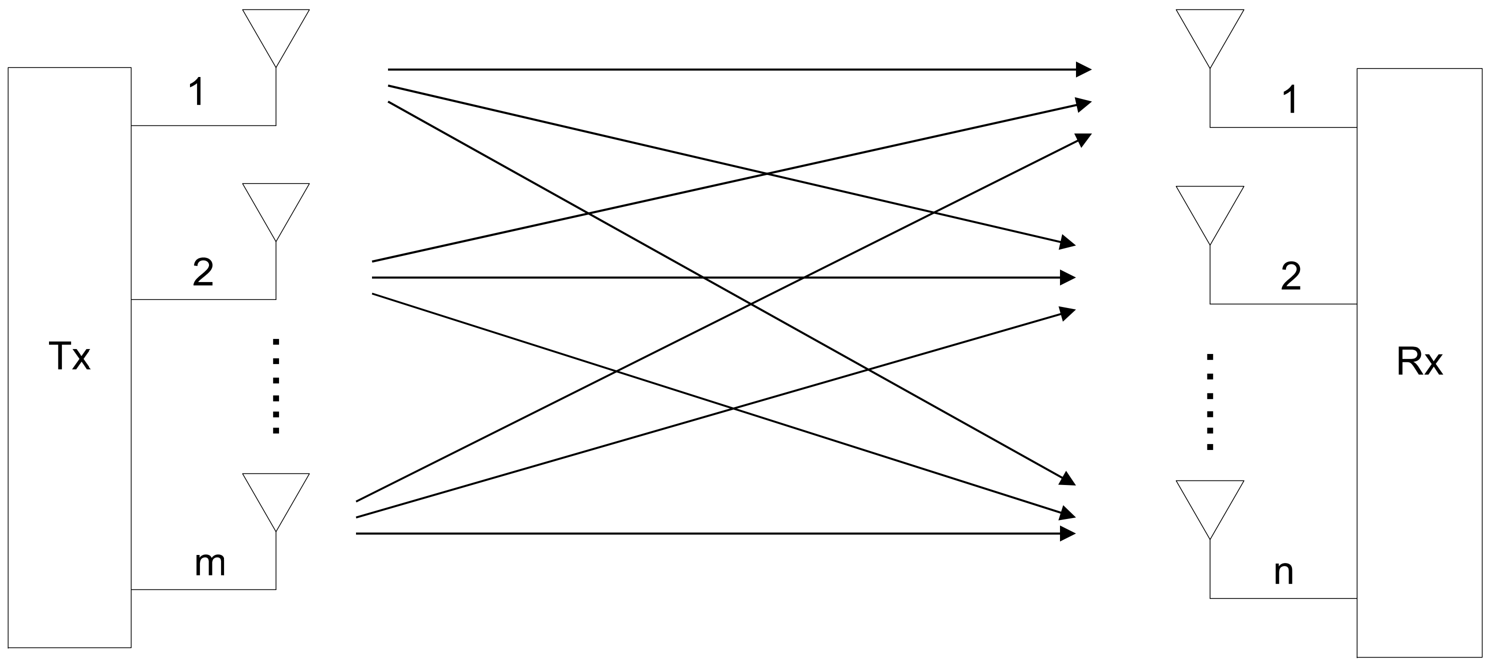
Diversity order = L

Rate: L/T

- No full rate STBC when L>2

$$\begin{array}{c|c} & T \rightarrow \\ \hline L \downarrow & \begin{bmatrix} s_1 & -s_2^* & -s_3^* & 0 \\ s_2 & s_1^* & 0 & -s_3^* \\ s_3 & 0 & s_1^* & s_2^* \end{bmatrix} \end{array}$$

Multiple-Input-Multiple-Output (MIMO)



MIMO channel: $$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{nm} \end{bmatrix}$$

Diversity in MIMO Channels

- What is the maximum diversity order provided by an n by m MIMO channel?

  mn

- How to achieve the full diversity gain?

Transmitter	Receiver
➢ Transmit s simultaneously at all antennas ? Full rate	
➢ Turn on one antenna at each time slot, repetition ? Rate=1/m	Maximum Ratio Combining
➢ Orthogonal STBC ? Rate=m/T	
➢ Transmit $\frac{\mathbf{h}_k^*}{\|\mathbf{h}_k\|}s, \quad k \in \{1,\ldots,n\}$? Full rate	

Summary

- Detection in Fading Channels
  - Poor performance due to deep fade $P_e \sim SNR^{-1}$
- Use Diversity to Overcome Fading
  - Time Diversity
    - Repetition code + Interleaving
  - Space Diversity
    - Receive diversity
    - Transmit diversity
    - Diversity in MIMO channels
  - Frequency Diversity ?

Frequency Diversity

- Frequency Diversity in Frequency-selective Fading Channel
- Equalization, DSSS and OFDM

Frequency-Selective Fading Channel

- Consider a slow frequency-selective fading channel.

  1) the channel response has a finite number of taps $L>1$;

  2) the channel taps do not vary over $N>L$ symbol times.

$$y[m]=\sum_{l=0}^{L-1} h_l s[m-l]+z[m] \qquad m=1,\ldots,N.$$

How many copies of each symbol can be obtained at the receiver over such a frequency-selective channel?

Transmission over Frequency-Selective Fading Channel

N transmit symbols: $s[1], s[2], \ldots, s[N]$

L channel taps: $h_0, h_1, \ldots, h_{L\text{-}1}$ $\Downarrow$

$$y[m] = \sum_{l=0}^{L-1} h_l s[m-l] + z[m]$$

At time slot 1: $y[1] = \sum_{l=0}^{L-1} h_l s[1-l] + z[1] = h_0 s[1] + z[1]$

At time slot 2: $y[2] = \sum_{l=0}^{L-1} h_l s[2-l] + z[2] = h_0 s[2] + h_1 s[1] + z[2]$

At time slot 3: $y[3] = \sum_{l=0}^{L-1} h_l s[3-l] + z[3] = h_0 s[3] + h_1 s[2] + h_2 s[1] + z[3]$

…… ……

At time slot L: $y[L] = \sum_{l=0}^{L-1} h_l s[L-l] + z[L] = h_0 s[L] + \cdots + h_{L-1} s[1] + z[L]$

…… ……

At time slot m: $y[m] = \mathbf{h}\mathbf{x}[m] + z[m]$, $\mathbf{x}[m] = [s[m], s[m-1], \ldots, s[m-L+1]]'$

$\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]$

Transmission over Frequency-Selective Fading Channel

$$\mathbf{y} = \mathbf{h}\mathbf{X} + \mathbf{z}$$

$$\mathbf{y} = [y[1], y[2], ..., y[N+L-1]]$$
$$\mathbf{X} = [\mathbf{x}[1], \mathbf{x}[2], ..., \mathbf{x}[N+L-1]]$$

L delayed replicas

ISI (Inter-Symbol Interference)

$$\mathbf{X} = \begin{bmatrix} s[1] & s[2] & \cdots & \cdots & s[L] & s[L+1] & \cdots & \cdots & s[N+L-1] \\ 0 & s[1] & s[2] & \cdots & \cdots & s[L] & s[L+1] & \cdots & s[N+L-2] \\ 0 & 0 & s[1] & s[2] & \cdots & \cdots & s[L] & \cdots & s[N+L-3] \\ \vdots & \vdots & & \ddots & \ddots & & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & s[1] & s[2] & \cdots & \cdots & s[N] \end{bmatrix}$$

- Frequency diversity is intrinsic in frequency-selective channels.
- Inter-Symbol Interference occurs, i.e., the delayed replicas of previous symbols interfere with the current symbols, if a symbol is sent every symbol time.

How to mitigate ISI and achieve full frequency diversity?

To Achieve Full Frequency Diversity I

- Transmit one symbol every L symbol times

$$\mathbf{X} = \begin{bmatrix} s[1] & 0 & \cdots & 0 & s[2] & 0 & \cdots & 0 & \cdots \\ 0 & s[1] & \cdots & 0 & 0 & s[2] & \cdots & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\ 0 & 0 & \cdots & s[1] & 0 & 0 & \cdots & s[2] & \cdots \end{bmatrix}$$

- Full diversity gain
- Low complexity
- Low efficiency

To Achieve Full Frequency Diversity II

- Time-domain Equalization

$$\mathbf{X}=\begin{bmatrix} s[1] & s[2] & \cdots & \cdots & s[L] & s[L+1] & s[L+2] & \cdots & s[N+L-1] \\ 0 & s[1] & s[2] & \cdots & \cdots & s[L] & s[L+1] & \cdots & s[N+L-2] \\ 0 & 0 & s[1] & s[2] & \cdots & \cdots & s[L] & \cdots & s[N+L-3] \\ \vdots & \vdots & \ddots & \ddots & \ddots & & & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & s[1] & s[2] & s[3] & \cdots & s[N] \end{bmatrix}$$

At time slot m: $y[m]=\mathbf{h}\mathbf{x}[m]+z[m]$

$\mathbf{h}=[h_0,h_1,\ldots,h_{L-1}]$

$\mathbf{x}[m]=[s[m],s[m-1],\ldots,s[m-L+1]]'$

$m=1,\ldots,N+L-1$: $\mathbf{y}=\mathbf{h}\mathbf{X}+\mathbf{z}$

$\mathbf{y}=[y[1],y[2],\ldots,y[N+L-1]]$

$\mathbf{X}=[\mathbf{x}[1],\mathbf{x}[2],\ldots,\mathbf{x}[N+L-1]]$

How to solve $\mathbf{s}=[s[1],s[2],\ldots,s[N]]$ based on $\mathbf{y}$?

To Achieve Full Frequency Diversity II

- Time-domain Equalization
  - MLSD (maximum likelihood sequence detection)

$$\max_{\mathbf{s}} \Pr\{\mathbf{y} \mid \mathbf{s}\}$$

$$\max_{\mathbf{s}} \Pr\{\mathbf{y} \mid \mathbf{s}\} \Rightarrow \max_{\mathbf{X}} \Pr\{\mathbf{y} \mid \mathbf{X}\} = \max_{\mathbf{X}} \prod_m \Pr\{y[m] \mid \mathbf{x}[m]\}$$

$$\Rightarrow \min_{\mathbf{X}} -\log \Pr\{\mathbf{y} \mid \mathbf{X}\} = \min_{\mathbf{X}} -\sum_m \log \Pr\{y[m] \mid \mathbf{x}[m]\}$$

At time slot m: $\mathbf{x}[m] = [s[m], s[m-1], \ldots, s[m-L+1]]'$

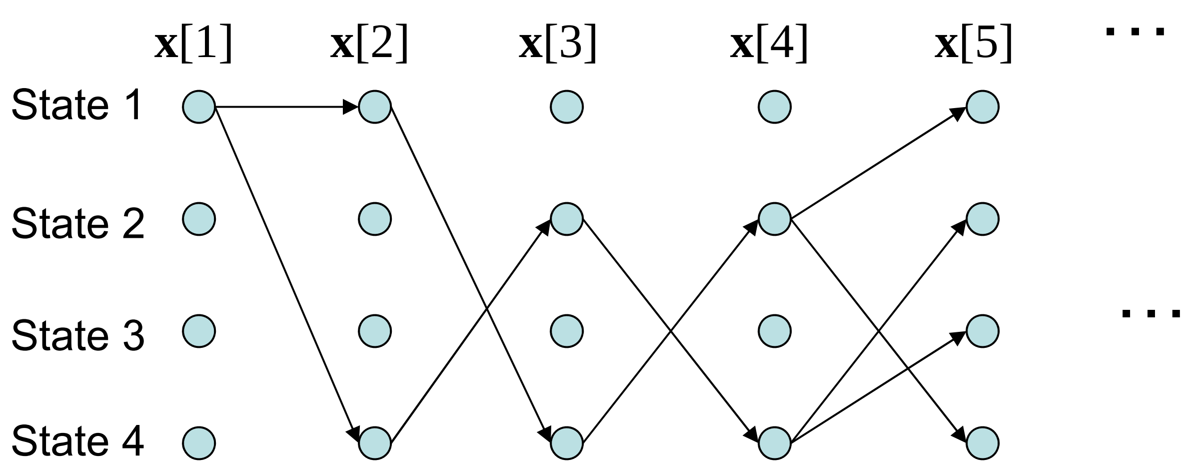
can be described using a finite state machine!

Number of states = M^L

M: constellation size for each symbol s[m]

To Achieve Full Frequency Diversity II

- Time-domain Equalization
  - MLSD - Viterbi algorithm $\min_{\mathbf{x}} -\sum_m \log \Pr\{y[m] \,|\, \mathbf{x}[m]\}$



At each stage and each state, only the best path is kept.

Optimality principle: if the optimal path to state s1 at stage m goes through state s2 at stage m-1, then the part of the path up to stage m-1 must be the optimal path to state s2.

To Achieve Full Frequency Diversity II

- Time-domain Equalization
  - Linear Equalizer $\quad \mathbf{K}\mathbf{y} = \mathbf{K}\mathbf{H}\mathbf{s} + \mathbf{K}\mathbf{z}$

$$\mathbf{H} = \begin{bmatrix} h_0 & & & \\ \vdots & \ddots & & \\ h_{L-1} & \cdots & h_0 & \\ & h_{L-1} & \cdots & h_0 \\ & & \ddots & \vdots \\ & & & h_{L-1} \end{bmatrix}$$

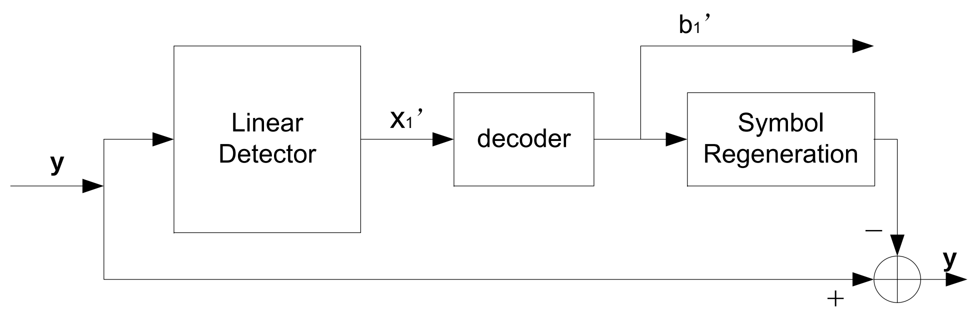
- Zero-forcing (ZF) $\quad \mathbf{K} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$

- Minimum Mean Squared Error (MMSE)

The j-th row of **K**: $\quad \mathbf{k}_j = \mathbf{h}_j^* \cdot \left(\sum_{i \neq j} \mathbf{h}_i \mathbf{h}_i^* + \sigma^2 \mathbf{I}_n \right)^{-1} \qquad j=1, \ldots, n$

the i-th column of **H**

  - Decision-Feedback Equalizer (DFE)



Successive Interference Cancellation (SIC)

To Achieve Full Frequency Diversity II

- Time-domain Equalization (eg. GSM)

  - MLSD
    - Viterbi algorithm

  - Optimal (minimizing the sequence error)
  - Full diversity gain
  - Complexity level O(M^L)

  - Linear Equalizer
    - ZF, MMSE
  - DFE

  - Not optimal
  - Full diversity gain
  - Low complexity O(L^a)

Complexity at receiver side!

To Achieve Full Frequency Diversity III

- DSSS (Direct Sequence Spread Spectrum)

Symbol   Chip

$$\mathbf{X}=\begin{bmatrix} s[1]u[1] & s[1]u[2] & \cdots & s[1]u[L] & \cdots & s[1]u[n] & s[2]u[1] & s[2]u[2] & s[2]u[3] \\ s[0]u[n] & s[1]u[1] & \cdots & s[1]u[L-1] & \cdots & s[1]u[n-1] & s[1]u[n] & s[2]u[1] & s[2]u[2] \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ s[0]u[n\text{-}L] & s[0]u[n\text{-}L\text{-}1] & \cdots & s[1]u[1] & \cdots & s[1]u[n-L+1] & \cdots & \cdots & s[1]u[n] \end{bmatrix}$$

$n=\dfrac{\text{symbol time}}{\text{chip time}}$: processing gain (spreading gain) $\quad n>>L$

- Negligible ISI
- Simple receiver

$\mathbf{u}^{(l)}=[\underbrace{0,\ldots,0}_{l\text{ zeros}},u[1],\ldots,u[n],\underbrace{0,\ldots,0}_{L\text{-}l\text{ zeros}}]$ $\quad l$ - shifted version of $\mathbf{u}=[u[1],\ldots,u[n]]$

Low correlation of pseudo-noise sequence: $(\mathbf{u}^{(l)})(\mathbf{u}^{(l')})^{*}\approx 0 \quad \text{if } l\neq l'$

To Achieve Full Frequency Diversity III

- DSSS (eg. IS-95, WCDMA, CDMA2000, …)

$$
\begin{array}{c}
\mathbf{X} = \begin{bmatrix}
s[1]u[1] & s[1]u[2] & \cdots & s[1]u[L] & \cdots & s[1]u[n] & 0 & 0 & 0 \\
0 & s[1]u[1] & \cdots & s[1]u[L-1] & \cdots & s[1]u[n-1] & s[1]u[n] & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & s[1]u[1] & \cdots & s[1]u[n-L+1] & \cdots & \cdots & s[1]u[n]
\end{bmatrix}
\end{array}
\begin{array}{l}
s[1]\mathbf{u}^{(0)} \\
s[1]\mathbf{u}^{(1)} \\
\\
s[1]\mathbf{u}^{(L-1)}
\end{array}
$$

At the receiver:
$$\mathbf{y} = \mathbf{hX} + \mathbf{z} = \sum_{l=0}^{L-1} h_l s[1]\mathbf{u}^{(l)} + \mathbf{z} \qquad \mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]$$

$$\left(\mathbf{u}^{(l')}\right)^* \qquad \left(\mathbf{u}^{(l')}\right)^* \quad l' = 0, \ldots, L-1$$

$$\Downarrow$$

$$r^{(l)} = h_l s[1] + \tilde{z}^{(l)}, \quad l = 0, \ldots, L-1$$

Rake Receiver is nothing more than a maximal ratio combiner

- Full diversity gain
- Low complexity
- Low efficiency
- Security
- Interference mitigation
- High requirement on synchronization

To Achieve Full Frequency Diversity IV

- OFDM (Orthogonal Frequency Division Multiplexing)

$$d[0], d[1], \ldots, d[N_c-1] \Rightarrow \underbrace{d[N_c-L+1], d[N_c-L+2], \ldots, d[N_c-1],}_{\text{Cyclic Prefix}} d[0], d[1], \ldots, d[N_c-1]$$

$$\begin{array}{cccccccc} & & & & \mathbf{d}_L & \mathbf{d}_{L+1} & & \mathbf{d}_{Nc+L\text{-}1} \end{array}$$

$$\mathbf{X} = \left[\begin{array}{cccc|cccc} d[N_c-L+1] & d[N_c-L+2] & \cdots & d[N_c-1] & d[0] & d[1] & \cdots & d[N_c-1] \\ \times & d[N_c-L+1] & \cdots & d[N_c-2] & d[N_c-1] & d[0] & \cdots & d[N_c-2] \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots \\ \times & \times & \cdots & \times & d[N_c-L+1] & d[N_c-L+2] & \cdots & d[N_c-L] \end{array}\right]$$

- No interference from the other block.
- Rows are cyclic shifts of each other!

At the receiver: $\mathbf{y} = \mathbf{hX} + \mathbf{z}$ $\qquad \mathbf{y} = [y_1, \ldots, y_{L-1}, y_L, \ldots, y_{N_c+L-1}], \ \mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]$

Ignoring the first L-1 receive symbols,

$$y[m] = \mathbf{hd}_m + z[m] = \sum_{l=0}^{L-1} h_l d[(m-L-l) \bmod N_c] + z[m] \qquad m = L, L+1, \ldots, N_c+L-1.$$

To Achieve Full Frequency Diversity IV

- OFDM (eg. Wi-Fi (802.11 a, g, n), WiMAX, …)

$$y[m]=\sum_{l=0}^{L-1} h_l d[(m-L-l) \bmod N_c]+z[m] \qquad m=L, L+1, \ldots, N_c+L-1.$$

Let $\hat{\mathbf{h}}=[h_0, h_1, \ldots, h_{L-1}, 0, \ldots, 0]$ $\Downarrow$ $\hat{\mathbf{y}}=[y_L, \ldots, y_{N_c+L-1}]$

$$\hat{y}[n]=\sum_{l=0}^{N_c-1} \hat{h}_l d[(n-l) \bmod N_c]+\hat{z}[n] \qquad n=0,1, \ldots, N_c-1.$$

$$\Downarrow$$

$$\hat{\mathbf{y}}=\hat{\mathbf{h}} * \mathbf{d}+\hat{\mathbf{z}} \qquad \mathbf{d}=[d[0], \ldots, d[N_c-1]]'$$

* denotes the cyclic convolution.

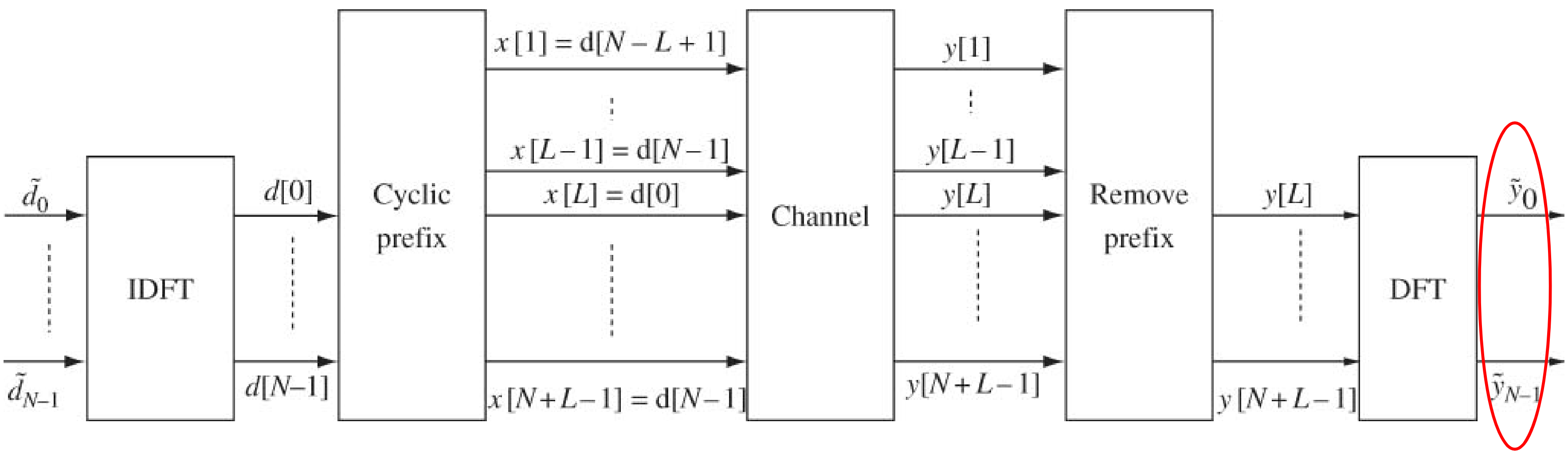
$$\mathrm{DFT}(\hat{\mathbf{h}} * \mathbf{d})_n=\sqrt{N_c}\, \mathrm{DFT}(\hat{\mathbf{h}})_n \cdot \mathrm{DFT}(\mathbf{d})_n \qquad \Downarrow$$

$$\tilde{y}_n=\tilde{h}_n \tilde{d}_n+\tilde{z}_n \qquad n=0,1, \ldots, N_c-1.$$

$$\tilde{h}_n=\sum_{l=0}^{L-1} h_l \exp\left(\frac{-j 2 \pi n l}{N_c}\right) \qquad \tilde{d}_n=\frac{1}{\sqrt{N_c}} \sum_{m=0}^{N_c-1} d[m] \exp\left(\frac{-j 2 \pi n m}{N_c}\right)$$

To Achieve Full Frequency Diversity IV

- OFDM (eg. Wi-Fi (802.11 a, g, n), WiMAX, …)



- Convert the data symbols in frequency domain (in different subcarriers) into time domain using IDFT
- Add cyclic prefix to remove ISI (so that the subcarriers become orthogonal after going through the channel)
- Convert the receive signals into frequency domain to recover the data symbols

$$\tilde{y}_n = \tilde{h}_n \tilde{d}_n + \tilde{z}_n \qquad n = 0, 1, \ldots, N_c - 1.$$

N_c parallel sub-channels

To Achieve Full Frequency Diversity IV

- OFDM (eg. Wi-Fi (802.11 a, g, n), WiMAX, …)

1. Can we get N_c–fold diversity gain by sending the same symbol over N_c subcarriers?

2. How to collect full frequency diversity gain?

3. Can N_c be arbitrarily large?

- Full diversity gain
- No ISI (by virtue of cyclic prefix)
- Flexible resource allocation

Summary

- Detection in Fading Channels
  - Poor performance due to deep fade $P_e \sim SNR^{-1}$
- Use Diversity to Overcome Fading
  - Time Diversity
    - Repetition code + Interleaving
  - Space Diversity
    - Receive diversity, transmit diversity, MIMO
  - Frequency Diversity
    - Equalization — High complexity, entirely at the receiver
    - DSSS — Low complexity, shared by transmitter and receiver
    - OFDM — moderate complexity, shared by transmitter and receiver

will be revisited in multiuser scenario