On the Capacity of Distributed Antenna Systems

Lin Dai

City University of Hong Kong

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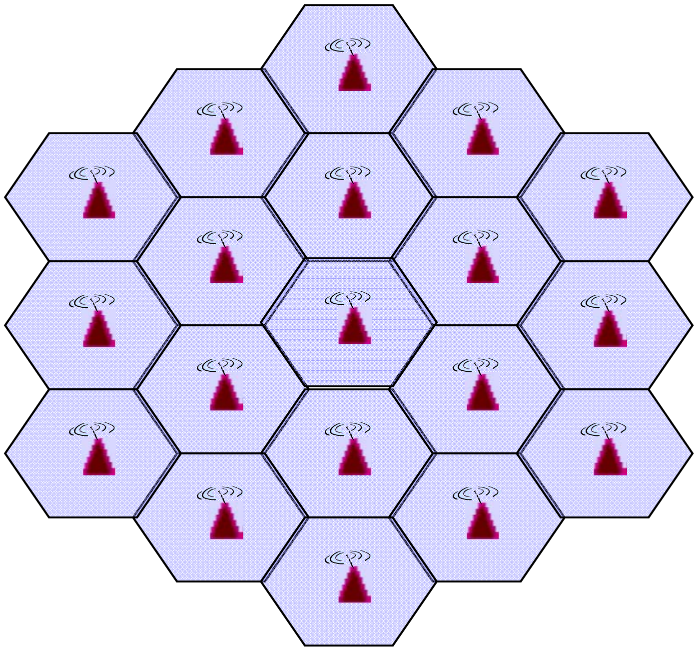
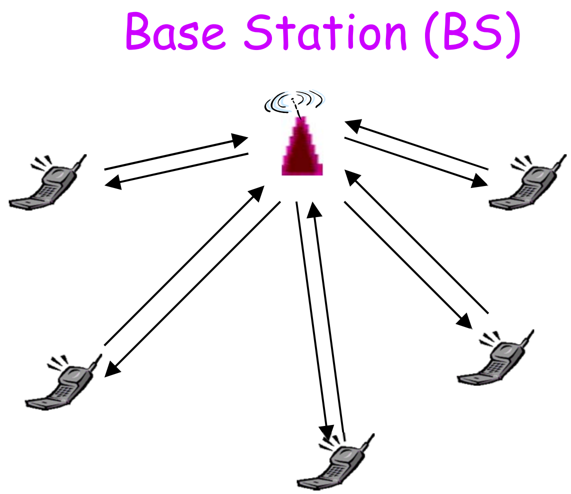
Cellular Networks (1)



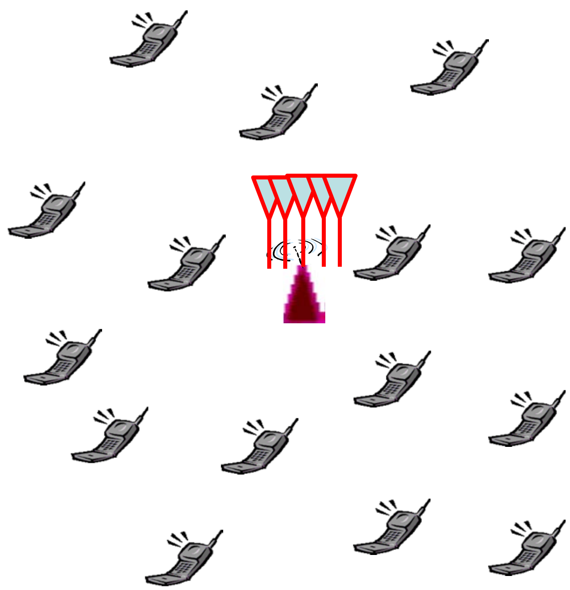


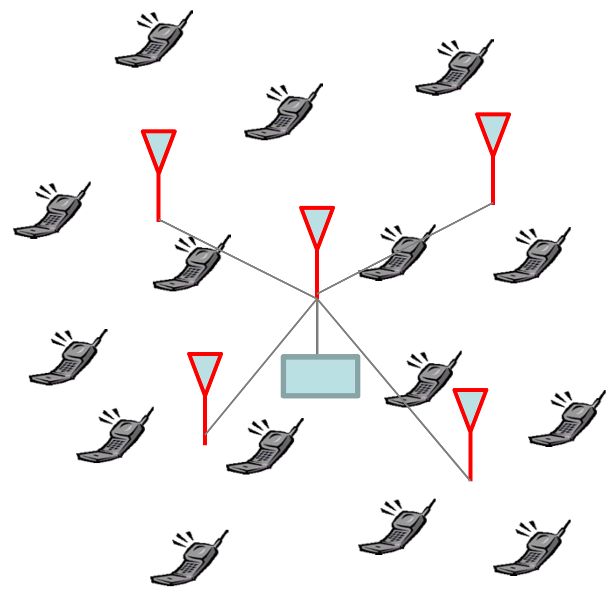
Growing demand for high data rate

⇩

Multiple antennas at the BS side

Cellular Networks (2)





	Co-located BS antennas	Distributed BS antennas
Implementation cost	Lower	
Sum rate		Higher?

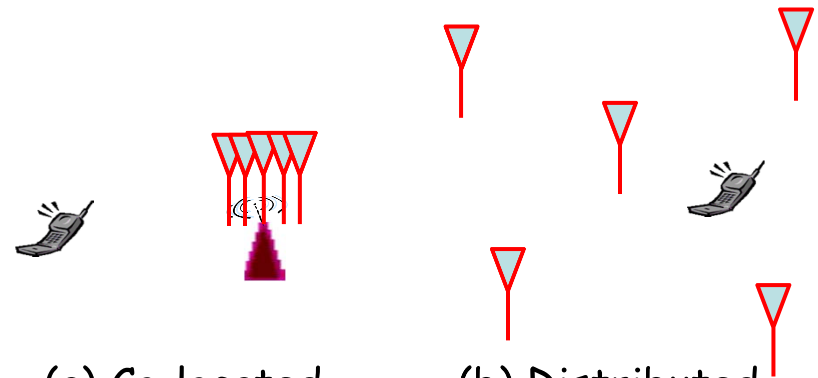
A little bit of History of Distributed Antenna Systems (DASs)

- Originally proposed to cover the dead spots for indoor wireless communication systems [Saleh&etc'1987].
- Implemented in cellular systems to improve cell coverage.
- Recently included into the 4G LTE standard.
- Key technology for C-RAN and 5G?

- Multiple-input-multiple-output (MIMO) theory has motivated a series of information-theoretic studies on DASs.

Single-user SIMO Channel

- Single user with a single antenna.
- L>1 BS antennas.
- Uplink (user->BS).



(a) Co-located BS Antennas

(b) Distributed BS Antennas

- Received signal: $\mathbf{y} = \mathbf{g} \cdot s + \mathbf{z}$

  $s \sim \mathcal{CN}(0, \bar{P})$: Transmitted signal

  $\mathbf{z} \in C^{L\times 1}$: Gaussian noise

  $z_i \sim \mathcal{CN}(0, N_0), \quad i = 1, \ldots, L.$

- Channel gain : $\mathbf{g} = \boldsymbol{\gamma} \circ \mathbf{h}$

  $\boldsymbol{\gamma} \in C^{L\times 1}$: Large-scale fading

  $\gamma_i \sim \mathrm{Log} - \mathcal{CN}(d_i^{-\alpha/2}, \sigma_i^2), \quad i = 1, \ldots, L.$

  $\mathbf{h} \in C^{L\times 1}$: Small-scale fading

  $h_i \sim \mathcal{CN}(0,1), \quad i = 1, \ldots, L.$

Single-user SIMO Channel

- Single user with a single antenna.
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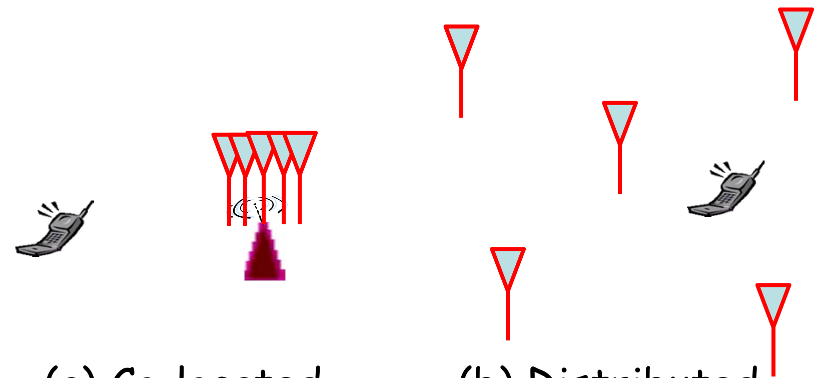


(a) Co-located BS Antennas

(b) Distributed BS Antennas

- Ergodic capacity without channel state information at the transmitter side (CSIT):

$$C_o = \mathrm{E}_{\mathbf{h}}\left\{\log_2\left(1+\frac{\bar{P}\|\mathbf{g}\|^2}{N_0}\right)\right\}$$

- Ergodic capacity with CSIT:

$$C_w = \max_{P:\, \mathrm{E}_{\mathbf{h}}[P]=\bar{P}} \mathrm{E}_{\mathbf{h}}\left\{\log_2\left(1+\frac{P\|\mathbf{g}\|^2}{N_0}\right)\right\}$$

Function of large-scale fading vector γ

Co-located Antennas versus Distributed Antennas

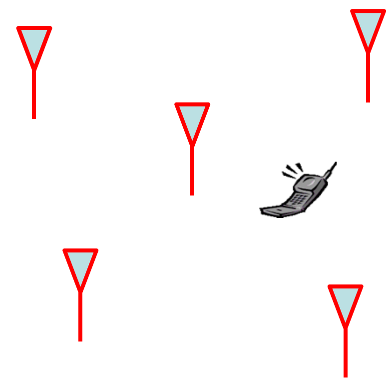
- With co-located BS antennas:
  - ✓ Ergodic Capacity without CSIT:

$$C_o^C = \mathrm{E}_{\mathbf{h}}\left\{\log_2\left(1+\tfrac{1}{L}\mu\|\mathbf{h}\|^2\right)\right\}$$

μ is the average received SNR:

$$\mu = \bar{P}\cdot\|\boldsymbol{\gamma}\|^2 / N_0$$

- With distributed BS antennas:
  - ✓ Distinct large-scale fading gains to different BS antennas.
  - ✓ Ergodic Capacity without CSIT:



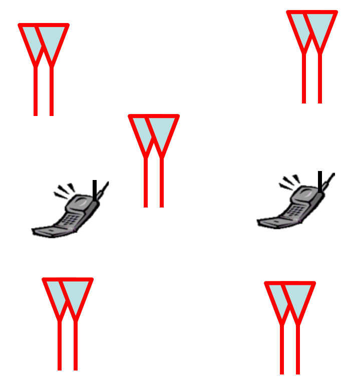
$$C_o^D = \mathrm{E}_{\mathbf{h}}\left\{\log_2\left(1+\mu\|\tilde{\mathbf{g}}\|^2\right)\right\}$$

Normalized channel gain: $\tilde{\mathbf{g}} = \boldsymbol{\beta}\circ\mathbf{h}$

$$\boldsymbol{\beta} = \frac{\boldsymbol{\gamma}}{\|\boldsymbol{\gamma}\|}$$

Capacity of DAS

- For given large-scale fading vector γ:
  - ✓ [Heliot&etc'11]: Ergodic capacity without CSIT
    - A single user equipped with N co-located antennas.
    - BS antennas are grouped into L clusters. Each cluster has M co-located antennas.
    - Asymptotic result as M and N go to infinity and M/N is fixed.
  - ✓ [Aktas&etc'06]: Uplink ergodic sum capacity without CSIT
    - K users, each equipped with $N\beta_k$ co-located antennas.
    - BS antennas are grouped into L clusters. Each cluster has $N\lambda_l$ co-located antennas.
    - Asymptotic result as N goes to infinity and β_k and λ_l are fixed.



- Implicit function of γ (need to solve fixed-point equations)
- Computational complexity increases with L and K.

Capacity of DAS

- With random large-scale fading vector γ:

Average ergodic capacity (i.e., averaged over γ)

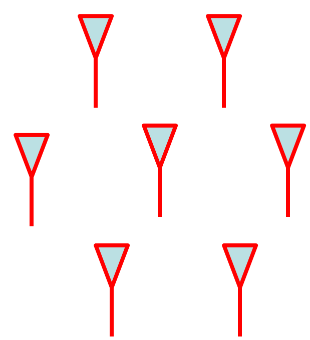
- Single user
- Without CSIT

✓ [Roh&Paulraj'02], [Zhang&Dai'04]: The user has identical access distances to all the BS antennas. $\gamma_i \sim \mathrm{Log}-\mathcal{CN}(1, \sigma^2),\ i = 1, \ldots, L.$

✓ [Zhuang&Dai'03]: BS antennas are uniformly distributed over a circular area and the user is located at the center. $\gamma_i = \rho_i^{-\alpha/2},\ i = 1, \ldots, L.$

✓ [Choi&Andrews'07], [Wang&etc'08], [Feng&etc'09], [Zhu'11], [Lee&ect'12]: BS antennas are regularly placed in a circular cell and the user has a random location.

$$\gamma_i \sim \mathrm{Log}-\mathcal{CN}(d_i^{-\alpha/2}, \sigma_i^2),\ i = 1, \ldots, L.$$



Computational complexity increases with the number of BS antennas!

Questions to be Answered

- How to characterize the sum capacity of DAS when there are a large number of BS antennas and users?
  - ✓ Large-system analysis using random matrix theory.
  - ✓ Bounds are desirable.
- How to conduct a fair comparison with the co-located case?
  - ✓ K randomly distributed users with a fixed total transmission power.
  - ✓ Decouple the comparison into two parts: 1) capacity comparison and 2) transmission power comparison for given average received SNR.
- What is the effect of CSIT on the comparison result?

Outline

- Single-cell comparison
- Multi-cell comparison
- DAS with virtual cells

[1] L. Dai, "A Comparative Study on Uplink Sum Capacity with Co-located and Distributed Antennas," IEEE J. Sel. Areas Commun., 2011.

[2] L. Dai, "An Uplink Capacity Analysis of the Distributed Antenna System (DAS): From Cellular DAS to DAS with Virtual Cells," IEEE Trans. Wireless Commun., 2014.

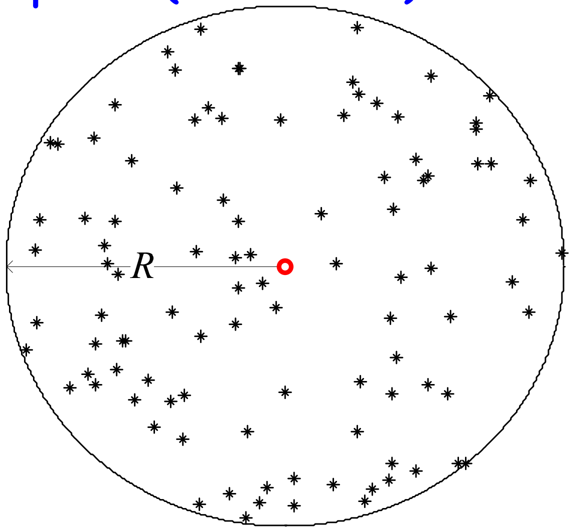
Part I. Single-Cell Comparison

- System model and preliminary analysis
- Uplink ergodic sum capacity
- Average transmission power per user

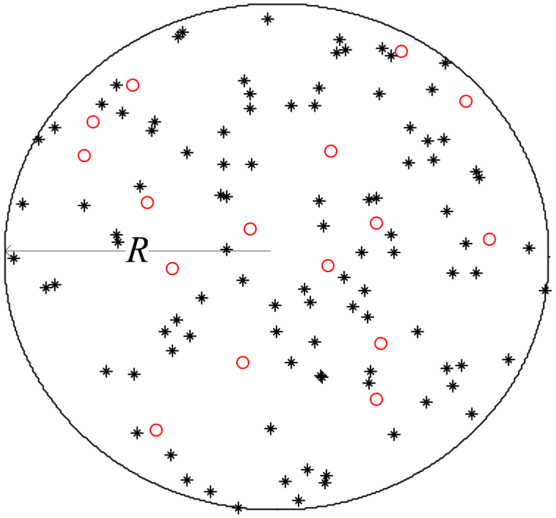
System Model and Preliminary Analysis

Assumptions

- K single-antenna users are uniformly distributed within a circular cell.
- L BS antennas are either co-located at the center of the cell, or uniformly distributed over the cell.
- Uplink (user->BS).



(a) Co-located Antennas (CA)



(b) Distributed Antennas (DA)

Random BS antenna layout!

*: user

o: BS antenna

Uplink Ergodic Sum Capacity

- Received signal:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{g}_k s_k + \mathbf{z}$$

- Uplink power control:

$$\bar{P}_k \cdot \|\boldsymbol{\gamma}_k\|^2 = P_0, \quad k = 1, \ldots, K.$$

$s_k \sim \mathcal{CN}(0, \bar{P}_k)$: Transmitted signal

$\mathbf{z} \in C^{L\times 1}$: Gaussian noise
$z_i \sim \mathcal{CN}(0, N_0), \quad i = 1, \ldots, L.$

$\mathbf{g}_k = \boldsymbol{\gamma}_k \circ \mathbf{h}_k$: Channel gain

$\boldsymbol{\gamma}_k \in C^{L\times 1}$: Large-scale fading
$\gamma_{i,k} = d_{i,k}^{-\alpha/2}, \quad i = 1, \ldots, L.$

$\mathbf{h}_k \in C^{L\times 1}$: Small-scale fading
$h_{i,k} \sim \mathcal{CN}(0,1), \quad i = 1, \ldots, L.$

Ergodic capacity without CSIT

$$C_{sum\_o} = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{1}{N_0}\sum_{k=1}^{K} \bar{P}_k \mathbf{g}_k \mathbf{g}_k^\dagger\right)\right\}$$

$$= \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{P_0}{N_0}\sum_{k=1}^{K} \tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k^\dagger\right)\right\}$$

Ergodic capacity with CSIT

$$C_{sum\_w} = \max_{\substack{P_k:\, \mathrm{E}_{\mathbf{H}}[P_k] = \bar{P}_k \\ k=1,\ldots,K}} \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{1}{N_0}\sum_{k=1}^{K} P_k \mathbf{g}_k \mathbf{g}_k^\dagger\right)\right\}$$

$$= \max_{\substack{\tilde{P}_k:\, \mathrm{E}_{\mathbf{H}}[\tilde{P}_k] = P_0 \\ k=1,\ldots,K}} \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{1}{N_0}\sum_{k=1}^{K} \tilde{P}_k \tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k^\dagger\right)\right\}$$

More about Normalized Channel Gain

- Normalized channel gain vector:

$$\tilde{\mathbf{g}}_k = \boldsymbol{\beta}_k \circ \mathbf{h}_k$$

$\mathbf{h}_k \in C^{L\times 1}$: Small-scale fading

$\boldsymbol{\beta}_k = \frac{\boldsymbol{\gamma}_k}{\|\boldsymbol{\gamma}_k\|} \in C^{L\times 1}$: Normalized Large-scale fading

$\|\boldsymbol{\beta}_k\| = 1.$

✓ With CA: $\boldsymbol{\beta}_k = \frac{1}{\sqrt{L}}\mathbf{1}_{L\times 1}.$

$$\|\tilde{\mathbf{g}}_k\|^2 \to 1 \text{ if } L \to \infty.$$

The channel becomes deterministic with a large number of BS antennas L!

✓ With DA: $\beta_{i,k} \neq \beta_{j,k},\ i \neq j.$

- With a large L, it is very likely that user k is close to some BS antenna l_k^*: $\boldsymbol{\beta}_k \approx \mathbf{e}_{l_k^*}$

$$e_l = \begin{cases} 1 & \text{if } l = l_k^* \\ 0 & \text{otherwise} \end{cases}$$

$$\|\tilde{\mathbf{g}}_k\|^2 \approx |h_{l_k^*,k}|^2$$

Channel fluctuations are preserved even with a large L!

More about Normalized Channel Gain

- Theorem 1. For n=1,2,...,

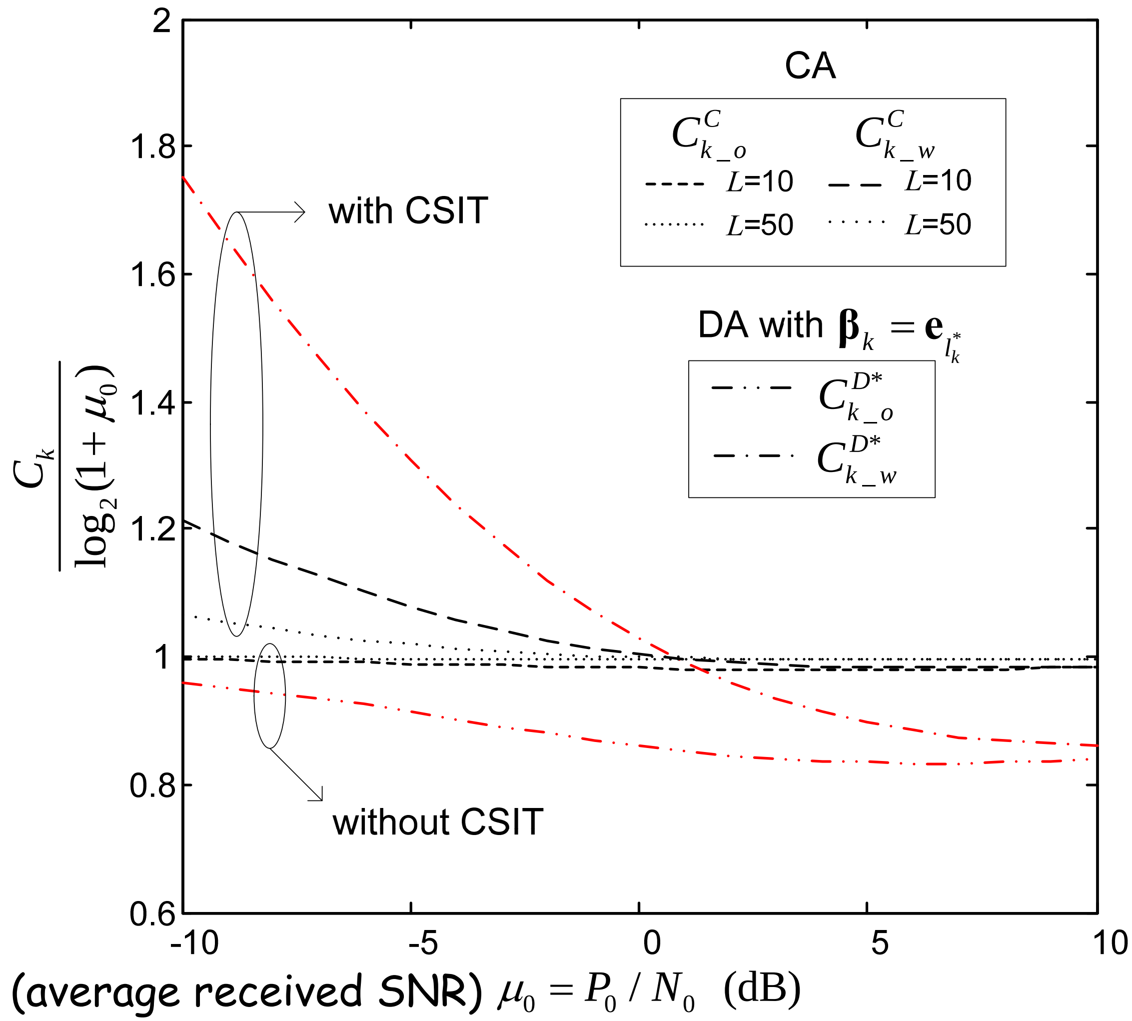
$$\min_{\boldsymbol{\beta}_k:\,\|\boldsymbol{\beta}_k\|=1} \mathrm{E}_{\mathbf{h}_k}\left[\|\tilde{\mathbf{g}}_k\|^{2n}\right] = \frac{(n+L-1)!}{L^n(L-1)!},$$ which is achieved when $\boldsymbol{\beta}_k = \frac{1}{\sqrt{L}}\mathbf{1}_{L\times 1}$.

$$\max_{\boldsymbol{\beta}_k:\,\|\boldsymbol{\beta}_k\|=1} \mathrm{E}_{\mathbf{h}_k}\left[\|\tilde{\mathbf{g}}_k\|^{2n}\right] = n!,$$ which is achieved when $\boldsymbol{\beta}_k = \mathbf{e}_{l_k^*}$.

✓ Channel fluctuations are minimized when $\boldsymbol{\beta}_k = \frac{1}{\sqrt{L}}\mathbf{1}_{L\times 1}$,
maximized when $\boldsymbol{\beta}_k \approx \mathbf{e}_{l_k^*}$.

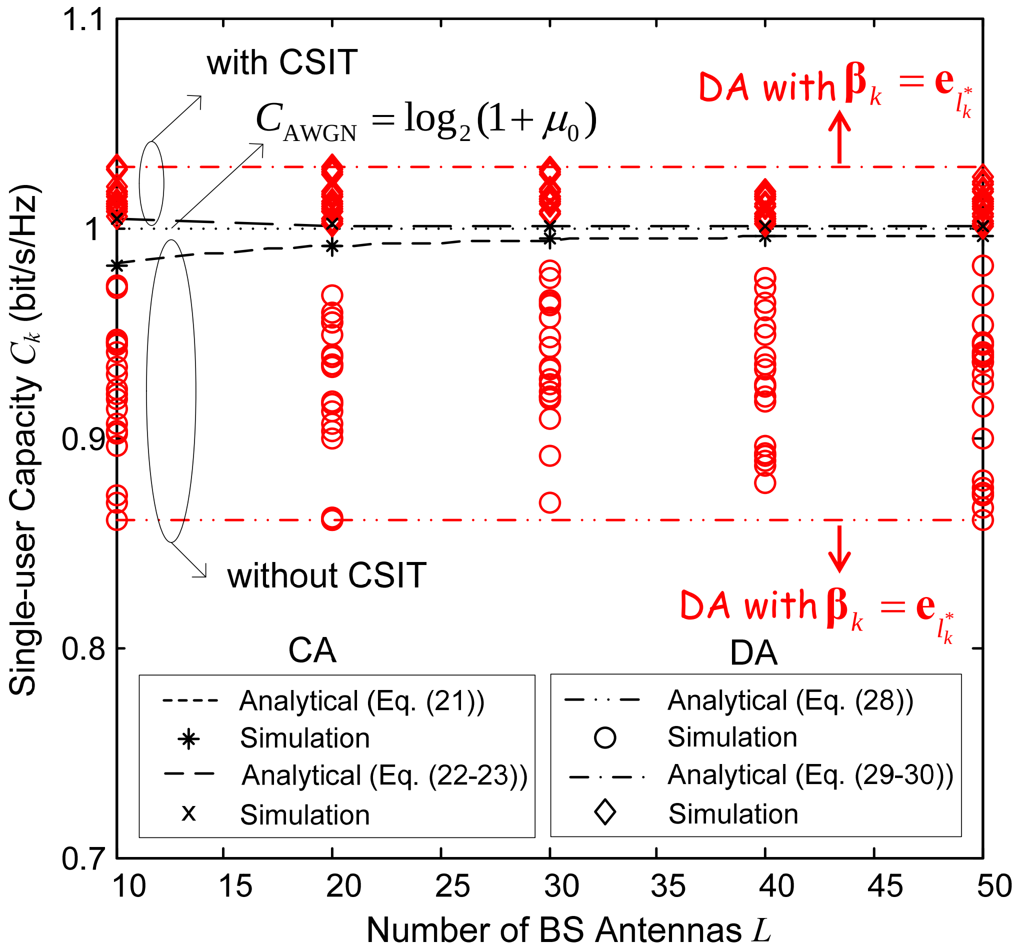
✓ Channel fluctuations are undesirable when CSIT is absent,
desirable when CSIT is available.

Single-user Capacity (1)



- Without CSIT
  - ✓ $C_{k\_o} / C_{AWGN} < 1$ -- Fading always hurts if CSIT is absent!
  - ✓ $C^C_{k\_o} / C_{AWGN}$ quickly approaches 1 as L grows.
  - ✓ $C^C_{k\_o} > C^{D*}_{k\_o}$
- With CSIT (when μ_0 is small)
  - ✓ $C_{k\_w} / C_{AWGN} > 1$ at low μ_0 --"Exploit" fading
  - ✓ $C^{D*}_{k\_w} > C^C_{k\_w}$

Single-user Capacity (2)



The average received SNR $\mu_0 = 0\text{dB}$.

- Without CSIT
  - ✓ A higher capacity is achieved in the CA case thanks to better diversity gains.
- With CSIT
  - ✓ A higher capacity is achieved in the DA case thanks to better waterfilling gains.

Uplink Ergodic Sum Capacity

Uplink Ergodic Sum Capacity without CSIT

- Sum capacity without CSIT:

$$C_{sum\_o} = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{P_0}{N_0}\sum_{k=1}^{K}\tilde{\mathbf{g}}_k\tilde{\mathbf{g}}_k^{\dagger}\right)\right\} = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \mu_0\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\dagger}\right)\right\}$$

$$\tilde{\mathbf{G}} = \mathbf{B}\circ\mathbf{H}$$
$$\mathbf{B} = [\boldsymbol{\beta}_1,\ldots,\boldsymbol{\beta}_K]$$
$$\mathbf{H} = [\mathbf{h}_1,\ldots,\mathbf{h}_K]$$

- Sum capacity per antenna (with K>L):

$$C_{L\_o} = \frac{1}{L}\mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \mu_0\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\dagger}\right)\right\}$$
$$= \frac{1}{L}\mathrm{E}_{\mathbf{H}}\left\{\sum_{l=1}^{L}\log_2\left(1+\mu_0\lambda_l\right)\right\}$$

where $\{\lambda_l\}$ denotes the eigenvalues of $\tilde{\mathbf{G}}\tilde{\mathbf{G}}^{\dagger}$.

More about Normalized Channel Gain

- Theorem 2. As $K, L \to \infty$ and $K/L \to \upsilon$,

$$\mathrm{E}_{\mathbf{H}}[\lambda] \to \upsilon, \quad \text{and} \quad \mathrm{E}_{\mathbf{H}}[\lambda^2] \to 2\upsilon + \upsilon^2 - \mathcal{B}_\infty, \quad \text{with } 0 \le \mathcal{B}_\infty \le \upsilon.$$

$$\mathcal{B}_\infty = \upsilon \quad \text{when} \quad \mathbf{B} = \frac{1}{\sqrt{L}} \mathbf{1}_{L \times K}.$$

$$\mathcal{B}_\infty = 0 \quad \text{when} \quad \mathbf{B} = [\mathbf{e}_{l_1^*}, \ldots, \mathbf{e}_{l_K^*}].$$

✓ With CA: $\mathbf{B} = \frac{1}{\sqrt{L}} \mathbf{1}_{L \times K}$.

$$f_\lambda^C(x) = \frac{1}{2\pi x} \sqrt{((\sqrt{\upsilon}+1)^2 - x)(x - (\sqrt{\upsilon}-1)^2)}$$

as $K, L \to \infty$ and $K/L \to \upsilon$.

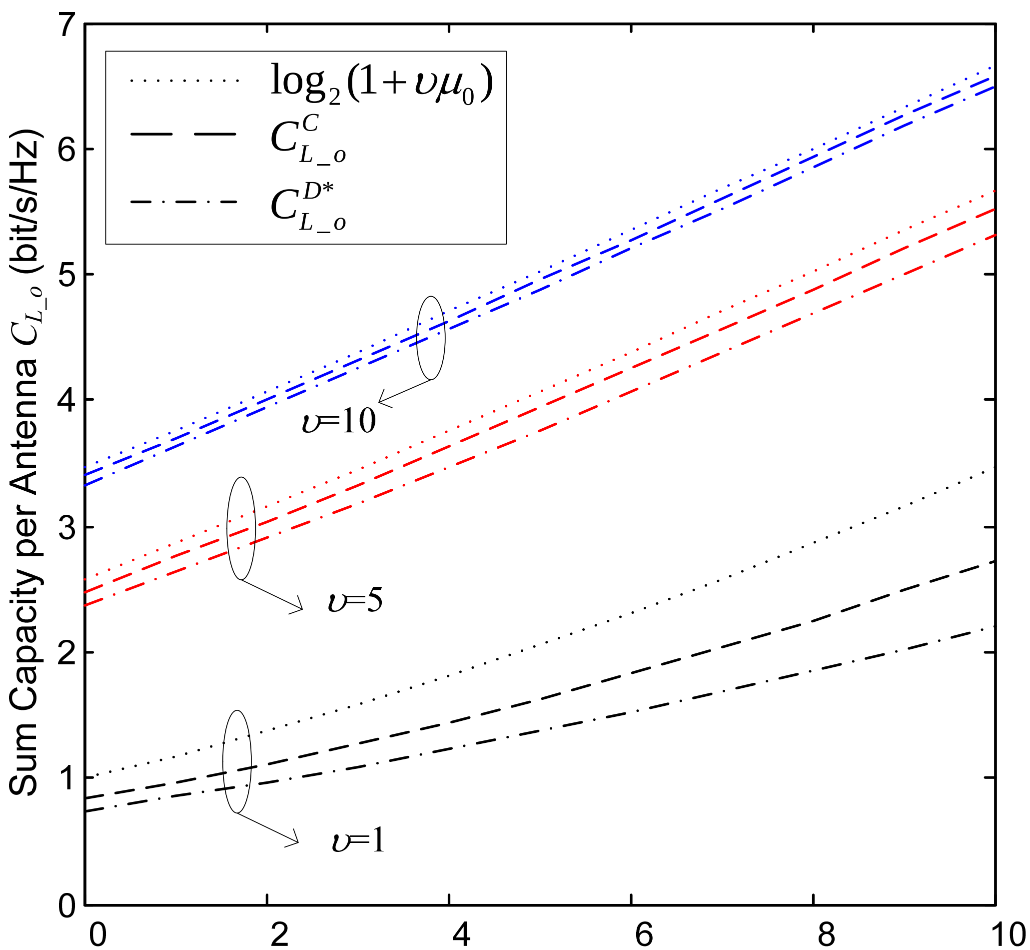
[Marcenko&Pastur'1967]

✓ With DA and $\mathbf{B} = [\mathbf{e}_{l_1^*}, \ldots, \mathbf{e}_{l_K^*}]$:

$$f_\lambda^{D*}(x) = \upsilon e^{-(x+\upsilon)} \sum_{k=0}^{\infty} \frac{(x\upsilon)^k}{k!(k+1)!}$$

as $K, L \to \infty$ and $K/L \to \upsilon$.

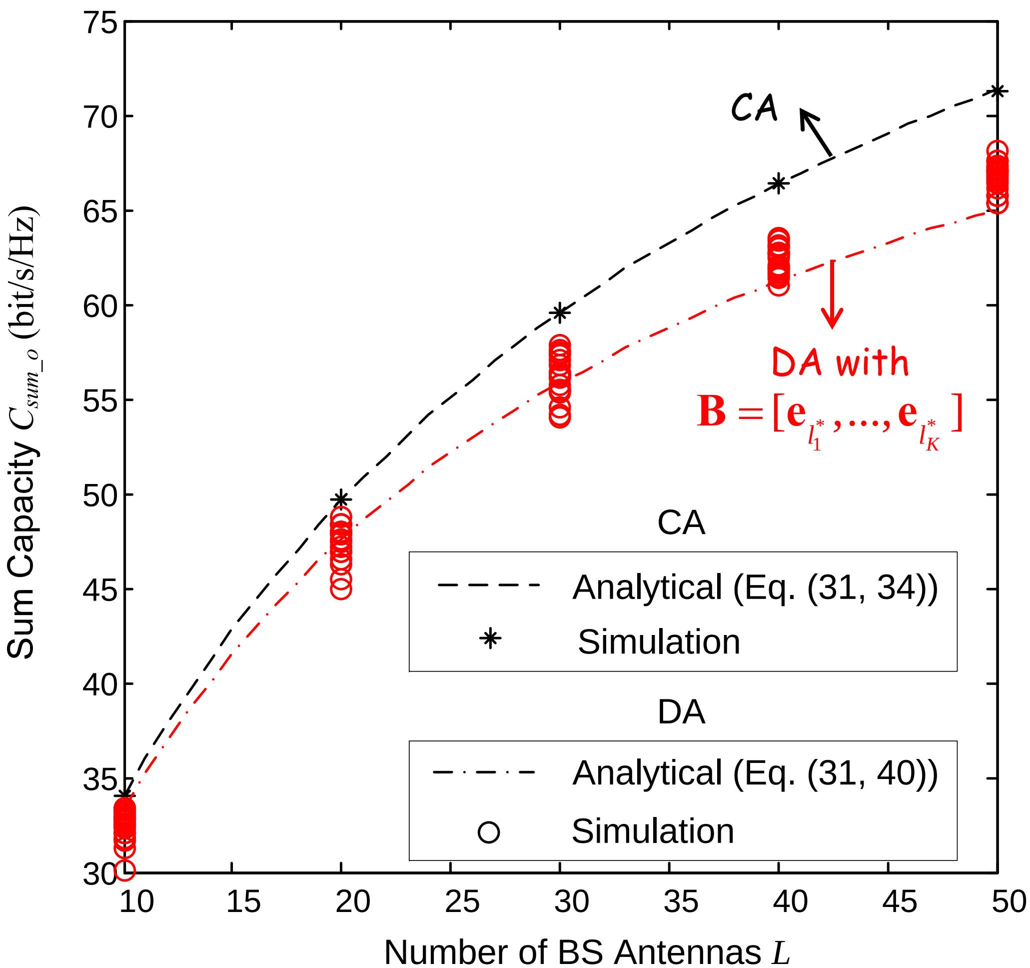
Sum Capacity without CSIT (1)



(average received SNR) $\mu_0 = P_0 / N_0$ (dB)

- ✓ $C^C_{L\_o} > C^{D*}_{L\_o}$
- ✓ Gap diminishes when υ is large -- the capacity becomes insensitive to the antenna topology when the number of users is much larger than the number of BS antennas.

Sum Capacity without CSIT (2)



- ✓ A higher capacity is achieved in the CA case thanks to better diversity gains.
- ✓ $C_{sum\_o}^{D*}$ serves as an asymptotic lower-bound to $C_{sum\_o}^{D}$.

The average received SNR $\mu_0 = 0\text{dB}$. The number of users K=100.

Uplink Ergodic Sum Capacity with CSIT

- Sum capacity with CSIT:

$$C_{sum\_w} = \max_{\substack{\tilde{P}_k:\, \mathrm{E}_{\mathbf{H}}[\tilde{P}_k]=P_0 \\ k=1,\ldots,K}} \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_L + \frac{1}{N_0}\sum_{k=1}^{K} \tilde{P}_k \tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k^{\dagger}\right)\right\} \qquad \tilde{P}_k = P_k \left\|\boldsymbol{\gamma}_k\right\|^2$$

✓ With CA:

The optimal power allocation policy:

$$\tilde{P}_k^* = N_0\left(\frac{1}{\zeta} - \frac{1}{\tilde{\mathbf{g}}_k^{\dagger}\left(\mathbf{I}_L + \sum_{j\neq k}\tilde{P}_j^*\tilde{\mathbf{g}}_j\tilde{\mathbf{g}}_j\right)^{-1}\tilde{\mathbf{g}}_k}\right)^{+}$$

where ζ is a constant chosen to meet the power constraint $\mathrm{E}_{\mathbf{H}}\left\{\tilde{P}_k\right\} = P_0$, k=1,…, K. [Yu&etc'2004]

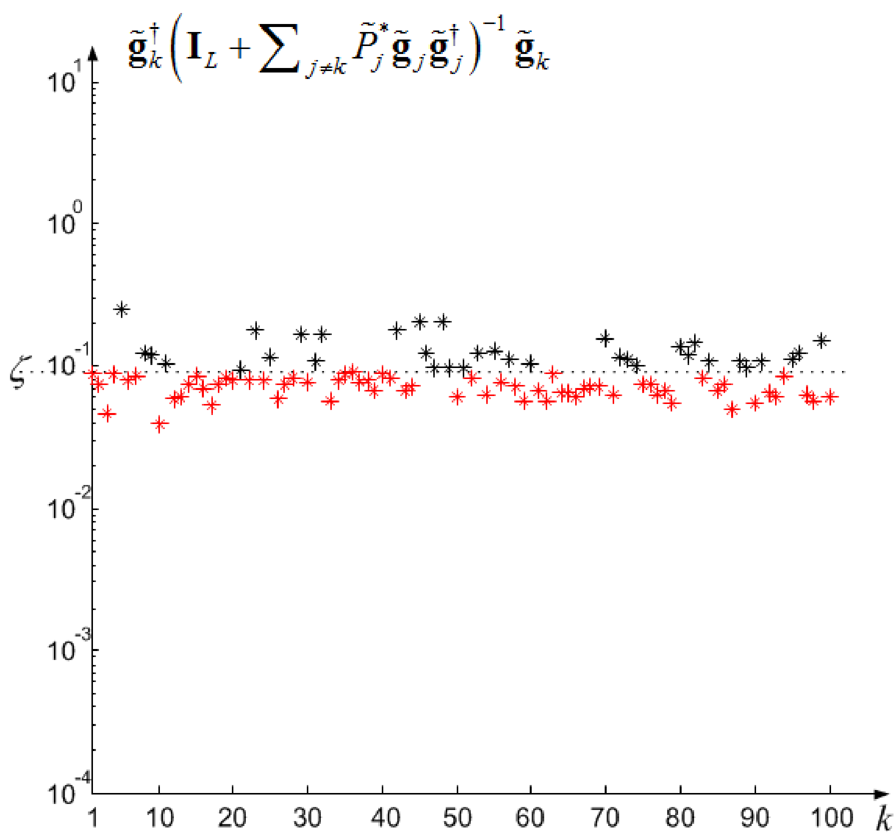
✓ With DA and $\mathbf{B} = [\mathbf{e}_{l_1^*}, \ldots, \mathbf{e}_{l_K^*}]$:

The optimal power allocation policy:

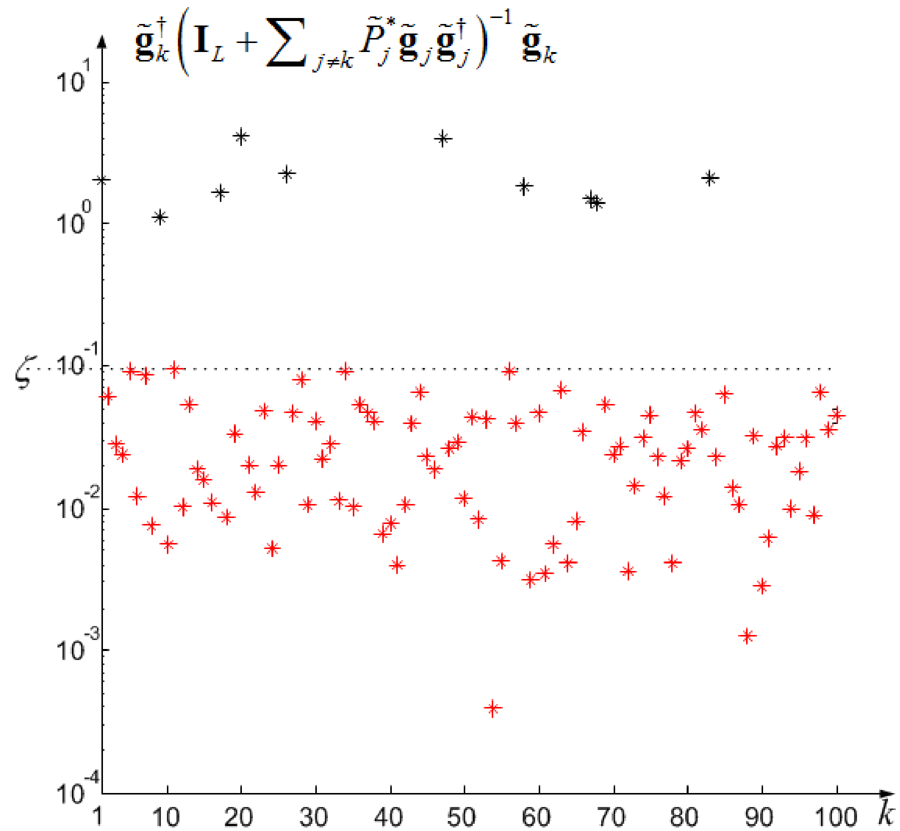
$$\tilde{P}_k^* = \begin{cases} N_0\left(\frac{1}{\zeta} - \frac{1}{|h_{i,k_i^*}|^2}\right)^{+} & k = k_i^* = \arg\max_{k\in\mathcal{K}_i} |h_{i,k}|^2 \\ 0 & k \neq k_i^* \end{cases}$$

i=1,…,L, where ζ is a constant chosen to meet the sum power constraint $\mathrm{E}_{\mathbf{H}}\left\{\sum_{k=1}^{K}\tilde{P}_k\right\} = KP_0$.

Signal-to-Interference Ratio (SIR)



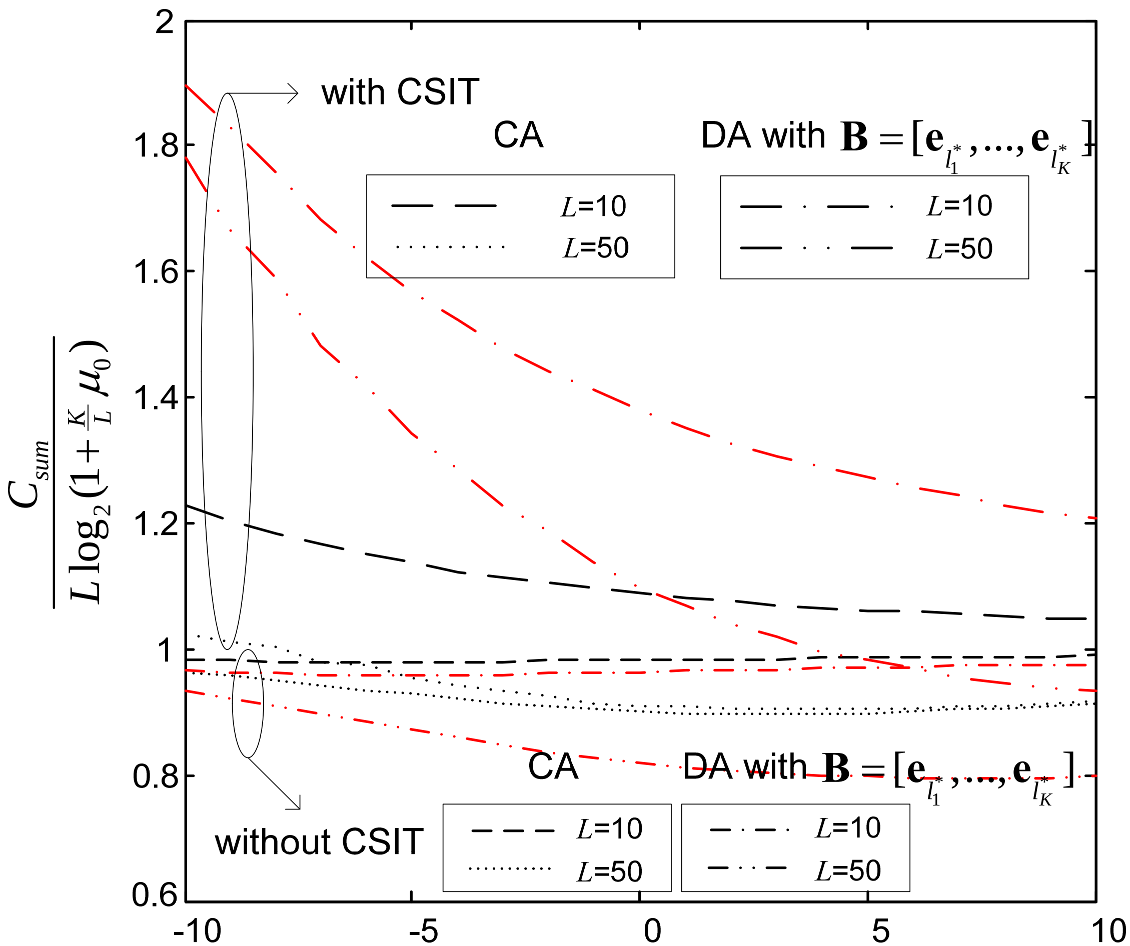
(a) CA



(b) DA with $\mathbf{B} = [\mathbf{e}_{l_1^*}, \ldots, \mathbf{e}_{l_K^*}]$

The received SNR $\mu_0 = 0\text{dB}$. The number of users K=100. The number of BS antennas L=10.

Sum Capacity (1)

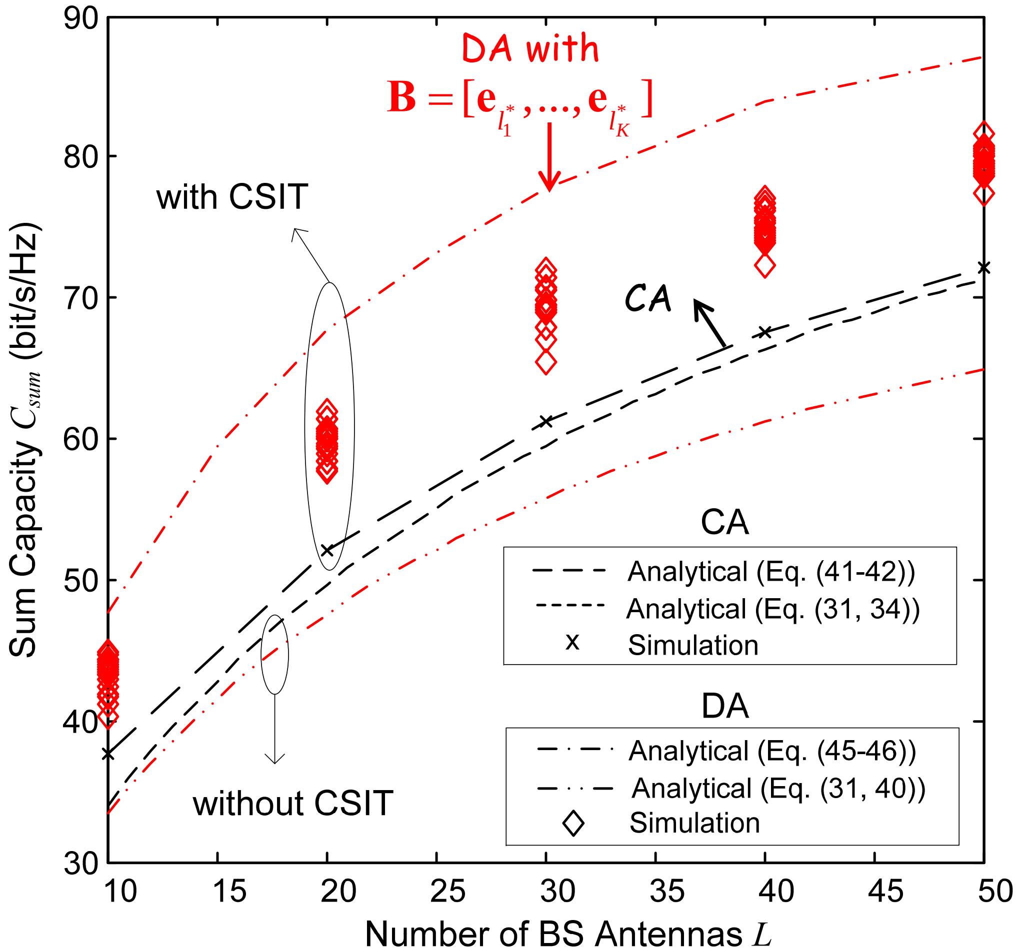


(average received SNR) $\mu_0 = P_0 / N_0$ (dB)

The number of users K=100.

- Without CSIT
  - ✓ $C^{C}_{sum\_o} > C^{D*}_{sum\_o}$
  - ✓ Gap between $C^{C}_{sum\_o}$ and $C^{D*}_{sum\_o}$ is enlarged as L grows (i.e., due to a decreasing K/L).
- With CSIT
  - ✓ $C^{D*}_{sum\_o} > C^{C}_{sum\_o}$ even at high SNR (i.e., thanks to better multiuser diversity gains)

Sum Capacity (2)



- Without CSIT
  - ✓ A higher capacity is achieved in the CA case thanks to better diversity gains.
- With CSIT
  - ✓ A higher capacity is achieved in the DA case thanks to better waterfilling gains and multiuser diversity gains.

The average received SNR $\mu_0 = 0\text{dB}$. The number of users K=100.

Average Transmission Power per User

Average Transmission Power per User

- Transmission power of user k: $\overline{P}_k = \dfrac{P_0}{\|\boldsymbol{\gamma}_k\|^2}, \quad k = 1, \ldots, K.$
- Average transmission power per user:

$$\mathcal{P} \triangleq \frac{1}{K}\sum_{k=1}^{K}\overline{P}_k \xrightarrow{K\to\infty} \int_0^{\infty}\frac{P_0}{x} f_{\|\boldsymbol{\gamma}_k\|^2}(x)dx$$

✓ With CA:

- Users are uniformly distributed in the circular cell. BS antennas are co-located at cell center.

$$\|\boldsymbol{\gamma}_k\|^2 = L\rho_k^{-\alpha}$$

$$f_{\rho_k}(x) = \frac{2x}{R^2}$$

$$\Rightarrow \quad \mathcal{P}^C = \frac{2P_0}{\alpha+2}\cdot\frac{R^\alpha}{L}$$

✓ With DA:

- Both users and BS antennas are uniformly distributed in the circular cell.

What is the distribution of $\|\boldsymbol{\gamma}_k\|^2$?

Minimum Access Distance

- With DA, each user has different access distances to different BS antennas. Let

$$d_k^{(1)} \le d_k^{(2)} \le \cdots \le d_k^{(L)}$$

denote the order statistics obtained by arranging the access distances $d_{1,k}, \ldots, d_{L,k}$.

- $\|\gamma_k\|^2 = \sum_{l=1}^{L} \left(d_{l,k}\right)^{-\alpha} > \left(d_k^{(1)}\right)^{-\alpha}$ for L>1.

- An upper-bound for average transmission power per user with DA:

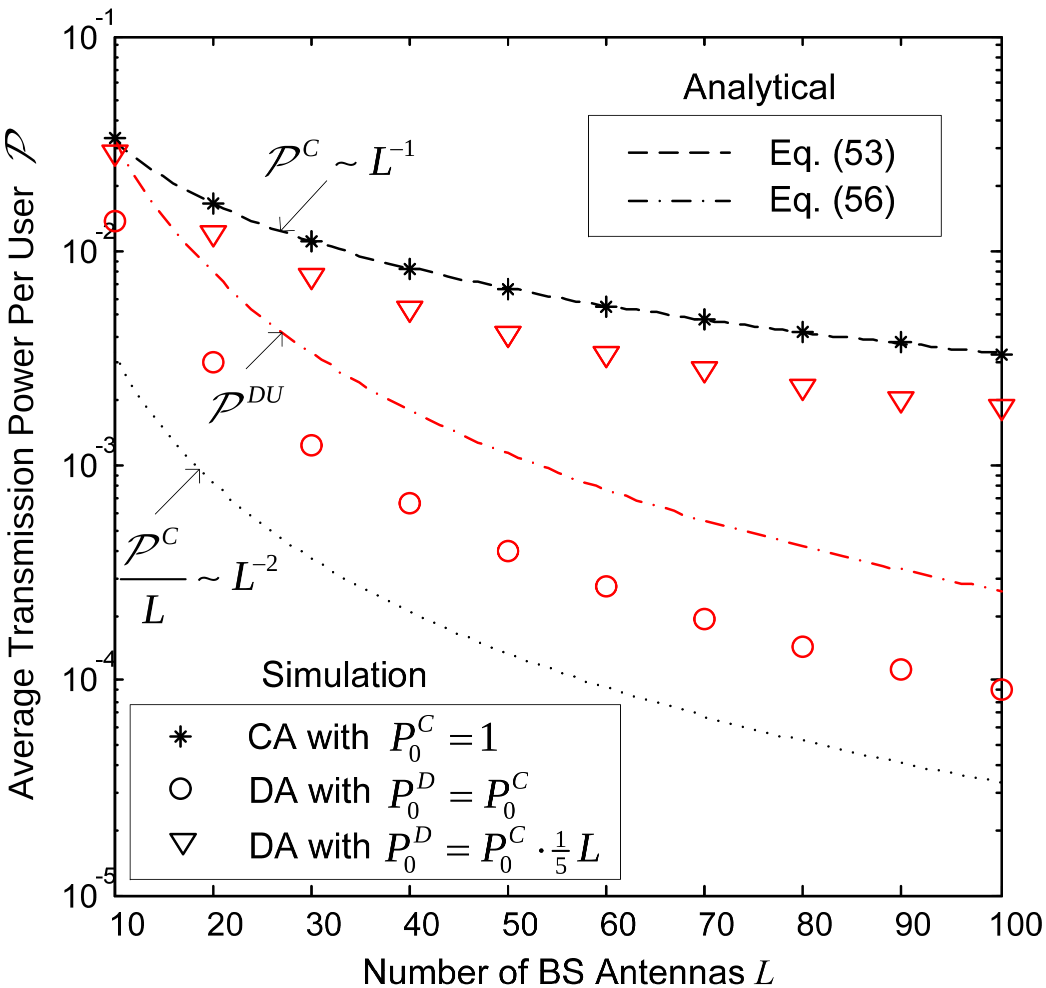
$$\mathcal{P}^D < \mathcal{P}^{DU} = \int_0^R f_{\rho_k}(y) \int_0^{R+y} \frac{P_0}{x^{-\alpha}} \cdot f_{d_k^{(1)}|\rho_k}(x\,|\,y)\,dx\,dy$$

$$f_{\rho_k}(y) = \frac{2y}{R^2}$$

$$f_{d_k^{(1)}|\rho_k}(x\,|\,y) = L(1 - F_{d_{l,k}|\rho_k}(x\,|\,y))^{L-1} f_{d_{l,k}|\rho_k}(x\,|\,y)$$

$$f_{d_{l,k}|\rho_k}(x\,|\,y) = \begin{cases} \frac{2x}{R^2} & 0 \le x \le R - y \\ \frac{2x}{\pi R^2} \arccos \frac{x^2 + y^2 - R^2}{2xy} & R - y < x \le R + y \end{cases}$$

Average Transmission Power per User



Path-loss factor α=4.

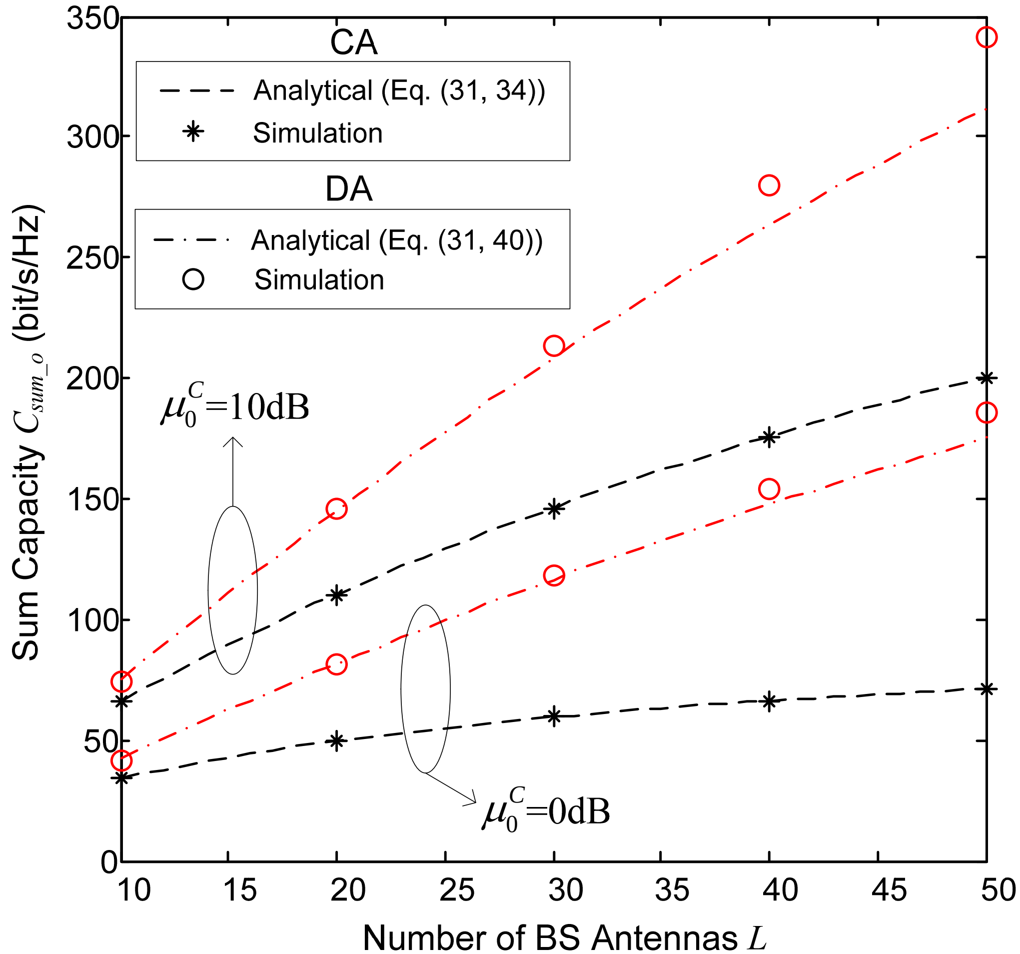
- ✓ CA: $\mathcal{P}^C = O\left(L^{-1}\right)$
- ✓ DA: $\mathcal{P}^{DU} = O\left(L^{-\alpha/2}\right)$

  (path-loss factor α>2)

For given received SNR, a lower total transmission power is required in the DA case thanks to the reduction of minimum access distance.

- ✓ With α=4, $\mathcal{P}^C \approx \mathcal{P}^D$ if $P_0^D = P_0^C \cdot \frac{1}{5} L$.

Sum Capacity without CSIT



$\mu_0^D = \mu_0^C \cdot \frac{1}{5} L$. The number of users K=100.

- For fixed K and μ_0^C ($\mu_0^D = \mu_0^C \cdot \frac{1}{5} L$ such that $\mathcal{P}^C \approx \mathcal{P}^D$)

✓ $C_{sum\_o}^C < L \log_2(1 + \mu_0^C K / L) \xrightarrow{L \to \infty} \mu_0^C K \log_2 e$

✓ $C_{sum\_o}^D = O(L)$

Given the total transmission power, a higher capacity is achieved in the DA case. Gains increase as the number of BS antennas grows.

Summary

- A comparative study on the uplink ergodic sum capacity with co-located and distributed BS antennas is presented by using large-system analysis.
  - A higher sum capacity is achieved in the DA case. Gains increase with the number of BS antennas L.
  - Gains come from 1) reduced minimum access distance of each user; and 2) enhanced channel fluctuations which enable better multiuser diversity gains and waterfilling gains when CSIT is available.
- Implications to cellular systems:
  - With cell cooperation: capacity gains achieved by a DAS over a cellular system increase with the number of BS antennas per cell thanks to better power efficiency.
  - Without cell cooperation: lower inter-cell interference with DA?

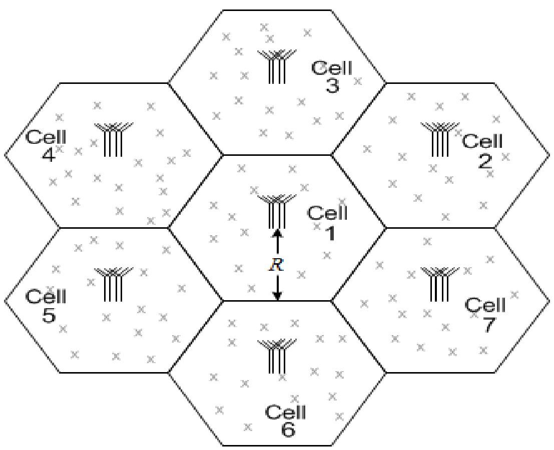
Part II. Multi-Cell Comparison

- System model and preliminary analysis
- Uplink ergodic sum capacity
- Sum rate with orthogonal access

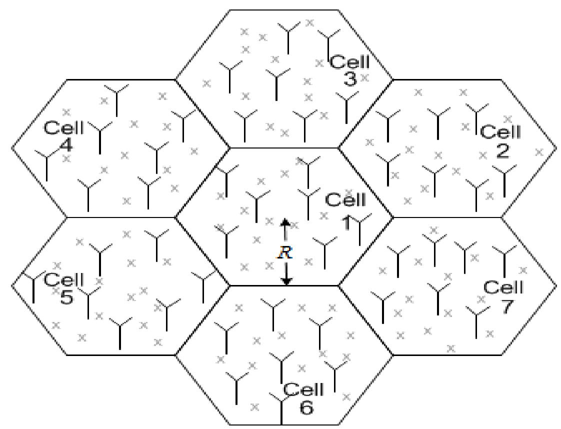
System Model and Preliminary Analysis

Assumptions

- A total number of 7 cells share the same frequency band. No cooperation is adopted among BSs.
- Kc single-antenna users are uniformly distributed within each cell.
- Lc BS antennas are either co-located at the center of each cell, or uniformly distributed over each cell.
- No CSIT.



(a) Co-located Antennas (CA)



(b) Distributed Antennas (DA)

*: user

o: BS antenna

Signal Model

- Received signal of BS 1:

Inter-cell interference

$$\mathbf{y}_{\mathcal{B}_1} = \sum_{k\in\mathcal{K}_1} \mathbf{g}_{\mathcal{B}_1,k} s_k + \sum_{m=2}^{7} \sum_{k\in\mathcal{K}_m} \mathbf{g}_{\mathcal{B}_1,k} s_k + \mathbf{z}_{\mathcal{B}_1}$$

$s_k \sim \mathcal{CN}(0, \bar{P}_k)$: Transmitted signal

$\mathbf{z} \in C^{L_c \times 1}$: Gaussian noise

$z_i \sim \mathcal{CN}(0, N_0), \quad i = 1, \ldots, L_c.$

$\mathbf{g}_k = \boldsymbol{\gamma}_k \circ \mathbf{h}_k$: Channel gain

$\boldsymbol{\gamma}_k \in C^{L_c \times 1}$: Large-scale fading

$\gamma_{i,k} = d_{i,k}^{-\alpha/2}, \quad i = 1, \ldots, L_c.$

$\mathbf{h}_k \in C^{L_c \times 1}$: Small-scale fading

$h_{i,k} \sim \mathcal{CN}(0,1), \quad i = 1, \ldots, L_c.$

- Uplink power control:

For user $k \in \mathcal{K}_m, \quad \bar{P}_k \cdot \left\| \gamma_{\mathcal{B}_m,k} \right\|^2 = P_0, \quad m = 1, \cdots, 7.$

Inter-cell Interference Density

- Inter-cell interference: $\mathbf{u}_{\mathcal{B}_1} = \sum_{m=2}^{7} \sum_{k \in \mathcal{K}_m} \mathbf{g}_{\mathcal{B}_1,k} s_k$

  With a larger number of interfering users, $\mathbf{u}_{\mathcal{B}_1}$ can be modeled as a complex Gaussian random vector with zero mean and covariance matrix $\Sigma_{\mathbf{u}}$, where $\Sigma_{\mathbf{u}}$ is an $L_c \times L_c$ diagonal matrix with diagonal entries

$$\sigma_l^2 = \sum_{m=2}^{7} \sum_{k \in \mathcal{K}_m} \left|\gamma_{l,k}\right|^2 \bar{P}_k$$

- Inter-cell interference density of BS antenna $l \in \mathcal{B}_1$

$$\eta_l^C \triangleq \frac{\sigma_l^2}{6K_c P_0} = \frac{1}{6K_c} \sum_{m=2}^{7} \sum_{k \in \mathcal{K}_m} \frac{\left|\gamma_{l,k}\right|^2}{\left\|\gamma_{\mathcal{B}_m,k}\right\|^2}$$

$$= \frac{1}{6K_c} \sum_{m=2}^{7} \sum_{k \in \mathcal{K}_m} \frac{\left\|\mathbf{r}_l^B - \mathbf{r}_k^U\right\|^{-\alpha}}{\sum_{n \in \mathcal{B}_m} \left\|\mathbf{r}_n^B - \mathbf{r}_k^U\right\|^{-\alpha}}$$

$\mathbf{r}_l^B$: position of BS antenna l; $\mathbf{r}_k^U$: position of user k.

Inter-cell Interference Density

- Theorem 1. The inter-cell interference density of BS antenna $l \in \mathcal{B}_1$ in cellular systems with the CA layout

$$\eta_l^{CC} \overset{K_c \to \infty}{\to} \tilde{\eta}_l^{CC} = \frac{\Upsilon(\alpha)}{L_c}$$

where $\Upsilon(\alpha) = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R \rho^{1+\alpha} (\rho^2 + 4R^2 + 4R\rho \sin\theta)^{-\alpha/2} d\rho d\theta.$

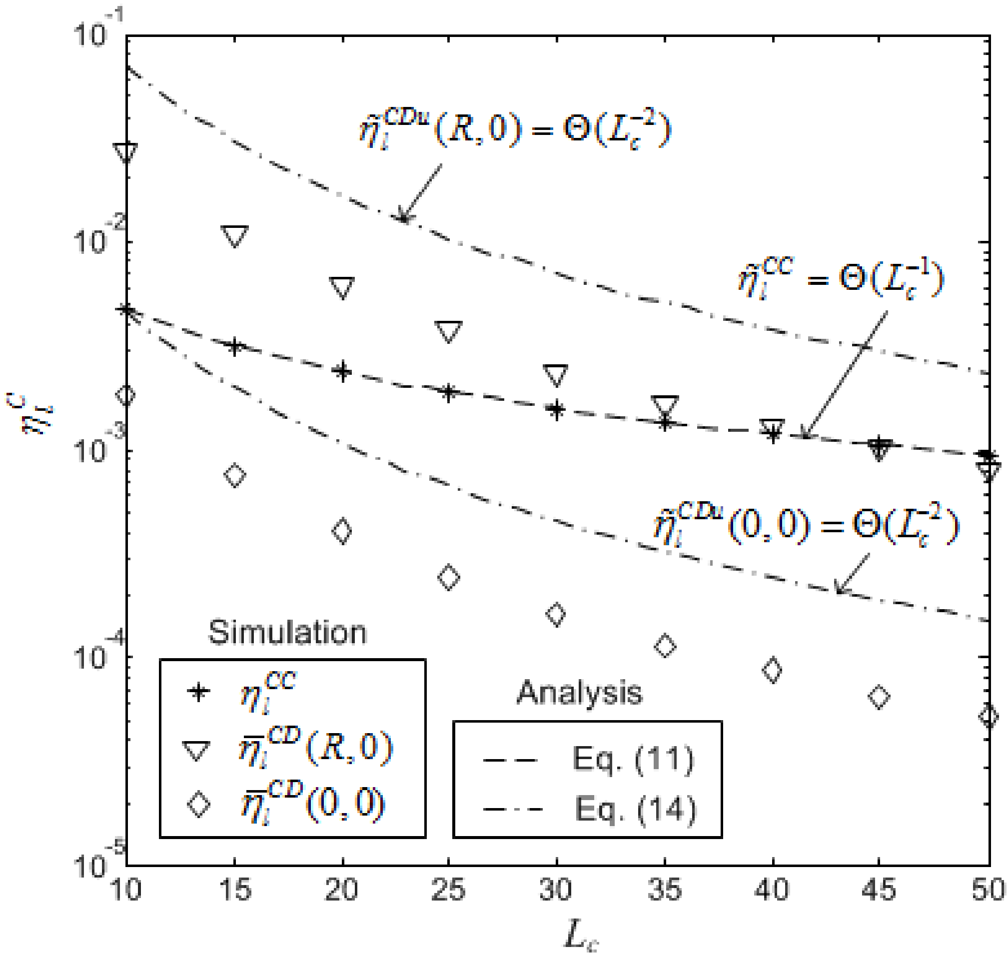
- Theorem 2. The average inter-cell interference density of BS antenna $l \in \mathcal{B}_1$ in cellular systems with the DA layout is upper-bounded by

$$\bar{\eta}_l^{CDu}(r_l, \phi_l) \overset{K_c \to \infty}{\to} \tilde{\eta}_l^{CDu}(r_l, \phi_l) = \frac{1}{6\pi R^2} \sum_{m=2}^{7} \int_0^{2\pi} \int_0^R \rho(\rho^2 + \Delta x_m^2 + \Delta y_m^2 - 2\rho(\Delta x_m \cos\theta + \Delta y_m \sin\theta))^{-\frac{\alpha}{2}} \cdot \int_0^{\rho+R} x^\alpha f_{d_k^{(1)}}(x; \rho, R, L_c) dx d\rho d\theta$$

where $\Delta x_m = r_l \cos\phi_l - 2R\cos(m \cdot \frac{\pi}{3} - \frac{\pi}{2}), \ \ \Delta y_m = r_l \sin\phi_l - 2R\sin(m \cdot \frac{\pi}{3} - \frac{\pi}{2}).$

✓ $\bar{\eta}_l^{CD} = \mathbb{E}_{\{\mathbf{r}_n^B\}_{n \in \cup_{m=2}^{7} \mathcal{B}_m}} \{\eta_l^{CD}\}$

Inter-cell Interference Density



Path-loss factor α=4.

- ✓ CA: $\tilde{\eta}_l^{CC} = \Theta\left(L_c^{-1}\right)$

  DA: $\tilde{\eta}_l^{CDu} = \Theta\left(L_c^{-\alpha/2}\right)$

  (path-loss factor α>2)

- ✓ $\bar{\eta}_l^{CD}$ decreases at a higher rate than η_l^{CC} as the number of BS antennas per cell Lc increases.

- ✓ With the DA layout, the inter-cell interference density significantly varies with the position of the BS antenna.

Uplink Ergodic Sum Capacity

Uplink Ergodic Sum Capacity

- Normalized sum capacity:

$$C_k^C = \frac{1}{K_c}\mathbb{E}_{\mathbf{H}_{\mathcal{B}_1,\mathcal{K}_1}}\left\{\sum_{l\in\mathcal{B}_1}\log_2(1+\mu_l^C\lambda_l^C)\right\}$$

$\{\lambda_l^C\}$: eigenvalues of $\tilde{\mathbf{G}}_{\mathcal{B}_1,\mathcal{K}_1}\tilde{\mathbf{G}}^\dagger_{\mathcal{B}_1,\mathcal{K}_1}$

$\mu_l^C = \frac{1}{N_0/P_0+6K_c\eta_l^C}$: average received SINR of BS antenna $l \in \mathcal{B}_1$

- As $K_c, L_c \to \infty$ and $K_c/L_c \to \upsilon$:

✓ Asymptotic normalized sum capacity with CA:

$$\tilde{C}_k^{CC} = \log_2(1+\tilde{\mu}_l^{CC} - \tfrac{1}{4}\mathcal{F}(\tilde{\mu}_l^{CC},\upsilon)) - \frac{\log_2 e}{4\upsilon\tilde{\mu}_l^{CC}}\mathcal{F}(\tilde{\mu}_l^{CC},\upsilon)$$
$$+\frac{1}{\upsilon}\log_2(1+\upsilon\tilde{\mu}_l^{CC} - \tfrac{1}{4}\mathcal{F}(\tilde{\mu}_l^{CC},\upsilon))$$

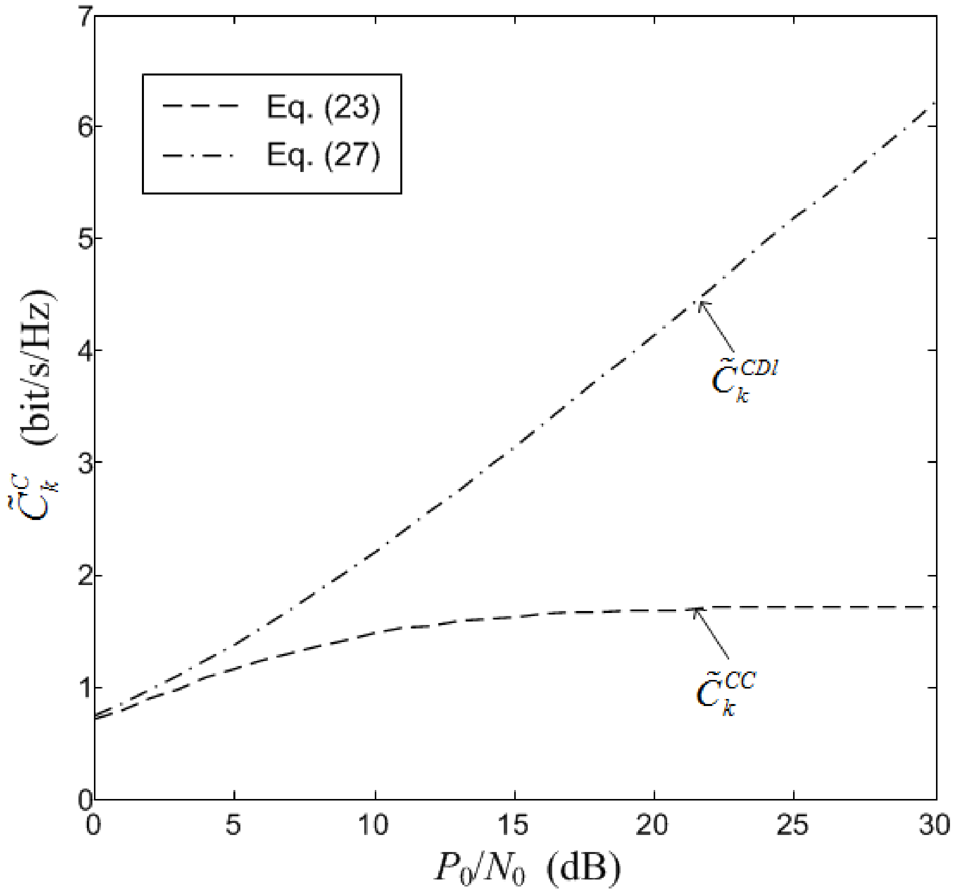
where $\tilde{\mu}_l^{CC} = \frac{1}{N_0/P_0+6\upsilon\Upsilon(\alpha)}$

✓ An asymptotic lower-bound of the normalized sum capacity with DA:

$$\tilde{C}_k^{CDl} = \frac{1}{\upsilon}\int_0^\infty \log_2\left(1+\tilde{\mu}_l^{CDl}x\right)$$
$$\cdot \upsilon e^{-(x+\upsilon)}\sum_{k=0}^\infty \frac{(x\upsilon)^k}{k!(k+1)!}dx$$

where $\tilde{\mu}_l^{CDl} = \frac{P_0}{N_0}$

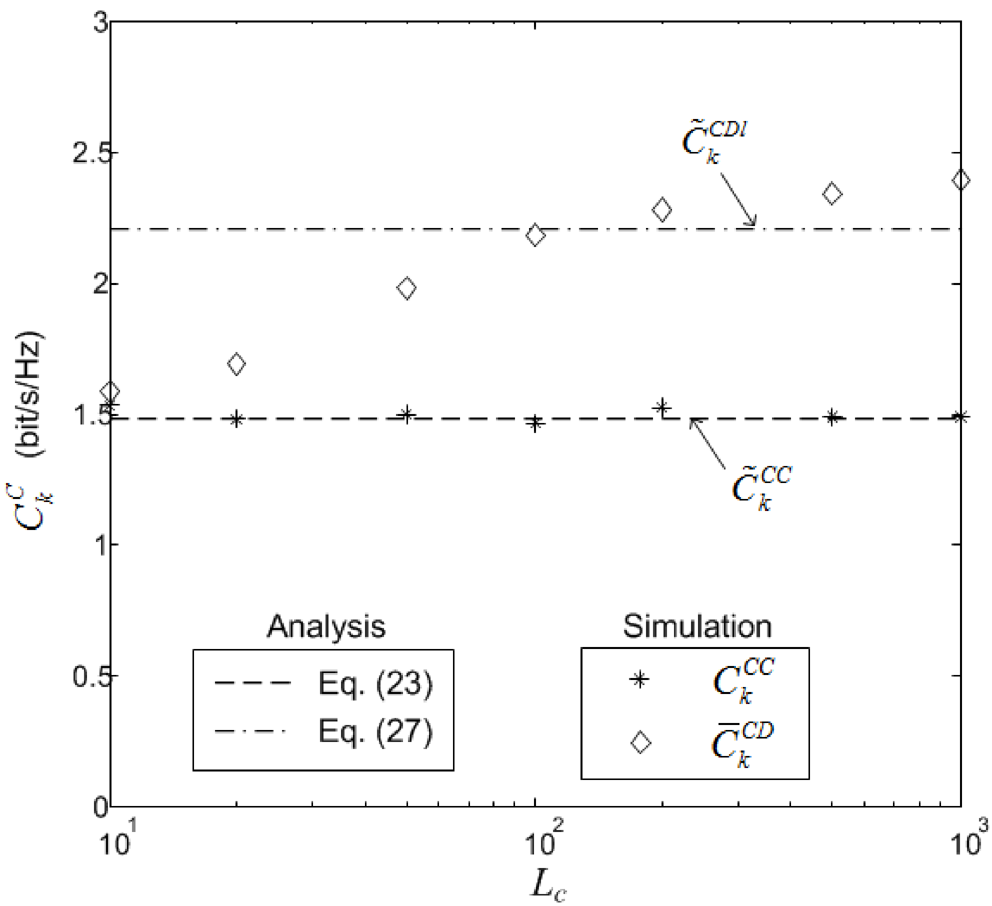
Uplink Ergodic Sum Capacity



Path-loss factor $\alpha = 4$. $\upsilon = 1$.

- As P_0/N_0 increases:
  - ✓ CA: $\tilde{C}_k^{CC}$ converges to a function of υ.
  - ✓ DA: $\tilde{C}_k^{CDI}$ grows unboundedly.
- Substantial gains can be achieved in the DA case owing to the improvement in the inter-cell interference density.

Uplink Ergodic Sum Capacity



Path-loss factor $\alpha = 4$. $\upsilon = 1$.

- Simulation results verify that:
  - ✓ CA: $\tilde{C}_k^{CC}$ serves as a good approximation.
  - ✓ DA: $\tilde{C}_k^{CDl}$ is an asymptotic lower-bound.
- Substantial gains can be achieved in the DA case owing to the improvement in the inter-cell interference density.

Sum Rate with Orthogonal Access

Sum Rate with Orthogonal Access

- Normalized sum rate with orthogonal access:

$$R_k^C = \frac{1}{K_c}\mathbb{E}_{\mathbf{h}_{\mathcal{B}_1,k}}\left\{\log_2\left(1+\sum_{l\in\mathcal{B}_1} K_c\mu_l^C|\tilde{g}_{l,k}|^2\right)\right\}$$

✓ $R_k^C < R_k^{Cu} = \frac{1}{K_c}\log_2(1+\frac{K_cP_0}{N_0}) \to 0$ as $K_c, L_c \to \infty$ and $K_c/L_c \to \upsilon$.

✓ $R_k^C \ll C_k^C$ for large K_c and L_c: A significant tradeoff has to be made between complexity and performance if each cell has a large number of BS antennas and users.

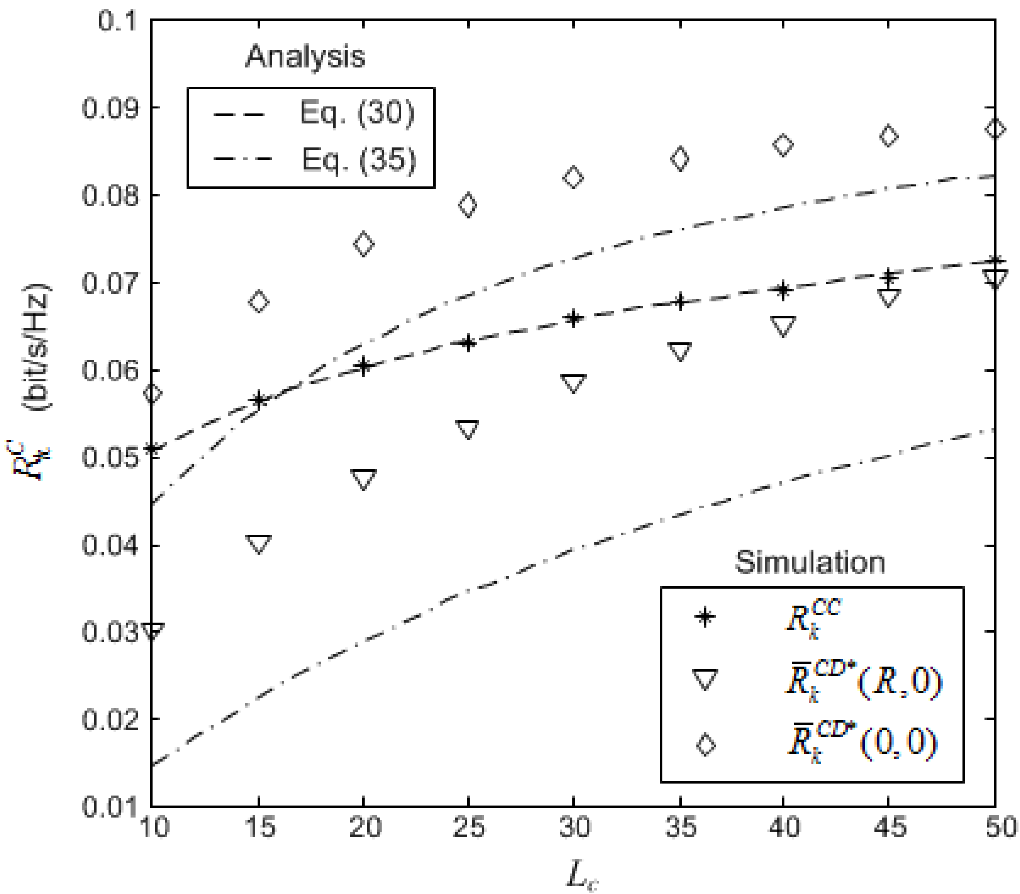
- For large $\frac{P_0}{N_0}$ and K_c:

✓ CA: $R_k^{CC} \approx \frac{1}{K_c}\int_0^\infty \frac{x^{L_c-1}e^{-x}}{(L_c-1)!}\log_2\left(1+\frac{x}{6L_c\tilde{\eta}_l^{CC}}\right)dx$

✓ DA: $\bar{R}_k^{CDl*} \approx \frac{\log_2 e}{K_c}\exp\left(6\tilde{\eta}_l^{CDu}\right)E_1\left(6\tilde{\eta}_l^{CDu}\right)$

✓ Both R_k^{CC} and $\bar{R}_k^{CDl*}$ logarithmically increase with L_c because $\tilde{\eta}_l^{CC} = \Theta(L_c^{-1})$ and $\tilde{\eta}_l^{CDu} = \Theta\left(L_c^{-\alpha/2}\right)$.

Sum Rate with Orthogonal Access



Path-loss factor $\alpha = 4. K_c = 100.$

- On average, a much higher rate is achieved in the DA case when the number of BS antennas per cell Lc is large.
- With the DA layout, the BS antennas at the cell edge suffer from much higher inter-cell interference than those at the cell center, thus leading to degraded rate performance for cell-edge users.

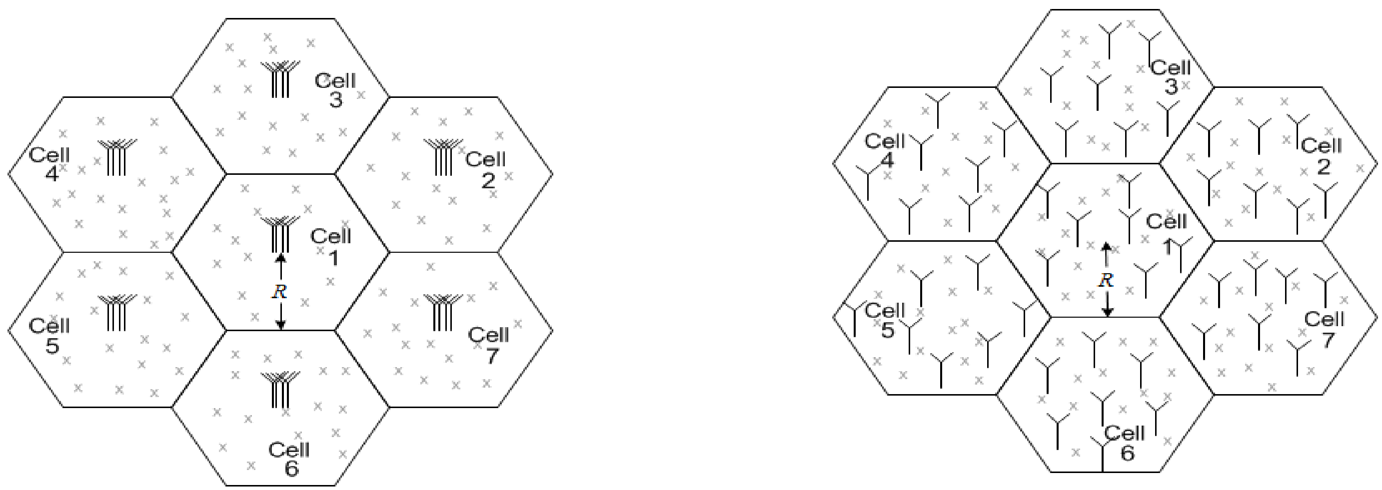
Summary

- In cellular systems, the inter-cell interference density decreases as the number of BS antennas per cell increases, but at different rates for CA and DA.
  - A higher sum capacity is achieved in the DA case owing to the improvement in the inter-cell interference density.
- With the DA layout, the inter-cell interference density significantly varies with the position of the BS antenna.
  - Uplink rate performance is greatly degraded at the cell-edge due to intensified inter-cell interference density.
- When the number of BS antennas per cell is large, there exists a huge gap between the uplink sum capacity and the sum rate with orthogonal access regardless of which BS antenna layout is adopted.

Part III. DAS with Virtual Cells

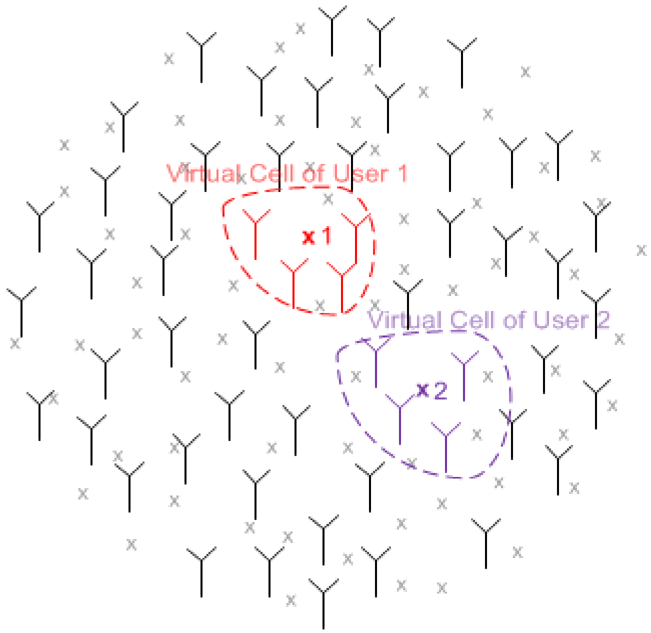
- Virtual cell
- Inter-cell interference density
- Sum capacity and sum rate with orthogonal access

To Cellular or Not to Cellular?



- By splitting a large area into small ones, there are always a certain number of users/BS antennas located at the border and closer to the neighboring cells.
- With distributed BS antennas, the geographic division of cells becomes less justified.

Virtual Cell



- Each user chooses a few surrounding BS antennas as its virtual cell [1-3], i.e., its own serving BS antenna set.
- Different from the conventional cellular structure where cells are divided according to the coverage of BS antennas, here the virtual cell is formed in a user-centric manner.

- For user k, define its virtual cell $\mathcal{V}_k$ as a set of BS antennas with the largest large-scale fading gains to this user.

[1] L. Dai, Researches on Capacity and Key Techniques of Distributed Wireless Communication Systems, *Ph.D. Dissertation*, Tsinghua University, Beijing, Dec. 2002.
[2] L. Dai, S. Zhou, and Y. Yao, ``Capacity with MRC-based Macrodiversity in CDMA Distributed Antenna Systems," in *Proc. IEEE Globecom*, pp. 987--991, Nov. 2002.
[3] L. Dai, S. Zhou, and Y. Yao, ``Capacity Analysis in CDMA Distributed Antenna Systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2613--2620, Nov. 2005.

Signal Model

- Received signal of Virtual Cell $\mathcal{V}_k$:

Inter-cell interference

$$\mathbf{y}_{\mathcal{V}_k} = \sum_{j\in\mathcal{K}_{\mathcal{V}_k}} \mathbf{g}_{\mathcal{V}_k,j} s_j + \sum_{j\notin\mathcal{K}_{\mathcal{V}_k}} \mathbf{g}_{\mathcal{V}_k,j} s_j + \mathbf{z}_{\mathcal{V}_k}$$

$\mathcal{K}_{\mathcal{V}_k}$: The set of users whose signals are jointly processed with user k's at virtual cell $\mathcal{V}_k$.

- $\mathcal{K}_{\mathcal{V}_k}$ is defined as the set of users whose virtual cells are overlapped with $\mathcal{V}_k$, i.e., $j \in \mathcal{K}_{\mathcal{V}_k}$ iff $\mathcal{V}_j \bigcap \mathcal{V}_k \neq \emptyset$.
- Uplink power control: $\bar{P}_k \cdot \left\|\gamma_{\mathcal{V}_k,k}\right\|^2 = P_0.$
- $|\mathcal{V}_k| = V.$

Inter-cell Interference Density

Inter-cell Interference Density

- Inter-cell interference density of BS antenna $l \in \mathcal{V}_k$

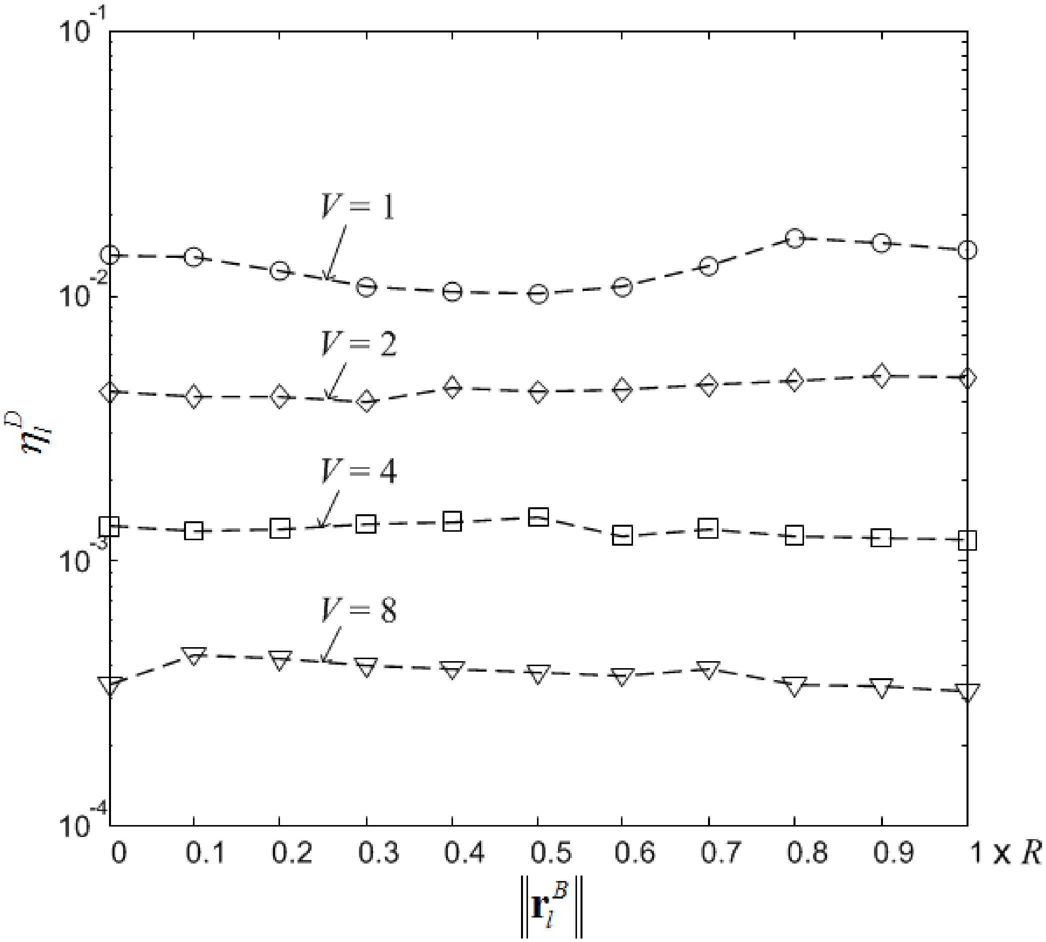
$$\eta_l^D = \frac{1}{K-|\mathcal{K}_{\mathcal{V}_k}|}\sum_{j\notin\mathcal{K}_{\mathcal{V}_k}} \frac{|\gamma_{l,j}|^2}{\left\|\gamma_{\mathcal{V}_j,j}\right\|^2} = \frac{1}{K-|\mathcal{K}_{\mathcal{V}_k}|}\sum_{j\notin\mathcal{K}_{\mathcal{V}_k}} \frac{\left\|\mathbf{r}_l^B-\mathbf{r}_j^U\right\|^{-\alpha}}{\sum_{n\in\mathcal{V}_j}\left\|\mathbf{r}_n^B-\mathbf{r}_j^U\right\|^{-\alpha}}$$

Compared to the cellular system:

$$\eta_{l\in\mathcal{B}_1}^C = \frac{1}{K-|\mathcal{K}_1|}\sum_{j\notin\mathcal{K}_1} \frac{\left\|\mathbf{r}_l^B-\mathbf{r}_j^U\right\|^{-\alpha}}{\sum_{n\in\mathcal{B}_m}\left\|\mathbf{r}_n^B-\mathbf{r}_j^U\right\|^{-\alpha}_{j\in\mathcal{K}_m}}$$

- Key difference of η_l^D and η_l^C lies in the division of cells:
  - ✓ $\mathcal{K}_1$ consists of users who fall into the cell centered at BS 1.
  - ✓ $\mathcal{K}_{\mathcal{V}_k}$ consists of users whose virtual cells are overlapped with $\mathcal{V}_k$.

Inter-cell Interference Density



Path-loss factor α=4. Total number of users K=700. Total number of BS antennas L=70.

Average Inter-cell Interference Density

- Average inter-cell interference density

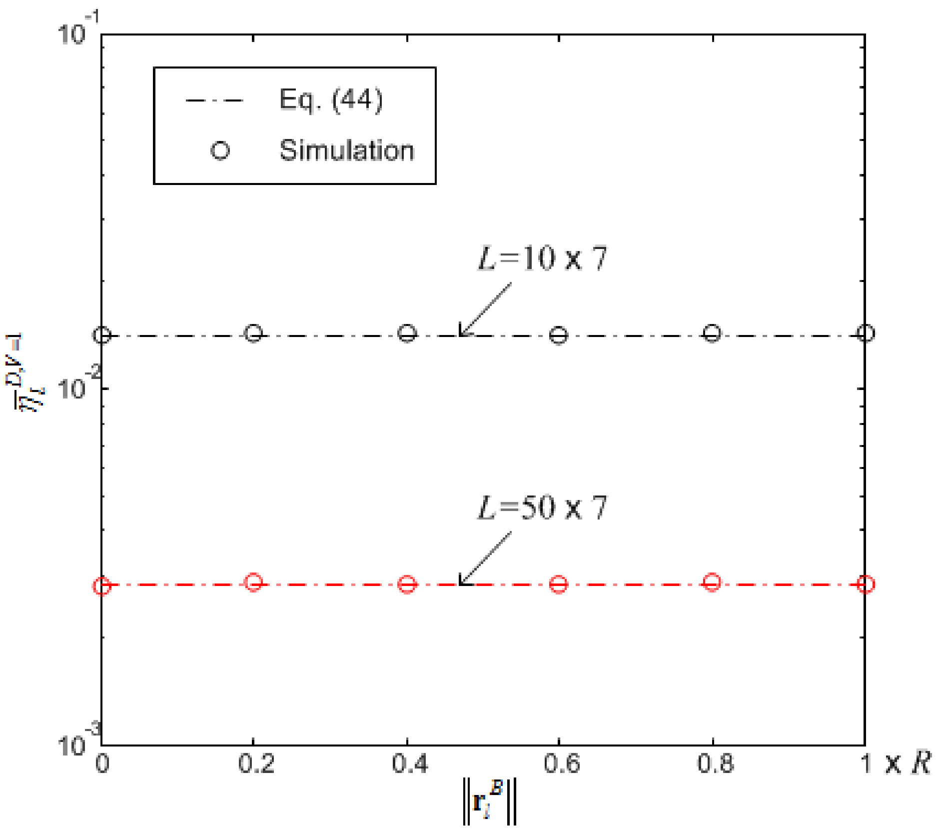
$$\bar{\eta}_l^D = \mathbb{E}_{\{\mathbf{r}_n^B\}_{n\in\mathcal{L}\backslash\mathcal{V}_k}}\{\eta_l^D\}$$

- Theorem 3. The average inter-cell interference density of BS antenna l at $\mathbf{r}_l^B = (r_l, \phi_l)$ in DASs with V=1 is

$$\bar{\eta}_l^{D,V=1} \overset{K\to\infty}{\to} \tilde{\eta}_l^{D,V=1}(r_l) = \frac{1}{\pi R_0^2}\int_0^{2\pi}\int_0^{R_0}\rho(\rho^2+r_l^2-2\rho r_l\cos\theta)^{-\frac{\alpha}{2}} \cdot \int_0^{\sqrt{\rho^2+r_l^2-2\rho r_l\cos\theta}} x^\alpha f_{d_k^{(1)}}(x;\rho,R_0,L-1)dxd\rho d\theta.$$

✓ $\tilde{\eta}_l^{D,V=1}(0) = \frac{2}{R_0^2}\int_0^{R_0}\rho^{1-\alpha}\int_0^{\rho}x^\alpha f_{d_k^{(1)}}(x;\rho,R_0,L-1)dxd\rho = \Theta(L^{-1})$

Average Inter-cell Interference Density



Path-loss factor α=4.

- ✓ $\tilde{\eta}_l^{D,V=1} = \Theta\left(L^{-1}\right)$

  For cellular systems:

  $\tilde{\eta}_l^{CC} = \Theta\left(L_c^{-1}\right)$

  $\tilde{\eta}_l^{CDu} = \Theta\left(L_c^{-\alpha/2}\right)$

- ✓ Compared to cellular systems with the DA layout, the average inter-cell interference density $\bar{\eta}_l^{D,V=1}$ with V=1 has a smaller decreasing rate with the number of BS antennas L because the interfering area increases with L.

- ✓ The average number of intra-cell users $\bar{\kappa}_l^{V=1} \approx K / L.$

Sum Capacity and Sum Rate with Orthogonal Access

Uplink Ergodic Sum Capacity with V=1

- Normalized sum capacity:

$$C_k^D = \frac{1}{\kappa_l}\int_0^\infty \frac{x^{\kappa_l-1}e^{-x}}{(\kappa_l-1)!}\log_2\left(1+\mu_l^D x\right)dx$$

κ_l : number of intra-cell users

$\mu_l^D = \frac{1}{N_0/P_0+(K-\kappa_l)\eta_l^{D,V=1}}$: average received SINR of BS antenna $l \in \mathcal{V}_k$

- Lower-bound for the average normalized sum capacity $\bar{C}_k^D$:

$$\bar{C}_k^{Dl} = \frac{1}{\bar{\kappa}_l^{V=1}}\int_0^\infty \frac{x^{\bar{\kappa}_l^{V=1}-1}e^{-x}}{(\bar{\kappa}_l^{V=1}-1)!}\cdot\log_2\left(1+\frac{1}{N_0/P_0+K\bar{\eta}_l^{D,V=1}}x\right)dx$$

✓ For large $\frac{P_0}{N_0}$, K and L :

$$\bar{C}_k^{Dl} \approx \frac{L}{K}\int_0^\infty \frac{x^{K/L-1}e^{-x}}{(K/L-1)!}\log_2\left(1+\frac{1}{K\tilde{\eta}_l^{D,V=1}(0)}x\right)dx$$

Sum Rate with Orthogonal Access with V=1

- Normalized sum rate with orthogonal access:

$$R_k^D = \frac{\log_2 e}{\kappa_l} \exp\left(\frac{1}{\kappa_l \mu_l^D}\right) E_1\left(\frac{1}{\kappa_l \mu_l^D}\right)$$

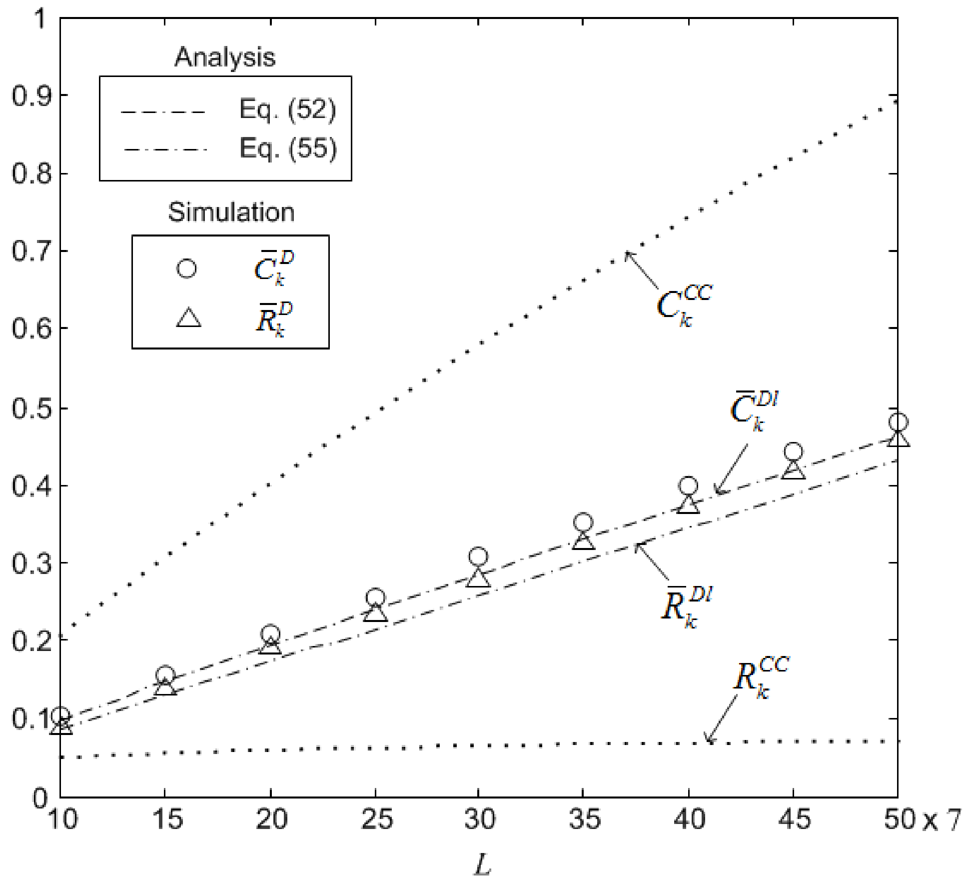
- Lower-bound for the average sum rate $\bar{R}_k^D$:

$$\bar{R}_k^{Dl} = \frac{\log_2 e}{\bar{\kappa}_l^{V=1}} \exp\left(\frac{N_0/P_0 + K\bar{\eta}_l^{D,V=1}}{\bar{\kappa}_l^{V=1}}\right) \cdot E_1\left(\frac{N_0/P_0 + K\bar{\eta}_l^{D,V=1}}{\bar{\kappa}_l^{V=1}}\right)$$

  ✓ For large $\frac{P_0}{N_0}$, K and L :

$$\bar{R}_k^{Dl} \approx \frac{L \log_2 e}{K} \exp(L\tilde{\eta}_l^{D,V=1}(0)) E_1(L\tilde{\eta}_l^{D,V=1}(0))$$

Sum Capacity and Sum Rate



Path-loss factor $\alpha = 4$. $K = 700$.

- Both $\bar{C}_k^D$ and $\bar{R}_k^D$ linearly increase with the number of BS antennas L.
- A small gap between $\bar{C}_k^D$ and $\bar{R}_k^D$ is observed ---- by the use of virtual cell, the average number of users served by each BS antenna decreases as L increases!
- With V=1, the DAS suffers from severe inter-cell interference.

Implications to Cutting-edge Cellular Technologies

- Cellular Systems with Small Cells
  - Equivalent to a DAS with V=1.
  - Sum capacity is lower than that of cellular systems with co-located BS antennas due to strong inter-cell interference.
  - To improve the sum capacity:
    - Cooperation should be adopted among BSs, and
    - The cooperative BS set should be formed in a user-centric manner ---- virtual cell.

Implications to Cutting-edge Cellular Technologies

- pCell Technology of Artemis Networks
  - Selling point: the average user rate ↓ as the number of users ↑ in cellular networks, while with pCell Technology the average user rate remains constant as the number of users ↑.

With orthogonal access:

What's the secret?

✓ Cellular networks

$$R_k^C < R_k^{Cu} = \frac{1}{K_c}\log_2(1+\frac{K_cP_0}{N_0}) \rightarrow \underline{0} \text{ as } K_c, L_c \rightarrow \infty \text{ and } K_c/L_c \rightarrow \upsilon.$$

CA: $$R_k^{CC} \approx \frac{1}{K_c}\int_0^{\infty}\frac{x^{L_c-1}e^{-x}}{(L_c-1)!}\log_2\left(1+\frac{x}{6L_c\tilde{\eta}_l^{CC}}\right)dx$$

✓ DASs with virtual cells

$$\bar{R}_k^{Dl} \approx \frac{L\log_2 e}{K}\exp(L\tilde{\eta}_l^{D,V=1}(0))E_1(L\tilde{\eta}_l^{D,V=1}(0)) \rightarrow \underline{\frac{1}{\upsilon}eE_1(1)\log_2 e}$$

as $K, L \rightarrow \infty$ and $K/L \rightarrow \upsilon.$

Summary

- In a DAS, if each user chooses a few surrounding BS antennas to form its virtual cell:
  - A uniform inter-cell interference density can be achieved.
  - Each BS antenna serves a declining number of users as the density of BS antennas increases, indicating good network scalability.
  - A small gap between the sum capacity and the sum rate with orthogonal access with V=1 is observed, which is in sharp contrast to cellular systems where a significant tradeoff between performance and complexity has to be made when the number of BS antennas is large.

- The size of virtual cell V is a crucial system parameter.
  - How does the sum capacity vary with V?

Thank you!

Any Questions?

Slides can be downloaded at my homepage: http://www.ee.cityu.edu.hk/~lindai