Hermitian Precoding for Distributed MIMO Systems

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Abstract—In this paper, we consider a distributed MIMO communication network in which multiple transmitters cooperatively send common messages to a single receiver. In this scenario, it is usually costly to acquire full channel state information at the transmitters (CSIT), i.e., every transmitter perfectly knows the overall channel state information (CSI) of the network. Hence, we assume individual CSIT (I-CSIT), i.e., each transmitter only knows its own CSI. We propose a novel precoding technique, named Hermitian precoding, to enhance the system performance under the constraint of I-CSIT. We show that the proposed scheme can perform close to the system capacity with full CSIT. This reveals that the amount of CSI required at the transmitters can be significantly reduced without considerably compromising performance.

I. INTRODUCTION
Consider a distributive communication system in which multiple transmitters located at different places cooperatively send common messages to a single receiver. This scenario arises, e.g., in the following two applications.

- In a cooperative cellular system, several adjacent base stations (BSs) simultaneously serve a mobile terminal (MT) in the downlink transmission [1]. Such a scheme is referred to as coordinated multi-point (CoMP) [2]-[4] in long term evolution (LTE) standardization documents [5].
- In a parallel relaying system, multiple relays decode and forward (DF) messages from a common source to a common destination [6]-[8].

Intensive work has been focused on this scenario under various assumptions on the availability of channel state information at the transmitters (CSIT). In [7]-[8] no CSIT is assumed and distributed space-time coding is proposed for efficient transmission. In [1]-[5] full CSIT is assumed, i.e., every transmitter perfectly knows all the channel state information (CSI). With full CSIT, beamforming is performed jointly among the transmitters to enhance the system performance significantly.

However, in practice, it is very costly to acquire full CSI at all transmitters. This problem is especially serious in multiple-input multiple-output (MIMO) systems in which multiple antennas are employed in transmission. The resulting overall system may have a very large dimension. Distributing all these coefficients to all transmitters can be a daunting task.

In this paper, we study the situation of individual CSIT (I-CSIT), in which each transmitter has the CSI of its own link but not other’s. This setting can significantly decrease the overhead for distributing CSI among transmitters. We develop a Hermitian precoding technique for efficient transmission with I-CSIT. We show analytically that the proposed precoding technique is optimal in the sense of the rate maximization under the I-CSIT assumption, provided that the other transmitters are a priori fixed to the same precoding structure. Numerical results demonstrate that the proposed scheme with I-CSIT performs close to the system capacity with full CSIT.

II. SYSTEM MODEL
A. System Model
We describe the system model for the distributed MIMO system, in which $K$ transmitters send common messages to a single receiver. Each transmitter $k$, $k = 1, \ldots, K$, is equipped with $N$ antennas and the receiver is equipped with $M$ antennas. The received signal at the receiver can be expressed as

$$r = \sum_{k=1}^{K} H_k x_k + n$$

where $r$ is an $M$-by-$1$ received signal vector, $H_k$ is an $M$-by-$N$ channel transfer matrix for the link between the transmitter $k$ and the receiver, $x_k$ is an $N$-by-$1$ signal vector sent by the transmitter $k$, and $n \sim \mathcal{CN}(0, \sigma^2 I)$ is an additive white Gaussian noise vector. Moreover, each transmitter $k$ has an individual power constraint of

$$\mathbb{E}\left[\|x_k\|^2\right] \leq P_k, \quad k = 1, \ldots, K,$$

where $P_k$ is the maximum transmission power of transmitter $k$, $\|\cdot\|$ stands for the Euclidean norm of a vector. Perfect CSI at the receiver is always assumed, i.e., the receiver knows all $H_k$.

B. Haar Matrix

Definition 1: A random square matrix $U$ is called a Haar matrix if it is uniformly distributed on the set of all the unitary matrices (of the same size as $U$) [9].

Property 1: [p.25, 9] A Haar matrix $U$ is unitarily invariant, i.e., for any fixed unitary matrix $T$ independent of $U$, the statistical behavior of $TU$ or $UT$ is the same as $U$.

Property 2: [p.25, 9] Consider a random Hermitian matrix $B$ given by $B = UAU^H$, with $U$ being a Haar matrix and $A$ being a random diagonal matrix. Assume that $U$ and $A$ are independent of each other. Then $B$ is unitarily invariant, i.e., for any unitary matrix $T$ independent of $B$, $TTB$ has the same distribution as $B$. Also, $B^*$ has the same distribution as $B$.

1 A Haar matrix is also called an isotropic matrix in [10]. More detail about Haar matrices can be found in [9], [11].
C. Properties of \{H_k\}

Now we describe some useful properties of \{H_k\}. Throughout this paper, the entries of \(H_k\), \(k = 1, \ldots, K\), are assumed to be i.i.d. random variables drawn from \(CN(0, \sigma_k^2)^2\) Let the singular value decomposition (SVD) of \(H_k\) be

\[
H_k = U_k D_k V_k^H,
\]

where \(U_k\) and \(V_k\) are unitary matrices, and \(D_k\) is an \(M\)-by-\(N\) diagonal matrix\(^3\) with non-negative diagonal elements arranged in the descending order. Then

Property 3: The matrices \(U_k\) and \(D_k\) are independent.

Property 4: \(U_k\) is a Haar matrix [9] [11].

Based on Property 4, the properties of Haar matrix in Subsection B hold for \(U_k\).

III. PROBLEM FORMULATION WITH I-CSIT

In this section, we consider the situation of I-CSIT in which each transmitter \(k\) only knows its own channel \(H_k\).

A. Modeling of the Transmit Signals

Recall that all the transmitters share the same message to be sent to a single receiver. Yet, the transmitters still have the freedom to use the same codebook or different codebooks in channel coding. Hence the transmitted signals \(\{x_k\}\) may be either correlated or uncorrelated. We use \(c_k\) to represent the correlated signal component shared by all the transmitters, and \(e_i\) to represent the uncorrelated signal component for each transmitter \(k\) (for \(k = 1, \ldots, K\), where \(\{e_i\}_{i=0}^{C}\) is \(M\)-by-1 random vectors with the entries independently drawn from \(CN(0, 1)\). By definition, \(E[c_k c_k^H] = \mathbb{I}\) and \(E[e_j e_j^H] = 0\), \(\forall k, j = 0, 1, \ldots, K, k \neq j\). Then, the transmitted signal of transmitter \(k\) can be expressed as

\[
x_k = F_k e_k + G_k e_k, \quad k = 1, \ldots, K,
\]

where \(F_k\) and \(G_k\) are \(N\)-by-\(M\) precoding matrices that is adaptive to the channel. With (4), we rewrite (1) as

\[
r = \sum_{k=1}^{K} H_k (F_k e_k + G_k e_k) + n.
\]

We focus on the design of \(\{F_k\}\) and \(\{G_k\}\) in the following.

B. Problem Formulation

We are interested in the following distributive precoder:

\[
F_k = f_k(H_k), \quad G_k = g_k(H_k), \quad k = 1, \ldots, K
\]

where “distributive” means that the precoders of each transmitter \(k\) only depends on the local CSI of transmitter \(k\), i.e., \(f_k(\cdot)\) and \(g_k(\cdot)\) are functions of \(H_k\) only (but not functions of \(H_{k'}, k' \neq k\)). This ensures that the precoding scheme in (6) can be realized under the I-CSIT assumption.

The distributive precoding functions \(\{f(\cdot)\}\) and \(\{g(\cdot)\}\) should be optimized to maximize the average achievable rate of the system in (5) under the individual transmitter power constraints in (2). This problem can be formulated as

\[
\text{max} \ E \left[ \log \det \left( I + \frac{1}{\sigma^2} \left( \sum_{k=1}^{K} H_k f_k(H_k) + \sum_{k=1}^{K} |H_k| g_k(H_k) \right) \right) \right]
\]

s.t. \(\text{tr} \left( |f_k(H_k)|^2 + |g_k(H_k)|^2 \right) \leq P_k, \forall H_k, k = 1, 2, \ldots, K\)

where \(|A|^2\) is a shorthand of \(AA^H\) for any matrix \(A\). The expectation in (7) is taken over the joint distribution of \(\{H_k\}\) and the power constraint is imposed on every possible realization of \(H_k\), \(k = 1, \ldots, K\).

C. Locally Optimal Solution

The problem in (7) is difficult to handle in general since jointly optimizing \(\{f(\cdot)\}\) and \(\{g(\cdot)\}\) is required. In what follows, we focus on a locally optimal solution defined below.

Definition 2: A set of precoding functions \(\{f(\cdot)\}\) and \(\{g(\cdot)\}\) is locally optimal if, \(\forall k = 1, \ldots, K\),

\[
\arg \max_{\{F_k, G_k\}} \ E \left[ \log \det \left( \left| H_k F_k + \sum_{k=1}^{K} |H_k| f_k(H_k) \right|^2 + |H_k G_k|^2 + \sum_{k=1}^{K} |H_k| g_k(H_k) \right) \right] \right] H_k
\]

where the expectation is taken over \(\{H_k\}\) for \(k' = 1, \ldots, K, k' \neq k\).

In other words, any particular transmitter \(k\) deviating from its own precoding strategy leads to performance degradation.

IV. HERMITIAN PRECODING

A. Hermitian Precoding

Recall the SVD of \(H_k\) in (3) as \(H_k = U_k D_k V_k^H\). As \(U_k\) and \(V_k\) are unitary, the precoders in (6) can always be written in the following form: for any \(k = 1, \ldots, K\),

\[
F_k = f(H_k) = V_k W_k U_k^H, \quad G_k = g(H_k) = V_k^H \Sigma_k \Sigma_k^H
\]

where \(W_k\) and \(\Sigma_k\) can be arbitrary \(N\)-by-\(M\) matrices except that they only depend on \(H_k\) (due to the I-CSIT assumption). We have the following result with the proof given in Appendix.

Theorem 1: The optimal \(\{F_k\}\) and \(\{G_k\}\) to (8) are given by (9) with \(\{W_k\}\) and \(\{\Sigma_k \Sigma_k^H\}\) being real and diagonal matrices.

Theorem 1 gives the optimal structure of the precoders for the problem defined in (8).

B. Further Discussions

With Theorem 1, we still need to optimize \(\{W_k\}\) and \(\{\Sigma_k\}\) for power allocation based on available CSI. Due to space limitation, we only consider a simple choice of \(W_k\) and \(\Sigma_k\) as

\[
W_k = \sqrt{P_k/M} I \quad \text{and} \quad \Sigma_k = 0.
\]

More advanced power allocation strategies are discussed in [12]. The reason for \(\Sigma_k = 0\) is intuitively explained as follows.

Consider the multiple-input single-output (MISO) case (i.e., \(N \geq M = 1\)). The channels \(\{H_k\}\) reduce to vectors \(\{h_k\}\) and allow the expression of \(H_k = \|h_k\| v_k^H\), where \(v_k = h_k^H/\|h_k\|\), for \(k = 1, \ldots, K\). Then, it is easy to see that the optimal precoding strategy to the problem in (7) is given by

\[
F_k = f(h_k) = \sqrt{P_k} v_k, \quad \text{and} \quad G_k = g(h_k) = 0, \quad k = 1, \ldots, K.
\]

Correspondingly, the received signal can be expressed as

\[\text{max} \ E \left[ \log \det \left( I + \frac{1}{\sigma^2} \left( \sum_{k=1}^{K} H_k f_k(H_k) + \sum_{k=1}^{K} |H_k| g_k(H_k) \right) \right) \right]

s.t. \(\text{tr} \left( |f_k(H_k)|^2 + |g_k(H_k)|^2 \right) \leq P_k, \forall H_k, k = 1, 2, \ldots, K\)
\[ r = \sum_{k=1}^{K} \sqrt{P_k} \| h_k \|_2 c_k + n \]  

(12)

where, \( c_k \) and \( n \) are respectively the scalar versions of \( r \), \( c_k \), and \( n \). In (12), we see that the signals from different transmitters are coherently superimposed on each other. Based on the above observation, it is reasonable to expect that \( G_k = 0 \) (or \( \Sigma = 0 \)) is also a good choice for the MIMO cases.

From the above, our proposed precoding strategy is

\[ F_k = f_k (H_k) = V_k W_k U_k^H \]  

(13)

With (13), the received signal in (5) can be rewritten as

\[ r = \sum_{k=1}^{K} U_k D_k W_k U_k^H c_k + n. \]  

(14)

Clearly, each term \( U_k D_k W_k U_k^H \) in (14) is a positive semi-definite Hermitian matrix. From Weyl Theorem [13], for any positive semi-definite matrices \( A \) and \( B \), we have \( A \) (or \( B \)) \( \leq A + B \). Hence, our proposed precoding strategy in (13) increases the eigen-values of the overall equivalent channel, which provides an intuitive explanation of the related gain. This can be seen as the effect of coherent transmission.

V. NUMERICAL RESULTS

In this section, numerical results are provided to evaluate the performance of the proposed Hermitian precoding technique in distributed MIMO channels. Some alternative schemes are listed below for comparison. For simplicity of discussion, we always assume \( N = M \).

(i) Hermitian Precoding with Equal Power Allocation (HP-EPA):

\[ F_k = \sqrt{P_k} MV_k U_k^H \]  

and \( G_k = 0 \).

This is obtained by substituting \( W_k = \sqrt{P_k} / M I \) in (13).

(ii) Hermitian Precoding with Channel Inverse (HP-CI):

\[ F_k = \beta V_k D_k^{-1} U_k^H \]  

and \( G_k = 0 \).

This is obtained by substituting \( W_k = \beta D_k^{-1} \) in (13) with \( \beta \) a scalar chosen to meet the power constraint.

(iii) No CSIT with fully Correlated Signaling (No-CSIT-CS):

\[ F_k = \sqrt{P_k} / M I \]  

and \( G_k = 0 \).

(iv) No CSIT with Independent Signaling (No-CSIT-IS):

\[ F_k = 0 \]  

and \( G_k = \sqrt{P_k} / M I \).

(v) Individual Water-Filling (IWF):

\[ F_k = V_k W_k \]  

and \( G_k = 0 \).

Here \( V_k \) is obtained from the SVD \( H_k = U_k D_k V_k^H \) and \( W_k \) (that is diagonal) is obtained by water-filling over \( D_k \). In other words, IWF applies standard water-filling [14] at each individual transmitter.

Note that the only difference between (i) and (ii) is the choice of \( W_k \). They both have the Hermitian precoding structure. On the other hand, the choices (iii)-(v) do not involve the Hermitian precoding structure, and thus lack the coherent transmission effect discussed in Section IV.C.

Fig.1 illustrates the performance of various precoding schemes with \( K = 3, N = 16, \) and \( M = 8 \). It is seen that HP-EPA performs very close to the upper bound obtained by assuming full CSIT (as studied in [3]). This implies that the potential performance loss due to the I-CSIT assumption is marginal. We also see that HP-EPA significantly outperforms other alternatives. Note that the performance of No-CSIT-CS, No-CSIT-IS and IWF is relatively poor in Fig. 1. The reason is that they cannot achieve the coherent effect provided by the Hermitian precoding structure.

Fig.2 studies the impact of the numbers of transmitters on the performance of Hermitian precoding. The numbers of antennas are set to \( M = 4 \) and \( N = 16 \). The performance curves with full-CSIT and No-CSIT-IS are also included for comparison. Again, the gap between the Hermitian precoding scheme and the full-CSIT capacity is very small. Compared with the curve of No-CSIT-IS, the power gain of the proposed precoder is about 9 dB for \( K = 2 \), 12 dB for \( K = 4 \), 15 dB for \( K = 8 \), and 17 dB for \( K = 12 \). Roughly speaking, this power gain increases linearly with \( K \).

VI. CONCLUSIONS

In this paper, we consider efficient transmission over distributed MIMO channels with I-CSIT. A Hermitian precoding technique is proposed to exploit the available CSIT. We show analytically that the proposed Hermitian precoding technique is optimal for each transmitter under the I-CSIT
assumption when the other transmitters are a priori fixed to the same precoding structure. Numerical results show that the proposed Hermitian precoding scheme with I-CSIT suffers marginal performance loss compared with the system with full CSIT in various settings. This indicates that the amount of CSI required at the transmitter sides can be significantly reduced without considerably compromising the system performance.

APPENDIX: PROOF OF THEOREM 1

For notational simplicity, we only consider the case of $K = 2$. The extension of the proof to the case of $K > 2$ is trivial. Since all transmitters experience the same channel distribution, these two transmitters are symmetric. Then, the problem in (8) reduces to

$$
\max_{\{A, W_i\}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( [HF + H_F F_i] + [HG_i F + H_{G_i} F_i] \right) \right) H_i \right)
$$

where $F_2 = f_2(H_2) = V_2 W_2 U_2^H$, and $G_2 = g_2(H_2) = V_2 \Sigma_2 U_2^H$. The expectation is taken over $H_2$. Due to the local optimality defined in Section III.C that other transmitters are a priori fixed to the desired structure, $W_2$ and $\Sigma_2$ are real diagonal matrices and independent of $V_2$ and $U_2$. Furthermore, note that $H_{F_i} = U_i D_i U_i^H$ and $H_{G_i} = U_i D_i U_i^H$ are Hermitian, and that $U_i$ is a Haar matrix. From Property 2 in Section 11.B, $V_i H_i U_i$ and $U_i H_i G_i U_i$ have the same distribution as $H_{F_i}$ and $H_{G_i}$, respectively. Thus, with the individual CSI (i.e., $H_i = U_i D_i V_i^H$) of the transmitter 1, we rewrite the objective function in (15) as

$$
\mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( |D_i| H_{F_i} A^H W_i^H |D_i| + |D_i F_i^H G_i F_i| + |D_i H_{F_i}^H H_i| \right) \right)
$$

where $V_i = V_i H_i^H U_i$ from (9) and $A_i$ is defined as

$$
A_i = V_i H_{F_i}^H G_i F_i W_i^H V_i
$$

From (16) and (17), the problem in (15) can be rewritten as an optimization problem for $A_i$ and $W_i$ as

$$
\max_{\{A_i, W_i\}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( |D_i| H_{F_i} A^H W_i^H |D_i| + |D_i F_i^H G_i F_i| + |D_i H_{F_i}^H H_i| \right) \right)
$$

where $X \leq Y$ means that $Y-X$ is positive semi-definite, and

$$
\mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

The following lemmas are proven in Subsection A-C. Let $(A_i)_{\text{diag}}$ be the diagonal matrix formed by the diagonal of $A_i$.

**Lemma 1:** The maximum in (18) is upper-bounded as

$$
\max_{A_i, W_i \in \{A_i(4), \ldots \}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

**Lemma 2:** For any positive semi-definite matrix $A_i$ and any matrix $W_i$, we have

$$
\mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

**Lemma 3:** The optimal solution to the problem below is achieved at real-valued $A_i$ and $W_i$.

$$
\max_{A_i, W_i \in \{A_i(4), \ldots \}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

From Lemma 1-3, we have

$$
\max_{\{A_i, W_i\} \in \{A_i(4), \ldots \}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

Comparing (23a) with (23b), we conclude that the optimal $A_i$ and $W_i$ to the problem in (18) should be real diagonal matrices. Finally, from (9) and (17), we have

$$
\mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

**Proof of Lemma 1**

Let $B$ be a Hermitian matrix. Consider any matrix $A$ satisfying $A A^H \leq B$, i.e., $B - A A^H$ is positive semi-definite. The diagonal elements of $B - A A^H$ are non-negative, i.e., $B_{ii} - (\sum_{j \neq i} |A_{ij}|^2) \geq 0$, for $i = 1, \ldots, N$. Thus, $B_{ii} - |A_{ii}|^2 \geq 0$, for $i = 1, \ldots, N$, or equivalently, $(A_{\text{diag}} A_{\text{diag}}^H \leq (B_{\text{diag}} \leq (B_{\text{diag}}).$ The above reasoning implies that

$$
\max_{\{A, W_i\} \in \{A_i(4), \ldots \}} \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

Comparing (23a) with (23b), we conclude that the optimal $A_i$ and $W_i$ to the problem in (18) should be real diagonal matrices. Finally, from (9) and (17), we have

$$
\mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

**Proof of Lemma 2**

We first consider the case of $M = N$. Let $\overline{U}_i$ be an M-by-M diagonal matrix with the diagonal elements being ±1. There are in total $2^M$ different such matrices indexed from $l = 1$ to $2^M$. Then (cf., [15])

$$
\overline{U}_i \overline{A} \overline{U}_i = (A_{\text{diag}})
$$

where $A$ be an arbitrary M-by-M matrix.

For the function defined in (19), we note that both $H_{F_i}$ and $I + F_i^H G_i F_i$ are Hermitian matrices, and that $U_2$ is a Haar matrix. From Property 2 in Section 11.B, $U_i H_i F_i U_i$ and $U_i H_i G_i U_i$ have the same distribution as $H_{F_i}$ and $I + F_i^H G_i F_i$, respectively. Thus, (19) can be written as:

$$
\phi (A_i, W_i) = \mathbb{E} \log\det \left( I + \frac{1}{\sigma} \left( \sum_{i} \phi (A_i, W_i) \right) \right)
$$

where $A_i$ is an arbitrary M-by-M matrix.
\[
\begin{aligned}
&= \varphi(\bar{U}, A\bar{U}, \bar{U}W\bar{U}) \\
&= \varphi(\bar{U}, A\bar{U}, \bar{U}W\bar{U})
\end{aligned}
\]  
(26)

Define
\[
\psi(A, W_i) = I + \frac{1}{\sigma^2}(D_i H_i F_i \bar{U}) (\bar{U} W_i H_i F_i)^* (H_i^T D^*_i F^*_i H^*_i) H_i
\]

Note that \( \log \det(\cdot) \) is concave and \( \psi(A, W_i) \) is an affine function of \( A_i \) and \( W_i \). As composition with affine mapping preserves concavity [16], \( \varphi(A, W_i) = \mathbb{E} \left[ \log \det (\psi(A, W_i)) \right] \) is concave in \((A, W_i)\). Thus, we have
\[
\varphi(A, W_i) = \frac{1}{2^m} \sum_{i=1}^{2^m} \varphi(A, W_i) = \frac{1}{2^m} \sum_{i=1}^{2^m} \varphi(\bar{U}, A\bar{U}, \bar{U}W\bar{U})
\]
where step (a) follows from (26), step (b) follows from the Jensen’s inequality, and step (c) utilizes (25). Thus, (21) holds for the case of \( M = N \).

We now discuss the case of \( M > N \). We replace (25) with
\[
\frac{1}{2^m} \sum_{i=1}^{2^m} \varphi(\bar{U}, A\bar{U}, \bar{U}W\bar{U}) = \left( A_{\text{diag}} \right)
\]
where \( A \) is an arbitrary \( M \times N \) matrix, \( U_{\text{left}} = \bar{U}_i \), and \( U_{\text{right}} \) is the \( N \times N \) principle submatrix of \( U_{\text{left}} \) with index set \( \{1, \ldots, N\} \). The other reasoning literally follows the case of \( M = N \), except for some minor modifications. The treatment for \( M < N \) is similar. This proves the proof of Lemma 2.

C. Proof of Lemma 3

By definition in (17), \( A_i \) is Hermitian. This implies that any diagonal \( A_i \) is real-valued. Thus, we only need to consider \( W_i \). We first show that, for any diagonal matrix \( A_i \) and \( W_i \),
\[
\varphi(A, W_i) = \varphi(A, W_i) \quad (28)
\]
where \((\cdot)^* \) represents the conjugate operation.

Since \( \det(A) = \det(A^*) \) for any Hermitian matrix \( A \), we have
\[
\varphi(A, W_i) = \mathbb{E} \left[ \log \det \left( I + \frac{1}{\sigma^2}(D_i H_i F_i)^* (H_i^T D^*_i F^*_i H^*_i) H_i \right) \right]
\]
(29)

From Property 2 in Section II.B, \((H_i F_i)^* \) and \((I + F_i G_i)^* \), have the same distribution as \( H_i F_i \) and \( I + F_i G_i \), respectively. Thus, we obtain
\[
\varphi(A, W_i) = \mathbb{E} \left[ \log \det \left( I + \frac{1}{\sigma^2}(D_i H_i F_i)^* (H_i^T D^*_i F^*_i H^*_i) H_i \right) \right]
\]
(30)

Then
\[
\varphi(A, W_i) = \frac{1}{2} \left( \varphi(A, W_i) + \varphi(A, W_i^*) \right)
\]  
(31)

where step (a) follows from (30), and (b) from the Jensen’s inequality (as \( \varphi(A_i, W_i) \) is concave). The equality in (31) is achieved when \( W_i \) is a real-valued matrix.

Consider the optimization problem in (22). For any diagonal matrix \( W_i \) satisfying \( W_i H_i^T \leq A_i \), we obtain \( \text{Re}(W_i) \leq \text{Re}(A_i) \). This implies that \( \text{Re}(W_i) \) falls into the feasible region of (22) if \( W_i \) does. Together with (31), we conclude that the optimal \( W_i \) to (22) is real-valued.

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