Abstract—Interleave-division multiple access (IDMA) is a multiple access scheme that has been considered in several recent proposals for the 5th generation cellular system. In this letter, based on evolution analysis, we show that the performance of IDMA can be enhanced using the transfer function matching principle. Such matching can be realized by superposition coded modulation, power control, repetition coding and zero padding. Zero padding together with cyclic shifting also leads to reduced implementation complexity. Our analysis is based on additive white Gaussian noise channels and we show by simulations that the matching techniques can also provide impressive performance in fading channels.

Index Terms—IDMA, evolution technique, system design

I. INTRODUCTION

Interleave-division multiple access (IDMA) [1], [2] is inspired by the success of low-density parity-check (LDPC) codes [3]. Recently, IDMA has been discussed for the 5th generation (5G) cellular system [4], [5].

For LDPC codes, decoding performance can be optimized by matching the transfer functions of local decoders [3]. An IDMA receiver also involves two local processors named as, respectively, an elementary signal estimator (ESE) and a decoder (DEC). (See Sect. II.) As shown in [6], the performance of IDMA can be improved by tuning an underlying LDPC code for better matching between ESE and DEC. There are, however, some obstacles for this strategy. First, in 5G, the LDPC code used has already been specified [7] so other alternatives, instead of altering code structure, should be used for system optimization. Second, there is also a lack of efficient matching method when high order modulation is involved for high rate applications. Third, matching for multi-user systems is generally a difficult problem. Very limited progress is made in this direction.

In this letter, we consider IDMA system design in high sum-rate situations. We first derive the achievable rate for IDMA using the matching principle. We show that, with perfect matching, IDMA is potentially capacity approaching in non-fading additive white Gaussian noise (AWGN) channels. We outline several practical matching techniques including modulation, power control, repetition coding and zero padding. Incidentally, we also show that zero padding together with cyclic shifting can reduce the implementation cost related to user-specific interleaving in IDMA. Our analysis is based on AWGN channels and we will provide experimental results for fading channels. We will show that the proposed techniques can provide noticeable performance enhancement.

II. SYSTEM MODEL AND EVOLUTION ANALYSIS

A. Transmitter Principles

Consider a $K$-user up-link multiple access system with received symbols:

$$y(j) = \sum_{k=1}^{K} h_k x_k(j) + \eta(j), j = 1, 2, \ldots, J,$$  \hspace{1cm} (1)

where $h_k$ is the channel coefficient of user $k$, $x_k(j)$ a transmitted symbol, $\eta(j)$ a complex AWGN sample with mean zero and variance $\sigma^2$, and $J$ the frame length. We assume an underlying orthogonal frequency division multiplexing (OFDM) layer that resolves the inter-symbol interference problem. We also assume quasi-static channels that remains unchanged over a frame.

The principle of IDMA is illustrated graphically in Fig. 1. The graph is randomized with user-specific interleaving, which is illustrated in Fig. 1 by the shuffled edge connections between $\{e_k(j)\}$ and $\{x_k(j)\}$. Fig. 1 can be seen as a graphic extension of a single-user LDPC code to a multi-user system. The randomness resulting from interleaving reduces short cycles in the graph, which facilitates low-cost message passing decoding. More details can be found in [1], [8].

Fig. 1 involves a user-specific interleaver for each user. This interleaver can be combined with the inherent interleaver in the LDPC code involved. This is equivalent to the scheme in [9], in which each user employs a unique interleaver for its LDPC code. Later we will show that such an interleaver can be realized by cyclic shifting (see Fig. 5 below), which further reduces the hardware implementation cost for IDMA.

B. Receiver Principles

We divide an iterative detector for the system in Fig. 1 into two local processors: an ESE and a DEC. The iterative process is outlined below.

Initialization: Assume that the modulation constellation of $\{x_k(j)\}$ is with zero mean and unit average power. Then $E(x_k(j))$ and $\text{Var}(x_k(j))$ are respectively initialized to 0 and 1, $\forall k, j$.

ESE Operations: We rewrite (1) as

$$y(j) = h_k x_k(j) + \zeta_k(j),$$  \hspace{1cm} (2a)
Fig. 1. A factor graph of a 2-user IDMA system with LDPC coding. $J = 8$. $(\alpha_k(j), j = 1, 2, \ldots, J)$ is a codeword that is interleaved and modulated to produce $\{x_k(j)\}$. Circles represent variables and squares constraints. Three types of constraints are presented: a white square for an LDPC coding constraint, a square with “+” for a modulation constraint and a square with “×” for a multiple access constraint defined in (1).

where

$$
\zeta_k(j) = y(j) - h_k x_k(j) = \sum_{k' \neq k}^{K} h_{k,k'} x_{k'}(j) + \eta(j) \tag{2b}
$$

includes the interference and noise seen by user $k$. We model $\zeta_k(j) \sim CN(\mu_k(j), \nu_{\zeta,k}(j))$ using Gaussian approximation (GA). Given a priori $\{\mathbb{E}(x_k(j))\}$ and $\{\text{Var}(x_k(j))\}$, we can evaluate $\mu_k(j)$ and $\nu_{\zeta,k}(j)$ and then estimate $x_k(j)$ based on (2a). For example, for binary phase shift keying (BPSK), each $x_k(j) \in \{-1, +1\}$ and the estimation outputs are log-likelihood ratios (LLRs):

$$
\text{LLR}(x_k(j)) = \frac{\Pr(x_k(j) = +1)}{\Pr(x_k(j) = -1)} = 2h_k \frac{y(j) - \mu_k(j)}{\nu_{\zeta,k}(j)}. \tag{3}
$$

Similar results can be obtained for quadrature phase shift keying (QPSK) and other modulations.

**DEC Operations:** The DEC is further divided into $K$ constituent decoders $\{\text{DEC }1, \text{DEC }2, \ldots, \text{DEC }K\}$, one for each user. The LLRs in (3) are used as the inputs to the DEC. Assume soft-output decoding, by which $\{\mathbb{E}(x_k(j))\}$ and $\{\text{Var}(x_k(j))\}$ are updated. For an LDPC or a turbo code, such decoding follows the standard procedures in [3], [10], [11].

**Iterative Process:** The DEC outputs $\{\mathbb{E}(x_k(j))\}$ and $\{\text{Var}(x_k(j))\}$ are fed back to the ESE. Then (3) is re-evaluated and the iterative process continues.

### C. Evolution Analysis

Let $v_k$ be the average of $\text{Var}(x_k(j))$ over $j$ and $\text{snr}_k$ the average signal-to-noise-ratio (SNR) related to $\{\text{LLR}(x_k(j))\}$ in (3). From [1], $\text{snr}_k$ can be expressed as a function:

$$
\text{snr}_k = \mathbb{E}(\{x_k(j)^2\})/\mathbb{E}(\nu_{\zeta,k}(j)) \equiv \phi_k(v_1, \ldots, v_{k-1}, v_{k+1}, \ldots, v_K). \tag{4}
$$

Now let the input-output relationship of DEC $k$ be characterized by a function $v_k = \psi_k(\text{snr}_k)$ that can be generated numerically. The behavior of the iterative process can be characterized by the following recursions (initialized to $v_k = 1$):

**ESE:** $\text{snr}_k = \phi_k(v_1, \ldots, v_{k-1}, v_{k+1}, \ldots, v_K), ~ \tag{5a}$

**DEC:** $v_k = \psi_k(\text{snr}_k). \tag{5b}$

### III. IDMA Design Techniques Based on Evolution Analysis

#### A. Matching Principle

For an LDPC or a turbo code, it is known that the overall performance can be optimized by matching the transfer functions of the two local decoders [3]. This matching principle was later extended to other iterative systems [12], [13].

Following this principle, a symmetric IDMA system can be optimized by matching $\phi$ and $\psi$ in (6). The matching condition is given by

$$
\psi(z) = \phi^{-1}(z), \tag{7}
$$

where $\phi^{-1}$ is the reverse function of $\phi$. The analytical treatments on general situations are beyond the scope of this letter. We will only provide derivation for AWGN channels and rely on simulation results for fading channels below.
B. Optimality of IDMA in AWGN Channels

Consider an AWGN channel in which \( h_k = 1 \) in (1) for all users. Assume the same code for all users. In this case, (6a) is given by

\[
\text{snr} = \phi(v) = \frac{1}{(K-1)v + \sigma^2}.
\]  

(8)

Following the minimum mean square error (MMSE)-SNR relationship developed in [12], [13], [14], the achievable rate of each user is given by

\[
R = \int_{0}^{\infty} \text{mmse}(\text{snr}) d\text{snr},
\]  

(9)

where \( \text{mmse} \) is the MMSE at the output of DEC \( k \) with \( \text{snr} \) at its input. Then, following [13], we have

\[
\text{mmse}(\text{snr}) = (\text{snr} + (\psi(\text{snr}))^{-1})^{-1},
\]  

(10)

which includes the contributions of both \textit{a priori} information (related to \( \text{snr} \)) and extrinsic one (related to \( \psi(\text{snr}) \)). From (7) and (8), we have

\[
\begin{aligned}
\psi(\text{snr}) &= 1, & 0 \leq \text{snr} < 1/(K-1 + \sigma^2), \\
\psi(\text{snr}) &= \frac{1-\sigma^2\text{snr}}{K-1}, & \text{snr} \geq 1/(K-1 + \sigma^2).
\end{aligned}
\]  

(11)

Here the first equation in (11) results from the constraint \( \text{Var}(x_i(j)) \leq 1 \) so that \( \psi(\text{snr}) \leq 1 \). Substituting (10) and (11) into (9) and with some straightforward manipulations, we have the achievable rate per user

\[
R = K^{-1}\log(1 + K/\sigma^2).
\]  

(12)

For \( K \) users, the sum-rate is

\[
R_{\text{sum}} = KR = \log(1 + K/\sigma^2),
\]  

(13)

which achieves the \( K \)-user channel capacity. This demonstrates the optimality of IDMA under perfect matching.

Eqn. (13) extends the conclusions in [13] from single-user systems to multi-user ones in AWGN channels. With practical coding constraints, it can be difficult to design perfectly matched \( \phi \) and \( \psi \). Also, the situation is much more complicated if the received powers are random variables due to fading. Detailed analytical discussions on the matching principle for fading channels are beyond the scope of this letter. Nevertheless, the matching condition in (7) suggests a way for performance optimization. Below, we will discuss some empirical results along this direction.

C. Design Strategies Based on the Matching Principle

The followings are some strategies to shape either \( \phi \) or \( \psi \), so as to improve matching.

1) A standard approach is to shape \( \psi \) by adjusting the degree polynomials for an LDPC code [3], [6], [11]. This method requires different code structures for different situations, which may cause difficulty in hardware implementation. In this letter, we will examine options with fixed codes.

2) Properly designed modulation and labeling methods can be used to shape \( \psi \) in the high rate region. In particular, with superposition coded modulation (SCM), two or more layers of modulated codes (with, say, QPSK modulation) are scaled to different power levels and then linearly superimposed, as demonstrated in [15].

3) Other simple options include zero padding (ZP) and repetition coding. With ZP, some users are silent on some symbols. Then (8) becomes

\[
\text{snr} = \phi(v) = \frac{1}{(K'-1)v + \sigma^2},
\]  

(14)

where \( K' \) is the non-zero symbols transmitted simultaneously (assume that ratio of zero symbols is uniform over all resources). ZP can shape \( \phi \) to a certain extend and can also lead to reduced multi-user detection complexity. Repetition coding does not provide coding gain. However, it can be useful in multi-user systems to shape \( \psi \).

IV. Numerical Results

We follow the 3GPP NR requirement [7] of frame length of 864 symbols with information bits \( J_{\text{info}} = 616 \) per user. We consider AWGN first. Fig. 3 shows the \( \phi \) and \( \psi \) functions in AWGN channels. \( \phi \) is the ESE function of \( K = 4 \). \( \psi(1) \) is the DEC function of the 3GPP NR LDPC code and QPSK modulation. We can clearly see poor matching between \( \psi(1) \) and \( \phi \). \( \psi(2) \) is the DEC function of a two-layer SCM scheme with the 3GPP NR LDPC code followed by a rate-1/4 repetition coding. (The power levels of the two SCM layers are obtained by exhaustive searching.) We can see a staircase shape of \( \psi(2) \), which is due to the different convergence behaviors of different SCM layers with different powers: the higher-power layer leads the convergence in the first stage (see Fig. 3) and then the lower-power layer follows in the second stage. Clearly, such staircase shape improves the matching between \( \psi(2) \) and \( \phi \).

Fig. 4 shows the frame error rates (FERs) of different schemes in a 4-user AWGN system. Schemes 1 and 2 correspond to \( \psi(1) \) and \( \psi(2) \) in Fig. 3 respectively. We can see that scheme 1 does not work at all, while scheme 2 works well due to better matching. Let \( M \) be the number of the SCM layers. Although increasing \( M \) may improve matching in theory, there is a limit in practice since the value of \( M \) will influence the coding gain of each layer under a given \( J_{\text{info}} \). We observed that, for \( J_{\text{info}} = 616 \), \( M = 2 \) represents a best compromise.

We now consider fading channels, in which the fluctuations of channel gains provide more diversity to support more users.
ZP can be used to reduce receiver complexity. A cyclic shifted ZP (CSZP) scheme is shown in Fig. 5. Each user is active only in half of a frame (i.e., with 50% ZP) and the active half of a frame for user k is cyclically shifted by an amount of \(\lfloor (k - 1)/K \rfloor \) where J is the frame length. Assume that a random interleaver is used in the underlying LDPC code. The user-specific shifting in Fig. 5 provides an equivalent realization for the user-specific interleaving in Fig. 1, since a shifted version of a random interleaver can be seen as a different random interleaver [8], [16]. Fig. 5 also involves a two-layer SCM scheme. To distinguish the two layers of each user, each layer is further independently and cyclically shifted within its active frame duration. Such shifting technique, both user-specific as well layer-specific, significantly reduces the hardware implementing cost for interleavers in IDMA.

Fig. 6 shows the FERs with SCM and CSZP in the TDL-C fading channels [17]. Two antennas are assumed at the receiver. We consider the following two settings:

- Case 1: \(J_{\text{info}} = 616\), \(K = 10\) and sum-rate = 7.13;
- Case 2: \(J_{\text{info}} = 496\), \(K = 12\) and sum-rate = 6.89.

The power levels of the two SCM layers given in the caption of Fig. 6 are obtained by exhaustive searching. Iterative linear MMSE detection [8] is used at the receiver. As reference, Fig. 6 also includes the performances of sparse code multiple access (SCMA) [17]. The sparse codebook in SCMA is an equivalent form of ZP in IDMA. The performance advantage of IDMA over SCMA mainly attributes to the freedom of curve shaping using SCM.

In conclusion, in this letter we examined several design techniques for IDMA following the transfer function matching principle. We showed that IDMA is capacity approaching in AWGN channels under perfect matching. We also showed by simulations that the matching techniques for IDMA can provide impressive performance in fading channels.

REFERENCES

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