Matrix methods for the design of transconductor ladder filters

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Abstract: Matrix-based methods for the design of transconductor ladder filters are presented. These allow the realisation of any bandpass or lowpass prototype using only one or two values of transconductance. The new methods are illustrated by experimental results from a 1 MHz elliptic lowpass filter, a 400 kHz elliptic bandpass filter, and a PLL frequency-control loop, all fabricated in a 1 µm CMOS process.

1 Introduction

In recent years, much research has been directed towards the development of continuous-time transconductor filters [1-7] as an alternative to switched-capacitor (SC) filters [8], particularly in the frequency range 100 kHz to 10 MHz. Although many linear transconductor circuits have been presented in the literature [9-12], less progress has been evident in the development of filter structures that are well suited to transconductor realisation.

A significant problem has been how to design ladder filters without recourse to ratioed transconductances. Ratioed transconductors are undesirable because the transistors which determine the value of a particular transconductor can vary in size within only a small range without suffering from poor matching in one extreme or producing significant parasitic capacitance and high power consumption in the other. Moreover, it is inconvenient for a designer to have to produce a different set of ratioed transconductors for each new filter design. This problem is specific to transconductor filters as the corresponding variables in RC and SC filters (resistors and sampling capacitors, respectively) can be scaled relatively freely.

Most methods used to derive active RC and SC filters from passive prototypes have been based, explicitly or otherwise, on the simulation of nodal voltages and inductor currents [13, 14]. Examples of such filters are leapfrog and coupled-biquad ladders, as well as circuits obtained by simulating inductors using gyrators [15]. These methods have been applied successfully to the design of lowpass transconductor ladders but they cannot generally be applied to bandpass ladders without the use of ratioed transconductor inputs. This is because, when the voltages of a coupled-biquad bandpass filter are scaled for dynamic range, the summing coefficients between biquads take values which are lower than the coefficients within each biquad by a factor typically close to the fractional bandwidth of the filter. The conventional coupled-biquad bandpass structure can only be used for transconductor ladders having an all-pole response of moderate selectivity [16]. The problem described above is compounded for highly selective filters which require large transconductance ratios, and for prototypes containing inductor loops as these lead to noninteger ratios that cannot be implemented by combinations of a unit transconductance [3].

In this paper, we present matrix-based methods for the design of transconductor ladder filters [17, 18], which can be applied to many more response types than conventional techniques. Ladder filters are considered to be preferable to those formed from cascaded biquad stages, the latter typically having much greater passband sensitivity. The objective of this work is to be able to realise any passive ladder as a canonical transconductor filter using only a single value of transconductance, or a small number of values in simple integer ratios.

Similar matrix methods have already been developed for the design of switched capacitor and active RC filters [19, 20]. As well as formalising the design procedure and providing a framework for computer-aided design tools, the use of matrices has facilitated the discovery of superior active filter structures which are not intuitively obvious. The same advantages are found for transconductor filters.

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The design procedure can be summarised as follows:

(i) A set of equations (derived by Kirchhoff's laws) which describe the passive prototype are combined to form a matrix equation

\[ J = (G + sC + s^{-1}F)V \]

where \( V \) is a vector representing the nodal voltages (and/or branch currents), \( J \) is a vector representing the input current source and \( G, C, \) and \( F \) are matrices whose elements are simple algebraic combinations of the passive component values.

(ii) The nodal voltages and branch currents of the prototype can be scaled if required by performing simple multiplication operations upon eqn. 1.

(iii) The second-order matrix equation is decomposed into two first-order design equations by the introduction of a vector of auxiliary variables, \( X \). A large number of decompositions are possible, of which we present those most useful for transconductor filters. The choice of decomposition for a particular filter design is dictated by the type of building block available and the nature of the desired response.

(iv) To form the active filter, each row of each design equation is implemented by a first-order transconductor/capacitor section.

(v) Finally, the filter is scaled in frequency by the appropriate choice of transconductor and capacitor unit values.

2 Matrix representation of the passive prototype ladder

For a given passive prototype ladder, various forms of the second-order matrix equation (eqn. 1) can be constructed, depending on the choice of variables used to form the vector \( V \). We use the terms \( V \)-representation, \( I \)-representation and \( VI \)-representation to refer to the use of nodal voltages, branch currents and mixed variables, respectively, in \( V \). The choice of representation is governed mainly by two factors. First, the order of the matrices (which equals the number of variables in \( V \)) should be kept to a minimum so that the resulting active circuit is canonical, i.e. has one integrator per pole of the desired transfer function. Secondly, a representation should be chosen which leads to the matrices \( G, C, \) and \( F \) being as sparse as possible, as the sparsity of these matrices is reflected in the complexity of interconnect in the resulting active circuit.

Each RLC ladder has a 'minimum inductance' and a 'minimum capacitance' version. Identical matrices are obtained if the \( V \)-representation is used for the former and the \( I \)-representation for the latter. However, the correct representation must be used for a particular prototype to ensure canonicity. Fig. 1 shows the minimum inductance version of a fifth-order elliptic lowpass prototype with 0.28 dB passband ripple and 60.5 dB stopband attenuation. In the \( V \)-representation this is described by the matrices

\[ J = \begin{pmatrix} \frac{V_o}{R} \\ 0 \\ 0 \end{pmatrix}, \quad V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}, \quad G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 + C_4 & -C_4 \\ 0 & -C_4 & C_4 + C_3 \end{pmatrix}, \quad F = \begin{pmatrix} \frac{1}{L_2} + \frac{1}{L_2} - \frac{1}{L_2} \\ -\frac{1}{L_2} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} - \frac{1}{L_4} \\ 0 & -\frac{1}{L_4} + \frac{1}{L_4} + \frac{1}{L_5} - \frac{1}{L_5} \end{pmatrix} \]

and

\[ \Gamma = \begin{pmatrix} 1 & -1 & 0 \\ -L_2 & L_2 & 0 \\ -L_2 & L_2 & 0 \end{pmatrix} \]

In general, where the LC pairs in a ladder are parallel, it is best to use the \( V \)-representation. As an example, Fig. 2 shows a sixth-order elliptic bandpass prototype with 0.1 dB passband ripple and 50 dB stopband attenuation. In the \( V \)-representation this is described by the matrices

\[ J = \begin{pmatrix} \frac{V_o}{R} \\ 0 \\ 0 \end{pmatrix}, \quad V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}, \quad G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 + C_4 & -C_4 \\ 0 & -C_4 & C_4 + C_3 \end{pmatrix}, \quad F = \begin{pmatrix} \frac{1}{L_2} + \frac{1}{L_2} - \frac{1}{L_2} \\ -\frac{1}{L_2} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} - \frac{1}{L_4} \\ 0 & -\frac{1}{L_4} + \frac{1}{L_4} + \frac{1}{L_5} - \frac{1}{L_5} \end{pmatrix} \]

Another example is the asymmetric Chebyshev bandpass ladder shown in Fig. 3. It will be demonstrated in Section 4 that the complexity of a symmetric bandpass transconductor ladder can be reduced if the condition \( C = F \) is satisfied. This is the
case if the bandpass prototype is obtained by transformation of each component of a lowpass ladder individually \([13, 14]\) and if the \(VI\)-representation is used. For example, the sixth-order elliptic ladder shown in Fig. 4 is obtained by transforming each component of a third-order elliptic prototype. In the \(VI\)-representation, this ladder is described by

\[
J = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0
\end{pmatrix}, \quad G = \frac{1}{R}
\]

\[
C = \begin{pmatrix}
L_1 & 0 & -C_4 & 0 \\
0 & -L_2 & 0 & 0 \\
-C_4 & 0 & C_4 + C_5 & 0 \\
0 & 0 & 0 & C_3
\end{pmatrix}
\]

and

\[
\Gamma = \begin{pmatrix}
\frac{1}{L_1} & 0 & -\frac{1}{L_4} & 0 \\
0 & -\frac{1}{C_4 R^2} & 0 & 0 \\
-\frac{1}{L_4} & 0 & \frac{1}{L_4 + L_5} & 0 \\
0 & 0 & 0 & \frac{1}{L_3}
\end{pmatrix}
\]

To maintain dimensional consistency, the current \(I_2\) is represented by the voltage variable \(V_{12}\), the multiplying factor being the termination resistance \(R\). In this \(VI\)-representation, the \(C\) and \(\Gamma\) matrices are identical because by definition \(L_i = 1/C_i\).

3 Transconductor-capacitor building blocks

The general first-order transconductor-capacitor building block has the transfer function

\[
V_{out} = \sum \frac{g_i V_i + s \sum C_j V_j}{sC}
\]

It is desirable that only one value of \(g_i\) be used in a particular filter, but where more than one value is used, they should be in low-integer ratios. Using a conventional transconductor only, eqn. 5 is implemented by the circuit shown in Fig. 5. In this case, the capacitors \(C_i\) represent bidirectional coupling paths \([21]\) when driven by internal nodes, as these nodes are all high-impedance. This can be a serious restriction, as many of the techniques available to maintain low-integer capacitor ratios in a filter \((\text{Section 4})\) rely on the use of unidirectional capacitive paths.

To obtain unidirectional capacitive coupling paths, a first-order stage with a low-impedance input and/or output is required. The most obvious realisation of this requires the addition of an opamp to create a virtual earth \((\text{Fig. 6})\). This is expensive in silicon area and current consumption, particularly as the opamp will need a very high bandwidth for video frequency operation. However, these problems are offset by the fact that the design of the transconductor itself can be somewhat simplified because it is only driving into the virtual earth and does not need a very high output impedance. In the extreme, the transconductor can be reduced to a pair of MOSFETs operating in triode mode, giving a so-called 'MOSFET-C' circuit \([22]\).

An alternative solution is to use a recently reported transconductor with low-impedance inputs \([23]\). The first-order section using this circuit is shown in Fig. 7.
The output current of the transconductor is the sum of the transconductance multiplied by the voltage at high-impedance input \( (V^+ + V^-) \), and the current entering the low impedance input \( (V^+ + V^-) \). The circuit diagram of a transconductor with low-impedance inputs is given in Fig. 8.

The output current of the transconductor is the sum of the transconductance multiplied by the voltage at high-impedance input \( (V^+ + V^-) \), and the current entering the low impedance input \( (2^+ + 2^-) \). The circuit diagram of a transconductor with low-impedance inputs is given in Fig. 8.

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![Fig. 8 Folded cascode transconductor with low-impedance inputs (a) and common-mode feedback circuit (b)](image)

**4 Matrix decompositions**

Five ways of obtaining an active ladder from the general second-order equation (eqn. 1) are now given, together with design examples. The first is recommended for lowpass and the remaining four for bandpass responses. For clarity, single-ended ladders are shown. However, practical transconductor circuits (including those presented in Section 5) are normally fully differential, for several reasons: to obtain linear transconductance functions, to allow the realisation of negative floating capacitors, and to maximise power supply rejection.

In each decomposition, a scaling factor \( g \) is introduced in the definition of the vector \( X \). This factor may be used to perform nodal voltage scaling between the \( X \) and \( V \) voltages in the transconductor ladder. The scaling of the voltages within each of \( X \) and \( V \) is determined by the design of the passive prototype. This may also be optimised by matrix techniques, as shown on pages 127–128 of Reference 18.

**4.1 Topological decomposition**

The \( \Gamma \) matrix is factorised as

\[
\Gamma = ADAT
\]

where \( D \) is a diagonal matrix whose elements are the reciprocals of the inductances in the prototype (assuming a \( V \)-representation), and \( A \) is a conventional incidence matrix. The auxiliary variables are defined by

\[
X = (sg)\Gamma^{-1}DATV
\]

where \( g \) is a scaling factor with the dimensions of conductance. Eqns. 6 and 7 are substituted into eqn. 1 to obtain

\[
CV = s^{-1}[J - GV - gAX]
\]

Substituting the matrices (eqn. 2a–e) into the design eqns. 7 and 8, and implementing each row with a conventional transconductor stage (Fig. 5) gives the active filter shown in Fig. 9, which is equivalent to a standard leapfrog ladder. Using transconductors with low-impedance inputs instead, we obtain the circuit shown in Fig. 10. The advantage of the first realisation is that there are fewer capacitors and the transconductors are simpler. The advantage of the second is that bottom plates of the floating capacitors can be connected to the low-impedance inputs, so the associated parasitic capacitances do not need to be estimated and subtracted from the grounded capacitors [4].

![Fig. 9 Lowpass elliptic ladder using conventional transconductor](image)

![Fig. 10 Lowpass elliptic ladder using transconductor with low-impedance inputs](image)
4.2 Left-inverse decomposition type 1 (LID1)
The auxiliary variables are defined by
\[ g_X = sCV \]  \hspace{1cm} (9)
where \( g \) is again a scaling factor with the dimensions of conductance, and eqn. 9 is substituted into eqn. 1, giving
\[ g_X = J - CY + s^{-1}fY \]  \hspace{1cm} (10)
The LID1 design equations are obtained by rearranging eqn. 9 and multiplying eqn. 10 by the inverse of \( f \):
\[ g^{-1}CV = s^{-1}X \]  \hspace{1cm} (11)
\[ g^{-1}X = -s^{-1}V + \Gamma^{-1}[J - GV] \]  \hspace{1cm} (12)
From eqns. 11 and 12, the main features of LID1 can be deduced. First, both integrated terms (i.e. those containing the factor \( s^{-1} \)) are vectors, so only a single value of transconductance is required in the active ladder. Secondly, both nonintegrated terms on the right-hand side of eqn. 12 are generally asymmetric, so they must be realised using unidirectional capacitive coupling paths. In other words, the dependence of \( V \) upon \( X \) is not the same as the dependence of \( X \) upon \( V \) in these terms, so each branch must be realised by a separate capacitor, rather than symmetric branches being realised by a single capacitor connected between the nodes concerned. A single capacitor can be used for symmetric nonintegrated branches arising from off-diagonal terms on the left-hand sides of eqns. 11 and 12, as long as first-order sections with high-impedance outputs are used.

Fig. 11 shows the LID1 realisation of the sixth-order elliptic prototype of Fig. 2. This was obtained by substituting the \( V \)-representation matrices, (eqn. 3a–e), into eqns. 11 and 12 and translating each row of each equation into a first-order section of the type shown in Fig. 7.

A general feature of inverse matrix decompositions is that the number of components and the density of interconnect may be high in the transconductor ladder if the sparsity of \( \Gamma \) (or \( C \)) is lost upon inversion. This is not a serious problem for the sixth-order elliptic ladder of Fig. 11, but for higher-order filters care should be taken with the choice of prototype. Essentially this means placing capacitors such that long chains of directly connected inductors are avoided. Some passive prototypes also exist whose \( \Gamma \) matrix may not be inverted at all, due to the determinant being zero. The only examples of such filters known to the authors are bandstop. A question yet to be investigated is to what extent an inverse matrix filter preserves the low-passband sensitivity properties of its passive ladder prototype.

4.3 Left-inverse decomposition type 2 (LID2)
The auxiliary variables are defined by
\[ o, X = sV \]  \hspace{1cm} (13)
where \( o, \) is a scaling factor with the dimensions of angular frequency, and eqn. 13 is substituted into eqn. 1, giving
\[ o, CX = J - GV - s^{-1}fV \]  \hspace{1cm} (14)
Rearranging eqn. 13 and multiplying eqn. 14 by \( f^{-1} \) gives the LID2 design equations
\[ o, f^{-1}CX = f^{-1}J - f^{-1}GV - s^{-1}V \]  \hspace{1cm} (15)
and
\[ V = o, s^{-1}X \]  \hspace{1cm} (16)
As in the other left-inverse decomposition, only one value of transconductance is required, together with unidirectional nonintegrated paths. The distinguishing feature of LID2 is the term \( f^{-1}C \), which gives the opportunity to obtain a relatively sparse transconductor ladder is a prototype can be used for which \( f = C \). As shown in Section 2, this condition is satisfied for a symmetric bandpass filter derived by the transformation of each component of a lowpass ladder individually. Fig. 12 shows the sixth-order bandpass elliptic transconductor ladder obtained by substituting eqns. 4a–e into eqns. 15.
4.4 Right-inverse decomposition (RID)

For the RID, $X$ is defined by

$$gX = s^{-1}TV$$  \hspace{1cm} (17)

This is substituted into eqn. 1 to give

$$J = (G + sC)V + gX$$  \hspace{1cm} (18)

The design equations are obtained by multiplying eqn. 17 by $Γ^{-1}$ and rearranging eqn. 18:

$$gΓ^{-1}X = s^{-1}V$$  \hspace{1cm} (19)

$$g^{-1}CV = s^{-1}[g^{-1}(J - GV) - X]$$  \hspace{1cm} (20)

Conventional transconductors can be used to implement eqns. 19 and 20 because the only nonintegrated terms are those arising from the offdiagonal elements of $Γ^{-1}$ and $C$, which represent bidirectional coupling paths. Neither $V$ nor $X$ is premultiplied before integration, so no unrealisable summing coefficients are introduced. To scale the filter correctly for dynamic range, a second (smaller) value of transconductance is usually required to realise the input branch and filter terminations. This use of a second transconductance value is acceptable because it can be chosen to be in integer ratio to the first and it is used only to represent the termination resistors which are the least sensitive components of the prototype. Such a realisation compares favourably with a coupled-biquad ladder in which high and/or noninteger transconductor ratios can occur throughout the filter.

Fig. 13 shows the RID transconductor ladder obtained from the sixth-order elliptic prototype of Fig. 2, using the $V$-representation.

4.5 Left-direct decomposition (LD)

The vector of auxiliary variables $X$ is defined by

$$gX = sCV$$  \hspace{1cm} (21)

and 16 and implementing each row of each design equation with the first-order section of Fig. 7.

Fig. 13 Elliptic bandpass ladder obtained by RI decomposition

Eqn. 21 is substituted into eqn. 1 to give

$$J = GV + gX + s^{-1}TV$$  \hspace{1cm} (22)

and the design equations are obtained by rearranging eqns. 21 and 22:

$$CV = s^{-1}gX$$  \hspace{1cm} (23)

$$X = -(sg^{-1}TV - g^{-1}GV + g^{-1}J)$$  \hspace{1cm} (24)

The principal features of the LD decomposition are as follows. First, the matrix $Γ$ should be diagonal to avoid the requirement for summing integrators which would imply the use of randomly ratioted transconductor values. Secondly, the damping and coupling branches (as represented by the term $g^{-1}GV$) are nonintegrated. These branches are unidirectional, as they describe a dependence of $X$ upon $V$ which is not matched by an identical dependence on $V$ upon $X$. Therefore, transconductors with low-impedance inputs (Fig. 7) or opamp-based integrators (Fig. 6) must be used in a left-direct filter.
shown in Fig. 9, scaled to
filter, which is a fully differential version of the filter
5.1 Elliptic lowpass filter
Table 1 summarises the specification and measured per-
enough for amplitude-control loops [3] to be required.
and one bandpass Chebyshev. Each of the bandpass
filters includes a phase-lock frequency-control loop, a
operating frequencies and selectivities of these filters are not high
description of which is also given here. The operating fre-
cencies and selectivities of these filters are not high
enough for amplitude-control loops [3] to be required.

5 Experimental results
To verify the methods described above, a set of high-
frequency transconductor ladder filters has been designed
and fabricated on a 1 μm double-poly double-metal
CMOS process. In this Section, results are given from
three filters: one lowpass elliptic, one bandpass elliptic,
and one bandpass Chebyshev. Each of the bandpass
filters includes a phase-lock frequency-control loop, a
description of which is also given here. The operating fre-
frequencies and selectivities of these filters are not high
enough for amplitude-control loops [3] to be required.

5.1 Elliptic lowpass filter
Fig. 15 shows the schematic of the experimental lowpass
filter, which is a fully differential version of the filter
shown in Fig. 9, scaled to a centre frequency of 1 MHz.
Table 1 summarises the specification and measured per-
frequency of 1 MHz.
Fig. 14 shows the LD ladder obtained from the sixth-
or order asymmetric Chebyshev prototype of Fig. 3, using
the 'V'-representation. Transconductors with low-
impedance inputs are used only for the termination capa-
citors.

Fig. 14 Chebyshev bandpass ladder obtained by LD decomposition

Left- and right-decomposition filters employ capacitive
and resistive damping, respectively, hence we can refer to
them as 'E-type' and 'F-type' circuits by analogy with the
terminations and terminology used for SC biquads [24].

Table 1: Specification and measured performance of
lowpass filter

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<th>Parameter</th>
<th>Designed</th>
<th>Measured</th>
</tr>
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<tbody>
<tr>
<td>Order</td>
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<td></td>
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<tr>
<td>Cutoff frequency</td>
<td>1 MHz</td>
<td></td>
</tr>
<tr>
<td>Stopband ripple</td>
<td>0.28 dB</td>
<td>0.4 dB</td>
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<tr>
<td>Stopband attenuation</td>
<td>60 dB</td>
<td>81.5 dB</td>
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<td>THD (200 mV rms input)</td>
<td>–67.7 dB</td>
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</tr>
<tr>
<td>Common mode rejection</td>
<td>71 dB</td>
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<tr>
<td>Power supply rejection</td>
<td>43 dB</td>
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<tr>
<td>Current consumption</td>
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Table 2: Specification and measured performance of elliptic
bandpass filter

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<td>Order</td>
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<tr>
<td>Centre frequency</td>
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<tr>
<td>Bandwidth</td>
<td>10%</td>
<td>10%</td>
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<tr>
<td>Stopband attenuation</td>
<td>50 dB</td>
<td>49.5 dB</td>
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<tr>
<td>Noise density in passband</td>
<td>920 nV/√Hz</td>
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<tr>
<td>Intermodulation distortion</td>
<td>–43.8 dB</td>
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<tr>
<td>Power supply rejection</td>
<td>40 dB</td>
<td></td>
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<tr>
<td>Current consumption</td>
<td>11.3 mA</td>
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Table 3: Specification and measured performance of
Chebyshev bandpass filter

<table>
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<td>Order</td>
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<tr>
<td>Centre frequency</td>
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<tr>
<td>Bandwidth</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Noise density in passband</td>
<td>920 nV/√Hz</td>
<td></td>
</tr>
<tr>
<td>Intermodulation distortion</td>
<td>–48.7 dB</td>
<td></td>
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<tr>
<td>Power supply rejection</td>
<td>46 dB</td>
<td></td>
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<tr>
<td>Current consumption</td>
<td>11.6 mA</td>
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</tr>
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</table>


and 18. The compact layout is achieved by dividing the
larger transistors into units 100 μm wide so that the tran-
sistors of each polarity can be assembled in rectangular
areas, supplied by the respective power lines. Between the
two sets of transistors lies a routing bus, which occupies
area that would have to be used anyway due to the
relatively large 'p-well to n+ diffusion' design rule. The
input and output ports run over the VDD line (to the
right in the photograph). Including the bias lines in the
central bus enables the transconductors to be butted
directly. A photomicrograph of the complete lowpass
filter is given in Fig. 19. The transconductors are laid out
in a single row, and the capacitor bank (with units of
0.5 pF) is shaped to have approximately the same length.
Another bus is used to provide efficient routing between
the transconductors and capacitors.

5.2 Elliptic bandpass filter
Fig. 20 shows the schematic of the experimental elliptic
bandpass filter, which is a fully differential version of the
filter shown in Fig. 13, scaled to a centre frequency of
400 kHz. Table 2 summarises the specification and meas-
ured performance. Fig. 21 shows the measured amplitude
response. A photomicrograph is given in Fig. 22. The
noise spike in the stopband at 1.9 MHz is breakthrough
from the control loop used to set the centre frequency
automatically with respect to a reference clock (see
below).

5.3 Chebyshev bandpass filter
Fig. 23 shows the schematic of the experimental Cheby-
shev bandpass filter. This is a fully differential version
of the low-direct filter shown in Fig. 14, scaled to a centre
frequency of 400 kHz. To the knowledge of the authors, it
is the first example of a transconductor ladder filter that
is capacitively terminated and has only a single value of
transconductance (100 μS). Table 3 summarises the spe-
cification and measured performance. Fig. 24 shows the
measured amplitude response. A photomicrograph is
given in Fig. 25.
Fig. 15 Fully differential lowpass transconductor ladder filter (bias lines omitted)

Fig. 16 Measured amplitude response of elliptic lowpass filter

Fig. 17 Schematic of double-input folded cascode transconductor

being that here a triangle-wave oscillator is used instead of a harmonic oscillator.

The triangle-wave oscillator is illustrated in Fig. 27. An analogue switch selects either $+\Delta V$ or $-\Delta V$ as the input to the transconductor and, respectively, $+\Delta V/2$ or $-\Delta V/2$ as one of the inputs of the comparator. If $+\Delta V$ is selected, the voltage across the capacitor will slew up linearly at a rate $\Delta V g / C$. When this voltage exceeds $\Delta V/2$, the comparator changes state so that the output of the analogue switch changes to $-\Delta V$. Then the output of the transconductor slews down until its value reaches $-\Delta V/2$ and the comparator changes state again. In each

5.4 Frequency-control loop

The frequency-control loop fabricated with the test filters is a phase-lock loop (Fig. 26). Based on a voltage-controlled oscillator. This is similar in principle to those described in References 9 and 22, the main difference

Fig. 18 Photomicrograph of double-input folded cascode transconductor

Fig. 19 Photomicrograph of elliptic lowpass filter

Fig. 20 Fully differential elliptic bandpass filter (bias lines omitted)

period, the transconductor output has to slew twice through a range of magnitude $\Delta V$, so the frequency of oscillation is

$$f_{\text{osc}} = \frac{\Delta V g/C}{2 \Delta V} = \frac{g}{2C} \quad (25)$$

The output of the comparator is, of course, a square wave at the same frequency.

The phase of the VCO output is compared to that of a reference square-wave clock by an XOR gate followed by a single-order lowpass filter. The result of the phase comparison is used as the control voltage for the transconductor in the VCO. When the PLL reaches lock, the

**Fig. 21** Measured amplitude response of elliptic bandpass filter

**Fig. 22** Photomicrograph of elliptic bandpass filter

**Fig. 23** Fully differential Chebyshev bandpass filter (bias lines omitted)
reference and VCO frequencies are identical, and so according to eqn. 25 the time constant of the transconductor and capacitor in the VCO are set. The same control voltage is used for each transconductor in the 'slave' filter. Therefore, within the accuracy of transconductor and capacitor matching, the frequency response of the filter is scaled with respect to the reference clock.

The operation of the control loop is illustrated in Fig. 28 which shows the measured amplitude response of the elliptic bandpass filter for three different values of clock frequency (865 kHz, 965 kHz and 1.065 MHz).

Fig. 24 Measured amplitude response of Chebyshev bandpass filter

Fig. 25 Photomicrograph of Chebyshev bandpass filter

Fig. 26 PLL control loop

Fig. 27 Triangle-wave voltage-controlled oscillator

6 Conclusions

A matrix-based methodology for the design of transconductor ladder filters has been presented. Many alternative design routes are possible, depending upon:

Table 4: Comparison of different representations of RLC prototype

<table>
<thead>
<tr>
<th>Name</th>
<th>Variables of prototype used</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>voltages</td>
<td>Best for a 'minimum inductance' prototype containing only parallel LC pairs</td>
</tr>
<tr>
<td>VI</td>
<td>voltages and currents</td>
<td>Best when prototype contains both parallel and series LC pairs</td>
</tr>
<tr>
<td>I</td>
<td>currents</td>
<td>Best for a 'minimum capacitance' prototype containing only series LC pairs</td>
</tr>
</tbody>
</table>

Table 5: Comparison of different decompositions of second-order matrix equation

<table>
<thead>
<tr>
<th>Name (acronym)</th>
<th>Defining equations</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topological (TD)</td>
<td>7, 8</td>
<td>Most applicable to lowpass filters. When conventional transconductors are used, gives standard leapfrog filters</td>
</tr>
<tr>
<td>Left-inverse type 1 (LID1)</td>
<td>11, 12</td>
<td>Applicable to any bandpass filter. Only one value of transconductance needed per filter. Requires transconductor stages with unidirectional nonintegrating paths</td>
</tr>
<tr>
<td>Left-inverse type 2 (LID2)</td>
<td>15, 16</td>
<td>Similar to LID1, but gives sparser active circuit if a prototype is used, for which the condition $\Gamma = C$ is satisfied</td>
</tr>
<tr>
<td>Right-inverse (RID)</td>
<td>19, 20</td>
<td>Applicable to any bandpass filter. Conventional transconductors may be used, with up to two values required</td>
</tr>
<tr>
<td>Left-direct (LD)</td>
<td>23, 24</td>
<td>Applicable to all-pole bandpass filters. Only one value of transconductance needed per filter. Requires transconductor stages with unidirectional nonintegrating paths</td>
</tr>
</tbody>
</table>
Table 6: Comparison of different first-order transconductor stages

<table>
<thead>
<tr>
<th>Active circuit used</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional transconductor (Fig. 5)</td>
<td>Best for VHF, due to simplicity. Unidirectional nonintegrating paths are not available. Parasitic input and output capacitance must be compensated for.</td>
</tr>
<tr>
<td>Opamp plus transconductor or MOSFET 'resistor' (Fig. 8)</td>
<td>Presence of low-impedance nodes makes unidirectional paths available. Opamp dominant pole must be much higher in frequency than filter poles.</td>
</tr>
<tr>
<td>Transconductor with low-impedance inputs (Fig. 7)</td>
<td>Combines good high-frequency performance of conventional transconductor with usefulness of unidirectional paths. But parasitic capacitances have first-order effect.</td>
</tr>
</tbody>
</table>

the prototype ladder is represented by a second-order matrix equation, the way this equation may be decomposed into first-order equations, and the type of transconductor stage that is used to realise the first-order equation. The choices available and their relative merits are summarised in Tables 4 to 6. The utility of the techniques described is demonstrated by results from three high-frequency CMOS transconductor ladder filters.

7 References