Sampling, Reconstruction and Quantization

EE4015 Digital Signal Processing

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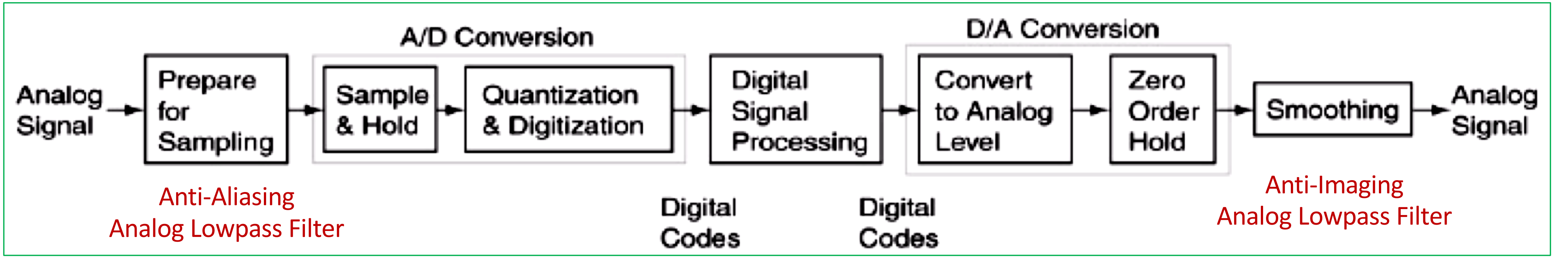
Content

- **ADC and DAC of DSP System**
- **Sampling of Continuous-Time Signals**
  - Model of the sampling process using modulation of the CT signal with impulse train (Time-domain)
  - Analysis of the sampling process in CTFT domain (Frequency Domain)
  - Nyquist Sampling Theorem and Anti-aliasing Filter
- **Reconstruction of Continuous-Time Signals**
  - Reconstruction Filter
- **Quantization of ADC and DAC**
- **2-D Sampling and Quantization (Optional = Not included in the Exam)**

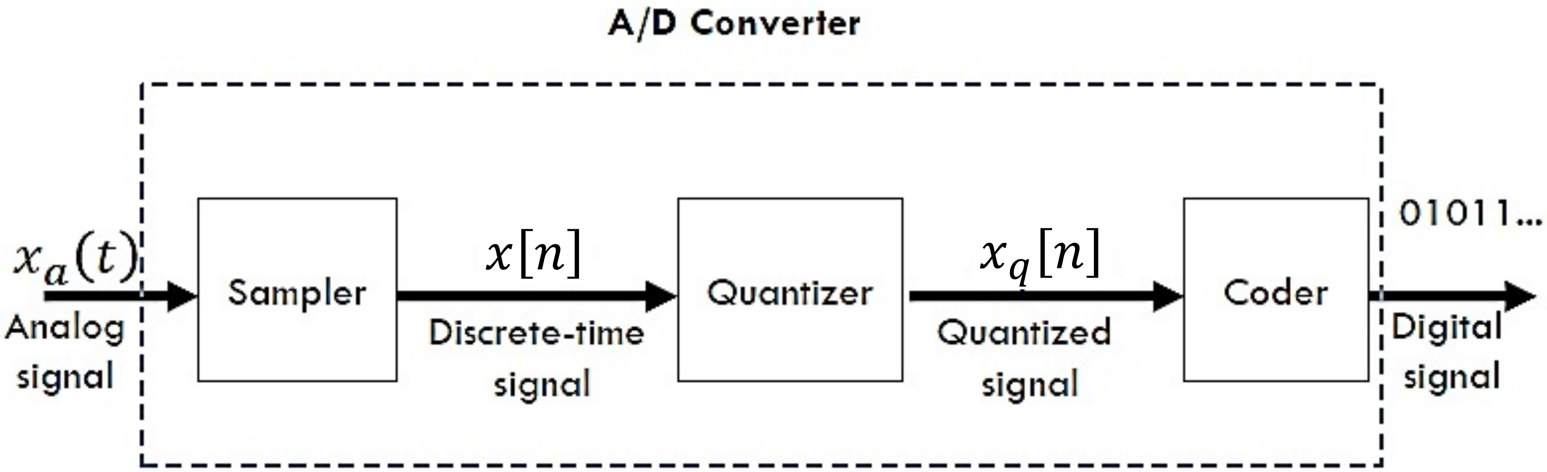
Typical Digital Signal Processing System

- A typical DSP system involving both sampling and reconstruction of analog signals
  - ADC (Analog-to-Digital Conversion) : Sampling of analog signals to generate discrete-time or digital signals
  - DAC (Digital-to-Analog Conversion) : Reconstruction of analog signals from digital signals



Analog to Digital (A/D) Conversion

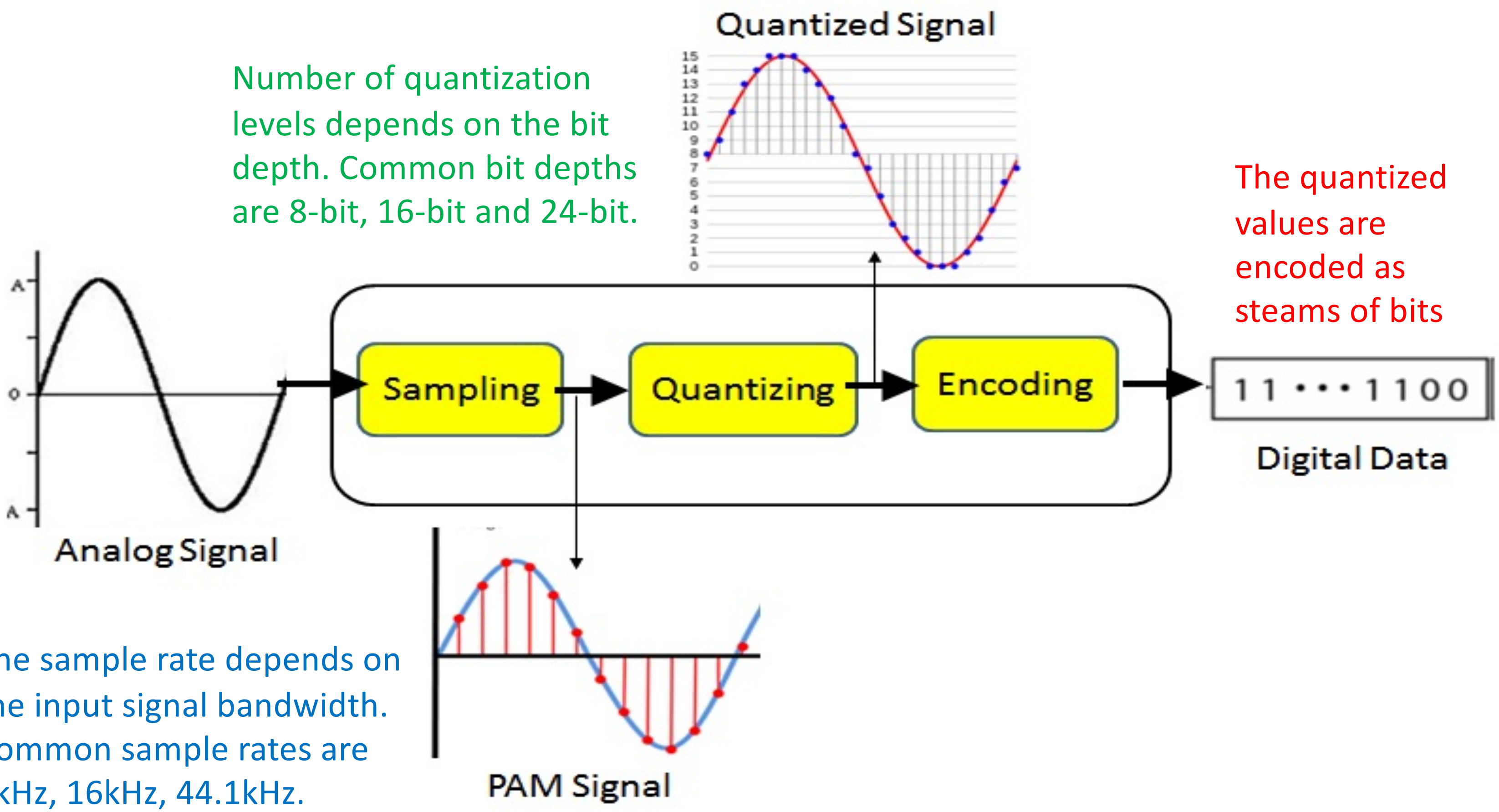
- Most signals of practical interest are analog in nature. Examples: Voice, Video, RADAR signals, Transducer/Sensor output, Biological signals, etc.
- In order to utilize those benefits, we need to convert our analog signals into digital. This process is called A/D conversion



Three Steps of A/D Conversion

- **Sampling**
  - Conversion from continuous-time and continuous-amplitude signal to discrete-time and continuous-amplitude signal
- **Quantization**
  - Conversion from discrete-time and continuous-amplitude signal to to discrete-time and discrete-amplitude signals
- **Encoding**
  - Conversion from a discrete-time and discrete-amplitude signal to an efficient digital data format.

Analog to Digital Conversion



Number of quantization levels depends on the bit depth. Common bit depths are 8-bit, 16-bit and 24-bit.

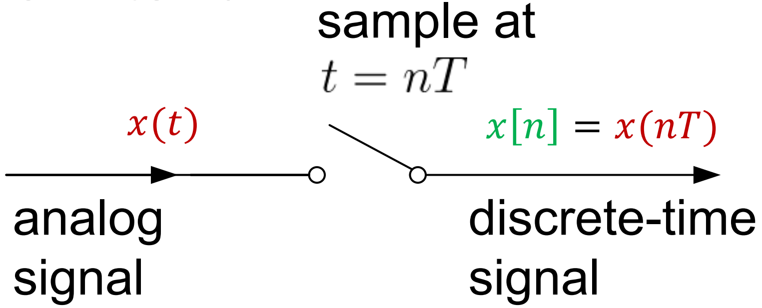
The quantized values are encoded as steams of bits

The sample rate depends on the input signal bandwidth. Common sample rates are 8kHz, 16kHz, 44.1kHz.

Sampling

Mathematical Modelling of Sampling

- Process of converting a continuous-time signal $x(t)$ into a discrete-time sequence $x[n]$
- $x[n]$ is obtained by extracting $x(t)$ every Ts where T is known as the sampling period or interval

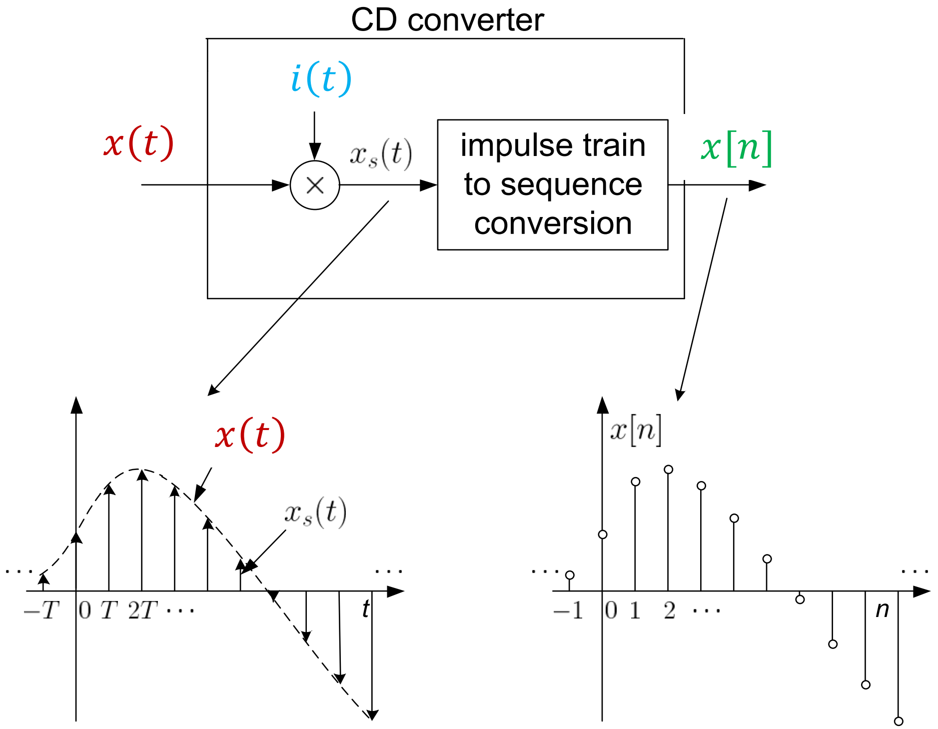


- Relationship between $x(t)$ and $x[n]$ is:

$$x[n] = x(t)\big|_{t=nT} = x(nT), \qquad n = \cdots, -2, -1, 0, 1, 2, \ldots$$

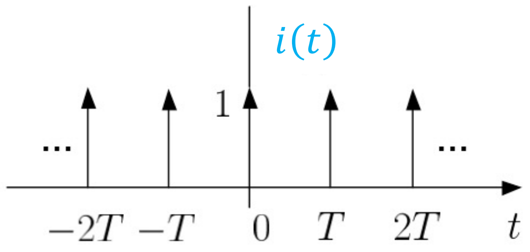
Continuous-Time to Discrete-Time (CD) Converter

- Conceptually, conversion of $x(t)$ to $x[n]$ is achieved by a continuous-time to discrete-time (CD) converter:



Continuous-Time Signal

Discrete-Time Signal



$i(t)$ is an impulse train with period *T*.

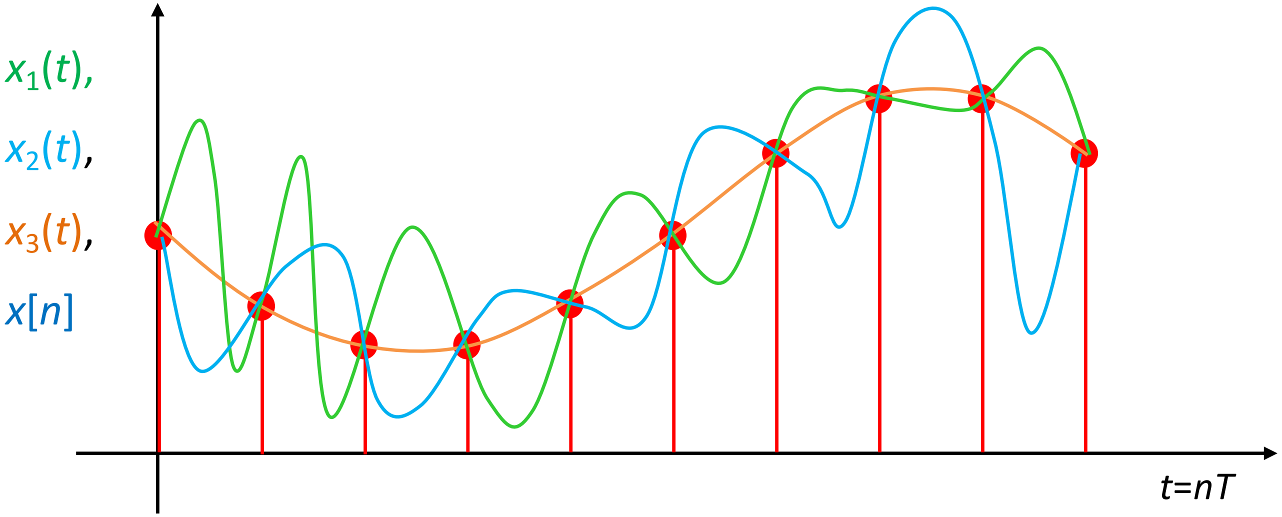
$$i(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

Relate Ω to ω

- To study both of continuous-time system and discrete-time system, we must understand the relationship between continuous-time angular frequency Ω $(rads/sec)$ and discrete-time angular frequency ω $(rads)$
- Let $x(t) = \mathrm{A}\cos(\Omega t + \phi)$ and $x[n] = \mathrm{A}\cos(\omega n + \phi)$
  - $x[n] = x(nT) = \mathrm{A}\cos(\Omega nT + \phi) = \mathrm{A}\cos(\omega n + \phi)$
  - $\omega = \Omega T$
  - DT frequency ω = (CT frequency Ω) x (Sampling Interval T)
  - The unit of ω is $rads = \frac{rads}{sec} \cdot sec$

What is the Major Concern on Sampling?

- A fundamental question is whether $x[n]$ can **uniquely represent** $x(t)$ or if we can use $x[n]$ to **reconstruct** $x(t)$
- It is because different analog signals, $x_1(t)$, $x_2(t)$, $x_3(t)$, can map to same sequence of $x[n]$ as shown on the figure below.

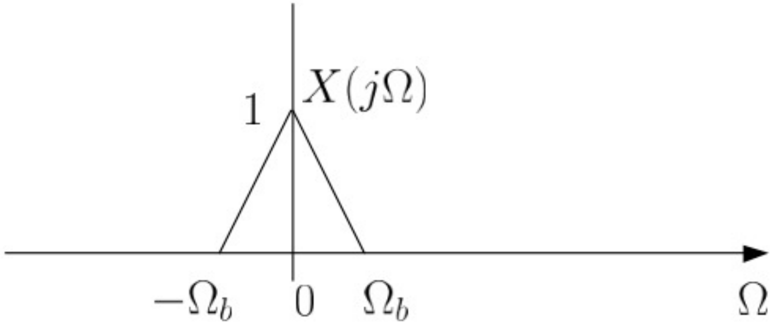


Sampling of Bandlimited Analog Signals

- For bandlimited analog signal, it is possible to uniquely represent in discrete-time $x[n]$, if the sampling period *T* is **sufficiently small**.

- The spectrum of **bandlimited signal** $x(t)$ can be defined as
  - $X(j\Omega) = 0 \quad \text{for } |\Omega| \geq \Omega_b$

  where Ω_b is called the bandwidth



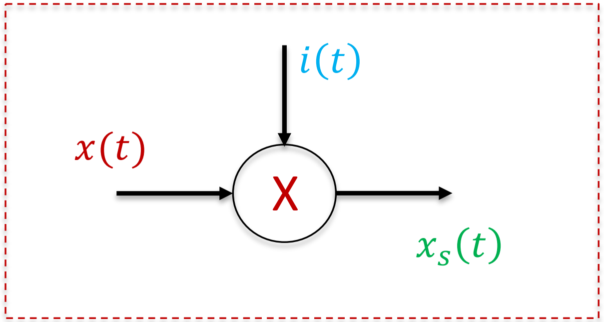
Spectrum of Bandlimited Signal

- To prove that and identify the maximum *T*, we need to study the sampling processing in Continuous-Time Fourier Transform domain.

Time-Domain Representation of Sampled Signal

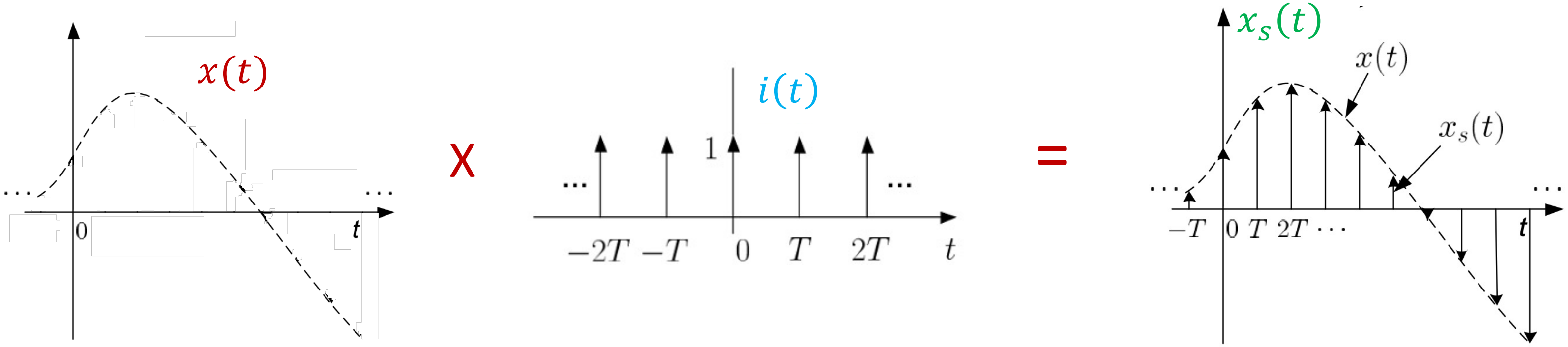
- In time-domain, the sampled signal is modeled as modulate the analog signal $x(t)$ with impulse train $i(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$

- The process can be represented as multiplying $x(t)$ by the impulse train $i(t)$

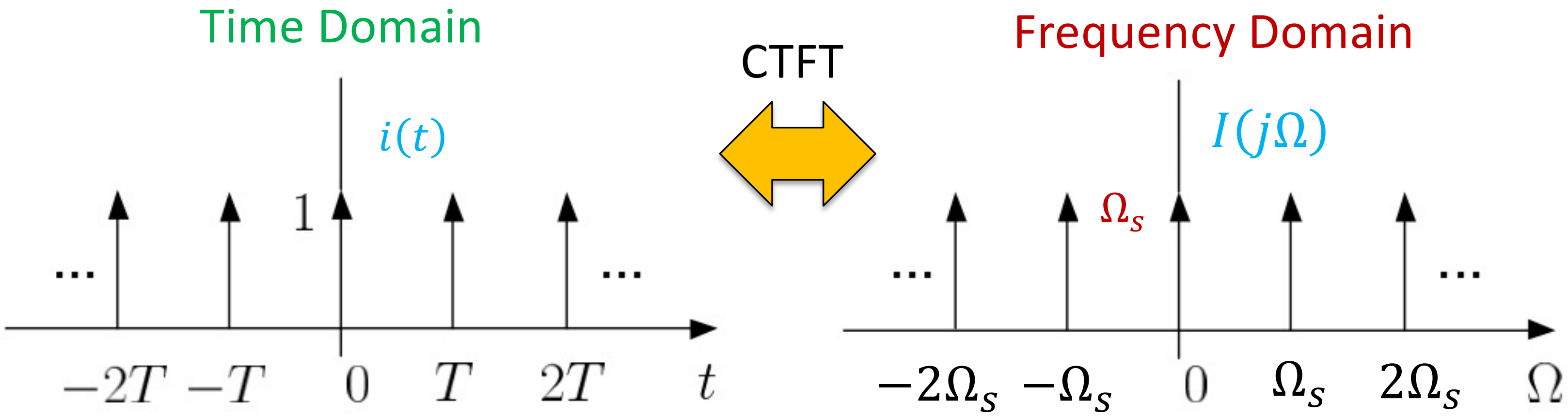


Modulation Model

$$x_s(t) = i(t)x(t) = x(t)\sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{k=-\infty}^{\infty} x[k]\delta(t - kT)$$



Frequency-Domain Representation of Impulse Train



$$i(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$I(j\Omega) = \Omega_s \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$$

Where the sample frequency $\Omega_s = \frac{2\pi}{T}$

Multiplication Property of CT Fourier Transform

- The multiplication operation of two continuous-time signals in time domain corresponds to convolution operation in Fourier Domain

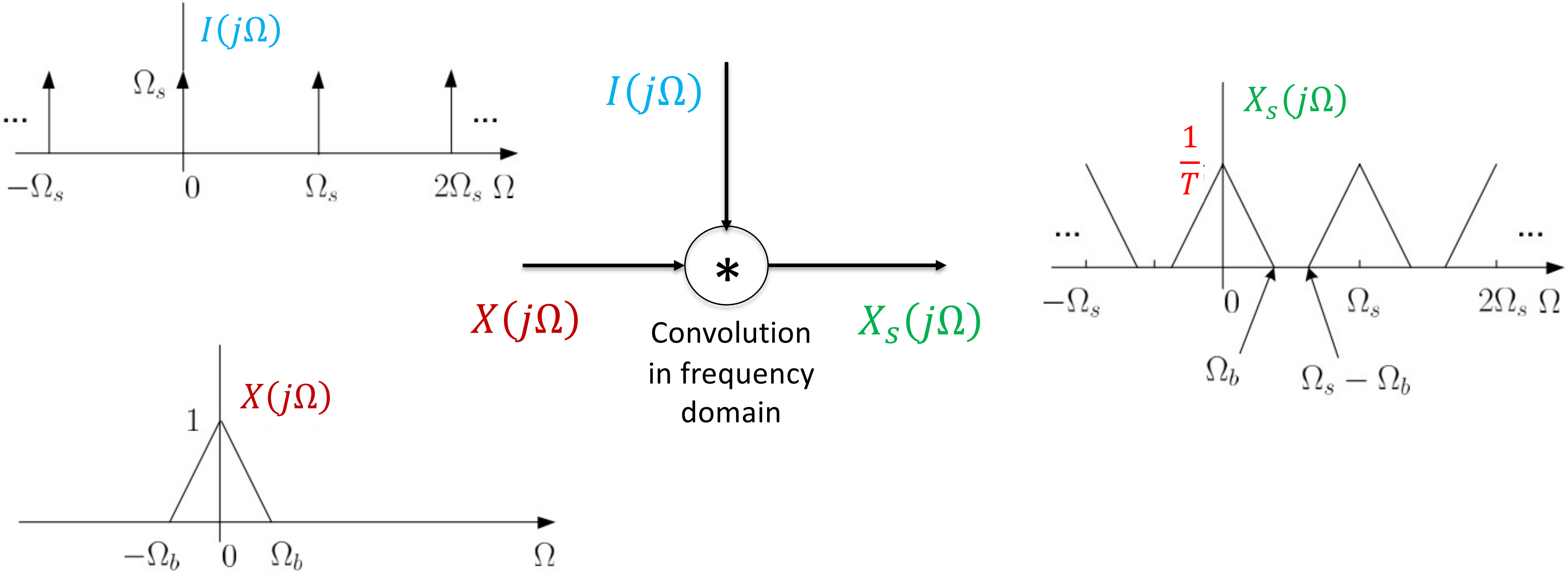
$$x_1(t)\cdot x_2(t) \leftrightarrow \frac{1}{2\pi}X_1(j\Omega) * X_2(j\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(j\tau)X_2(j(\Omega-\tau))d\tau$$

- Then,

$$x_s(t) = x(t)\cdot i(t) \quad \leftrightarrow \quad X_s(j\Omega) = \frac{1}{2\pi}X(j\Omega) * I(j\Omega)$$

Frequency-Domain Representation of Sampled Signal

$$x_s(t) = x(t) \cdot i(t) \quad \leftrightarrow \quad X_s(j\Omega) = \frac{1}{2\pi} X(j\Omega) * I(j\Omega)$$

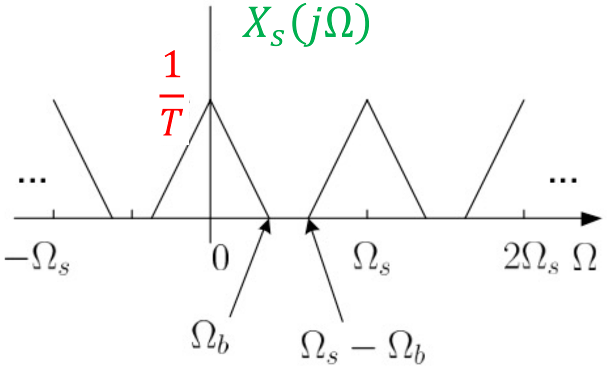


Convolution in frequency domain

Frequency-Domain Representation of Sampled Signal

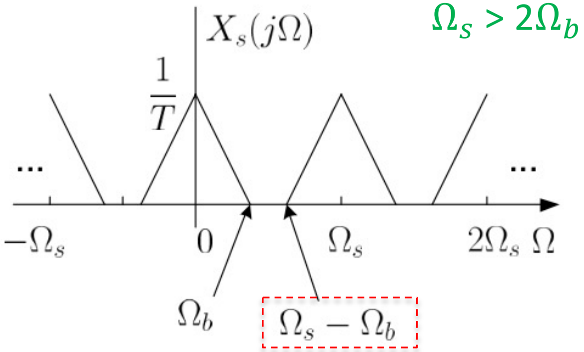
$$
\begin{aligned}
X_s(j\Omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} I(j\tau)X(j(\Omega-\tau))d\tau \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\Omega_s \sum_{k=-\infty}^{\infty}\delta(\tau-k\Omega_s)\right)X(j(\Omega-\tau))d\tau \\
&= \frac{1}{T}\sum_{k=-\infty}^{\infty}\left(\int_{-\infty}^{\infty} X(j(\Omega-\tau))\delta(\tau-k\Omega_s)d\tau\right) \\
&= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega-k\Omega_s))\left(\int_{-\infty}^{\infty}\delta(\tau-k\Omega_s)d\tau\right) \\
&= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\Omega-k\Omega_s))
\end{aligned}
$$

- This is the sum of infinite copies $X(j\Omega)$ scaled by $\frac{1}{T}=\frac{\Omega_s}{2\pi}$



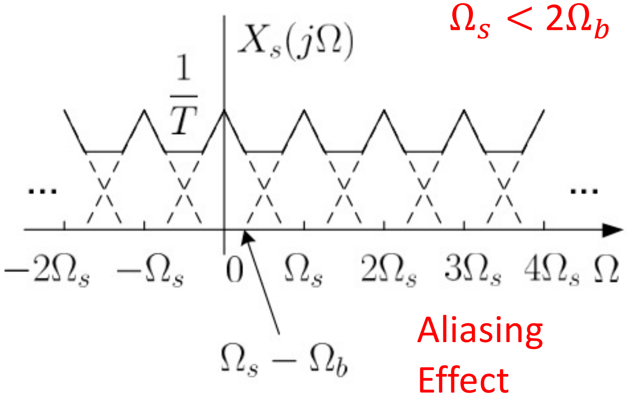
Sampling Frequency needs to sufficiently Large

- **When Ω_s is chosen sufficiently large** such that all copies of $X(j\Omega)/T$ do not overlap, that is , Ω_s- Ω_b> Ω_b or $\Omega_s > 2\Omega_b$, we can get $X(j\Omega)$ from $X_s(j\Omega)$



$\Omega_s > 2\Omega_b$

- However, **if Ω_s is NOT chosen sufficiently large** such that $\Omega_s < 2\Omega_b$, copies of $X(j\Omega)/T$ overlap, we cannot get $X(j\Omega)$ from $X_s(j\Omega)$

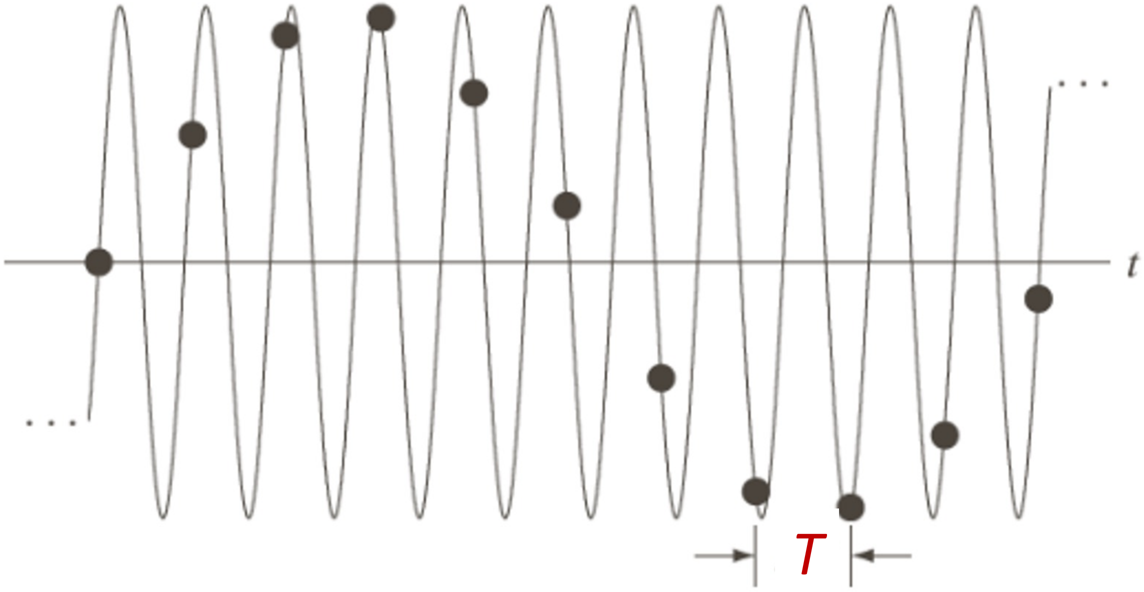


$\Omega_s < 2\Omega_b$

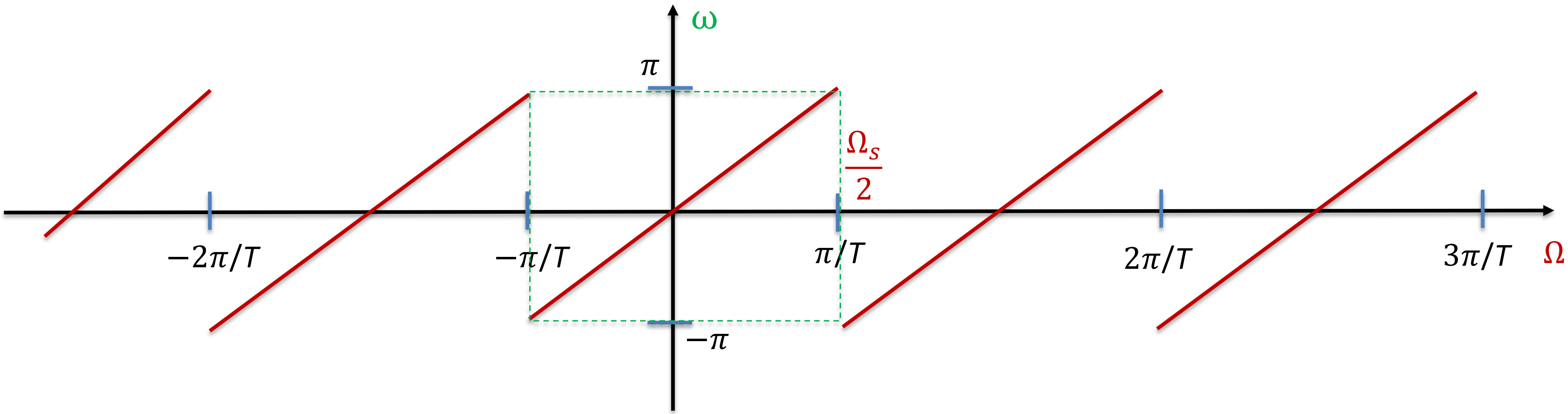
Aliasing Effect

Illustration of Aliasing

- The **under-sampled signal** (black dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal.
- The period of the sine wave is 2s, so the zero crossings of the horizontal axis occur every second *T* is the separation between samples.



Relationship Between CT Freq Ω and DT Freq ω

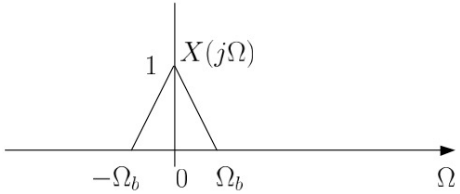


- **Aliasing** – CT sinusoids with frequency ($|\Omega| > \frac{\pi}{T} = \frac{\Omega_s}{2}$) appear at incorrect DT frequencies ω.
- Reconstruction – given DT sinusoid ω, cannot uniquely determine the CT sinusoid Ω unless we know $|\Omega| < \pi/\mathrm{T}$

https://www.youtube.com/watch?v=KuaannH5pnM&list=PLGI7M8vwfrFNzwXZcoaCamnZld7mXue5B&index=3

Nyquist Sampling Theorem (1928)

- Let $x(t)$ be a bandlimited CT signals with
  - $X(j\Omega) = 0 \quad \text{for } |\Omega| \geq \Omega_b$



- Then $x(t)$ is uniquely determined by its samples $x[n] = x(nT), \; n = \cdots, -2, -1, 0, 1, 2, \ldots,$ if
  - $\Omega_s = \frac{2\pi}{T} > 2\,\Omega_b$
- The bandwidth Ω_b is also known as the Nyquist frequency while $2\Omega_b$ is called the Nyquist rate and Ω_s must exceed $2\Omega_b$ in order to avoid aliasing.

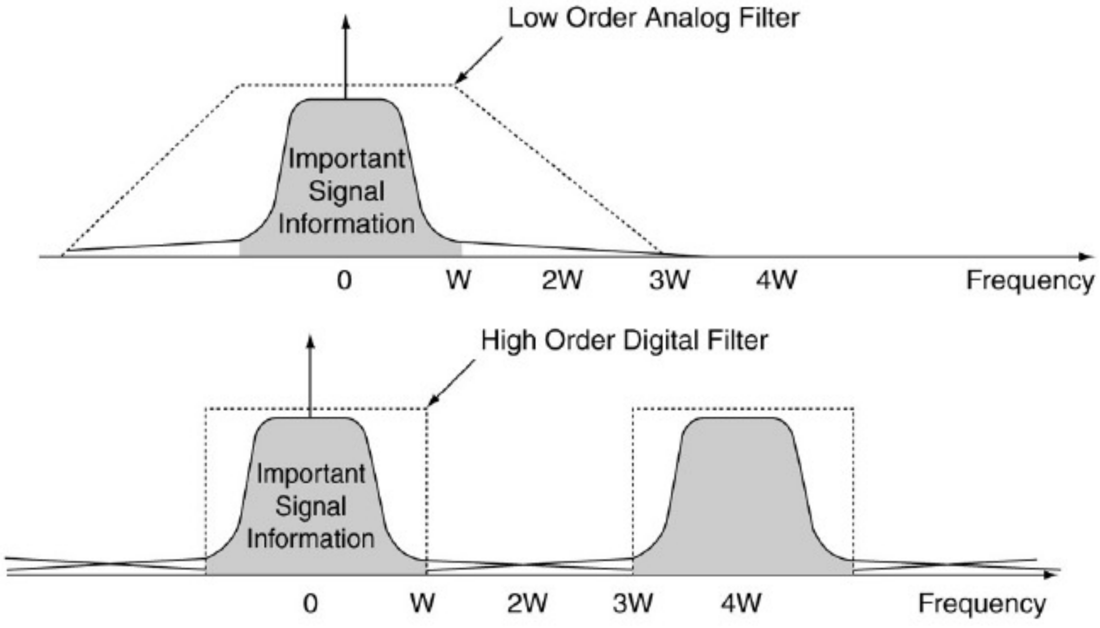
Over Sampling

- Over sampling is defined as sampling above the minimum Nyquist rate, that is, $\Omega_s > 2\Omega_{max}$ or $F_s > 2f_{\mathrm{max}}$.

- Over sampling is useful because it creates space in the spectrum that can reduce the demands on the analog anti-aliasing filter.

Over Sampling Example

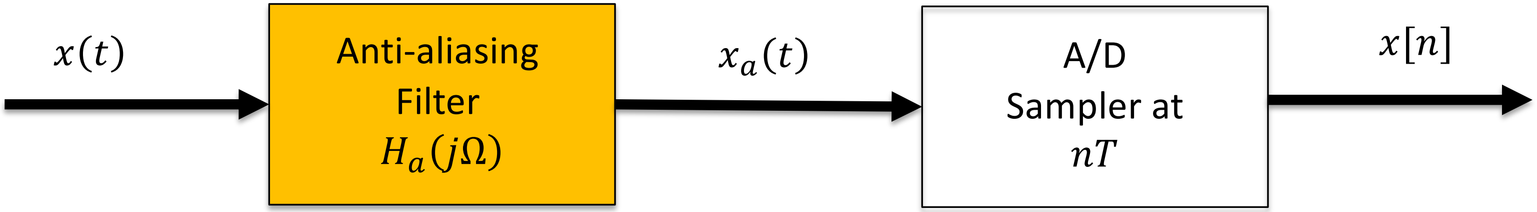
- In the example below, 2x oversampling means that a low order analog filter is adequate to keep important signal information intact after sampling.
- After sampling, higher order digital filter can be used to extract the information.



(b) Digital Filtering After Sampling

Practical Sampling with Anti-Aliasing Filter

- Many signals are not bandlimited
- Noise is usually present
  - Use analog lowpass filter before sampling to guarantee the signal is bandlimited and this filter is called Anti-aliasing filter



Example : Generation of Discrete-Time Signals

Assume we have a DSP system with a sampling time interval of 125 microseconds.

(a) Convert each of following analog signal $x(t)$ to a discrete-time signal $x[n]$:

1. $x(t) = 10e^{-5000t}u(t)$
2. $x(t) = 10\sin(2000\pi t)\,u(t)$

(b) Determine and plot the sample values from each obtained discrete-time function.

Example : Generation of Discrete-Time Signals

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(b) Determine and plot the sample values from each obtained discrete-time function.

Solution

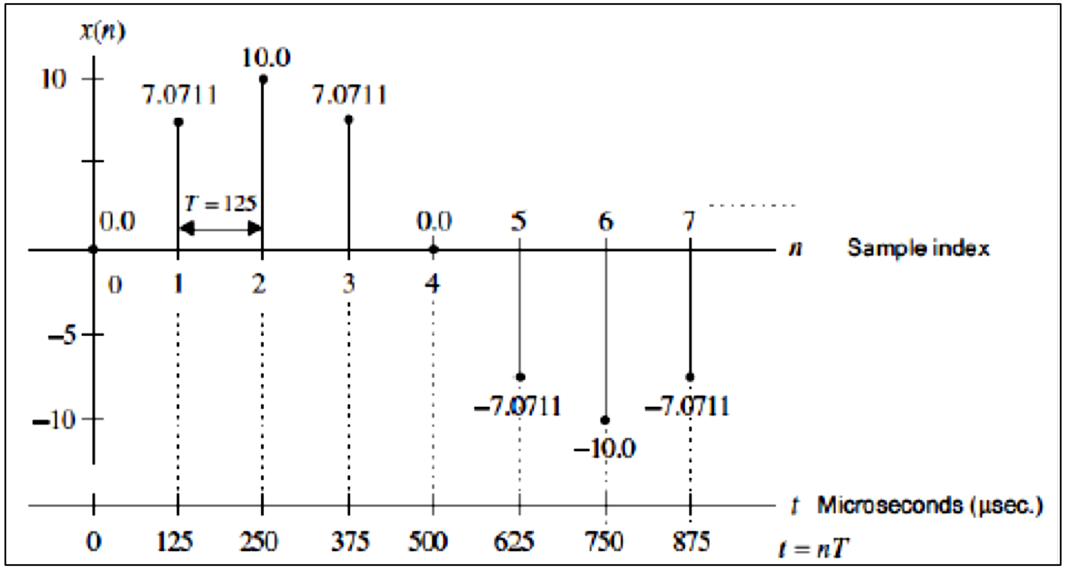
(a)

1. $x[n] = x(nT) = 10e^{-5000\times 0.000125n}u(nT) = 10e^{-0.625n}u[n]$

2. $x[n] = x(nT) = 10\sin(2000\pi\times 0.000125n)\,u(nT) = 10\sin(0.25\pi)\,u[n]$

Example : Generation of Discrete-Time Signals

(b) The first eight sample values for part (2) are calculated and plotted in the Figure.



Typical bandwidths and Sampling Frequency

Application	$f_{max} = \Omega_{max}/2\pi$	$F_s = \Omega_s/2\pi$
Biomedical	< 500 Hz	1 kHz
Telephone speech	< 4 kHz	8 kHz
Music - Compact Disc (CD)	< 20 kHz	44.1 kHz
Digital Audio Tape (DAT)	< 20 kHz	48 kHz
Hi-Res Audio	< 20 kHz	96, 192, 352.8 kHz
Ultrasonic	< 100 kHz	250 kHz
Radar	< 100 MHz	200 MHz

http://www.2l.no/hires/index.html?

Example 1

- Determine the Nyquist frequency and Nyquist rate for the CT signal $x(t)$ which has the form of:
  - $x(t) = 1 + \sin(2000\pi t) + \cos(4000\pi t)$

Solution

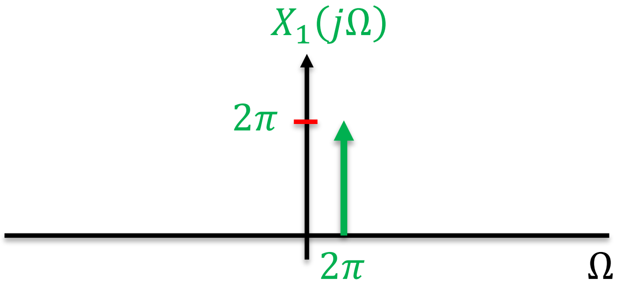
  - The continuous-Time angular frequencies are 0, 2000π and 4000π.
  - The Nyquist frequency is 4000π rads/sec or 2kHz
  - The Nyquist Rate is 8000π rads/sec or 4kHz

Example 2

- Complex exponential signals of $x_1(t) = e^{j2\pi}$ and $x_2(t) = e^{j18\pi}$, with sampling period of T = 0.1 sec.
- Draw the CTFT of $x_1(t)$, $x_2(t)$ and their sampled $x_{1s}(t)$ and $x_{2s}(t)$

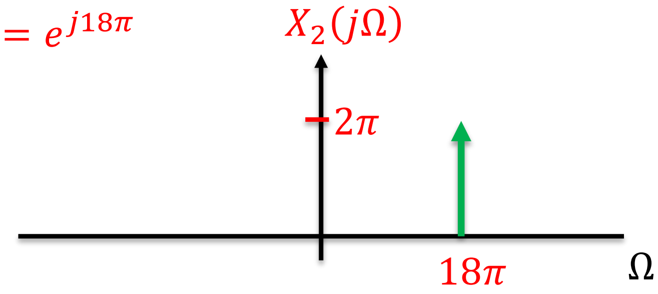
Solution

$x_1(t) = e^{j2\pi}$



$X_1(j\Omega) = 2\pi\delta(\Omega - 2\pi)$

$x_2(t) = e^{j18\pi}$



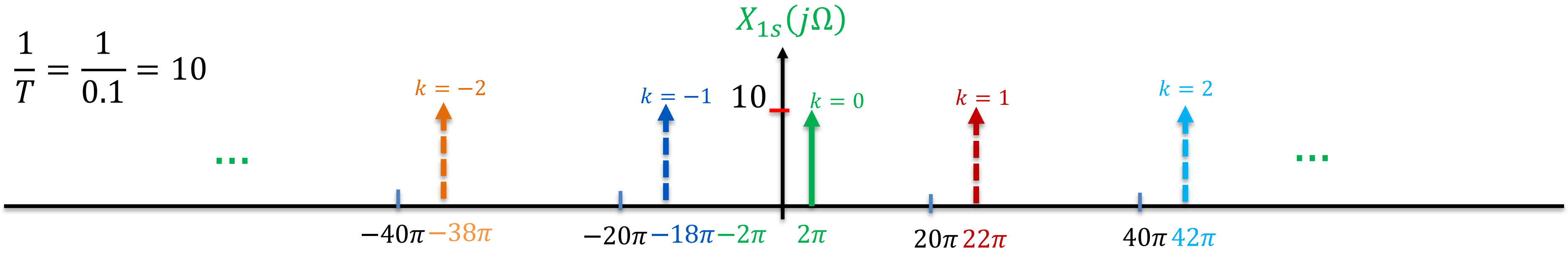
$X_2(j\Omega) = 2\pi\delta(\Omega - 18\pi)$

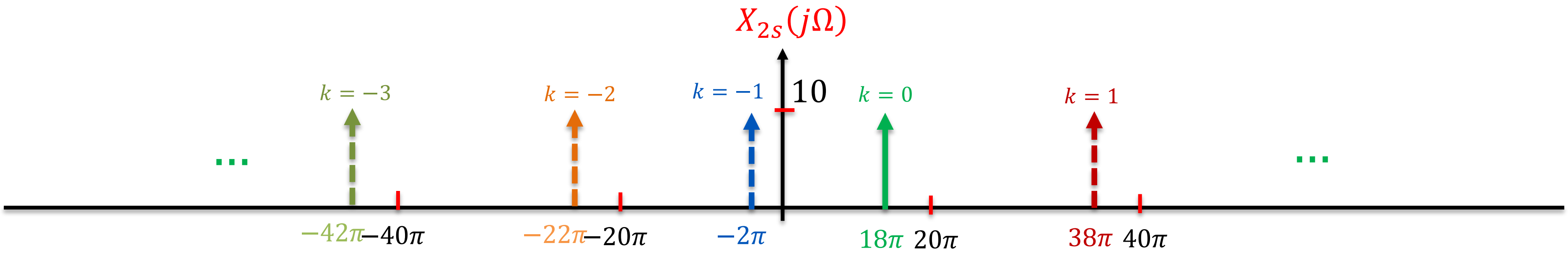
https://www.youtube.com/watch?v=duyQmhnse78&t=433s

Solution of Example 2

$$\Omega_s = \frac{2\pi}{0.1} = 20\pi$$

$$\frac{1}{T} = \frac{1}{0.1} = 10$$





https://www.youtube.com/watch?v=duyQmhnse78&t=433s

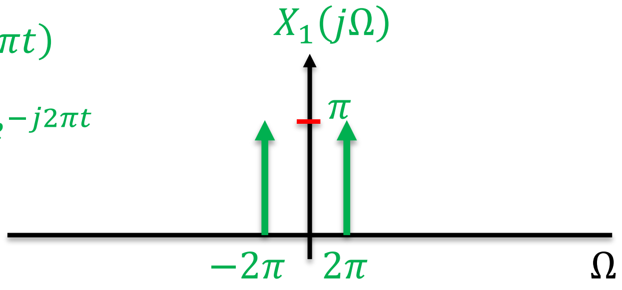
Example 3

- Sinusoids of $x_1(t) = \cos(2\pi t)$ and $x_2(t) = \cos(22\pi t)$, with sampling period of *T* = 0.1 sec.
- Draw the CTFTs of $x_1(t)$, $x_2(t)$ and their sampled $x_{1s}(t)$ and $x_{2s}(t)$

Solution

$$x_1(t) = \cos(2\pi t)$$

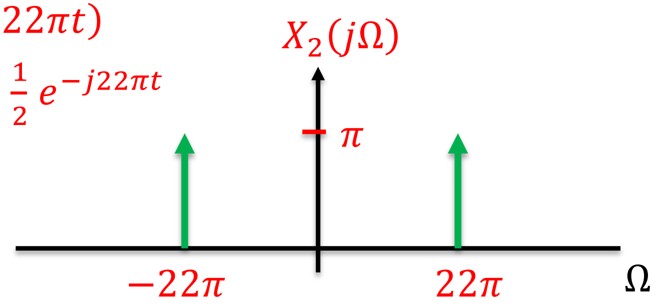
$$= \frac{1}{2}e^{j2\pi t} + \frac{1}{2}e^{-j2\pi t}$$



$$X_1(j\Omega) = \pi\ (\Omega + 2\pi) + \pi\delta(\Omega - 2\pi)$$

$$x_2(t) = \cos(22\pi t)$$

$$= \frac{1}{2}e^{j22\pi t} + \frac{1}{2}e^{-j22\pi t}$$



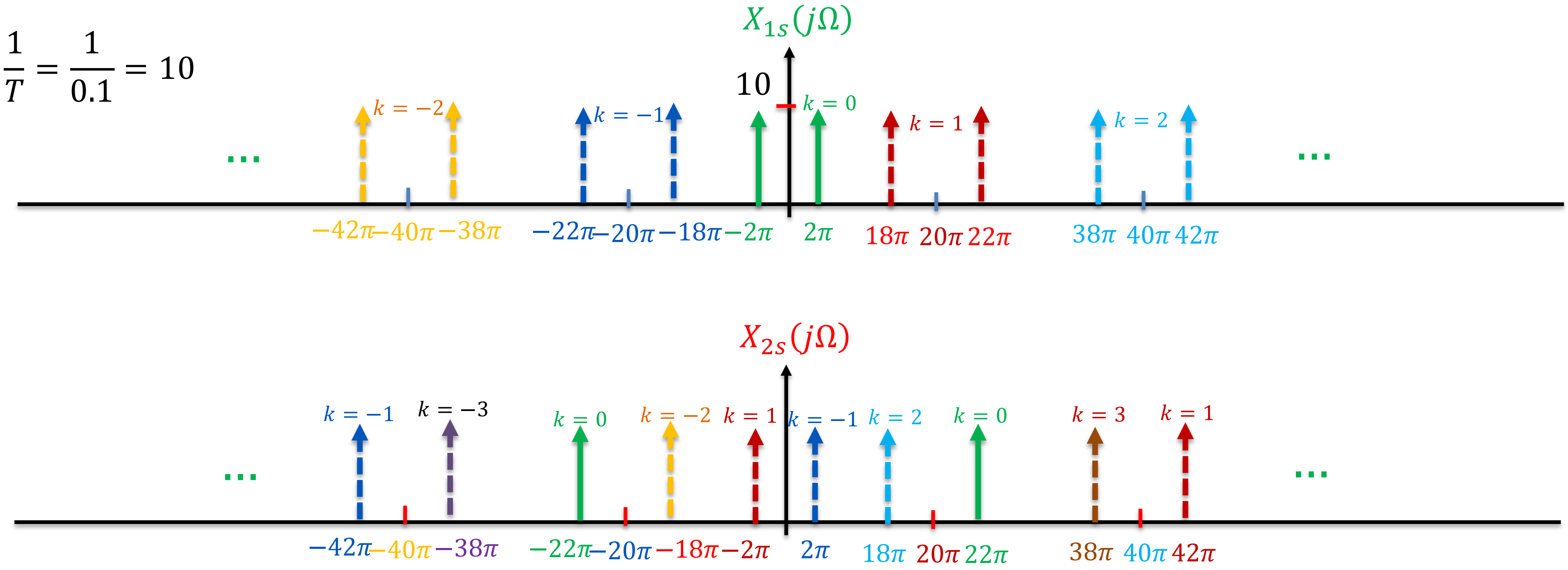
$$X_2(j\Omega) = \pi\ (\Omega + 22\pi) + \pi\delta(\Omega - 22\pi)$$

https://www.youtube.com/watch?v=duyQmhnse78&t=129s

Solution Example 3

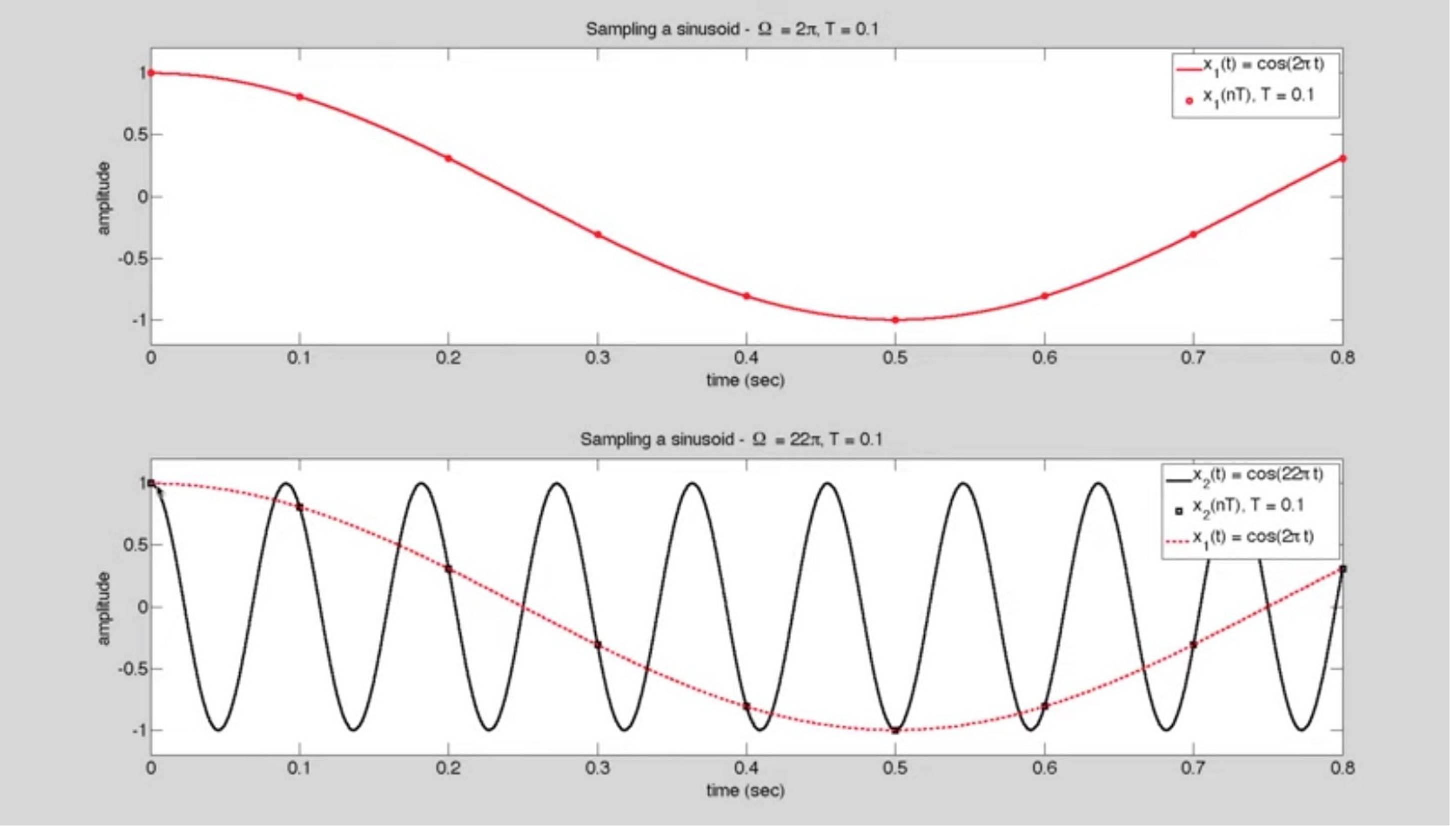
$$\Omega_s = \frac{2\pi}{0.1} = 20\pi$$

$$\frac{1}{T} = \frac{1}{0.1} = 10$$

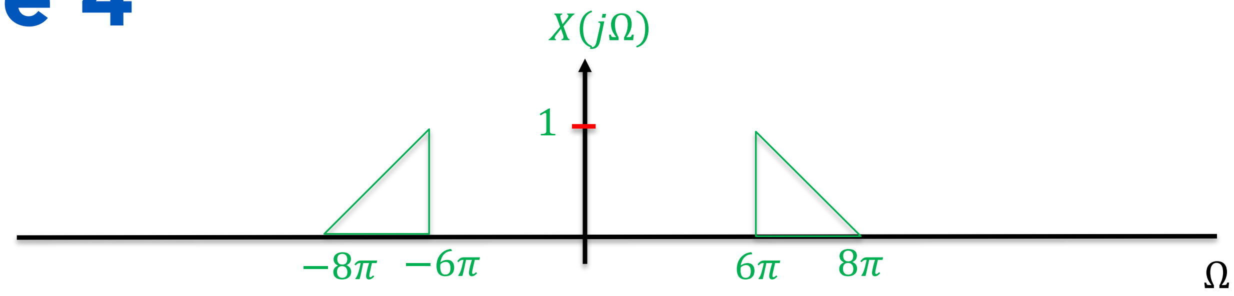


https://www.youtube.com/watch?v=duyQmhnse78&t=129s

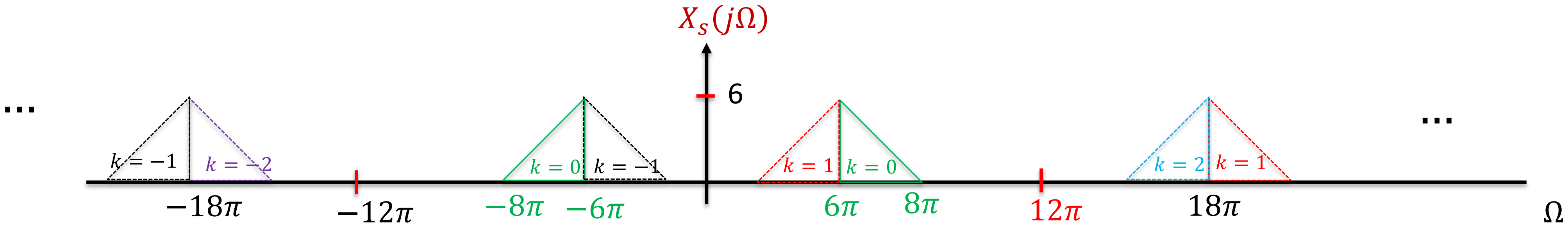
Reconstructed $x_1(t)$, $x_2(t)$



Example 4



- Sampling $x(t)$ at $T = \frac{1}{6}$ sec , => $\Omega_s = 12\pi$. Draw the spectrum $X_s(j\Omega)$ of the sampled signal.

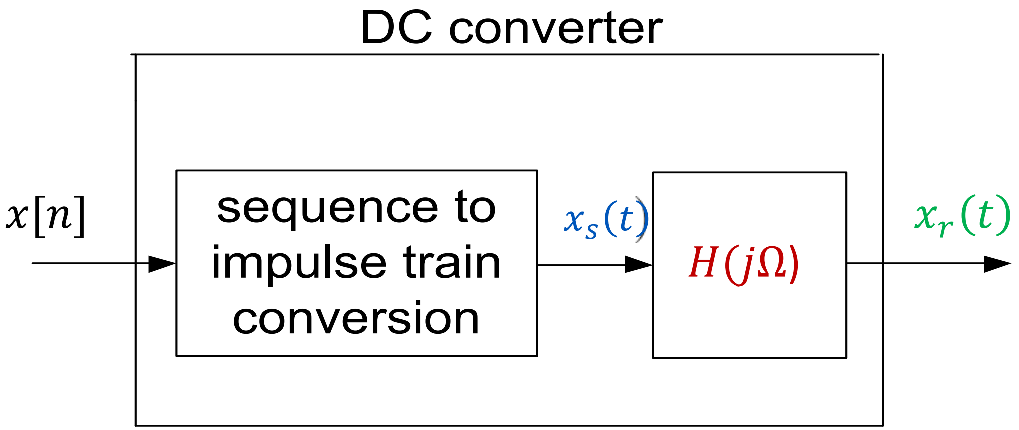


https://www.youtube.com/watch?v=duyQmhnse78&t=687s

Reconstruction

Analog Signals Reconstruction

- Process of transforming $x[n]$ back to analog signal $x_r(t)$ via Discrete-time to Continuous-time (DC) converter



DC converter

$x[n]$

sequence to impulse train conversion

$x_s(t)$

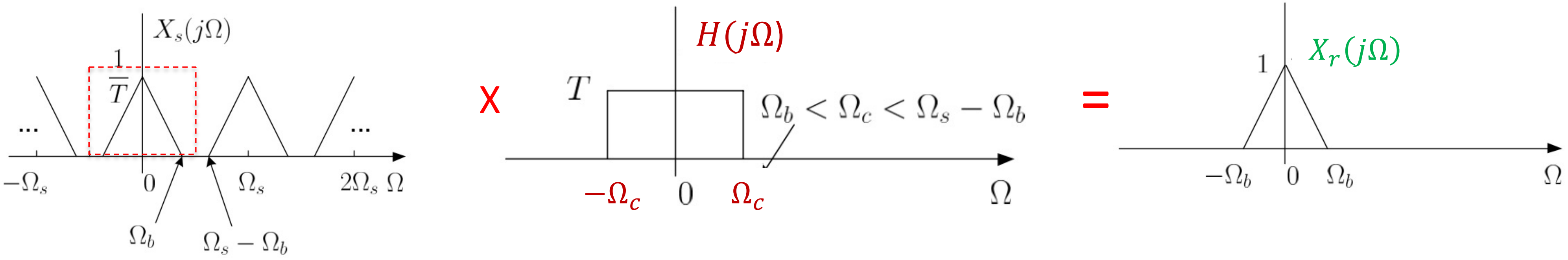
$H(j\Omega)$

$x_r(t)$

$$x_r(t) = \underbrace{x_s(t) * h(t)}_{\text{Convolution}} \quad \leftrightarrow \quad X_r(j\Omega) = \underbrace{X_s(j\Omega)\, H(j\Omega)}_{\text{Multiplication}}$$

Signal Reconstruction Model

- We know that the spectrum $X_s(j\Omega)$ of $x_s(t)$ is infinite copies $X(j\Omega)$ of the original bandlimited signal $x(t)$
- To reconstruction an analog signal $x_r(t)$, we can pass $x_s(t)$ through an ideal lowpass filter $H(j\Omega)$ with cut-off frequency Ω_c to remove all the replicated frequencies. This a multiplication operation in frequency domain.



The Specifications of the Lowpass Filter

$$H(j\Omega) = \begin{cases} T, & -\Omega_c < \Omega < \Omega_c \\ 0, & \text{otherwise} \end{cases}$$

- This is a lowpass filter. For simplicity, we set cut-off frequency Ω_c as the half of the sampling frequency Ω_s
  - $\Omega_c = \frac{\Omega_s}{2} = \frac{\pi}{T}$
- To obtain lowpass filter impulse response $h(t)$, we take inverse CTFT of $H(j\Omega)$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\Omega)e^{j\Omega t}d\Omega = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} Te^{j\Omega t}d\Omega = \frac{T\sin(\pi t/T)}{\pi t} = \text{sinc}\left(\frac{t}{T}\right)$$

The Reconstructed Analog Signal

- In time domain, we can obtain the reconstructed analog signal $x_r(t)$ by convolution between $x_s(t)$ and $h(t)$

$$\begin{aligned}
x_r(t) &= x_s(t) * h(t) \\
&= \left(\sum_{k=-\infty}^{\infty} x[k]\delta(t-kT) \right) * h(t) \\
&= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k]\delta(\tau-kT)h(t-\tau)d\tau \\
&= \sum_{k=-\infty}^{\infty} x[k]h(t-kT) \\
&= \sum_{k=-\infty}^{\infty} x[k]\mathrm{sinc}\left(\frac{t-kT}{T}\right)
\end{aligned}$$

2D Sampling and Reconstruction

(Optional)

2D Continuous-Time Fourier Transform (2D CTFT)

- 2D Continuous-Time Fourier Transform and Inverse

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(xu+yv)}\,dx\,dy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(xu+yv)}\,du\,dv$$

$$\boxed{\begin{aligned}&\mathfrak{F}\{f(x,y)\} = F(u,v)\\&\mathfrak{F}^{-1}\{F(u,v)\} = f(x,y)\end{aligned}}$$

Sampling and Aliasing in 2D

- We often subsample an image
  - When we originally digitize it
  - When we shrink it
- We can reconstruct the image exactly from the samples if the samples are “dense” enough
- The sampling theorem says that the sampling rate must be more than twice the maximum frequency of the image (this is the “Nyquist rate”)
- If the sampling rate is lower, we can get errors in the reconstructed image (called “aliasing”)

Sampling in 2D

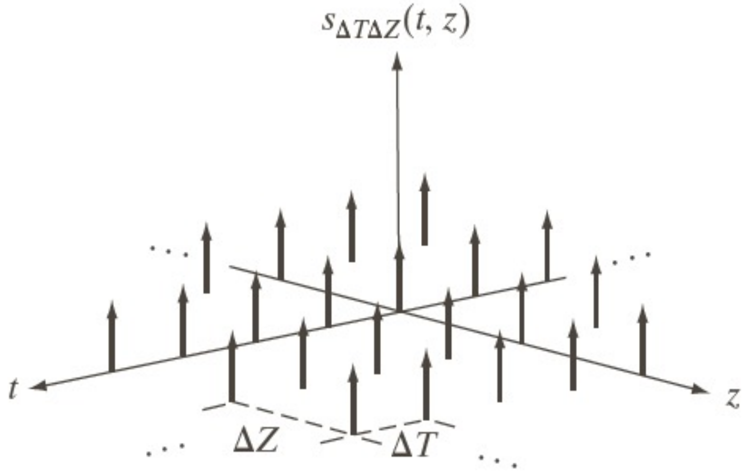
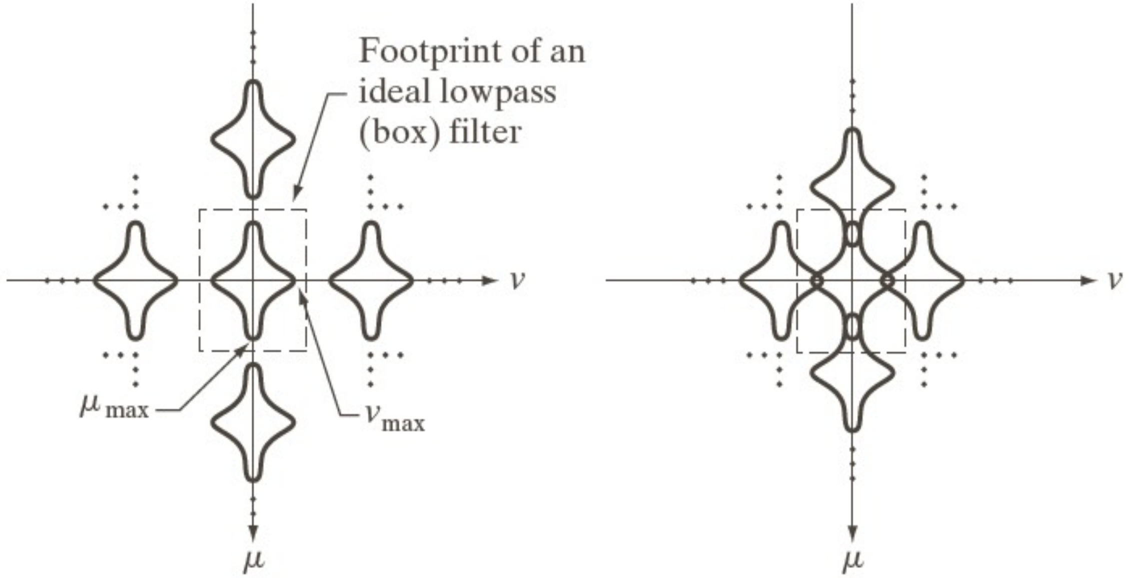


FIGURE 4.14
Two-dimensional impulse train.



a b

FIGURE 4.15
Two-dimensional Fourier transforms of (a) an over-sampled, and (b) under-sampled band-limited function.

Example of Aliasing (1)

a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3 × 3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

Example of Aliasing (2)

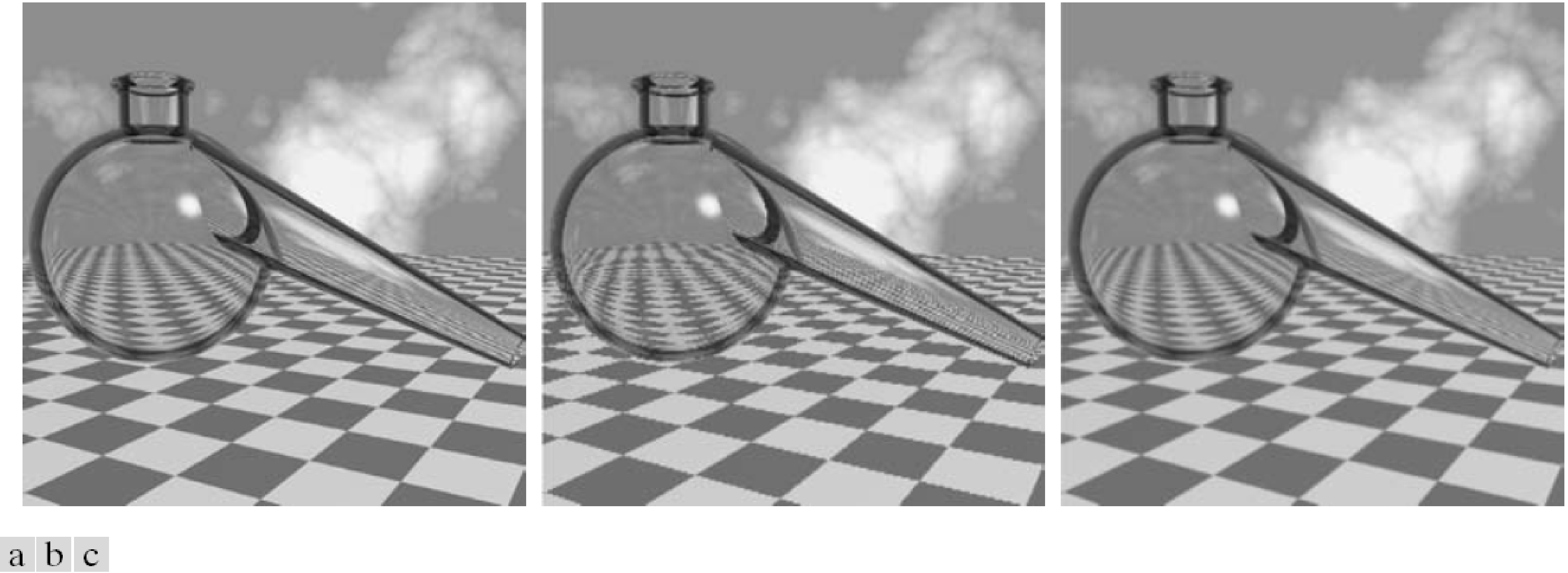


FIGURE 4.18 Illustration of jaggies. (a) A 1024 × 1024 digital image of a computer-generated scene with negligible visible aliasing. (b) Result of reducing (a) to 25% of its original size using bilinear interpolation. (c) Result of blurring the image in (a) with a 5 × 5 averaging filter prior to resizing it to 25% using bilinear interpolation. (Original image courtesy of D. P. Mitchell, Mental Landscape, LLC.)

Example of Aliasing (3)

Subsampling without low pass filtering

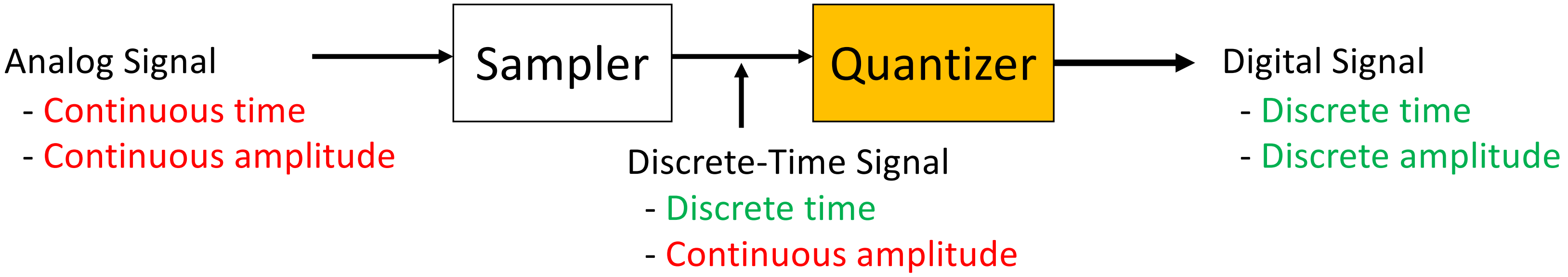
Original image

Subsampling after low pass filtering

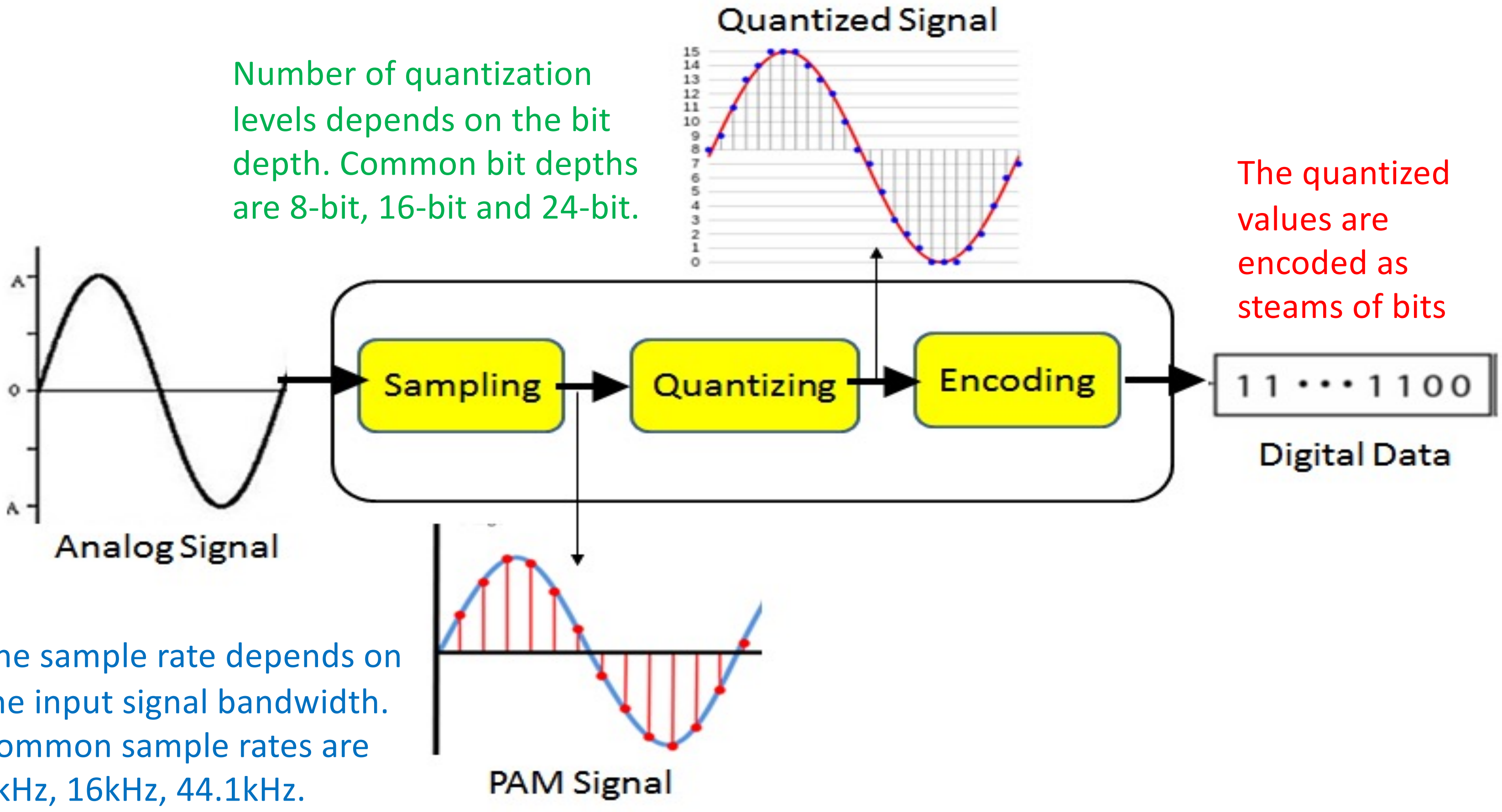
Quantization

Quantization

- After the sampling, the discrete-time continuous-amplitude signal still carry infinite information (can take any value) in terms of amplitude.
- **Quantization** is a non-linear transformation which maps continuous amplitude to a finite set of possible values.
- It is also the second step required by A/D conversion.



Analog to Digital Conversion



Number of quantization levels depends on the bit depth. Common bit depths are 8-bit, 16-bit and 24-bit.

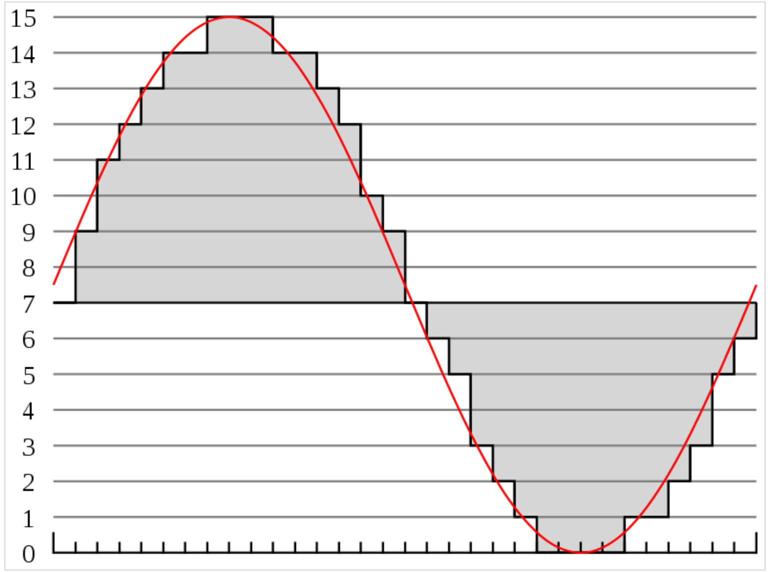
The quantized values are encoded as steams of bits

The sample rate depends on the input signal bandwidth. Common sample rates are 8kHz, 16kHz, 44.1kHz.

Quantizer

- The A/D converter chooses a quantization level for each analog sample.
- Number of levels of quantizer is equal to $L = 2^N$
- An N-bit converter chooses among 2^N possible quantization levels.
- The *N* is also called the bit depth which is the amplitude resolution
- Therefore, a 4-bit quantizer has 16 quantization levels (0, 1, 2, 3, … , 15) as shown on the right.

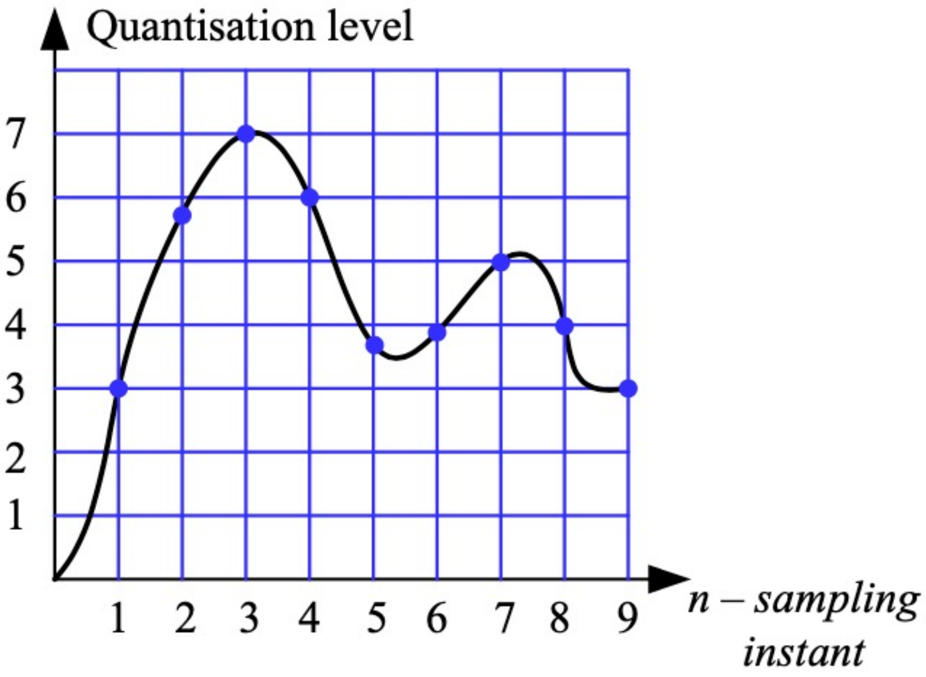
4-bit Quantization



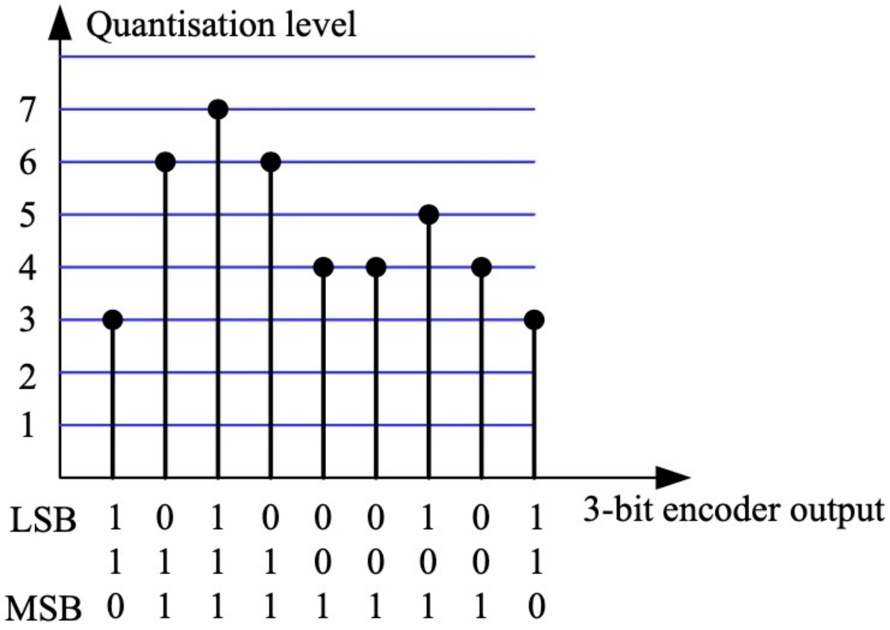
$N = 4, L = 2^4 = 16$

3-Bit Quantization of Discrete-Time Signals

Original Analog Signal



3-Bit Quantized Digital Signal



$N = 3, L = 2^3 = 8$

Quantization Step Size (Resolution)

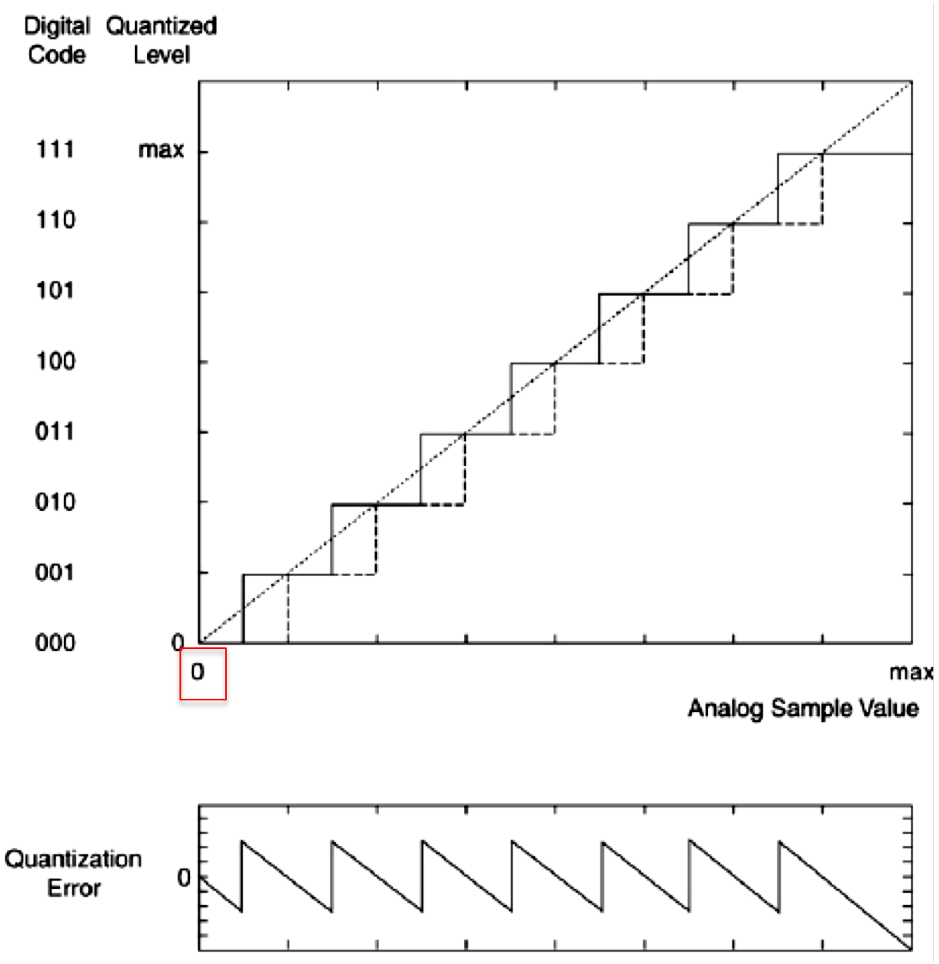
- The **quantization step size** or **resolution** is defined as as:

$$\Delta = \frac{R}{2^N} = \frac{R}{L}$$

  - R is the full-scale range of the amplitude $(Y_{max} - Y_{min})$
  - N is the number of bits (or bit depth)
- More quantization levels, a better resolution!
- The strength of the signal compared to that of the quantization errors is measured by dynamic range and signal-to-noise ratio.

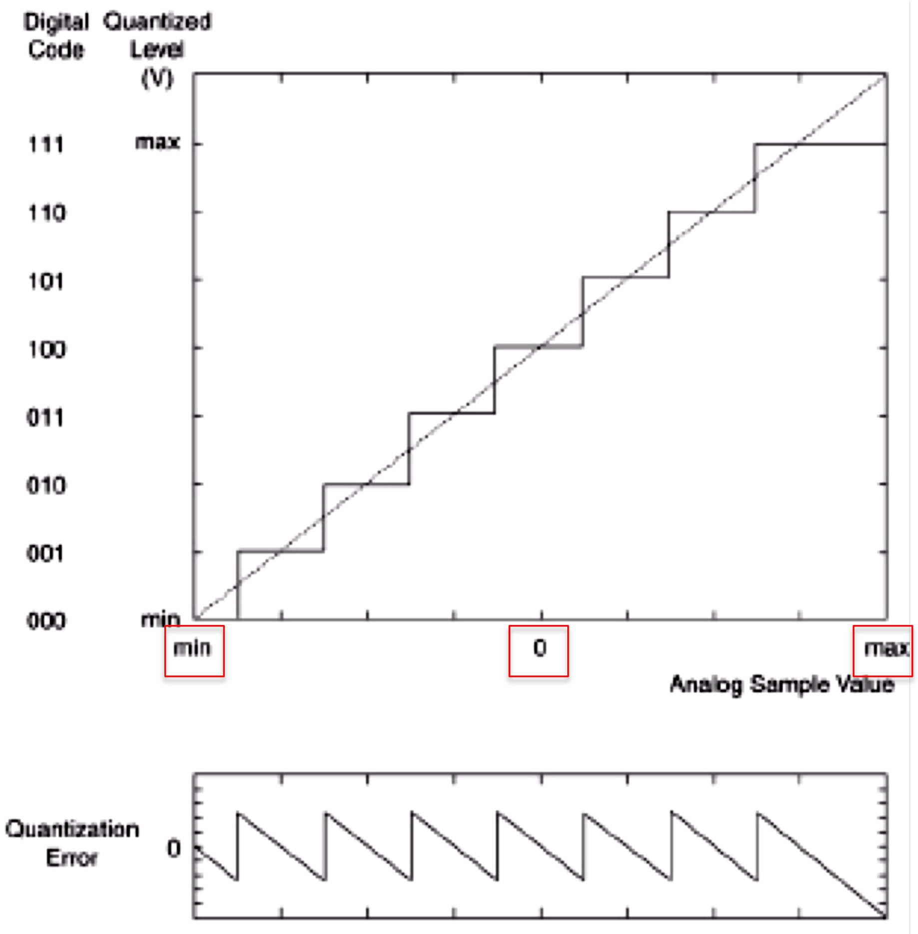
Unipolar Quantizer

- A unipolar quantizer deals with analog signals ranging from 0 volt to a positive reference voltage.
- Maximum Quantization Error = $\frac{\Delta}{2}$



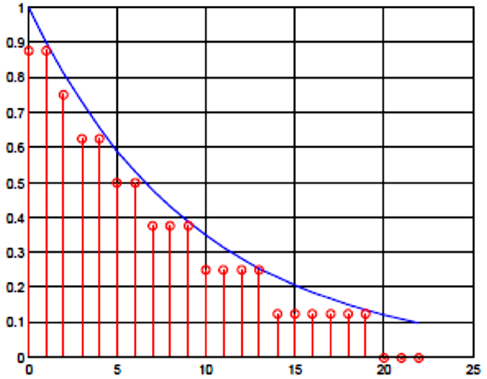
Bipolar Quantizer

- A bipolar quantizer deals with analog signals ranging from a negative reference and a positive reference voltage.
- Maximum Quantization Error = $\frac{\Delta}{2}$

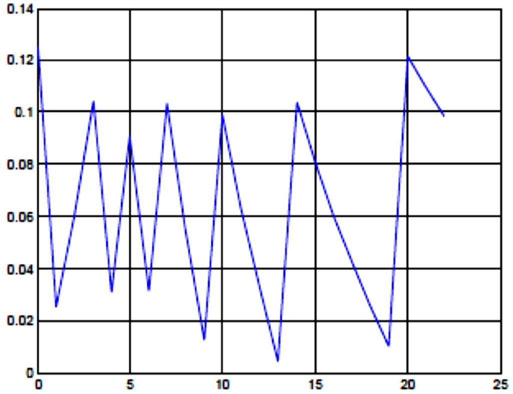


Reduce Quantization Error by Increase Bit Depth

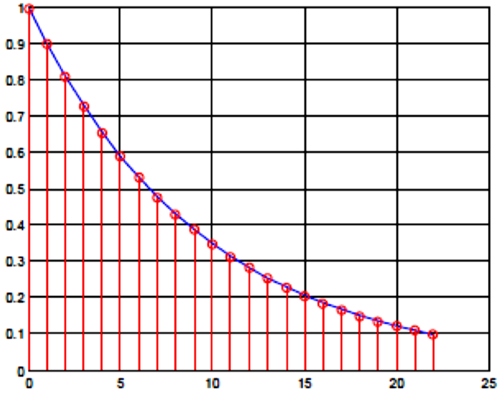
- Quantization error can be reduced, however, if the number of quantization levels is increased as illustrated in the figure



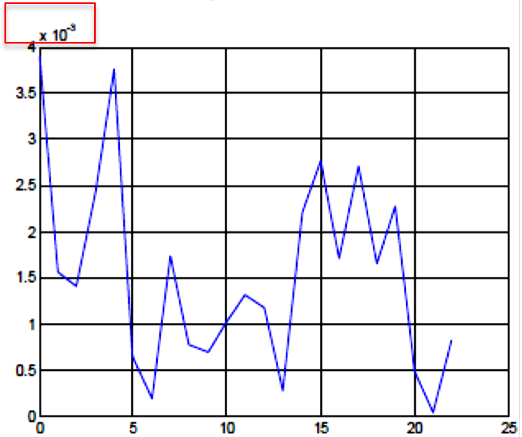
3-bit ADC



Quant. error



8-bit ADC

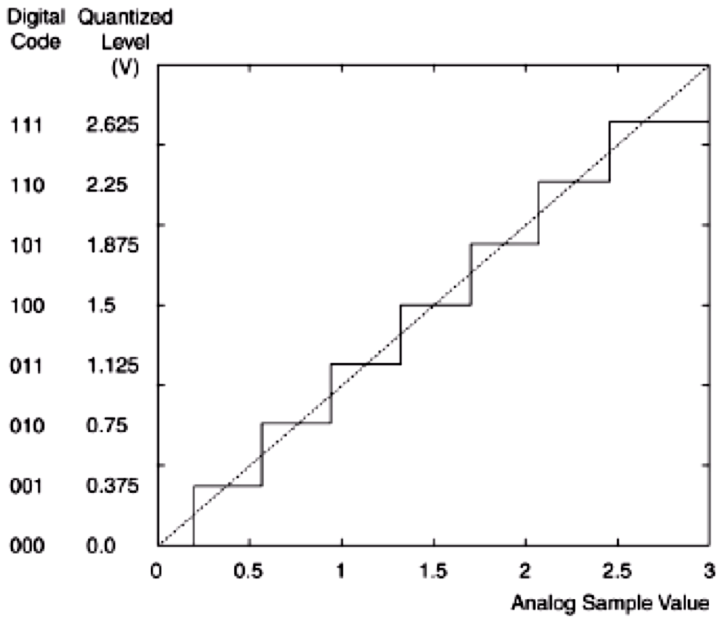


Quant. error

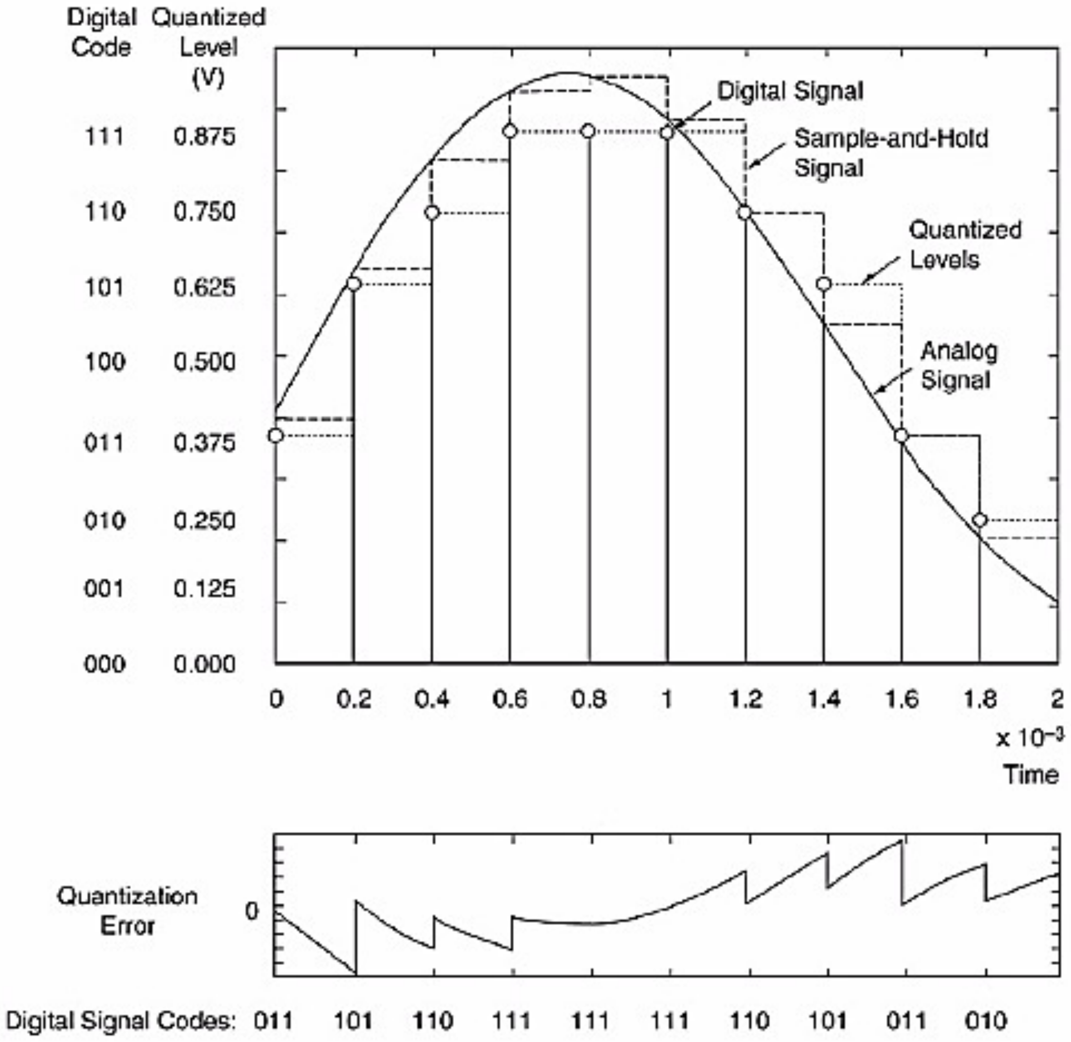
Quantization Example

Analog pressures are recorded using a pressure transducer as voltages between 0 and 3 V. The signal must be quantized using a 3-bit digital code. Indicate how the analog voltages will be covered to digital values.

- The quantization step size is
  - $\Delta = \frac{R}{2^N} = \frac{3-0}{2^3} = 0.375\ \mathrm{V}$
- The half of quantization step is
  - $\frac{\Delta}{2} = \frac{0.375}{2} = 0.1875\ V$



Three-bit A/D Conversion

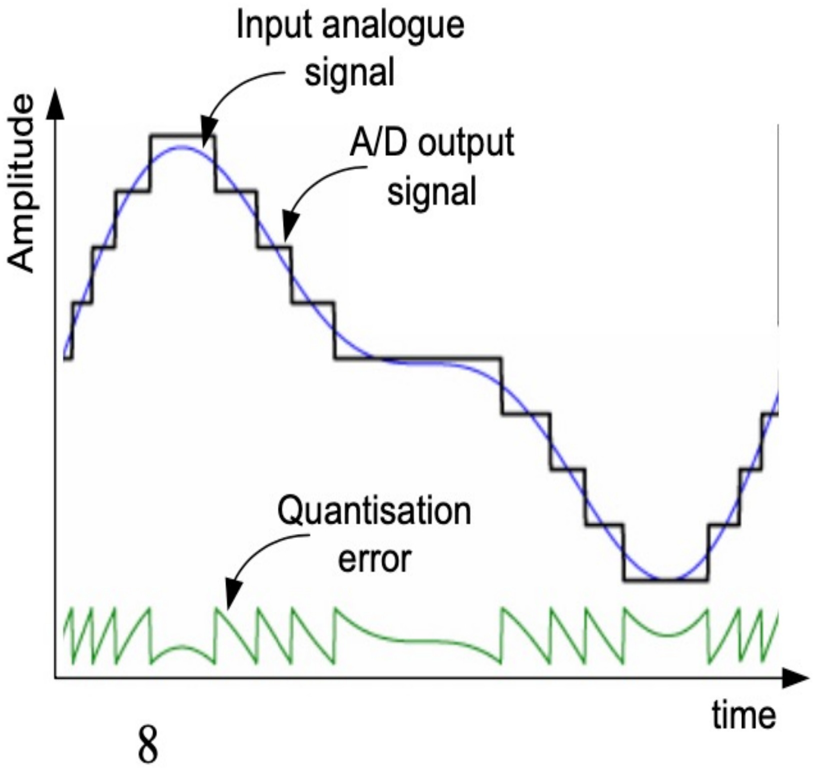


Quantization Error

- Suppose a quantizer operation given by Q(.) is performed on continuous-valued samples x[n] is given by Q(x[n]), then the quantization error is given by

$$\boxed{e_q[n] = x[n] - x_q[n]}$$

- This error caused by representing a continuous-valued signal (infinite set) by a finite set of discrete-valued levels.
- Most quantization errors are limited in size to half a quantization step Δ.



Signal-to-Quantization-Noise Ratio (SQNR)

- Provides the ratio of the signal power to the quantization noise (or quantization error)

$$Pq = \frac{1}{N}\sum_{n=0}^{N-1}\left(e_q[n]\right)^2 = \frac{1}{N}\sum_{n=0}^{N-1}\left(x[n]-x_q[n]\right)^2$$

- Mathematically,

$$SQNR_{dB} = 10\log_{10}\left(\frac{P_x}{P_q}\right)$$

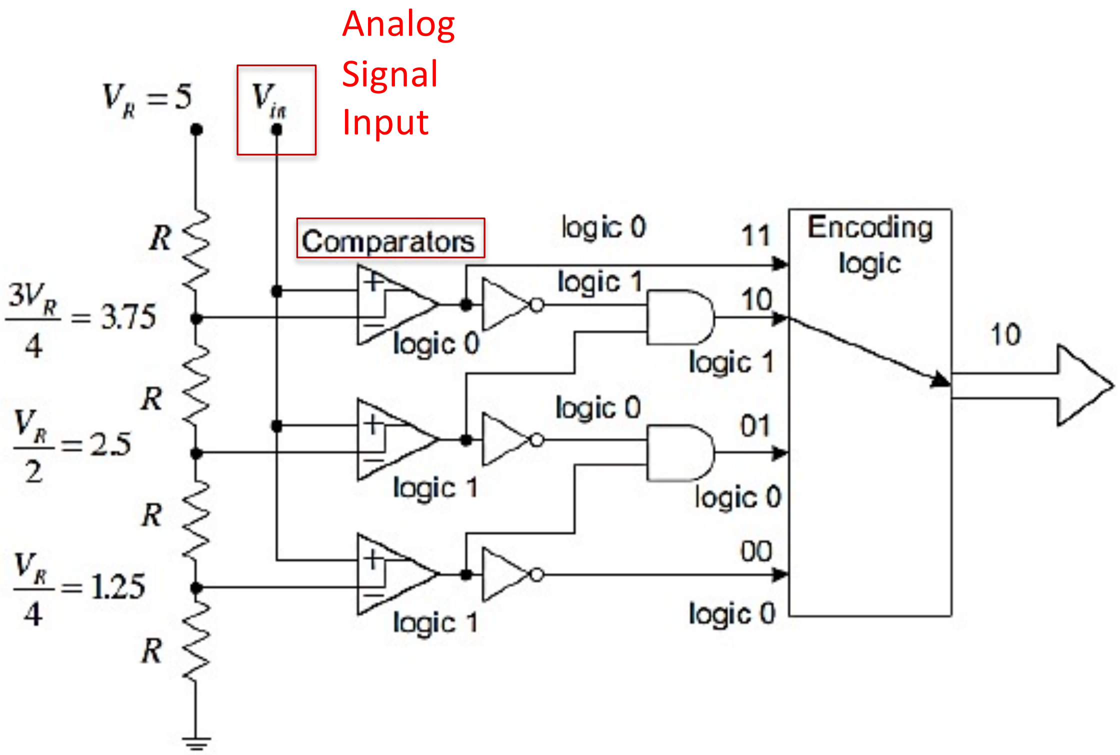
where

- P_x= Power of the signal ‘x’ (before quantization)
- P_q= Power of the error signal ‘x_q’

Dynamic Range

- Quantization errors can be determined by the quantization step.
  - Quantization errors can be reduced by increasing the number of bits used to represent each sample.
  - Unfortunately, these errors cannot be eliminated, and their combined effect is called quantization noise.
- The dynamic range of the quantizer is the number of levels it can distinguish in noise.
  - It is a function of the range of signal values and the range of error values, and is expressed in decibels, dB.
  - $Dynamic\ Range = 20log_2\left(\frac{R}{\Delta}\right)$

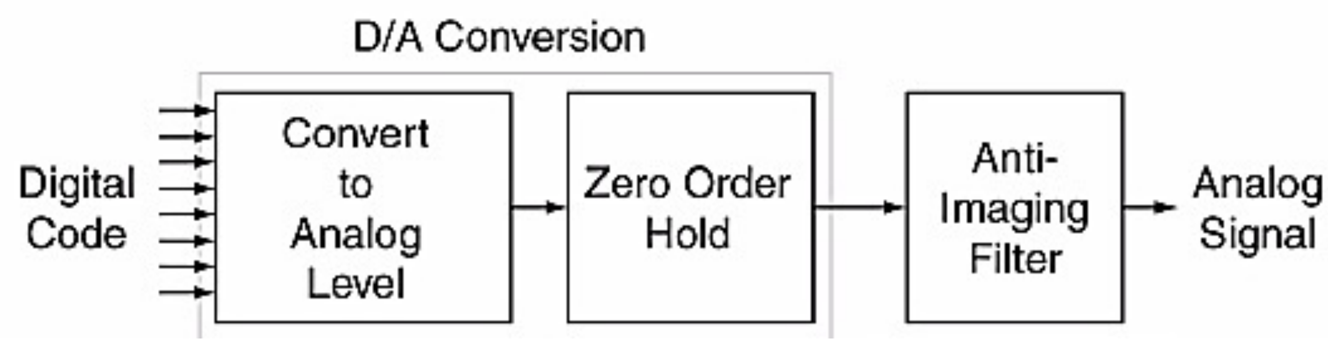
2-Bit Flash ADC (Optional)

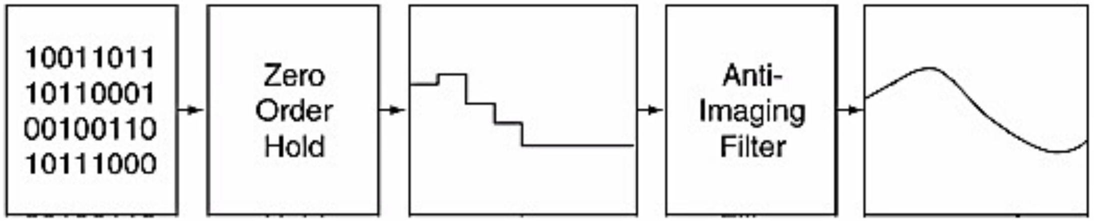


Digital-to-Analog (D/A) Conversion

- Once digital signal processing is complete, digital-to-analog (D/A) conversion must occur.
- This process begins by converting each digital code into an analog voltage that is proportional in size to the number represented by the code.
- This voltage is held steady through zero order hold until the next code is available, one sampling interval later.
- This creates a staircase-like signal that contains frequencies above W Hz.
- These signals are removed with a smoothing analog low pass filter, the last step in D/A conversion.

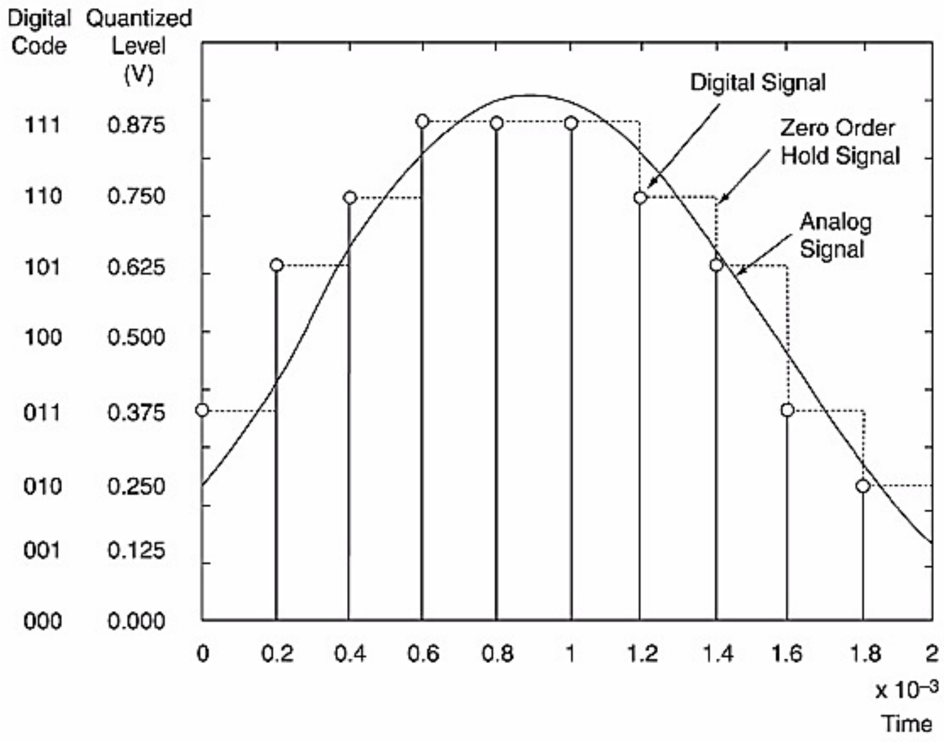
Block Diagram of D/A Conversion (Optional)





- In the frequency domain, the high frequency elements present in the zero-order hold signal appear as images, copies of the original signal spectrum situated around integer multiples of the sampling frequency.
- The smoothing analog filter removes these images and so is given the name of ***Anti-Imaging Filter***.
- Only the frequencies in the baseband, between 0 and $F_s/2$ Hz, remain.

3-Bit D/A Conversion



Audio Digital Signal Data Rate

- Data rate = sample rate * quantization (bit depth) * channel

- **Compare rates for Mono Speech Audio vs. CD Audio**
  - Mono Speech Audio Data Rate
    - 8000 samples/second * 8 bits/sample * 1 channel = 8 kBytes / second
  - CD Audio Data Rate
    - 44,100 samples/second * 16 bits/sample *2 channel = 176 kBytes / second ~= 10MB / minute

Raw Bitrate of Audio Signals

- Telephone Speech : 300Hz – 3.4kHz
- Wideband Speech : 50Hz – 7kHz (improved speech quality)
- **Wideband Audio** : 20Hz - 20kHz
  - Bitrate without Synchronization
    - Sample rate F_s : 44.1kHz for audio signal
    - Quantization (Bit Depth) : 16-bit/sample for each of the two stereo channels.
    - Total bitrate = 2 x 16 x 44100 = 1.41 Mbps (Mega bits per second)
  - Bitrate with Synchronization
    - Sync + error correction: 49 bits for every 16-bit audio sample.
    - Total bitrate = 1.41 x 49/16= 4.32 Mbps
  - If audio signals store uncompressed audio using the PCM (Pulse-Code Modulation), these files are large with around 10MB for one minute of a standard audio recording.

Hi-Res Audio – 2014

- Hi-Res Audio (High-Resolution Audio)
- Hi-Res Audio describes digital audio files have higher sample rate and bit dept than Compact Disc Digital Audio (CD-DA) of 44.1kHz/16-bit.
  - Hi-Res Sample Rate: 96kHz to 192kHz
  - Hi-Res Bit Depth: 24-bit
- Hi-Res audio is the favorite of audiophiles.
- Today, Hi-Res music and songs are available in special audio file formats (FLAC and ALAC), and widely are widely promoted by audio streaming services such as Tidal, Spotify, Moov, Apple Music.

Audio Coding Formats

- **Uncompressed Formats**
  - Raw representation of each signal sample using PCM encoding.
  - WAV (Waveform Audio File) : It uses LPCM and primarily for MS-Windows computers.
  - AIFF : It is a Mac format that's like WAV but using PCM encoding.
- **Lossless Compression Formats** (The decoded signal is same as original signal)
  - Use Inter-Sample Redundancy to compress with 40% to 60% size reduction
  - FLAC (Free Lossless Audio Coding) : License-Free for Hi Res Audio
  - ALAC (Apple Lossless Audio Coding) : Mainly use in iTune but now is opened
- **Lossy Compression Formats** (The decoded Signal is **NOT** same as original signal)
  - Use Psychoacoustic Properties to achieve higher compression
  - MP3 (MPEG-2 Layer III Audio) : Universal compressed audio format
  - AAC (Advanced Audio Coding) : Enhanced version of MP3 as a part of MPEG-2 Standard
  - MQA (Master Quality Authenticated) : Hi-Res Audio but has signal quality degradation.

Image Quantization

(Optional)

Image Gray Level Resolution

- Intensity level resolution refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail discernable in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

Intensity Level Resolution

256 grey levels (8 bits per pixel)

128 grey levels (7 bpp)

64 grey levels (6 bpp)

32 grey levels (5 bpp)

16 grey levels (4 bpp)

8 grey levels (3 bpp)

4 grey levels (2 bpp)

2 grey levels (1 bpp)

256 gray levels (8 bits per pixel)

128 gray levels (7 bits per pixel)

64 gray levels (6 bits per pixel)

32 gray levels (5 bits per pixel)

16 gray levels (4 bits per pixel)

8 gray levels (3 bits per pixel)

4 gray levels (2 bits per pixel)

2 gray levels (1 bits per pixel)

Summary

- An analog signal is continuous in both time and amplitude.
- The digital signal contains the digital values converted from the analog signal at the specified time instants.
- Analog-to-digital conversion requires an ADC unit and an analog anti-aliasing lowpass filter attached a head of the ADC unit to block the high-frequency components that ADC cannot handle.
- The digital signal can be manipulated using arithmetic. The manipulations may include digital filtering, calculation of signal frequency content, and son on.
- The digital signal can be converted back to an analog signal by sending the digital values to DAC to produce the corresponding voltage levels and applying a smooth filter (reconstruction filter) to the DAC voltage steps.