SPATIAL COEFFICIENT PARTITIONING FOR LOSSLESS WAVELET IMAGE CODING

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ABSTRACT

A novel coefficient partitioning algorithm is introduced for splitting the coefficients into two sets using spatial orientation tree data structure. By splitting the coefficients, the overall theoretical entropy is reduced due to the different probability distribution for the two coefficient sets. In spatial domain, it is equivalent to identifying smooth regions of the image. A lossless coder based on this spatial coefficient partitioning is described. Experimental results show that the new algorithm has a better coding performance than other wavelet based lossless image coder such as S+P and JPEG-2000.

1. INTRODUCTION

The state-of-the-art lossless image coding algorithms such as CALIC [1] and JPEG-LS [2] operate in spatial domain. However, in applications desiring fast preview of losslessly compressed images for archiving and progressive transmission, the multiresolution representation property of wavelet based coding algorithms is more attractive. Most wavelet based coding algorithms [3, 4] use an image model that natural images are well characterized as a linear combination of energy concentrated in both frequency and space, i.e. most of the energy of typical images is concentrated in low frequency information and the remaining high frequency energy components are spatially concentrated around edges. Coefficients are entropy coded using estimated probabilities conditioned on the context in which the coefficients are observed [5, 6]. The coding performance is comparable to most spatial domain image coders without relying on sophisticated design of offline parameters. This paper describes a preprocessing technique, called *spatial coefficient partitioning* (SCP), which can improve the coding performance of lossless wavelet image coders based on the properties of the wavelet domain image model. As high energy coefficients are usually clustered in the same spatial region, SCP tries to split the high frequency coefficients into two sets – one set with mainly low energy coefficients, and another set consisting of the remaining coefficients. Theoretically, by splitting the original coefficient set into two sets with different energy distribution, the overall entropy can be reduced. Experimental results show that image coder using the proposed preprocessing technique gives a better coding performance than that of the existing lossless wavelet image coders.

2. SPATIAL PARTITIONING IN WAVELET DOMAIN

A wavelet image decomposition provides a hierarchical data structure for representing images with each coefficient corresponding to a spatial region in the image. Figure 1 shows a 3-level wavelet decomposition of an image, together with a *spatial wavelet coefficient tree*, which is defined as the set of coefficients from different bands that represent the same spatial region in the image. Arrows in Figure 1 identify the parent-children dependencies in a tree. A *spatial orientation tree* is defined as the tree structured set of coefficients with the tree root started at one of the directional bands (i.e. LH, HL, and HH) at any level. We call it a *full depth spatial orientation tree* if the tree root starts at the highest level directional bands. In general, for n-level decomposition of a $d \times d$ image, the LL band has $d/2^n \times d/2^n$ coefficients. Each coefficient in LL band together with all its descendents forms a spatial wavelet coefficient tree corresponding to a $2^n \times 2^n$ spatial area of the original image. The three direct descendents of any LL band coefficient are the tree roots of three full depth spatial orientation trees. These three trees carry the high frequency information in three different orientations – horizontal, vertical and diagonal – of the corresponding spatial region.

2.1. Magnitude based partitioning

In wavelet representation, except in LL band, the coefficients measure the image's spatial intensity variation at dif-

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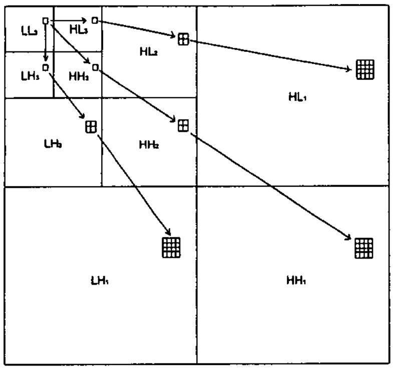

Fig. 1. Parent-child relationship in spatial wavelet coefficient tree of a 3-level wavelet decomposition.

ferent frequency ranges. The spatial activity level or intensity variation for spatial area is characterized by the magnitude or energy of the coefficients in corresponding spatial orientation trees. It is possible to differentiate between low and high activity areas by examining the coefficient magnitude. Another characteristic of common wavelet coefficient is that the magnitude distribution concentrates in the near zero magnitude range. Based on these observations, we define a binary partitioning function,

$$P_z(T_i) = \begin{cases} 0, & \text{More than 90\% coefficients in } T_i \\ & \text{have magnitude less than } z. \\ 1, & \text{Otherwise.} \end{cases} \tag{1}$$

to determine the activity level (either low or high) of a spatial orientation tree T_i with tree root at node i (coefficient c_i). For a given threshold, z, a coefficient is said to be significant if its magnitude is larger than z while it is insignificant if its magnitude is less than or equal to z. The function P_z classifies the activity level of the spatial area corresponding to T_i based on the population of insignificant coefficients. A tree T_i is said to be significant (respectively insignificant) if $P_z(T_i) = 1$ (respectively $P_z(T_i) = 0$). Thus, some trees with few percentages of large magnitude coefficients can also be classified as insignificant trees with a small threshold. By considering the whole wavelet representation as the coding source, splitting the original source into different subsources with different probability distribution has theoretical entropy gain [7]. Applying (1) to all full depth spatial orientation tree, i.e. $\{T_i\}$ with all node i from LH_n, HL_n, and HH_n bands in n-level wavelet decomposition, we can partition the original source, i.e. all high frequency wavelet coefficients, into two subsources. The entropy of the sum of the two subsources can be reduced if their probability distributions are different. Figure 2 shows the two magnitude distributions for the significant trees and the insignificant trees partitioned using (1). Obviously, the distributions for the two coefficients sets are very different. Equivalent in spatial domain, the image is partitioned into non-overlapping $2^n \times 2^n$ pixel blocks. Each image block is classified as either low activity block or high activity block. A separate binary map is required to carry the partition information for the trees. For a 5-level wavelet decomposition of 512×512 image, the number of binary bit required is only 768 bits or 0.003 bpp. This overhead data size is relatively small compared with the whole compressed image size.

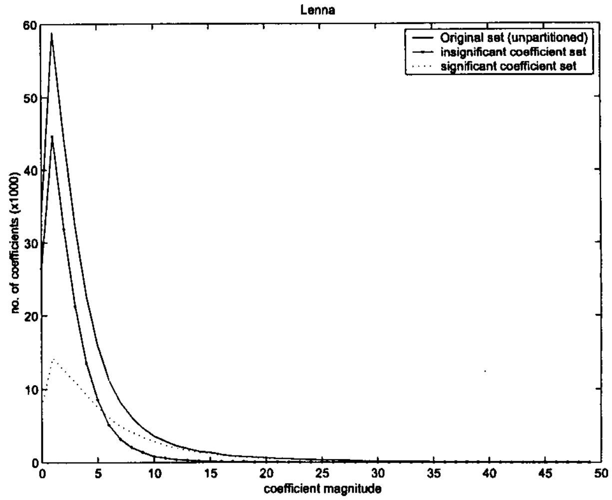

Fig. 2. Partitioned coefficient magnitude distributions of the image "Lenna".

2.2. Determining the partition threshold z

The partitioning function $P_z(T_i)$ in (1) classifies T_i with reference to a threshold value z. In this section, we describe a fast algorithm to find a suitable threshold value z for an image. Spatial partitioning classifies the full depth spatial orientation trees as either significant or insignificant depending on the percentage of insignificant coefficients in the trees. The purpose is to have as many insignificant trees as possible while the partitioning threshold should be as small as possible so as to minimize the dynamic range of the insignificant partition. The idea of the algorithm is to initially choose a high threshold to have an initial partition of significant and insignificant trees. Each insignificant tree with respect to the initial threshold is examined to find a more suitable individual threshold for that tree. Then, a new global threshold will be determined from these individual thresholds. The new threshold is used to partition all the trees again. This process repeats until the global threshold found converges. The iterative algorithm for determining the partition threshold z is given as follows.

Partition Threshold Algorithm

1. **Initialization:** Set $z^{(0)} \leftarrow |c_{max}|/2$ where c_{max} is the coefficient with maximum magnitude; set the iteration count $k \leftarrow 0$.
2. **Tree Classification:** Determine the partition map $m_i^{(k)}$ for each full depth spatial orientation tree by
$$m_i^{(k)} = P_{z^{(k)}}(T_i).$$
3. **Update threshold:** Find the number of insignificant tree, q. For each insignificant tree, find the individual tree threshold, $z_i^{(k)}$, such that 90 % of coefficients inside the tree have magnitude less than $z_i^{(k)}$. Update the threshold value by
$$z^{(k+1)} = \sum_{T_i \in I} z_i^{(k)}/q$$
where I is the set of insignificant trees.
4. **Convergence check:** If $\left|z^{(k+1)} - z^{(k)}\right| > 2$, increment the iteration count $k \leftarrow (k+1)$ and go back to Step 2 to partition all full depth spatial orientation trees using the new threshold. Else, $z = z^{(k+1)}$ is the final global partition threshold.

3. SPATIAL COEFFICIENT PARTITIONING ALGORITHM

In the proposed SCP algorithm, the magnitude and sign of each coefficient are encoded separately. The LL band coefficients are also separately entropy coded. The partition map m_i for $i \in \{LH_n, HL_n, HH_n\}$ classifies the full depth spatial orientation trees as either significant or insignificant trees. The partitioning threshold is determined as described in section 2.2. The coefficients are encoded in a low to high frequency band order so that parent coefficient is encoded before child coefficient. The magnitudes of the parent and the four nearest neighbour coefficients are used as conditioning context [6] for encoding coefficient magnitude using adaptive arithmetic coding [8]. For coefficients in insignificant trees, i.e $m_i = 0$, we use a two-pass coding method to encode the magnitude. For $|c_i| \leq z$, we use a one-to-one mapping to represent the insignificant magnitude as z is relatively small. A special symbol, SIG, is encoded if the coefficient in insignificant tree is significant, i.e. $|c_i| > z$. Coding of the exact magnitude for significant coefficients follows immediately after the coding of SIG symbol. For coefficients in significant trees, i.e. $m_i = 1$, coefficients are entropy coded. Coding of the sign follows immediately after the coding of coefficient magnitude. The exploitation of the spatial correlation of sign information is achieved via a set of sign conditioning contexts based on the signs of the adjacent coefficients, c_a, c_b and c_c as shown in Figure 3. An adaptive model numbered

$$K = 2s[a] + s[b] + s[c] \quad (2)$$

where

$$s[i] = \begin{cases} 0, & c_i \geq 0 \\ 1, & c_i < 0 \end{cases} \quad (3)$$

is determined for adaptive arithmetic coding of the sign of the coefficient under consideration.

s[a]	s[b]
s[c]	s[i]

Fig. 3. Signs of adjacent coefficients used by the arithmetic encoder in coding the signs of the current coefficients, $s[i]$.

The complete SCP algorithm is summarized as follows.

SCP Coding Algorithm

1. **Threshold determination:** Use the algorithm in section 2.2 to find the threshold value z for the wavelet image representation.
2. **Partition Map Coding:** Encode the partition map,
$$m_i = P_z(T_i),$$
for all $i \in \{LH_n, HL_n, HH_n\}$.
3. **Coefficient Coding:** Scan the coefficients, c_i in low-to-high frequency band order.
   (a) If $c_i \in LL$ band or $c_i \in T_i$ with $m_i = 1$: encode $|c_i|$ and then encode the sign of c_i.
   (b) If $c_i \in T_i$ with $m_i = 0$: encode $|c_i|$.
      i. If encoded symbol is "SIG" (i.e.$|c_i| > z$), encode $|c_i|$ and then encode the sign of c_i.
      ii. If encoded symbol is not "0" or "SIG", encode the sign of c_i.

4. EXPERIMENTAL INVESTIGATION

Experiments are performed on several standard 512 × 512 grey-scale images to test the proposed SCP algorithm. For comparison, the integer wavelet transform filters used is equivalent to that in S+P algorithm. A 5-level wavelet decomposition with the coarsest lowpass band of dimension 16 × 16 is used. Table 1 shows the experimental results for lossless coding of four standard images. The coding performance of other lossless coding schemes, S+P, JPEG-2000, JPEG-LS and CALIC, are also included for comparison. Both S+P and JPEG-2000 schemes are wavelet based schemes providing combined lossless and lossy coder using integer wavelet. JPEG-LS and CALIC are pure lossless coders operating in spatial domain. They are also included as reference schemes as they are two of the best performing schemes in lossless image coding. Table 1 shows that the proposed SCP algorithm can produce lossless rates competitive to the two best performing lossless schemes and constantly outperforms the other two multirate coders by, at most, 3%. Although SCP is inferior to CALIC in coding efficiency, SCP produces an embedded output due to the multiresolution wavelet representation used. Thus, SCP is suitable for storage and progressive transmission of images at different resolutions, from lossy to lossless.

The need to find a partition threshold in SCP algorithm does not impose a serious computational loading. The partition threshold algorithm can quickly converge to the final threshold value, after about 4 to 5 iterations for our testing images. For instance, for the image "Lenna", the intermediate thresholds $z^{(k)}$ during the iteration are $\{25, 16, 12, 9, 7\}$ and the corresponding number of insignificant trees are $\{766, 688, 579, 494, 349\}$. Table 2 gives the partition threshold z determined for four images using the partition threshold algorithm. Although the partition threshold values determined for the test images are very close, the corresponding percentages of insignificant coefficients vary greatly from 25% to 62% (also shown in Table 2). It is due to the different spatial activities of the images. Operating in wavelet domain, SCP algorithm can easily identify the spatial inactive regions.

Scheme	Barbara	Couple	Goldhill	Lenna
SCP	4.67	4.74	4.72	4.15
S+P	4.69	4.76	4.75	4.17
JPEG-2000	4.79	-	4.87	4.32
JPEG-LS	4.86	-	4.71	4.24
CALIC	4.63	-	4.63	4.12

Table 1. Coding performance of SCP algorithm (bpp).

Image	Threshold z	% Coeff. in insigni. trees
Barbara	8	39%
Couple	9	26%
Goldhill	10	25%
Lenna	7	62%

Table 2. Percentage of coefficients in insignificant trees.

5. CONCLUSIONS

A novel preprocessing technique, spatial coefficients partitioning (SCP), which utilizes tree structured data organization in partitioning wavelet coefficients in two sets of coefficients, is proposed. This partitioning technique can be applied to the existing lossless image coder to improve the coding performance. A lossless coder based on SCP is described. Experimental results show that SCP coder outperforms the classical wavelet lossless coder and is competitive with the current spatial domain lossless image coding. Thus, SCP algorithm is suitable for applications demanding progressive transmission of images at different resolutions, from lossy to lossless.

6. REFERENCES

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