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Naming Game with Multiple Hearers

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A B S T R A C T

A new model called Naming Game with Multiple Hearers (*NGMH*) is proposed in this paper. A naming game over a population of individuals aims to reach consensus on the name of an object through pair-wise local interactions among all the individuals. The proposed *NGMH* model describes the learning process of a new word, in a population with one speaker and multiple hearers, at each interaction towards convergence. The characteristics of *NGMH* are examined on three types of network topologies, namely ER random-graph network, WS small-world network, and BA scale-free network. Comparative analysis on the convergence time is performed, revealing that the topology with a larger average (node) degree can reach consensus faster than the others over the same population. It is found that, for a homogeneous network, the average degree is the limiting value of the number of hearers, which reduces the individual ability of learning new words, consequently decreasing the convergence time; for a scale-free network, this limiting value is the deviation of the average degree. It is also found that a network with a larger clustering coefficient takes longer time to converge; especially a small-word network with smallest rewiring possibility takes longest time to reach convergence. As more new nodes are being added to scale-free networks with different degree distributions, their convergence time appears to be robust against the network-size variation. Most new findings reported in this paper are different from that of the single-speaker/single-hearer naming games documented in the literature.

1. Introduction

Language is one of the most important aspects in human evolution, which demarcates human beings from other creatures in nature. With language, humans are enabled to exchange information and to express ideas, which in many ways have totally changed our world [1]. One of the most important scientific methods to study the evolution of language is to simulate the so-called "language game". A language game uses a simple mathematical model to reproduce the process involved in a linguistic pattern formation among individuals of a population [2,3], where the consensus of all individuals on a new word is of extreme importance [4,5]. This mathematical model is now commonly-known as the "naming game" model.

Naming game model describes linguistic conventions via pair-wise local interactions among individuals in a population [6]. Consensus emerges through local negotiations between pairs of individuals, so the topology of their communication network plays a central role in both the model and the process [7]. It has been verified that, compared to regular lattices, Watts–Strogatz (WS) small-world networks [8] and likewise Erdös–Rényi (ER) random-graph networks [9] as well as Barabási–Albert (BA) scale-free networks [10] can well facilitate the achievement of such consensus [11–13]. To analyze the role of each individual plays in the process, a modified model with weighted words is proposed in [14]. In this model, a tunable parameter is defined to govern the word weights based on the connectivity among the individuals, and an optimal value of the

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parameter is found leading to the fastest convergence. This indicates that certain hub nodes in a network favor the achievement of consensus. A negotiation strategy is proposed in [15,16] to address the role of geography on the dynamics of the naming game, which depends upon the geographical distance between two individuals, characterizing the correlation between the interaction strength and the geographical distance. Apart from the network topology, memory size is another important factor of consensus [17].

For the aforementioned naming game model, it is commonly assumed that there are two individuals in interaction at every time step of the evolutionary process, namely a speaker and a hearer. Clearly, it is not the case in the real human linguistic conversations where mostly conversations many hearers listening to the speaker, for example in TV shows and conference presentations. In this paper, therefore, a more realistic situation with multiple hearers is considered. That is, when a speaker calls a name, there are several hearers listening to it at the same time. The investigation here focuses on the convergence to the consensus state of a communication network over a population, subject to different network topologies.

More precisely, a new model called Naming Game with Multiple Hearers (*NGMH*) is proposed. For the new scenario with multiple hearers, one might have an impression that the convergence time would decrease as the number of hearers increases. This intuition has never been proved, however. The present paper shows the relationship of the convergence time of a network with the number of hearers, for several different network topologies. It will be shown that the convergence time decreases with the increasing of the number of hearers when the number of hearers is smaller than the average (node) degree of the network. With the additional increasing of hearers when the number of hearers is larger than the average degree, the variation of the convergence time is insignificant in homogeneous networks. But, for scale-free heterogeneous networks, increasing the number of hearers will significantly decrease the consensus time. It is also shown that the individual ability of learning new words is reduced as the number of hearers increases; that is, the maximal number of different words in the network is reduced as more hearers are being added into the population. Therefore, the number of hearers also plays a significant role in the linguistic evolution, which favors the network consensus at the cost of having more learners. For small-world networks, local convergence decreases the speed of global consensus. With a small rewiring probability, the cluster coefficient of such a network remains to be large, but some nodes do not learn any more new words from the others and, as a result, those nodes tend to the state of local convergence. Therefore, before the network reaching consensus, local convergence must be resolved in order to avoid being trapped at local minima. Thus, small rewiring probability leads to long consensus time in general. In summary, this study confirms that the network topology plays an important role in language consensus, and the number of hearers is another important factor in achieving the network convergence in naming games.

The rest of the paper is organized as follows. In Section 2, the new naming game model *NGMH* is introduced. Section 3 reports and analyzes the simulation results on *NGMH* over random-graph networks, small-world networks, and scale-free networks. Section 4 concludes the investigation.

2. The new naming game model

In this section, a new naming game model, Naming Game with Multiple Hearers (*NGMH*), is introduced.

Consider a population of individuals, called nodes hereafter, connected through a communication network in a certain topology. At each time step of the evolutionary process, a pair of nodes is picked at random, referred to as speaker and hearer respectively, and the speaker tells the hearer a new word from a vocabulary. With a certain probability or in a certain manner, the hearer learns and remembers the new word but then drops the other words from its memory. Thus, after a sufficiently large number of pair-wise iterations performed in this way, the memories of all nodes in the population may converge to having one same word. In this case, the network is said to converge to consensus in learning a new word. Roughly, this is the common scenario of most naming game models studied in the literature.

Compared with other naming game models, *NGMH* has multiple hearers in each time step; namely, there are one speaker and several hearers. In *NGMH*, a speaker is selected from the whole population, while hearers are selected from the neighborhood of the speaker, both randomly with a uniform distribution. Thus, there is a connection (called edge) between the speaker and each hearer, thus all nodes and edges together constitute a network. Fig. 1 illustrates the learning process according to the new model, which is summarized as follows.

NGMH model:

0. Start with a population of n nodes connected in a certain topology, along with an available vocabulary of sufficiently large size.
1. Initially, every node has empty memory.
2. At each time step, a speaker is randomly selected from the given population.
   - 2.1. If the memory of the speaker is empty, the speaker randomly picks a word from the vocabulary.
   - 2.2. If the memory of the speaker is not empty, the speaker randomly picks a word from its memory.
3. The speaker randomly chooses h hearers from its neighborhood in the networked population.
4. The process of learning about the new word is as follows:
   - 4.1. If the word was already in the memory of all hearers, the learning is successful. Then, the speaker and all hearers only keep this leaning word but drop all other words from their memories.

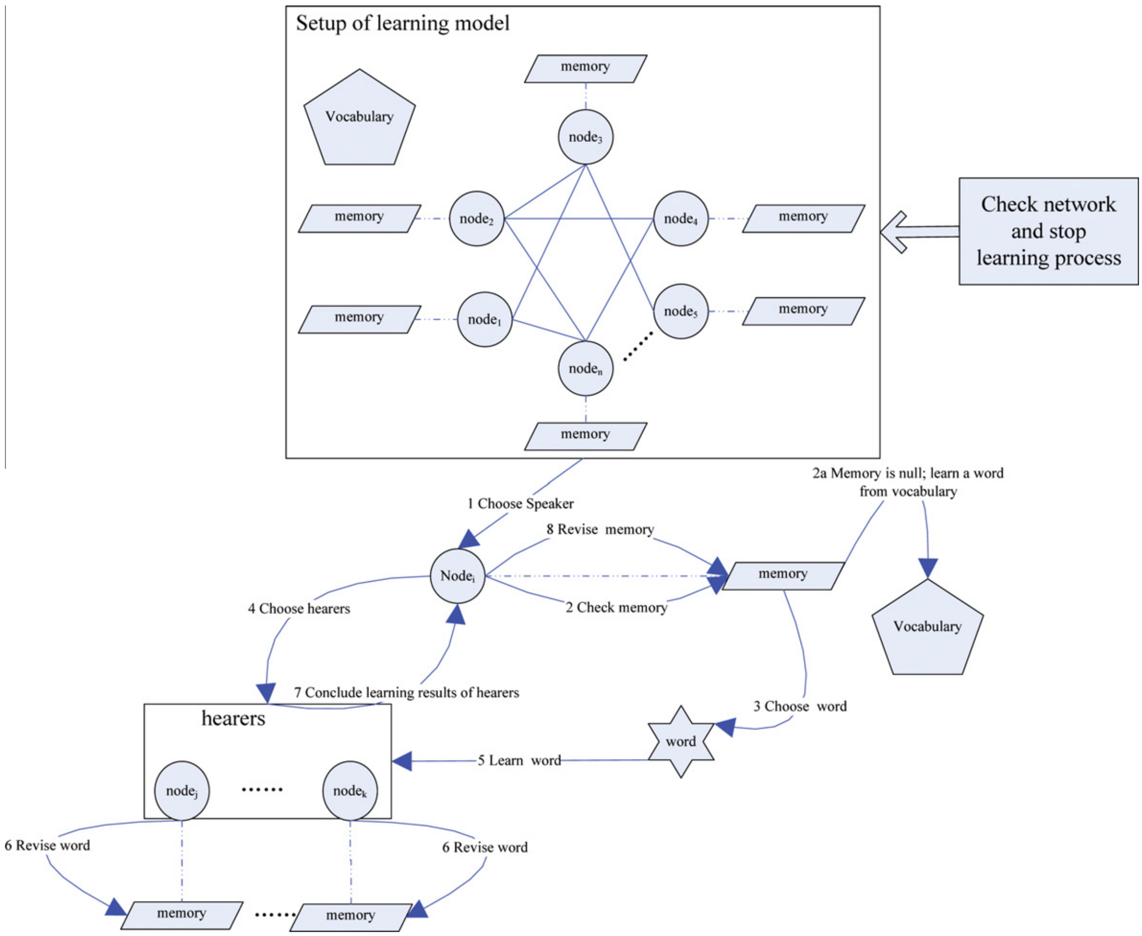

Fig. 1. Learning process of *NGMH*.

4.2. If the word was not in the memory of some hearers, learning fails. Then, those hearers with the learning word in their memories will keep this word and drop all the other words from their memories, while the other hearers add the new word into their memories.

5. The process stops when all nodes keep only one same word, or when the number of steps reaches a preset (large enough) threshold.

3. Analytic and experimental results and discussions

In this section, the learning process of *NGMH* is examined from several different aspects. Some analytic and many numerical experiments are carried out on different network models, namely, random-graph network, small-world network and scale-free network.

First, the ability of learning new words in the same network but with different numbers of hearers is investigated. Second, in order to study the effect of increasing the number of hearers on the convergence to consensus, differently increasing numbers of hearers in each time step are tested. Third, the convergence of various networks with different topological characteristics is studied. Two issues are considered here: one is the convergence time of different networks with the same number of nodes but different average degrees; the other is the dependence of the convergence time on the increasing number of nodes in the population on various networks of different topologies.

3.1. Simulation setup

In this section, 18 types of networks are used for simulation, as summarized in Table 1. For each type of network, 20 network realizations are tested, so every result shown in Table 1 is the averaged result of 20 realizations. To generate a random-graph network [9], starting with n isolated nodes, an edge is added between two nodes with a connection

Table 1
Details of network topologies.

Network	Number of nodes	Average degree	Average path length	Average clustering coefficient
Random-graph network with connection probability $P = 0.01$ (*Ran*-0.01)	2000	19.99	2.8357	0.0120
Random-graph network with connection probability $P = 0.02$ (*Ran*-0.02)	2000	39.98	2.4236	0.0238
Random-graph network with connection probability $P = 0.05$ (*Ran*-0.05)	2000	99.95	1.9676	0.0591
Random-graph network with connection probability $P = 0.1$ (*Ran*-0.1)	2000	199.9	1.9	0.1169
Random-graph network with connection probability $P = 0.2$ (*Ran*-0.2)	2000	399.8	1.8	0.2281
Random-graph network with connection probability $P = 0.5$ (*Ran*-0.5)	2000	999.5	1.5	0.5337
Small-word network with $k = 50$ rewrite possibility $RP = 0.01$ (*SW*-50-0.01)	2000	100	2.7630	0.7205
Small-word network with $k = 50$ rewrite possibility $RP = 0.05$ (*SW*-50-0.05)	2000	100	2.4892	0.6395
Small-word network with $k = 50$ rewrite possibility $PR = 0.1$ (*SW*-50-0.1)	2000	100	2.2831	0.5480
Small-word network with $k = 200$ rewrite possibility $RP = 0.01$ (*SW*-200-0.01)	2000	400	1.8831	0.7280
Small-word network with $k = 200$ rewrite possibility $RP = 0.05$ (*SW*-200-0.05)	2000	400	1.7999	0.6527
Small-word network with $k = 200$ rewrite possibility $RP = 0.1$ (*SW*-200-0.1)	2000	400	1.7999	0.5702
Scale-free with 11 initial nodes and 10 new edges each step (*BA*-10)	2000	19.8910	2.7219	0.0374
Scale-free with 16 initial nodes and 15 new edges each step (*BA*-15)	2000	29.7610	2.5282	0.0482
Scale-free with 21 initial nodes and 20 new edges each step (*BA*-20)	2000	39.5810	2.3672	0.0584
Scale-free with 26 initial nodes and 25 new edges each step (*BA*-25)	2000	49.3510	2.2407	0.0677
Scale-free with 31 initial nodes and 30 new edges each step (*BA*-30)	2000	59.0710	2.1449	0.0769
Scale-free with 41 initial nodes and 40 new edges each step (*BA*-40)	2000	78.3610	2.0301	0.0932

probability P. To generate a small-word network [8], starting from a ring-shaped network with n nodes, in which each node is connected to its $2K$ neighbors, one edge is reconnected to a random-picked node from the network with a rewiring probability RP. To generate a scale-free network [10], starting from a fully-connected network of initial nodes, one node is added into network which is connected to the existing nodes according to the so-called preferential attachment probability which is proportional to the degree of the existing node.

For the learning process, assume that

· there is no initial word in the memory of every node.
· the memory of each node is infinite (large enough).

These two conditions are common and indeed reasonable. If there are some words at the initial states of the network, the results will be affected by the number and distribution of these words, creating many different scenarios thereby making the comparison of various network topologies impossible. On the other hand, the objective of this work is the consensus patterns in *NGMH*, where the node's memory size is not a concern therefore is set to be infinite.

3.2. Verifying the stopping criterion

A network is said to arrive at the consensus state when all nodes keep a same word. It has been observed, as reported below, that with *NGMH* the words in the memories of all the nodes do not vary after all the nodes have reached the consensus state. If the learning process does not stop, the program needs to be stopped after the iteration has reached a threshold number of interaction steps. In simulation, this threshold number is set to be 400,000 time units, with 0.1 s each, which constitute a time interval of 40,000 s, long enough for statistical analysis. The convergence state of the network is then verified for two counts: the total number of words kept in the memories of all nodes, and the number of different words therein.

In this subsection, only the convergence state of the networks is examined, so only 6 different types of networks are used for testing: *Ran*-0.05, *Ran*-0.1, *SW*-50-0.05, *SW*-200-0.05, *BA*-15 and *BA*-20, as specified in Table 1, where the number of hearers is 5, 10, and 20, respectively. Fig. 2 illustrates the variation of the total number of words versus the running time. Fig. 3 shows the variation of the number of different words versus the running time. In both Fig. 2 and Fig. 3, the number of total words is stabilized at 2000 and the number of different words is 1, demonstrating that the network indeed has converged to consensus asymptotically.

3.3. Relationship between the maximum number of different words and the number of hearers

The number of different words kept in the memories of nodes shows the level of progress in the word-learning process. In this subsection, the maximum number max_{dw} of different words in a network is tested against the increasing number of hearers. It is found that the increase of hearers decreases the value of max_{dw} kept in the network. It is also found that, for a homogeneous network, its average degree determines the number of hearers where max_{dw} stops decreasing. In contrast, for a scale-free network, the number of hearers which decreases the value of max_{dw} is typically larger than the average degree, due to the heterogeneous nature of the network.

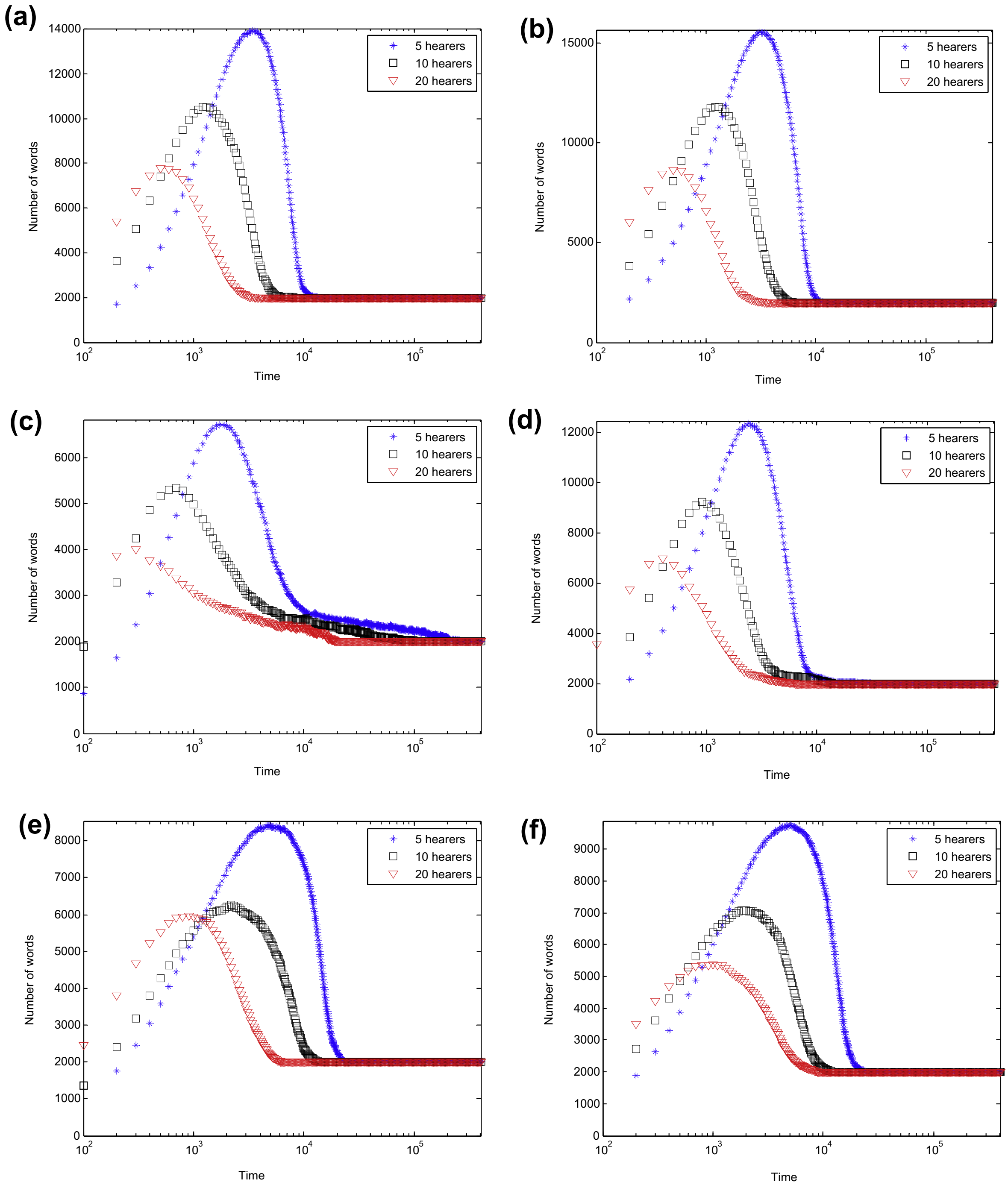

Fig. 2. Variation of total number of words: (a) *Ran*-0.05, (b) *Ran*-0.1, (c) *SW*-50-0.05, (d) *SW*-200-0.05, (e) *BA*-15 and (f) *BA*-20.

The simulation results of max_{dw} are plotted in Fig. 4 for random-graph networks, small-world network, and scale-free networks. In Fig. 4(a) and (b), the values of max_{dw} are displaced against the number of hearers for different networks in Table 1. It can be observed that the decreasing of the maximum value max_{dw} is saturated after the number of hearers becomes larger than the average degree. In Fig. 4(c), the values of max_{dw} is plotted against the number of hearers, for different scale-free networks shown in Table 1. In Fig. 4(c), the value of max_{dw} is also decreasing after the number of hearers becomes larger than the average degree of the network.

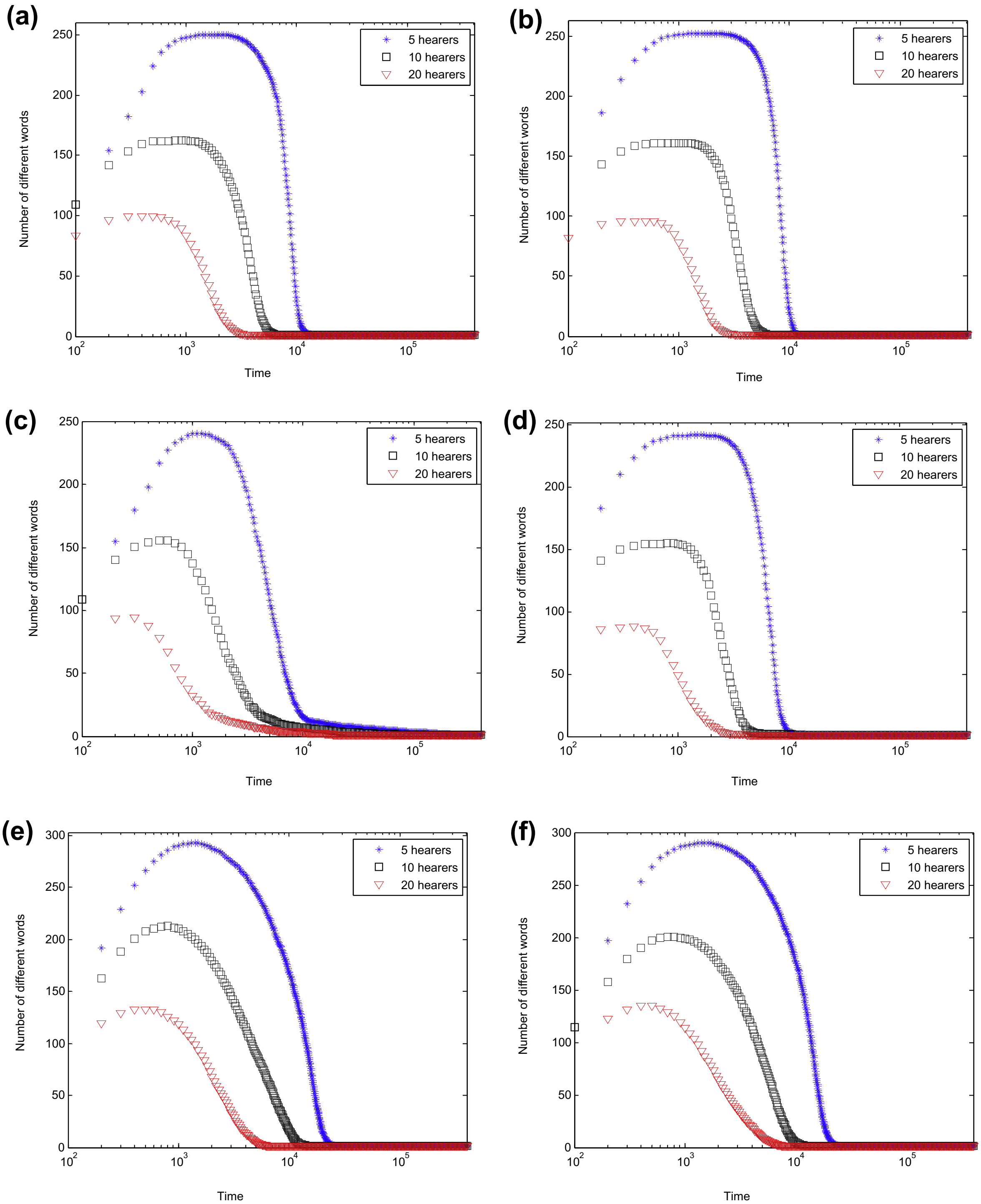

Fig. 3. Variation of the number of different words: (a) *Ran*-0.05, (b) *Ran*-0.1, (c) *SW*-50-0.05, (d) *SW*-200-0.05, (e) *BA*-15 and (f) *BA*-20.

3.4. Relationship between the number of hearers and the convergence time

In this subsection, it will be shown that the convergence time decreases as the number of hearers increases. Yet, it is also observed that when the number of hearers is larger than a threshold, further addition of hearers does not reduce the convergence time any further.

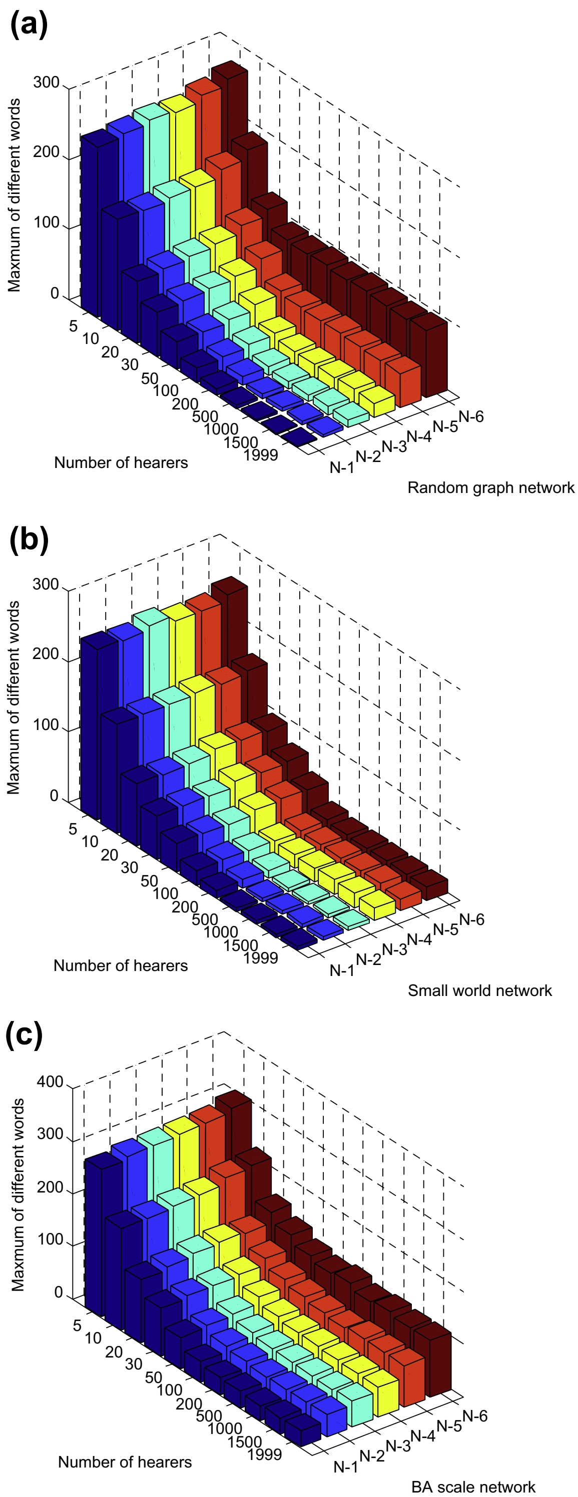

Fig. 4. Simulation results on the maximum number of different words: (a): N-1: *Ran*-0.5; N-2: *Ran*-0.2; N-3: *Ran*-0.1; N-4: *Ran*-0.05; N-5: *Ran*-0.02; N-6: *Ran*-0.01, (b) N-1: *SW*-200-0.01; N-2: *SW*-200-0.05; N-3: *SW*-200-0.1; N-4: *SW*-50-0.01; N-5: *SW*-50-0.05; N-6: *SW*-50-0.1, (c) N-1: *BA*-40; N-2: *BA*-30; N-3: *BA*-25; N-4: *BA*-20; N-5: *BA*-15; N-6: *BA*-10.

It is observed that, when the number of hearers is not larger than the average degree of the network, the running time to the convergence state decreases as the number of hearers increases. This is because, if there are a large number of hearers then the number of nodes which can learn the same word is large, so the success rate is high. Here, the success rate is defined as the successful learning steps divided by the total running steps on the time interval of one second (which, as mentioned above, means 10 time units in iteration).

In Fig. 5, it is shown that the variation of the success rate with a large number of hearers is quite significant. Fig. 6 shows the simulation results on the running time. In Fig. 6(a) and (b), the running time remains to be similar when the number of hearers is larger than the average degree of the network. For a scale-free network, in some range, the convergence time also decreases with the addition of hearers. According to the analysis given above in Sections 3.1, 3.3 and 3.4, the performance of convergence time against the number of hearers is similar with that of the maximum number of different words. So, this verifies that the number of hearers is important to the convergence performance of the network in learning.

3.5. Relationship between the average degree and the convergence time

In this subsection, with the same number of hearers fixed, the relationship between the average degree and the convergence time is investigated. It is found that the convergence time decreases with the increasing of the average degree of the network.

For random-graph networks and small-world networks, the clustering coefficient increases with the increasing of the average degree. For the nodes taking part at the same learning step, the possibility of their participating together in another learning step increases as the clustering coefficient increases. In a scale-free network, the speaker almost always has a small number of edges, since most nodes have few edges and the possibility for each node to become a speaker is the same. So, the exact number of hearers at one step is smaller than the defined number of hearers, when the number of edges that the speaker has is small. Thus, the exact number of hearers appears to increase with the increasing of the average degree. According to this analysis, the convergence time decreases with the increasing of the average degree.

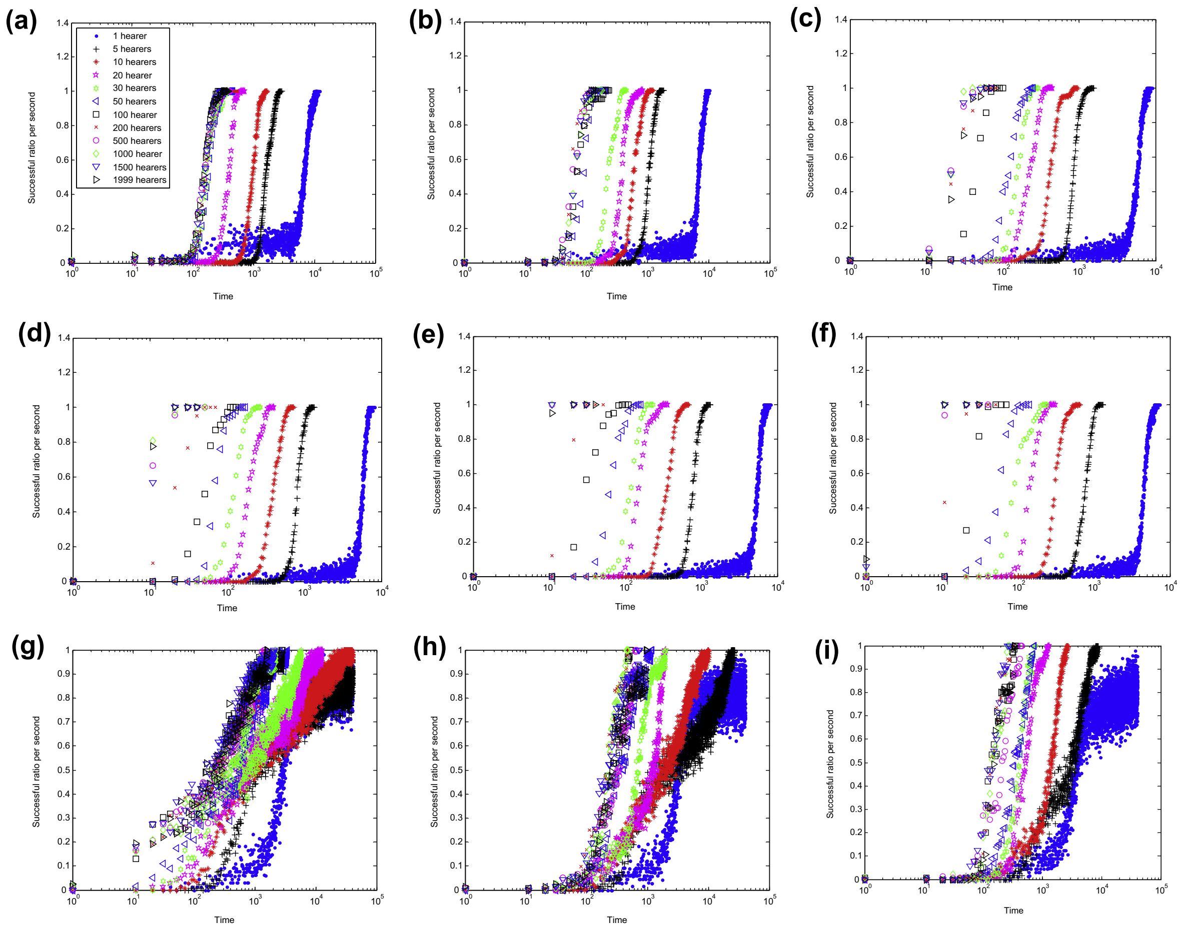

Fig. 5. Success rates on different networks: (a) *Ran*-0.01, (b) *Ran*-0.02, (c) *Ran*-0.05, (d) *Ran*-0.1, (e) *Ran*-0.2, (f) *Ran*-0.5, (g) *SW*-50-0.01, (h) *SW*-50-0.05, (i) *SW*-50-0.1, (j) *SW*-200-0.01, (k) *SW*-200-0.05, (l) *SW*-200-0.1, (m) *BA*-10, (n) *BA*-15, (o) *BA*-20, (p) *BA*-25, (q) *BA*-30, (r) *BA*-40. The labels in (b)–(r) are the same as in (a).

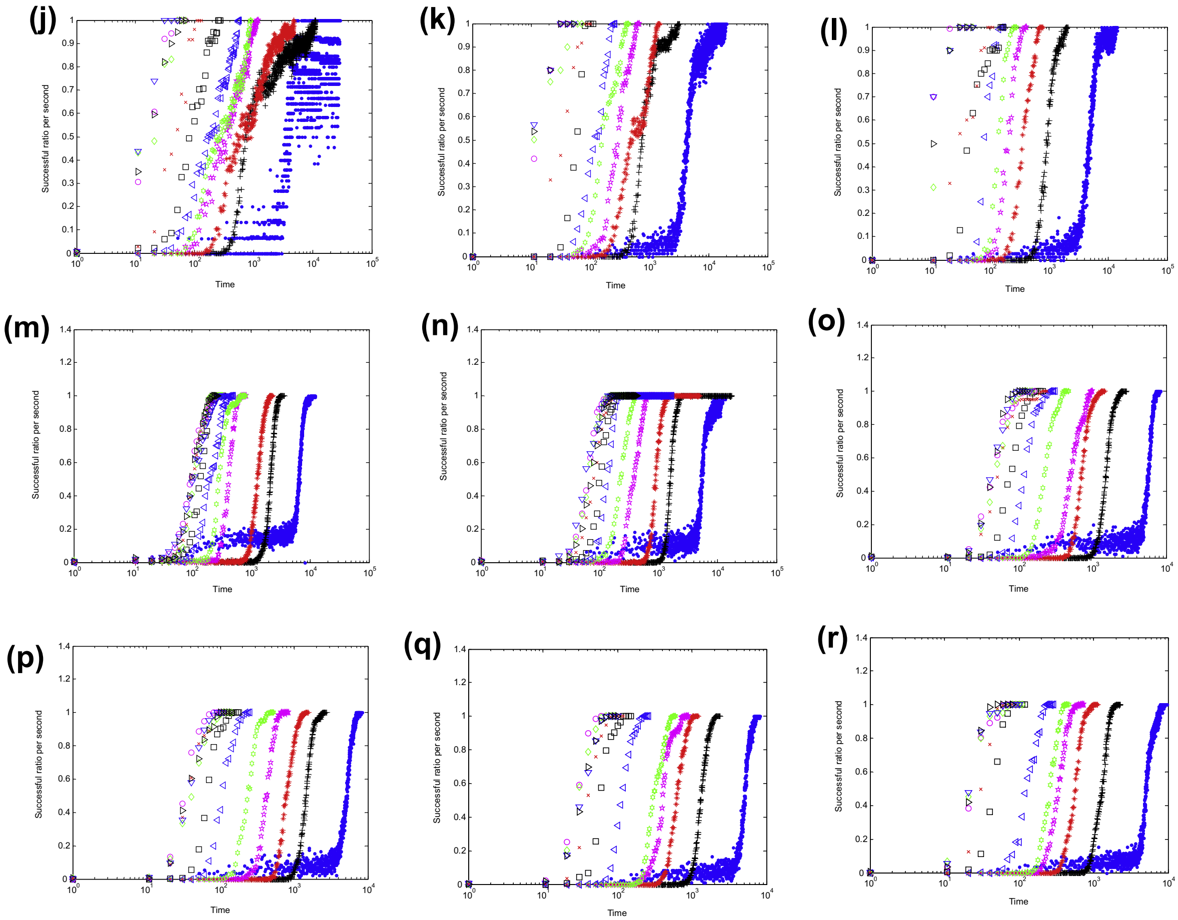

Fig. 5. (*continued*)

Fig. 6 shows the convergence time. In one network, the difference between the maximum and minimum of convergence time rapidly varies. In order to clearly show the variation as the number of hearers increases, the convergence time is plotted in the log scale. But the difference among different networks shown in Fig. 6 is not easy to discern when the number of hearers is small. In order to resolve this problem, Fig. 7 illustrates the "balanced time", which is calculated by subtracting the convergence time of the network with the largest average degree from its convergence time. In Fig. 7, it is clear that most values of the convergence time increases with the decreasing of the average degree.

For a small-world network, the rewiring probability also plays a significant role in the variation of the convergence time, as observed from Fig. 6(b) and Fig. 7(b). According to [16], the clustering coefficient of a small-world network is calculated by $C(rp) = (3/4) * [(K-2)/(K-1)] * (1-rp)^3$, where rp is the rewiring probability in generating the network. Here, $C(rp)$ is approximately equal to 0.75 when rp is small enough. For those networks with a small rewiring probability, certain nodes always take part at the same learning steps, implying that these nodes do not learn new words from other nodes. The local convergence of these nodes must be taken into account regarding the network global convergence. If the local convergence is strong, the network takes more learning steps to arrive at the convergence state. Fig. 8 shows the variation of the convergence time versus the rewiring probability. When the rewiring probability is larger than 0.1, the variation of the convergence time does not change much. On the other hand, the rewiring probability does not play a significant role in the maximum number of different words, as verified by Fig. 9.

Here, it is worth noting that the convergence time of information propagation is proportional to the distance diameter of the susceptible-infected-refractory model [18]. However, for *NGMH*, the relationship between average path length and convergence time is quite complicated. The transmitted information in [18] is unique, so its every step promotes the process to convergence. In contrast, the number of different words in *NGMH* experiences both increase and decrease, making information transmission on the network varied. And an unsuccessful learning step may hinder the process of consensus. The learning model of one speaker-multiple hearers intensifies the difficulty of success in learning. As a result, the process of consensus for *NGMH* depends not only on the information transmission but also on the agreement among the individuals.

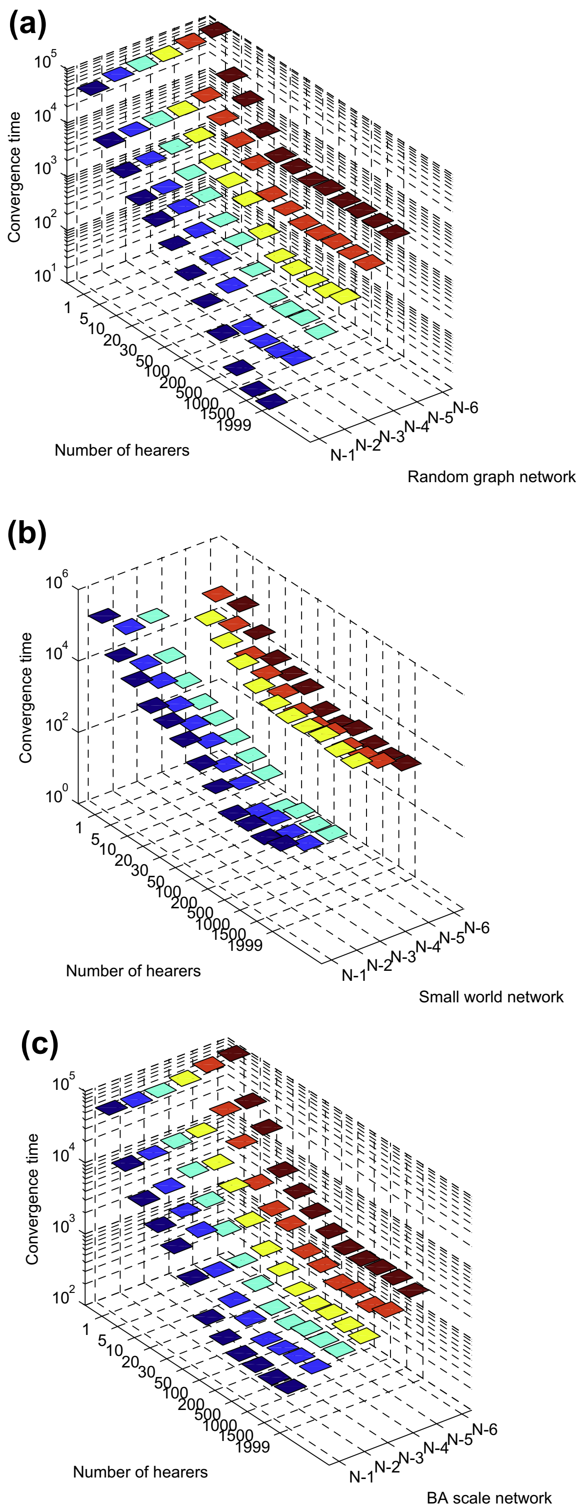

Fig. 6. Convergence time. (a): N-1: *Ran*-0.5; N-2: *Ran*-0.2; N-3: *Ran*-0.1; N-4: *Ran*-0.05; N-5: *Ran*-0.02; N-6: *Ran*-0.01, (b) N-1: *SW*-200-0.01; N-2: *SW*-200-0.05; N-3: *SW*-200-0.1; N-4: *SW*-50-0.01; N-5: *SW*-50-0.05; N-6: *SW*-50-0.1, (c) N-1: *BA*-40; N-2: *BA*-30; N-3: *BA*-25; N-4: *BA*-20; N-5: *BA*-15; N-6: *BA*-10.

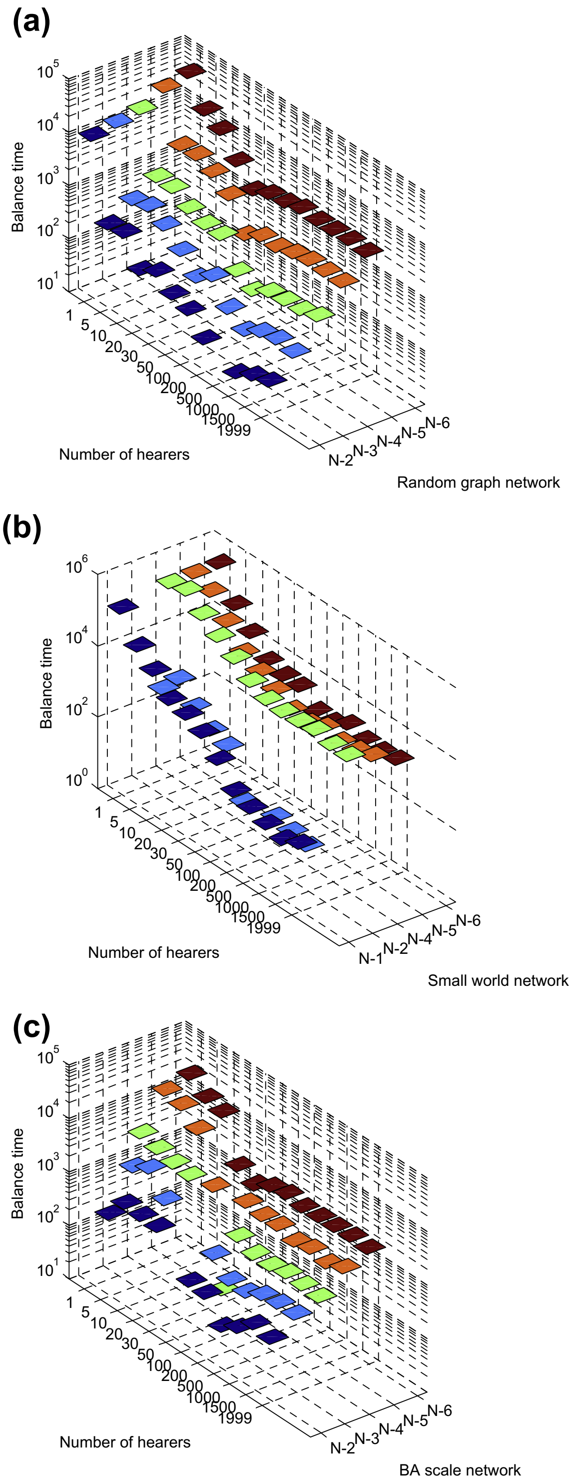

Fig. 7. Balanced time. (a): N-2: *Ran*-0.2; N-3: *Ran*-0.1; N-4: *Ran*-0.05; N-5: *Ran*-0.02; N-6: *Ran*-0.01,(b) N-1: *SW*-200-0.01; N-2: *SW*-200-0.05; N-4: *SW*-50-0.01; N-5: *SW*-50-0.05; N-6: *SW*-50-0.1, (c) N-2: *BA*-30; N-3: *BA*-25; N-4: *BA*-20; N-5: *BA*-15; N-6: *BA*-10.

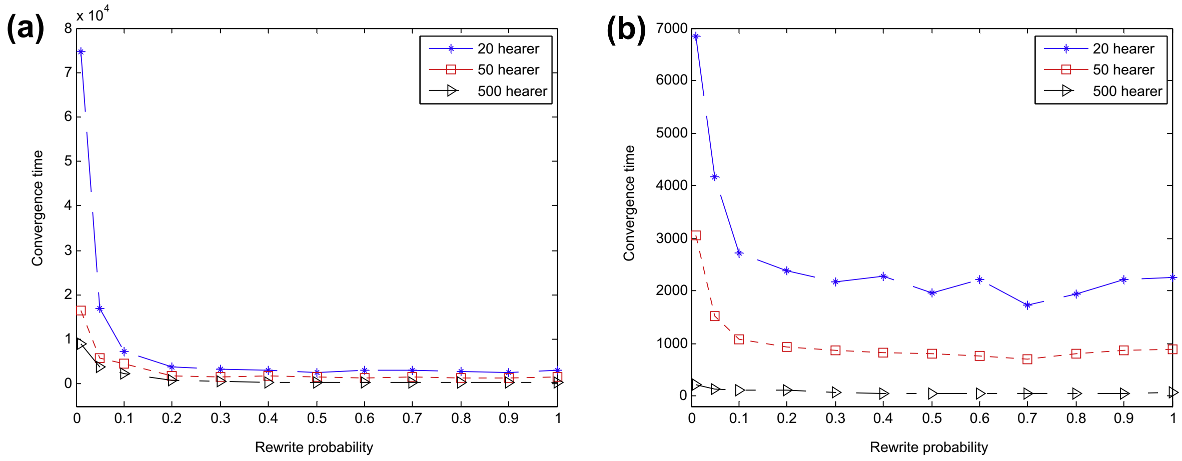

Fig. 8. Variation of the convergence time with the rewiring probability. The half neighbors of one node is (a) 50; (b) 200.

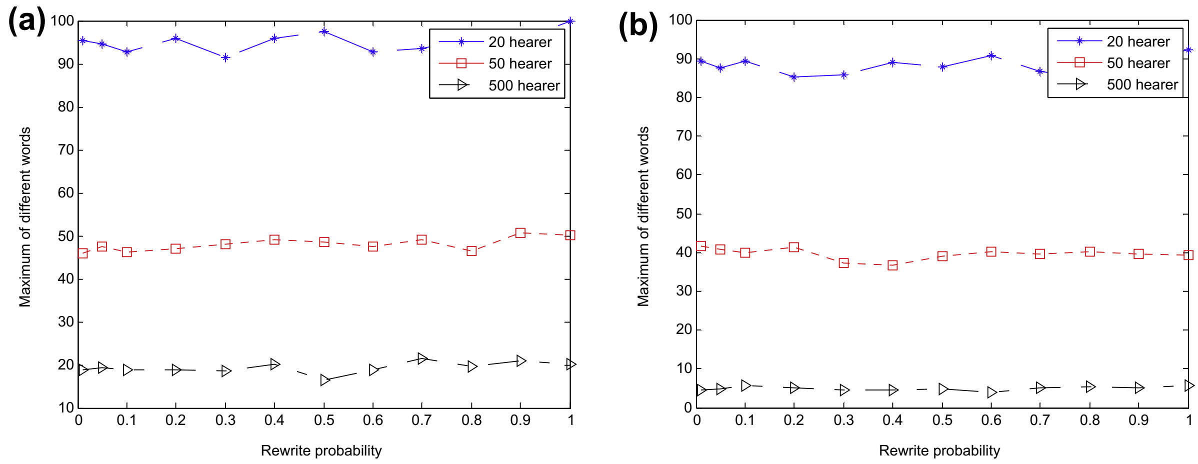

Fig. 9. Maximum number of different words with the rewiring probability. The half neighbors of one node is (a) 50; (b) 200.

3.6. Discussion about the total number of words

In Section 3.3, several important issues that play certain roles in the maximum number of different words were discussed. When the number of hearers is smaller than the average degree, the number of hearer determines the maximum number of different words. In comparison, the maximum number of total words is more complex. Table 2 gives the simulation results of the total number of words versus different numbers of hearers. For the same network, the minimum of total number shown in Table 2 is bolded. For random-graph networks, these minimum values rank from 2307.2 to 5874.75, and their maximal difference is 3400.3. For small-world networks, these minimum values rank from 2379.2 to 2778.8, and their maximal difference is 399.6. For scale-free networks, these minimum values rank from 3939.9 to 4986.6, and their maximal difference is 1046.7. With the increase of the average degree or the number of hearers, particular patterns of the variation about the maxima cannot be found. Table 3 shows the maximum number of total words with the increasing of the rewiring probability, showing that rewiring probability also plays a significant role in the maximum number of total words and that the pattern of variation for the total number of words is more complex than that for different words and that for the convergence time.

3.7. Relationship between the convergence time and the increase of the number of nodes

In the above subsection, the convergence time of a network with a fixed total number of nodes is discussed. Noticeably, the number of nodes also plays a role in the convergence time of a network. In this subsection, the variation of the convergence time with the increasing of the total number of nodes is investigated.

Table 2
Number of total words versus different numbers of hearers.

Maximum number of total words						
Random-graph networks						
Hearers	*Ran*-0.01	*Ran*-0.02	*Ran*-0.05	*Ran*-0. 1	*Ran*-0.2	*Ran*-0.5
1	9247.4	13047.6	16285.4	18049.4	18836.8	19741.2
5	7809.25	10907.45	13939.65	15560.65	16031.85	16913.05
10	5707.45	7951.35	10543.5	11825.05	12112.7	12857.75
20	**5874.75**	5658.05	7786.25	8662.7	9338.1	9756.85
30	6605.35	**5179**	6259.7	7158.3	7977.25	8124.4
50	6837.15	5922.1	4890	5621.8	6182.35	6383.8
100	6969.95	6258.25	4794.7	4182.2	4713.9	4951.45
200	6952.05	6161.6	**4557**	4042.75	3580.45	3733
500	6918.25	6323.1	4478.7	3686.5	3151.5	2883.5
1000	6926.25	6109.15	4900.8	**3663.45**	2981.35	2585.95
1500	6832.15	6305.6	4994.2	4017	3129.2	2405.5
1999	6800.25	6148.85	5070.6	3716.6	**2958.55**	**2307.2**
Small-world networks						
Hearers	*SW*-200-0.01	*SW*-200-0.05	*SW*-200-0. 1	*SW*-50-0.01	*SW*-50-0.05	*SW*-50-0. 1
1	15336	15671.4	16727.8	8793.4	9374.8	10210
5	11717.4	12370	13614.7	6244.6	6747	7753.625
10	8806.6	9239.1	10200.9	4816.5	5355.875	5909.625
20	6438.8	6985	7564	3840.5	4024.875	4407.875
30	5605.3	5842.1	6451.5	3301.5	3446.5	3825.125
50	4607.7	4896.7	5222.9	2830.75	3062.75	3259.875
100	3495.9	3885.9	3956	2420.25	2569.625	2870.75
200	2743.5	3000.2	3027.5	2405.75	2567.5	2814
500	2438.3	2567.9	2660	2412.75	2600.25	2816
1000	2405.3	2607	2774.3	2460.5	2667.625	2832.5
1500	**2393**	2639.9	**2622.9**	2411.875	2635.5	2791.75
1999	2448.6	**2493.8**	2683	**2379.2**	**2526.2**	**2778.8**
Scale-free networks						
Hearers	*BA*-10	*BA*-15	*BA*-20	*BA*-25	*BA*-30	*BA*-40
1	7219.8	9289	10638.8	11385.8	12488.4	13626
5	6675.8	8425.85	9763.75	10944.65	11607.4	12695
10	5353.1	6271.7	7089.2	8069.1	8460.45	9378.95
20	6092	5968.05	5399.25	5674.8	6051.85	6851.75
30	6307.55	6277.7	6102.35	5726.75	5217.65	5619.2
50	6039.15	6116.85	6034	5888.3	5877.05	5465.75
100	5566.95	5829.95	5417.3	5357.8	5314.3	5137.55
200	5463.1	5197.5	4812.9	4681.9	4814.65	4547.3
500	**4986.6**	4839.65	4671.25	**4382.05**	**4262.1**	4367.55
1000	5344.75	4997.15	**4411.55**	4522	4457.55	3991.5
1500	5015.55	**4674.8**	4564.15	4662.25	4571.9	**3939.95**
1999	5062.45	5093.2	4583.55	4501.35	4698.3	4142.75

Table 3
Maximum number of total words versus the rewiring probability.

Rewiring probability	$K = 200$			$K = 50$		
	20 Hearers	50 Hearers	500 Hearers	20 Hearers	50 Hearers	500 Hearers
0.01	6438.8	4607.7	2438.3	3840.5	2830.75	2412.75
0.05	6985	4896.7	2567.9	4024.875	3062.75	2600.25
0.1	7564	5222.9	2660	4407.875	3259.875	2816
0.2	8198.75	5783.375	2899.375	5446.75	3830.625	3306.125
0.3	8764.75	6035.375	2966.125	6310.875	4306.125	3937.125
0.4	9173.25	6131.5	2941.875	6977.125	4730.25	4416.125
0.5	8908	5784.375	3215.5	7493.625	4890	4244.75
0.6	9439.625	6370.125	2914.125	7681.125	4961.25	4502.125
0.7	8841.625	6022.75	3213.375	7862.375	5008.125	5140.75
0.8	8836.75	6643.375	3399.75	7796.125	5011.25	5208.625
0.9	9421.625	6309.875	2948.375	7824.5	5212.375	5514.25
1	9338.1	6182.35	3151.5	7786.25	4890	4478.7

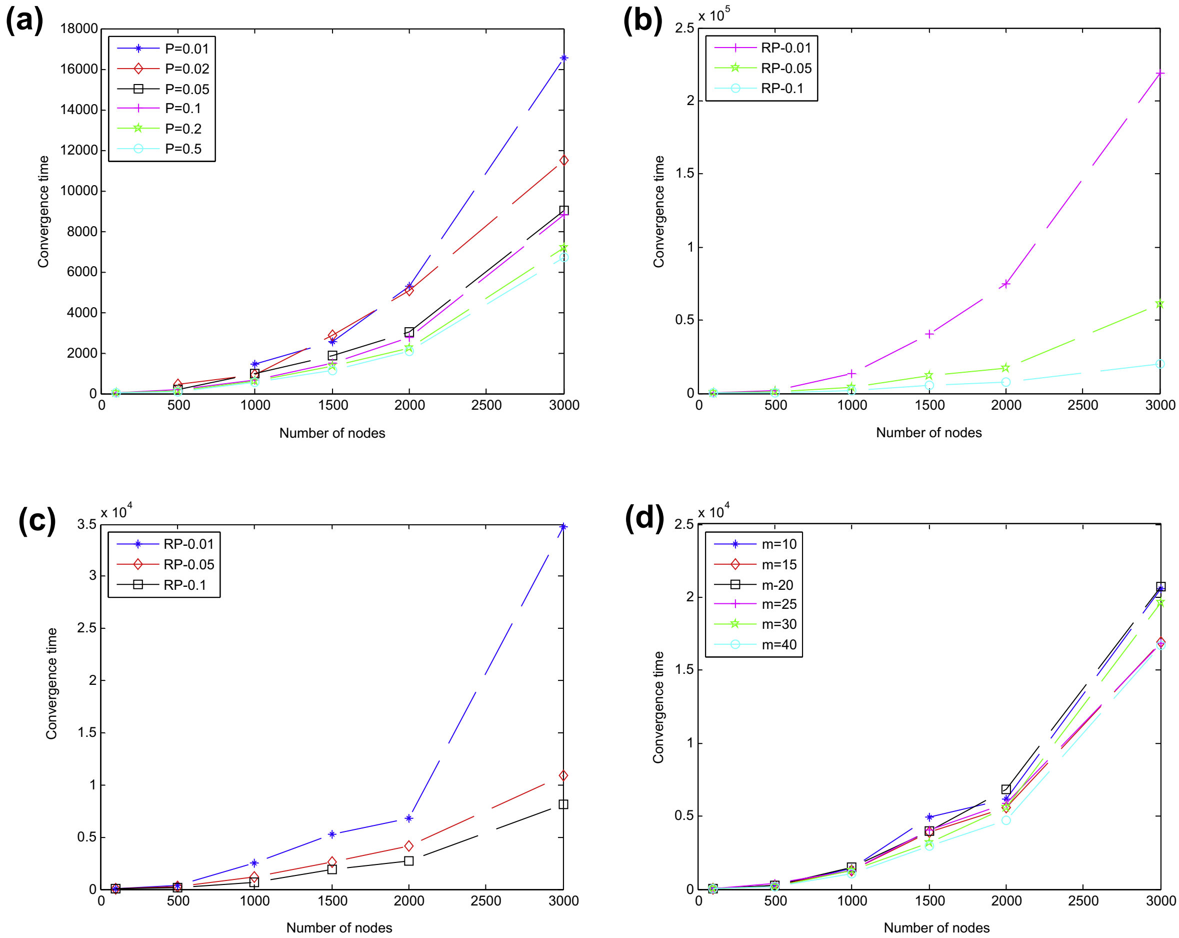

Fig. 10. Convergence time versus the total number of nodes.

Fig. 10 illustrates the variation of the convergence time against the increasing number of nodes. It is not necessary to employ multiple values of hearers, since the role of hearers has been discussed in the above subsections. So, the number of hearers is set to 20 in Fig. 10.

The possibility for a node to be a speaker is determined by the total number of nodes, and the possibility for a node to be a hearer is determined by the connection possibility in generating the network and by the number of existing hearers. So, in this subsection, the connection possibility in generating random-graph networks is tested with different network sizes. Similarly, for small-world networks, the average degree increases with the number of nodes, at a rate of 0.05 in Fig. 10(b) and of 0.2 in Fig. 10(c), which are the same as the networks described in Table 1. Since the degree distribution is a power-law, the setup of networks in Fig. 10(d) is the same as that used in the previous subsection.

In Fig. 10(a), the convergence time of a network with a small connection possibility varies significantly. In Fig. 10(b) and (c), for the network with a small rewiring possibility *rp*, the variation of the convergence time appears rapidly varying. Fig. 10(a)–(c) show that the average cluster coefficient is important to the convergence time of a network, especially when the total number of nodes is large. For the nodes at the same step, their possibilities to take part in learning at another same step are small when the average cluster coefficient is not very large. As a result, the improvement of the learning success is at a low rate. In contrast, in a small-work network with a large rewiring probability *rp*, the local convergence of the network becomes more and more significant as the total number of nodes increases, so the variation of the convergence time is also rapidly varying. Since the networks shown in Fig. 10(d) are of scale-free, the variation of the convergence time among different such networks with different number edges in generating networks turn out to be quite similar, as expected.

4. Conclusions

This paper has generalized the well-known one-speaker/one-hearer naming game model to a model with multiple hearers, referred to as Naming Game with Multiple Hearers (*NGMH*). In *NGMH*, there is a single speaker with multiple hearers at each time step throughout the evolutionary process, which is similar to the other naming games for studying social languages otherwise.

In this study, extensive simulations have been carried out and analyzed. Through studying the simulation results on *NGMH*, it is found that adding more hearers favor to reduce the convergence time of the network to consensus on learning a new word, particularly when the number of hearers is smaller than the average node degree of the network; yet it decreases the ability of individuals in learning new words in general. For a small-world network with a small rewiring possibility, local convergence is prominent, which leads the whole network to take longer running time to converge. It is also found that, for scale-free networks, their variations of the convergence time against the addition of nodes in the network are similar to each other, distinguishing themselves from the other network topologies.

It is believed that the new findings reported in this article can help, to some extent, enhancing our understanding of the language development and evolution processes.

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References

[1] Nowak MA. Philos Trans Roy Soc 2000;355(1403):1615.
[2] Nowak MA, Plotkin JB, Krakauer JD. J Theor Biol 1999;200:147.
[3] Steels, L., in: Proc. of PPSM – VI, Lecture notes in computer science Berlin: Springer; 2000.
[4] Briscoe T. Linguistic evolution through language acquisition: formal and computational models. Cambridge: Cambridge University Press; 1999.
[5] Hurford J, Knight C, Studdert-Kennedy M, editors. Approach to the evolution of human language. Cambridge: Cambridge University Press; 1999.
[6] Steels, L. Self-organizing vocabularies. In: Proc. Art. Life – V, Nara, Japan, 1996.
[7] Baronchelli, A., Dall'Asta, L., Barrat, A., Loreto, V. Eur. Phys. J., Special Topics 143 (2007) 233.
[8] Watts DJ, Strogatz SH. Nature 1998;393:440.
[9] Erdös P, Rényi A. Publ Math 1959;6:290.
[10] Barabási A-L, Albert R. Science 1999;286(5439):509.
[11] Dall'Asta L, Baronchelli A, Barrat A, Loreto V. Euro Phys Lett 2006;73:969.
[12] Dall'Asta L, Baronchelli A, Barrat A, Loreto V. Phys Rev E 2006;74:036105.
[13] Barrat A, Baronchelli A, Dall'Asta L, Loreto V. Chaos 2007;17:026111.
[14] Tang C-L, Lin B-Y, Wang W-X, Hu M-B, Wang B-H. Phys Rev E 2007;75:027101.
[15] Yang H-X, Wang W-X, Wang B-H. Phys Rev E 2008;77:027103.
[16] Liu R-R, Wang W-X, Lai Y-C, Chen G, Wang B-H. Phys Lett A 2011;375:363.
[17] Wang W-X, Lin B-Y, Tang C-L, Chen G. Eur Phys J B 2007;60:529.
[18] Huang L, Park K, Lai Y-C. Phys Rev E 2006;73:035103.