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Iterative learning control with initial rectifying action for nonlinear continuous systems

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Abstract: A new iterative learning control (ILC) method with initial rectifying action for nonlinear continuous multivariable systems is presented. Unlike general ILC techniques, the proposed ILC approach allows initial outputs of an ILC system at different iterations to fluctuate randomly around the initial value of the desired output. The proposed strategy includes an initial rectifying action of ILC on a very small initial time interval, and pursues the reference trajectory tracking beyond the initial time interval. The output tracking error beyond the initial time interval can be driven to a residual set whose size depends on the estimation error of input matrix. A numerical example is used to illustrate the effectiveness of the proposed ILC approach.

1 Introduction

Iterative learning control (ILC) is a versatile control technique to improve transient response of a system operating repetitively over a fixed time interval. One of the important features of ILC is that it requires less a priori knowledge about the controlled system in the course of design. This makes ILC increasingly important in control applications, such as robot manipulators and disk drive systems that are mostly designed for repetitive tasks. Until now, there have been a lot of ILC algorithms reported. But most of the existing ILC works assume that the initial outputs of an ILC system at different iterations were kept invariant [1, 2], which means that the ILC system must have a fixed initial error at different iterations. In some cases, a more stringent condition of zero initial error is even required [3–6]. All of these hypotheses are difficult to be implemented in practice, as locating operation repetitively in an ILC application can result in irregular drifts of initial outputs at different iterations to the initial value of the desired output. Clearly, the study of ILC problem applied to dynamical systems with non-fixed initial errors is essential.

The difficulty of ILC with non-fixed initial error mainly lies in the dynamical complexity of ILC in which initial

iterative error interacts with not only the system dynamics but also the iterative learning process. Lee and Bien [7] reported some novel undesirable phenomenon due to the mismatch in the initial conditions in ILC process. They pointed out that the control system could become unstable when the initial output at each iteration was different from the previous initial output. There are a few research papers [8–10] discussing the robustness of ILC algorithms to non-fixed initial errors, but the bound of robustness is normally too large and cannot be adjusted for practically tracking a reference trajectory. As a result, an initial rectifying action should be considered in ILC design. It is worth noting that there have been works [11, 12] incorporating an initial rectifying action into ILC design to achieve complete tracking of a reference trajectory over a specified interval, but they require that the initial error be a fixed value. They will not work when a random or non-fixed initial error is encountered. Hitherto, the research on ILC applied to dynamical systems with non-fixed initial errors still remains an open and important issue. Recently, a few studies [13–17] have been focussed on the ILC problem of dynamical systems with non-fixed initial errors. In [13], a method called ILC with multi-modal input was proposed to tackle the ILC problem for linear

continuous systems with non-fixed initial errors. But it is computationally complex because the determined input is, in fact, the synthesis of the multi-modal ILC input with varying initial condition. In [14], under a specific condition, an average operator-based PD-type ILC controller for linear continuous systems with non-fixed initial errors was investigated. In [17], an ILC controller for linear discrete systems with non-fixed initial errors was investigated using 2-D system theory. Despite its ability to handle variable initial conditions, it is only effective when the tracking time interval is sufficiently large. For ILC on nonlinear systems, Xu and Yan [15] discussed the inherent relationship between different initial conditions and the corresponding learning convergence (or boundedness) property under a Lyapunov-based ILC method. However, the proposed ILC technique [15] is only suitable for a class of simple first-order nonlinear continuous systems with unit input gain. Furthermore, a direct adaptive ILC approach based on a fuzzy neural network was presented in [16] for a class of nonlinear systems with non-fixed initial errors. Using the proposed adaptive ILC approach, the norm of state tracking error will asymptotically converge to a tunable residual set as iteration goes to infinite.

The main objective of this paper is to describe how an initial rectifying action can be combined into the conventional D-type ILC technique for nonlinear continuous multivariable (NCM) systems with non-fixed initial errors. Beyond the designated initial time interval, the established ILC technique is able to drive the output tracking error to a residual set whose size depends on the estimation error of input matrix. It is also important to note that a perfect reference trajectory tracking beyond the initial time interval can be achieved when an accurate knowledge on input matrix at the initial time interval is available.

The organisation of this paper is as follows. Section 2 presents the ILC problem formulation and the ILC rule with initial rectifying action. Section 3 investigates the learning convergence of the proposed ILC rule under non-fixed initial errors. And the simulation results are illustrated in Section 4. Finally, Section 5 concludes this paper.

2 Problem formulation and ILC rule with initial rectifying action

Consider the following class of NCM systems performing repetitive tasks over a fixed time interval $t \in [0, T]$

$$\dot{x}_k(t) = f(x_k(t), t) + B(t) \cdot u_k(t) \quad (1a)$$

$$y_k(t) = C(t) \cdot x_k(t) \quad (1b)$$

where k denotes the k th repetitive operation of the system; $x_k(t) \in R^n$, $u_k(t) \in R^m$ ($m \geq n$), and $y_k(t) \in R^p$ are the state, control input and output of the system, respectively; $B(t) \in R^{n \times m}$ and $C(t) \in R^{p \times n}$ are time-variant matrices; the nonlinear function $f(\cdot, \cdot) : R^n \times [0, T] \mapsto R^n$ is locally

Lipschitz in x_k , that is, for all $t \in [0, T]$ and k , there exists a constant L_f such that

$$\|f(x_{k+1}(t), t) - f(x_k(t), t)\| \leq L_f \cdot \|x_{k+1}(t) - x_k(t)\| \quad (2)$$

where the norm $\|\cdot\|$ will be defined later. An example of system (1) can be referred to the PM synchronous motor studied in [18].

Given a reference output trajectory $y_r(t)$ and an initial control input $u_0(t)$ at $t \in [0, T]$, an ILC design for system (1) is closely related to its boundary conditions $x_k(0)$ for $k = 0, 1, 2, \dots$. Unlike general ILC literatures, we assume that $x_k(0)$ is bounded and different for $k = 0, 1, 2, \dots$, and $y_k(0)$ fluctuates randomly around $y_r(0)$ within a bound in this study. Based on the given boundary conditions, our ILC objective is to iteratively determine a control input sequence $\{u_k(t)\}$, $t \in [0, T]$, such that as k goes to infinite, the output tracking error $y_r(t) - y_k(t)$ over a specified time interval $t \in [b, T]$ can be driven to a residual set whose size depends on the estimation error of input matrix $B(t)$, where b is a specified small real number between 0 and T . In many ILC applications, the model information on the controlled system, especially the input matrix $B(t)$, can be well known. In this case, our ILC design with the control objective can achieve an effective reference trajectory tracking.

The output tracking error in ILC process is denoted as

$$e_k(t) = y_r(t) - y_k(t) \quad (3)$$

When the initial ILC errors $e_k(0)$ for $k = 0, 1, 2, \dots$ are non-fixed, but bounded, it has been shown in [9] that a well-known D-type ILC rule

$$u_{k+1}(t) = u_k(t) + K(t)\dot{e}_k(t) \quad (4)$$

can ensure the boundedness of the output tracking error $e_k(t)$, $t \in [0, T]$ as k goes to infinite. In practice, this bound is usually too large to be applicable to track a reference trajectory. In this paper, a rectifying action is then combined into a D-type ILC rule (4), and the following ILC rule for control calculation is applied

$$u_{k+1}(t) = u_k(t) + K(t)\dot{e}_k(t) + \theta_b(t)\hat{B}(t)_R^{-1}X_k(0) \quad (5)$$

where $\hat{B}(t)$ denotes the estimation of $B(t)$; $\hat{B}(t)_R^{-1}$ is the right inverse of $\hat{B}(t)$; and

$$\theta_b(t) = \begin{cases} \frac{2}{b} \left(1 - \frac{t}{b}\right), & t \in [0, b) \\ 0, & t \in [b, T] \end{cases} \quad (6)$$

$$X_k(0) = \hat{B}(0)K(0)e_k(0) + x_k(0) - x_{k+1}(0) \quad (7)$$

For the convenience of convergence analysis of the proposed ILC approach, the following definitions and lemma on

norms are given

$$\begin{aligned} \|s\| &= \max_{1 \leq i \leq n} |s^{(i)}| \\ \|G\| &= \max_{1 \leq i \leq m} \left(\sum_{j=1}^n |g_{i,j}| \right) \\ \|q(t)\|_\lambda &= \sup_{t \in [0, T]} e^{-\lambda t} \|q(t)\|, \quad \lambda > 0 \end{aligned}$$

where $s = [s^{(1)} \ s^{(2)} \ \dots \ s^{(n)}]^T$ is a vector, $G = [g_{i,j}] \in R^{m \times n}$ is a matrix and $q(t)$, $t \in [0, T]$ is a real function vector/matrix. Specially, in this paper, we use

$$\|q(t)\|_\lambda|_{t \in [b, T]} = \sup_{t \in [b, T]} e^{-\lambda t} \|q(t)\| \quad (8)$$

to define λ -norm of $q(t)$ on the time interval $t \in [b, T]$.

On the relationship between the norm $\|\cdot\|$ and the λ -norm $\|\cdot\|_\lambda$ of a vector $x(t)$, $t \in [0, T]$, we have the following lemma 1.

Lemma 1:

$$\sup_{t \in [0, T]} \left(e^{-\lambda t} \int_0^t \|x(\tau)\| d\tau \right) \leq \frac{1}{\lambda} \|x(t)\|_\lambda \quad (9)$$

The proof of Lemma 1 is an immediate consequence of the norm $\|\cdot\|_\lambda$, and therefore is omitted.

3 Learning convergence under non-fixed initial errors

In this section, the convergence results of ILC rule (5) are provided in Theorem 1.

Theorem 1: For an NCM system (1), suppose that the reference output $y_r(t)$ is differentiable, and the initial ILC error $e_k(0)$ varies randomly within a bound. If there exists a matrix $K(t)$ to make

$$\sup_{t \in [0, T]} \|I - C(t)B(t)K(t)\| = \rho_0 < 1 \quad (10)$$

and the matrix $B(t)$ at $t \in [0, b]$ is right invertible; then, the ILC rule (5) can ensure

$$\limsup_{k \rightarrow \infty} \|e_k(t)\|_\lambda|_{t \in [b, T]} \leq \frac{\lambda M_C (M_E \cdot M_{\hat{X}} + M_0 \cdot M_{\hat{K}})}{(1 - \rho)(\lambda - L_f)} \quad (11)$$

where $B(t)\hat{B}(t)_R^{-1} = I + E(t)$, $\sup_{t \in [0, b]} \|E(t)\| \leq M_E$, $\sup_{t \in [0, T]} \|C(t)\| \leq M_C$, $\|\hat{B}(0) - B(0)\| \leq M_0$, $\sup_k \|X_k(0)\| \leq M_{\hat{X}}$, $\sup_k \|K(0)e_k(0)\| \leq M_{\hat{K}}$, $0 < \rho < 1$, and λ is a

bounded value. Especially, when the input matrix $B(t)$ at $t \in [0, b]$ is accurately known, we have $\lim_{k \rightarrow \infty} \|e_k(t)\|_\lambda|_{t \in [b, T]} = 0$.

Proof: From (1) and (5), we have

$$\begin{aligned} x_{k+1}(t) - x_k(t) &= \int_0^t \dot{x}_{k+1}(\tau) d\tau - \int_0^t \dot{x}_k(\tau) d\tau + [x_{k+1}(0) - x_k(0)] \\ &= \int_0^t [f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)] d\tau \\ &\quad + \int_0^t B(\tau)[u_{k+1}(\tau) - u_k(\tau)] d\tau + [x_{k+1}(0) - x_k(0)] \\ &= \int_0^t [f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)] d\tau \\ &\quad + \int_0^t B(\tau)K(\tau)\dot{e}_k(\tau) d\tau + \int_0^t \theta_b(\tau)B(\tau)\hat{B}(\tau)_R^{-1} d\tau \\ &\quad \cdot X_k(0) + [x_{k+1}(0) - x_k(0)] = \int_0^t [f(x_{k+1}(\tau), \tau) \\ &\quad - f(x_k(\tau), \tau)] d\tau + B(t)K(t)e_k(t) - B(0)K(0)e_k(0) \\ &\quad - \int_0^t \frac{d(B(\tau)K(\tau))}{d\tau} e_k(\tau) d\tau + \int_0^t \theta_b(\tau)[I + E(\tau)] d\tau \\ &\quad \cdot X_k(0) + [x_{k+1}(0) - x_k(0)] \quad (12) \end{aligned}$$

On the other hand, from (1) and (3)

$$\begin{aligned} e_{k+1}(t) &= y_r(t) - y_{k+1}(t) \\ &= e_k(t) - [y_{k+1}(t) - y_k(t)] \\ &= e_k(t) - C(t)[x_{k+1}(t) - x_k(t)] \quad (13) \end{aligned}$$

Substituting (12) into (13), we have

$$\begin{aligned} e_{k+1}(t) &= [I - C(t)B(t)K(t)]e_k(t) \\ &\quad - C(t) \int_0^t [f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)] d\tau \\ &\quad + C(t) \int_0^t \frac{d(B(\tau)K(\tau))}{d\tau} e_k(\tau) d\tau \\ &\quad - C(t) \int_0^t \theta_b(\tau)[I + E(\tau)] d\tau \cdot X_k(0) \\ &\quad + C(t)B(0)K(0)e_k(0) - C(t)[x_{k+1}(0) - x_k(0)] \quad (14) \end{aligned}$$

As $t \in [b, T]$, considering that $\int_0^t \theta_b(\tau) d\tau = \int_0^b \theta_b(\tau) d\tau = 1$

from (6), (12) and (14) can be, respectively, written as

$$\begin{aligned}
 & x_{k+1}(t) - x_k(t) \\
 &= \int_0^t [f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)] d\tau \\
 &+ B(t)K(t)e_k(t) - \int_0^t \frac{d(B(\tau)K(\tau))}{d\tau} e_k(\tau) d\tau \\
 &+ \int_0^b \theta_b(\tau)E(\tau)d\tau \cdot X_k(0) + [\hat{B}(0) - B(0)]K(0)e_k(0)
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 e_{k+1}(t) &= [I - C(t)B(t)K(t)]e_k(t) - C(t) \\
 &\times \int_0^t [f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)] d\tau \\
 &+ C(t) \int_0^t \frac{d(B(\tau)K(\tau))}{d\tau} e_k(\tau) d\tau - C(t) \\
 &\times \int_0^b \theta_b(\tau)E(\tau)d\tau \cdot X_k(0) - C(t)[\hat{B}(0) \\
 &- B(0)]K(0)e_k(0)
 \end{aligned} \tag{16}$$

Now, let us independently investigate the properties of (15) and (16) on the whole time interval $t \in [0, T]$. Taking the norms on two sides of (15) and (16), respectively, we have

$$\begin{aligned}
 \|x_{k+1}(t) - x_k(t)\| &\leq L_f \int_0^t \|x_{k+1}(\tau) - x_k(\tau)\| d\tau \\
 &+ M_1 \cdot \|e_k(t)\| + M_2 \int_0^t \|e_k(\tau)\| d\tau \\
 &+ M_E M_{\bar{X}} + M_0 M_{\bar{K}}
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 \|e_{k+1}(t)\| &\leq \rho_0 \|e_k(t)\| + M_C L_f \int_0^t \|x_{k+1}(\tau) - x_k(\tau)\| d\tau \\
 &+ M_C M_2 \int_0^t \|e_k(\tau)\| d\tau \\
 &+ M_C M_E M_{\bar{X}} + M_C M_0 M_{\bar{K}}
 \end{aligned} \tag{18}$$

where $\sup_{t \in [0, T]} \|B(t)K(t)\| \leq M_1$, $\sup_{t \in [0, T]} \|[d(B(t)K(t))/dt]\| \leq M_2$, and $M_E, M_0, M_C, M_{\bar{X}}, M_{\bar{K}}, \rho_0$ are defined as in Theorem 1.

Multiplying (17) and (18) by $e^{-\lambda t}$, respectively, where $\lambda > L_f$, we have

$$\begin{aligned}
 \|x_{k+1}(t) - x_k(t)\|_\lambda &\leq L_f \sup_{t \in [0, T]} \left(e^{-\lambda t} \int_0^t \|x_{k+1}(\tau) - x_k(\tau)\| d\tau \right) \\
 &+ M_1 \|e_k(t)\|_\lambda + M_2 \sup_{t \in [0, T]} \left(e^{-\lambda t} \int_0^t \|e_k(\tau)\| d\tau \right) \\
 &+ M_E M_{\bar{X}} + M_0 M_{\bar{K}}
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 \|e_{k+1}(t)\|_\lambda &\leq \rho_0 \|e_k(t)\|_\lambda + M_C L_f \\
 &\times \sup_{t \in [0, T]} \left(e^{-\lambda t} \int_0^t \|x_{k+1}(\tau) - x_k(\tau)\| d\tau \right) + M_C M_2 \\
 &\times \sup_{t \in [0, T]} \left(e^{-\lambda t} \int_0^t \|e_k(\tau)\| d\tau \right) + M_C M_E M_{\bar{X}} \\
 &+ M_C M_0 M_{\bar{K}}
 \end{aligned} \tag{20}$$

Applying (9) of Lemma 1 to (19) and (20), we can obtain the following inequality

$$\begin{aligned}
 \|x_{k+1}(t) - x_k(t)\|_\lambda &\leq \frac{\lambda M_1 + M_2}{\lambda - L_f} \|e_k(t)\|_\lambda \\
 &+ \frac{\lambda M_E M_{\bar{X}}}{\lambda - L_f} + \frac{\lambda M_0 M_{\bar{K}}}{\lambda - L_f}
 \end{aligned} \tag{21}$$

$$\begin{aligned}
 \|e_{k+1}(t)\|_\lambda &\leq \left(\rho_0 + \frac{M_C M_2}{\lambda} \right) \|e_k(t)\|_\lambda \\
 &+ \frac{M_C L_f}{\lambda} \|x_{k+1}(t) - x_k(t)\|_\lambda \\
 &+ M_C M_E M_{\bar{X}} + M_C M_0 M_{\bar{K}}
 \end{aligned} \tag{22}$$

Substituting (21) into (22), we obtain

$$\|e_{k+1}(t)\|_\lambda \leq \rho \|e_k(t)\|_\lambda + \frac{\lambda M_C M_E M_{\bar{X}}}{\lambda - L_f} + \frac{\lambda M_C M_0 M_{\bar{K}}}{\lambda - L_f} \tag{23}$$

where $\rho = \rho_0 + (M_C M_2 / \lambda) + (M_C L_f (M_1 + (M_2 / \lambda)) / (\lambda - L_f))$. When $\rho_0 < 1$, we can select λ to be sufficiently large such that $\rho < 1$. Equation (23) is a contraction, therefore

$$\limsup_{k \rightarrow \infty} \|e_k(t)\|_\lambda \leq \frac{\lambda M_C (M_E M_{\bar{X}} + M_0 M_{\bar{K}})}{(1 - \rho)(\lambda - L_f)} \tag{24}$$

On the other hand, from the definition of the λ -norm, $\limsup_{k \rightarrow \infty} \|e_k(t)\|_\lambda |_{t \in [b, T]} \leq \limsup_{k \rightarrow \infty} \|e_k(t)\|_\lambda$. Considering (24), we have

$$\limsup_{k \rightarrow \infty} \|e_k(t)\|_\lambda |_{t \in [b, T]} \leq \frac{\lambda M_C (M_E M_{\bar{X}} + M_0 M_{\bar{K}})}{(1 - \rho)(\lambda - L_f)} \tag{25}$$

Furthermore, if the input matrix $B(t)$ at $t \in [0, b]$ is accurately known, we have $M_E = M_0 = 0$. It can be directly derived from (25) that $\lim_{k \rightarrow \infty} \|e_k(t)\|_\lambda |_{t \in [b, T]} = 0$.

From the above deduction, it is important to note that (25) is obtained from the investigation of system (15) and (16) on the time interval $t \in [0, T]$. System (15) and (16) is not equivalent to the original system (12) and (14) at $t \in [0, T]$, but they are equivalent on the time interval $t \in [b, T]$. They should exhibit the same properties at $t \in [b, T]$. Therefore (25) is also suitable to system (12) and (14) at $t \in [b, T]$. Theorem 1 is proved. \square

Remark 1: Equation (25) shows that the ILC rule (5) is able to drive the ILC error at $t \in [b, T]$ into a bound. But, it is worth noting that after the parameter λ is selected, the bound is mainly decided by parameters M_E and M_0 , which are related to the estimation error with respect to the input matrix $B(t)$. Therefore compared with the robust effect of the D-type ILC rule (4), the bound can be controlled to a much smaller level when $B(t)$ is well estimated. This point will be better illustrated by the example presented in the next section.

Remark 2: From the ILC rule (5) and the definition (6) of $\theta_b(t)$, it is shown that the ILC rule (5) puts an initial rectifying action on a small initial time interval $[0, b]$ and pursues the reference trajectory tracking at $t \in [b, T]$. The function $\theta_b(t)$ specifies the initial rectifying action on $[0, b]$. A less value b in $\theta_b(t)$ may result in a larger control input. Thus, the selection of b should be done based on the trade-off between the resulting control input and the tracking error.

In addition, Theorem 1 requires that the input matrix $B(t)$ is right invertible and estimable. But the corresponding time interval is only a small initial time interval $[0, b]$, not the entire time interval $[0, T]$. Also, in order to achieve perfect tracking at $t \in [b, T]$ namely $\lim_{k \rightarrow \infty} \|e_k(t)\|_{\lambda} |_{t \in [b, T]} = 0$, we need the input matrix, $B(t)$, to be accurately known on the initial time interval $[0, b]$ only.

Remark 3: Regarding the selection of the learning gain matrix $K(t)$ in the ILC rule (5), Theorem 1 gives us a theoretical guideline that $K(t)$ should satisfy (10). In particular, let $K(t) = \alpha(\hat{C}(t)\hat{B}(t))^T [\hat{C}(t)\hat{B}(t)(\hat{C}(t)\hat{B}(t))^T]^{-1}$ if $\hat{C}(t)\hat{B}(t)$ is of full-row rank, where $\hat{C}(t)$ and $\hat{B}(t)$ are estimations to $C(t)$ and $B(t)$, respectively. We can find $\alpha \in (\varepsilon, 2 - \varepsilon)$ and $\varepsilon \in (0, 1)$ so that $\sup_{t \in [0, T]} \|I - C(t)B(t)K(t)\| = \rho_0 < 1$.

4 Illustrative example

Consider an ILC problem of the following NCM system

$$\begin{cases} \frac{d}{dt} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} = \begin{bmatrix} -0.5 \sin(x^{(1)}) + 0.8 \cos(x^{(2)}) \\ \sin(x^{(1)}x^{(2)}) \end{bmatrix} + B(t)u \\ y = C(t) \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} \end{cases} \quad (26)$$

where $B(t) = \begin{bmatrix} 2.5 & \sin(10t) \\ 0 & 1 - 0.3t \end{bmatrix}$, $C(t) = [0.5t + 0.1 \quad 1]$.

The desired output $y_r(t)$ is described by the equation

$$y_r(t) = 15(t - t^2), \quad t \in [0, 1] \quad (27)$$

In order to verify our ILC approach for NCM systems with non-fixed initial errors, we randomise the initial states of system (26) $x^{(1)}(0)$ and $x^{(2)}(0)$, which vary between -1 and 1 at different iterations of ILC process randomly, as shown in Table 1. The induced non-fixed initial ILC errors, $e_k(0)$, are also listed in Table 1.

Suppose that the accurate information on parameters $B(t)$ and $C(t)$ in system (26) is unavailable, and only their estimation $\hat{B}(t)$ and $\hat{C}(t)$ are given as $\begin{bmatrix} 2.3 & 0.7 \sin(10t) \\ 0.15 & 1.2 - 0.45t \end{bmatrix}$ and $[0.6t + 0.15 \quad 1.2]$, respectively. In the ILC process of system (26), we set the initial input of ILC as $u_0(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $t \in [0, 1]$, and $b = 0.05$. We pursue the tracking of the desired output $y_r(t)$ on the interval $t \in [0.05, 1]$. The accuracy of tracking is evaluated by the following maximum absolute error of tracking

$$EE = \sup_{t \in [b, 1]} |y_r(t) - y(t)|$$

To better illustrate the initial rectifying action of our

Table 1 Non-fixed initial states and initial errors of ILC tracking performance at different iterations

k	0	1	2	3	4	5	6	7
$x_k(0)$	$\begin{bmatrix} 0.36 \\ 0.41 \end{bmatrix}$	$\begin{bmatrix} 0.59 \\ 0.77 \end{bmatrix}$	$\begin{bmatrix} 0.42 \\ -0.81 \end{bmatrix}$	$\begin{bmatrix} -0.17 \\ 0.40 \end{bmatrix}$	$\begin{bmatrix} -0.31 \\ 0.84 \end{bmatrix}$	$\begin{bmatrix} 0.93 \\ 0.54 \end{bmatrix}$	$\begin{bmatrix} 0.73 \\ -0.75 \end{bmatrix}$	$\begin{bmatrix} 0.13 \\ 0.42 \end{bmatrix}$
$e_k(0)$	0.45	0.83	-0.77	0.38	0.81	0.63	-0.68	0.43
k	8	9	10	11	12	13	14	15
$x_k(0)$	$\begin{bmatrix} -0.38 \\ -0.84 \end{bmatrix}$	$\begin{bmatrix} 0.93 \\ 0.67 \end{bmatrix}$	$\begin{bmatrix} -0.60 \\ -0.33 \end{bmatrix}$	$\begin{bmatrix} 0.83 \\ 0.21 \end{bmatrix}$	$\begin{bmatrix} -0.71 \\ 0.15 \end{bmatrix}$	$\begin{bmatrix} -0.08 \\ -0.52 \end{bmatrix}$	$\begin{bmatrix} 0.27 \\ -0.78 \end{bmatrix}$	$\begin{bmatrix} -0.23 \\ 0.82 \end{bmatrix}$
$e_k(0)$	-0.88	0.76	-0.39	0.29	0.08	-0.53	-0.75	0.80

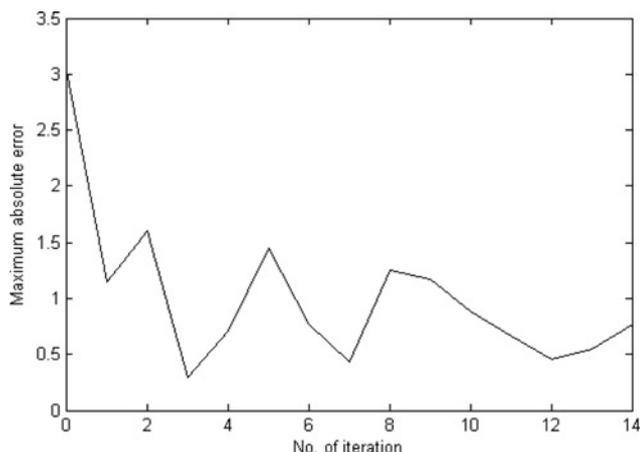


Figure 1 Maximum absolute error of tracking on the interval $t \in [0.05, 1]$ at different iteration numbers using the D-type ILC rule (4)

proposed ILC rule (5) by comparison, first, a D-type ILC rule (4) without a rectifying action is used. We set $K(t) = 0.6(\hat{C}(t)\hat{B}(t))^T[\hat{C}(t)\hat{B}(t)(\hat{C}(t)\hat{B}(t))^T]^{-1}$, which makes $\max_{t \in [0,1]} \|I - C(t)B(t)K(t)\| < 1$. Fig. 1 presents the situation of the tracking error index EE on the interval $t \in [0.05, 1]$ when the D-type ILC rule (4) is executed at different iteration numbers. In Fig. 1, it is shown that the ILC rule (4) is able to drive the ILC error into a bound. The robustness of the ILC rule (4) to non-fixed initial ILC errors is thus illustrated. But it is also noticed that the bound is surely too large for practical application.

Next, the above ILC rule (5), which includes an initial rectifying action, is applied. Fig. 2 shows how the tracking error index EE on the interval $t \in [0.05, 1]$ varies at different numbers of iteration. Compared with the results shown in Fig. 1, the ILC tracking error is bounded to a much lower level.

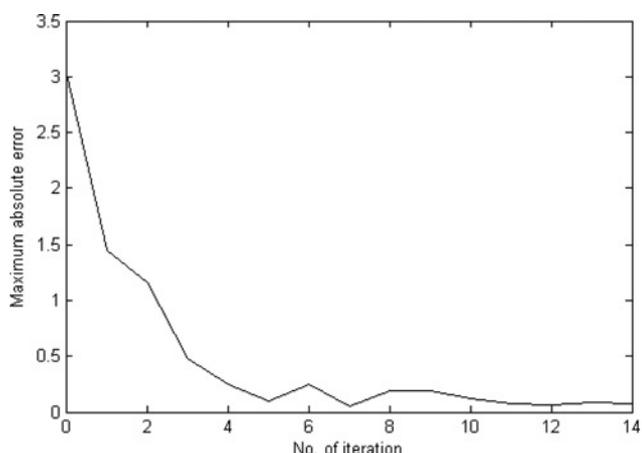


Figure 2 Maximum absolute error of tracking on the interval $t \in [0.05, 1]$ at different iteration number using the ILC rule (5)

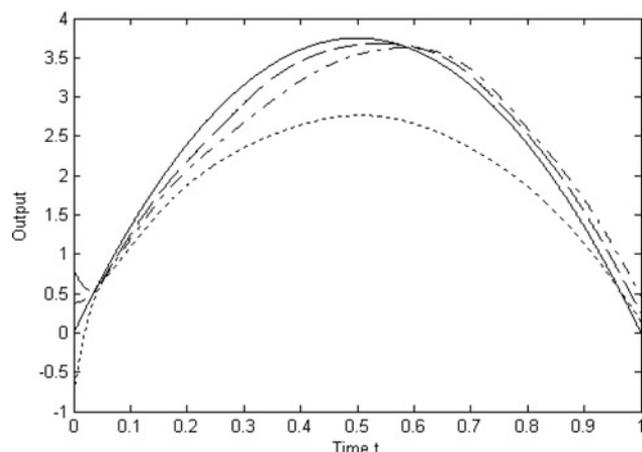


Figure 3 Tracking performance of the ILC system outputs on the interval $t \in [0, 1]$ at different iteration numbers using the ILC rule (5) with the known $B(t)$

(The dotted, dashed-dotted and dashed lines represent the system outputs as the ILC rule (5) is iteratively executed two, three and four times, respectively, and the solid line represents the desired output)

Finally, given that the input matrix $B(t)$ is accurately known on the initial time interval $[0,0.05]$, and the ILC rule (5) is used, Fig. 3 shows the tracking performance of the ILC system output on the interval $t \in [0, 1]$ when the ILC rule (5) is iteratively executed from the second to the fourth time. Also, Fig. 4 presents the situation of the tracking error index EE on the interval $t \in [0.05, 1]$ when the ILC rule (5) is executed at different iteration numbers. In Fig. 4, it is noticed that the ILC tracking error on the interval $t \in [0.05, 1]$ can be surely driven to zero by the ILC rule (5) with the known $B(t)$. A perfect tracking of the desired output $y_r(t)$ on the interval $t \in [0.05, 1]$ is achieved.

The example used in this paper illustrates that the proposed ILC approach for NCM systems with non-fixed

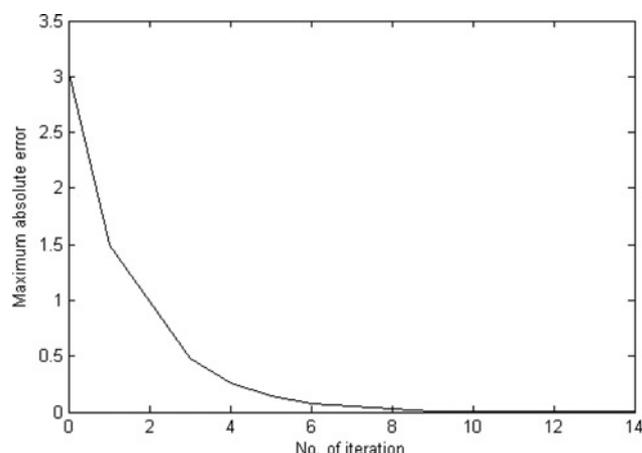


Figure 4 Maximum absolute error of tracking on the interval $t \in [0.05, 1]$ at different iteration numbers using the ILC rule (5) with the known $B(t)$

initial errors is very effective. It can overcome random initial errors of ILC systems and can successfully track the desired output trajectory beyond a small initial time interval after a small number of iterations.

5 Conclusion

This paper introduces an initial rectifying action into the conventional D-type ILC method for NCM systems. The established ILC technique allows the initial output of the ILC system at different iterations fluctuating randomly around the initial value of the desired output. The used ILC strategy is to set a very small initial time interval and to pursue the reference trajectory tracking beyond the initial time interval. The output tracking error beyond the initial time interval can be driven to a residual set whose size depends on the estimation error of input matrix. It is worth noting that when an accurate knowledge on the input matrix on the initial time interval is available, a perfect reference trajectory tracking beyond the initial time interval can even be achieved. Compared with the conventional ILC methods, the robustness of the proposed ILC system is greatly improved.

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