Iterative learning control for linear time-variant discrete systems based on 2-D system theory

X.-D. Li, J.K.L. Ho and T.W.S. Chow

Abstract: The two-dimensional (2-D) system theory iterative learning control (ILC) techniques for linear time-invariant discrete systems are extended to the cases of linear time-variant discrete systems. By exploiting the convergent property of 2-D linear time-variant discrete systems with only one independent variable, a kind of 2-D system theory ILC approach is presented for linear time-variant discrete systems. Sufficient conditions are given for convergence of the proposed ILC rules. Two numerical examples are used to validate the ILC procedures.

1 Introduction

Iterative learning control (ILC) has generated considerable interest since it was firstly introduced in 1984 by Arimoto *et al.* [1]. The objective of ILC is to use the repetitive nature of a process to progressively enhance the tracking performance. Using error measurements in a previous cycle, the control inputs are updated iteratively after each operation. These types of controller are able to deal with dynamic systems with imperfect knowledge of dynamics structures and/or parameters operating repetitively over a fixed time interval [2]. This makes ILC schemes particularly useful in applications with repetitive tasks such as robotic manipulators, disc-drive systems, IC wafer production, and steel-casting control [1, 3–6]. Until now there have been many ILC methods presented in the area of control systems [1, 2, 7–15], and the most widely used ILC method is the proportional–integral–derivative (PID) approach because it essentially forms a PID-like system. Although there are certain advantages contributed by the ILC control schemes, there are certain technical difficulties due to the two-dimensionality, as addressed in [16]. It is well known that amid the iterative learning process the interaction between the system dynamics and the iterative learning process poses an important and challenging issue in ILC research.

In recent years the theory of two-dimensional (2-D) system was successfully and widely introduced to the ILC approach [2, 8–13]. Owing to the two independent dynamic processes of the 2-D system, the 2-D model provides an excellent mathematical platform to describe both the dynamics of the control system and the behaviour of the learning iteration. Very promising results on ILC for linear multivariable systems have been obtained [2, 8, 9, 11–13]. Based on 2-D system theory, [2, 9, 11–13] investigated the ILC techniques applying to linear discrete multivariable systems. In [8] the ILC problem for linear continuous multivariable systems was addressed. But all these works focused only on the ILC problem of linear time-invariant systems in which the parameters of linear multivariable systems were invariant. Based on the assumption that the system parameters are invariant, they are clearly unable to deal with the cases of linear time-variant systems. It is also well known that linear time-variant systems, which simply regard linear time-invariant systems as a special case, have much wider application. And linear time-variant systems exhibit more complicated dynamics compared with the cases of the time-invariant case. Clearly the study on the ILC problem for linear time-variant systems is important. A convergent ILC scheme for linear time-variant discrete systems with a necessary and sufficient condition was recently developed in [17]. But the proposed ILC algorithm requires that the formed ILC systems always start their ILC cycles with zero initial error. This is a little more stringent for ILC of linear time-variant discrete systems.

The main objective of this paper is to extend the 2-D system theory ILC techniques for linear time-invariant discrete systems [9, 13] to the cases of linear time-variant discrete systems. The strategy largely depends on the convergent property of 2-D linear time-variant discrete systems with only one independent variable. Compared with the ILC algorithm for linear time-variant discrete systems in [17], our proposed ILC rules allow not only the ILC systems to have fixed initial errors but also deliver better performance. They can even drive the control error to zero for the desired output after only one learning iteration.

2 Preliminaries

To elaborate the ILC approaches for linear time-variant discrete systems, preliminaries are provided in this Section.

Lemma 1: For the Roessor-type model of 2-D linear time-variant discrete systems

$$\begin{pmatrix} \eta(t+1,k) \\ e(t,k+1) \end{pmatrix} = \begin{pmatrix} A1(t,k) & A2(t,k) \\ A3(t,k) & A4(t,k) \end{pmatrix} \begin{pmatrix} \eta(t,k) \\ e(t,k) \end{pmatrix} \quad (1)$$

where $\eta(t,k) \in R^{n_1}$, $e(t,k) \in R^{n_2}$, $A1(t,k) \in R^{n_1 \times n_1}$, $A2(t,k) \in R^{n_1 \times n_2}$, $A3(t,k) \in R^{n_2 \times n_1}$ and $A4(t,k) \in R^{n_2 \times n_2}$. Boundary conditions for (1) are given by

$$\eta(0,k) = 0 \text{ for } k = 0,1,2,\ldots \quad \text{and finite } e(t,0) \text{ for } t = 0,1,2,\ldots. \quad (2)$$

© IEE, 2005

IEE Proceedings online no. 20041125

doi: 10.1049/ip-cta:20041125

Paper first received 13th January and in revised form 27th May 2004

X.-D. Li and J.K.L. Ho are with the Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, 83 Tat Chee Avenue, Hong Kong

T.W.S. Chow is with the Department of Electronic Engineering, City University of Hong Kong

If

$$\sup_{t,k}\left\| \begin{pmatrix} 0 & 0 \\ A3(t,k) & A4(t,k) \end{pmatrix} \right\| < 1$$

($\|\cdot\|$ represents the matrix norm), then for each t, we have

$$\lim_{k\to\infty}\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = 0$$

Proof: The solution of (1) with the boundary condition (2) is given by [12]

$$\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = \sum_{i=0}^{t} T_{t,k}^{i,k}\begin{pmatrix}0\\ e(t-i,0)\end{pmatrix} \tag{3}$$

where the state transition matrix $T_{t,k}^{i,j}$ is defined as follows:

$$T_{t,k}^{i,j} = \begin{cases} I_{n_1+n_2} \text{ (the identity matrix of } n_1+n_2 \text{ order)} & \text{for } i=j=0 \\ A_{t-1,k}^{1,0}T_{t-1,k}^{i-1,j} + A_{t,k-1}^{0,1}T_{t,k-1}^{i,j-1} & \text{for } i\geq 0,\ j\geq 0\ (i+j\neq 0) \\ 0 \text{ (the zero matrix)} & \text{for } i<0 \text{ or } j<0 \text{ or } t<0 \text{ or } k<0 \end{cases}$$

and

$$A_{t,k}^{1,0} = \begin{pmatrix} A1(t,k) & A2(t,k) \\ 0 & 0 \end{pmatrix}, \quad A_{t,k}^{0,1} = \begin{pmatrix} 0 & 0 \\ A3(t,k) & A4(t,k) \end{pmatrix}$$

Simply following the definition of state transition matrix $T_{t,k}^{i,j}$, for $k\geq j>0$, obtains

$$T_{t,k}^{0,j} = A_{t,k-1}^{0,1}A_{t,k-2}^{0,1}\cdots A_{t,k-j}^{0,1} \tag{4}$$

and

$$T_{t,k}^{i,j} = \sum_{h=0}^{j} T_{t,k}^{0,h}A_{t-1,k-h}^{1,0}T_{t-1,k-h}^{i-1,j-h} \tag{5}$$

Proof of (5) is provided in the Appendix, Section 7. Let $\rho^{i,j} = \sup_{t,k}\|T_{t,k}^{i,j}\|$, $\rho^{1,0} = \sup_{t,k}\|A_{t,k}^{1,0}\|$ and $\rho^{0,1} = \sup_{t,k}\|A_{t,k}^{0,1}\|$, then from (4) and (5),

$$\rho^{i,j} \leq \sum_{h=0}^{j}(\rho^{0,1})^h\rho^{1,0}\rho^{i-1,j-h} = \sum_{h=0}^{\infty}(\rho^{0,1})^h\rho^{1,0}\rho^{i-1,j-h} \tag{6}$$

Therefore

$$\sum_{j=0}^{\infty}\rho^{i,j} \leq \left(\sum_{h=0}^{\infty}(\rho^{0,1})^h\rho^{1,0}\right)\sum_{j=0}^{\infty}\rho^{i-1,j} \tag{7}$$

Equation (7) is a recurrent inequality of $\sum_{j=0}^{\infty}\rho^{i,j}$ verse index i. Furthermore,

$$\sum_{j=0}^{\infty}\rho^{i,j} \leq \left(\sum_{h=0}^{\infty}(\rho^{0,1})^h\rho^{1,0}\right)^i\sum_{j=0}^{\infty}(\rho^{0,1})^j \tag{8}$$

According to the condition of lemma 1, $\rho^{0,1}<1$, thus $\sum_{h=0}^{\infty}(\rho^{0,1})^h = (1-\rho^{0,1})^{-1}$, and (8) can be written as

$$\sum_{j=0}^{\infty}\rho^{i,j} \leq (\rho^{1,0})^i(1-\rho^{0,1})^{-(i+1)} \tag{9}$$

For each i, $\sum_{j=0}^{\infty}\rho^{i,j}$ is convergent, therefore $\lim_{j\to\infty}\rho^{i,j}=0$. From (3)

$$\left\|\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix}\right\| \leq \sum_{i=0}^{t}\rho^{i,k}\left\|\begin{pmatrix}0\\ e(t-i,0)\end{pmatrix}\right\|, \text{ and } \left\|\begin{pmatrix}0\\ e(t-i,0)\end{pmatrix}\right\|$$

is finite, thus for each t we obtain

$$\lim_{k\to\infty}\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = 0.$$

Lemma 1 is proved.

Corollary 1: For the 2-D linear time-variant discrete system (1) with boundary conditions (2), if $\rho(A4(t,k))\leq p<1$, $t,k=0,1,2,\ldots$ ($\rho(\cdot)$ represents the spectral radius of matrix), then for each t

$$\lim_{k\to\infty}\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = 0$$

Proof: From the definition of $A_{t,k}^{0,1}$ we have $\rho(A_{t,k}^{0,1}) = \rho(A4(t,k))$. According to the relation between the matrix norm and the spectral radius of the matrix, for any given $\varepsilon>0$ there exists a kind of matrix norm such that

$$\left\|A_{t,k}^{0,1}\right\| \leq \rho\left(A_{t,k}^{0,1}\right) + \varepsilon = \rho(A4(t,k)) + \varepsilon \tag{10}$$

Therefore as $\rho(A4(t,k))\leq p<1$, $t,k=0,1,2,\ldots$, we take $0<\varepsilon<1-p$. As a result of (10), $\sup_{t,k}\|A_{t,k}^{0,1}\|<1$. From lemma 1,

$$\lim_{k\to\infty}\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = 0$$

for each t.

Remark: If the boundary conditions (2) in lemma 1 is $\eta(1,k)=0$ for $k=0,1,2,\ldots$ and finite $e(t,0)$ for $t=1,2,\ldots$, the solution of system (1) is

$$\begin{pmatrix}\eta(t,k)\\ e(t,k)\end{pmatrix} = \sum_{i=0}^{t-1} T_{t,k}^{i,k}\begin{pmatrix}0\\ e(t-i,0)\end{pmatrix}$$

From the proof of lemma 1 and corollary 1, the conclusions of both 1 are also correct for $t=1,2,\ldots$.

Lemma 1 and corollary 1 present sufficient conditions for convergence of the state vector in system (1) as only one independent variable k increases towards infinite. Compared with lemma 1, corollary 1 is sometimes more convenient in application.

3 ILC rules for linear time-variant discrete systems

Consider the ILC problem for linear time-variant discrete systems. A linear time-variant discrete system is represented by

$$x(t+1) = A(t)x(t) + B(t)u(t) \tag{11a}$$

$$y(t) = C(t)x(t) \tag{11b}$$

where $x(t)\in R^n$ is a state vector, $u(t)\in R^m$ is an input vector, $y(t)\in R^p$ is an output vector, and $A(t), B(t), C(t)$ are real time-variant matrices of appropriate dimensions that can be estimated. The following states the ILC problem that we are dealing with. Given system (11) with boundary condition $x(0)=x_0$ and reference output trajectory $y_r(t)$, $t=0,1,\ldots,N$, iteratively find an appropriate control input $u(t)$, $t=0,1,\ldots,N-1$, such that the system output follows the reference trajectory. Suppose that k denotes the learning iteration; a general ILC rule is given as

$$u(t,k+1) = u(t,k) + \Delta u(t,k), \quad t = 0,1,\ldots,N-1 \tag{12}$$

where Δu denotes modification of the control input. Sequentially system (11) can be modelled as the following 2-D time-variant form

$$x(t+1,k) = A(t)x(t,k) + B(t)u(t,k) \tag{13a}$$

$$y(t,k) = C(t)x(t,k) \tag{13b}$$

The boundary conditions for the 2-D system (13) are assumed to be

$$\begin{aligned} &x(0,k) = x_0 \quad \text{for } k = 0,1,2\ldots \text{ and} \\ &u(t,0) = u_0(t) \quad t = 0,1,\ldots,N-1 \end{aligned} \tag{14}$$

Our ILC objective is to find a suitable ILC rule (12) such that

$$\lim_{k\to\infty} y(t,k) = y_r(t) \text{ for } t = 1,2,\ldots,N$$

Denote

$$\eta(t,k) = x(t-1,k+1) - x(t-1,k) \tag{15}$$

and

$$e(t,k) = y_r(t) - y(t,k) \tag{16}$$

Using (12) and (13), we obtain for $t = 1,2,\ldots,N$

$$\begin{aligned} \eta(t+1,k) &= x(t,k+1) - x(t,k) \\ &= A(t-1)\eta(t,k) + B(t-1)\Delta u(t-1,k) \end{aligned} \tag{17}$$

$$\begin{aligned} &e(t,k+1) - e(t,k) \\ &\quad = -C(t)[x(t,k+1) - x(t,k)] \\ &\quad = -C(t)A(t-1)\eta(t,k) - C(t)B(t-1)\Delta u(t-1,k) \end{aligned} \tag{18}$$

Equations (17) and (18) may be rewritten in the compact form

$$\begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} = \begin{bmatrix} A(t-1) & 0 \\ -C(t)A(t-1) & I \end{bmatrix} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix} + \begin{bmatrix} B(t-1) \\ -C(t)B(t-1) \end{bmatrix} \Delta u(t-1,k) \tag{19}$$

Applying the following rule for the control calculation:

$$\Delta u(t,k) = K_1(t+1)\eta(t+1,k) + K_2(t+1)e(t+1,k) \tag{20}$$

one obtains for $t = 1,2,\ldots,N$ and $k \geq 0$ a control error system have the Roessor type model of 2-D linear time-variant discrete systems

$$\begin{aligned} &\begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} \\ &= \begin{bmatrix} A(t-1)+B(t-1)K_1(t) & B(t-1)K_2(t) \\ -C(t)A(t-1)-C(t)B(t-1)K_1(t) & I-C(t)B(t-1)K_2(t) \end{bmatrix} \\ &\quad \times \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix} \end{aligned} \tag{21}$$

The boundary conditions of the 2-D system (21) are $\eta(1,k) = 0$ for $k = 0,1,2,\ldots$ and $e(t,0)$ for $t = 1,2,\ldots,N$, which is finite. Let

$$\begin{aligned} &X(t) \\ &= \begin{bmatrix} 0 & 0 \\ -C(t)A(t-1)-C(t)B(t-1)K_1(t) & I-C(t)B(t-1)K_2(t) \end{bmatrix} \end{aligned} \tag{22}$$

The following theorem can be directly obtained from lemma 1.

Theorem 1: For a 2-D ILC model (13), if there exist matrixes $K_1(t)$ and $K_2(t)$ to make $\|X(t)\| < 1$, $t = 1,2,\ldots,N$, then the ILC rule

$$\begin{aligned} u(t,k+1) = {} & u(t,k) + K_1(t+1)[x(t,k+1) - x(t,k)] \\ & + K_2(t+1)e(t+1,k) \end{aligned} \tag{23}$$

can ensure $\lim_{k\to\infty} e(t,k) = 0$ for $t = 1,2,\ldots,N$.

Remark: The condition $\|X(t)\| < 1$, $t = 1,2,\ldots,N$, is robust with respect to small perturbations of the system parameters $A(t)$, $B(t)$ and $C(t)$. As a result, the ILC rule (23) is robust.

Theorem 1 provides an ILC approach for linear time-variant discrete systems. With the restriction of $\|X(t)\| < 1$, $t = 1,2,\ldots,N$, the matrices $K_1(t)$ and $K_2(t)$ in ILC rule (23) can be suitably selected to obtain some ILC rules with special properties. When $K_1(t) = -(C(t)B(t-1))^T[C(t)B(t-1)\,(C(t)B(t-1))^T]^{-1}C(t)\,A(t-1)$ and $K_2(t) = (C(t)\,B(t-1))^T[C(t)B(t-1)\,(C(t)B(t-1))^T]^{-1}$, it can be shown from (21) that always $e(t,1) = 0$ for $t = 1,2,\ldots,N$ no matter what $e(t,0)$ is. On the other hand, the matrix $(C(t)B(t-1))^T[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}$, which is the right inverse of matrix $C(t)B(t-1)$, exists IFF matrix $C(t)B(t-1)$ has full-row rank. Thus the following corollary has been proved.

Corollary 2: For a 2-D ILC model (13), if matrix $C(t)B(t-1)$ has full-row rank for $t = 1,2,\ldots,N$, the ILC rule (23) with $K_2(t) = (C(t)B(t-1))^T[(C(t)B(t-1)(C(t)B(t-1))^T)]^{-1}$ and $K_1(t) = -K_2(t)C(t)A(t-1)$ drives the control error to zero for the desired output at $t = 1,2,\ldots,N$ after only one learning iteration.

Undoubtedly, the ILC rule (23) with $K_2(t) = (C(t)B(t-1))^T[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}$ and $K_1(t) = -K_2(t)C(t)A(t-1)$ has the fastest iterative convergent rate but the current system state $x(t,k+1)$ is not available. Hence it is necessary to further explore the ILC rule (23). We can take a similar strategy with [9] to tackle this problem.

If the system matrices of system (11) are known, then from (13) and (23) we have

$$\begin{aligned} &x(t+1,k+1) \\ &\quad = A(t)x(t,k+1) + B(t)u(t,k+1) \\ &\quad = A(t)x(t,k+1) + B(t)[u(t,k) + K_1(t+1)(x(t,k+1) \\ &\qquad - x(t,k)) + K_2(t+1)e(t+1,k)] \\ &\quad = [A(t) + B(t)K_1(t+1)]x(t,k+1) + B(t)[u(t,k) \\ &\qquad - K_1(t+1)x(t,k) + K_2(t+1)e(t+1,k)]. \end{aligned} \tag{24}$$

Therefore we can apply the control

$$u^*(t) = u(t) - K_1(t+1)x(t) + K_2(t+1)e(t+1) \tag{25}$$

to the system

$$\hat{x}(t+1) = [A(t) + B(t)K_1(t+1)]\hat{x}(t) + B(t)u^*(t) \quad (26a)$$

$$\hat{y}(t) = C(t)\hat{x}(t) \quad (26b)$$

which is system (11) with a state feedback. Thus the output of the closed-loop system is identical with the reference output namely, $\hat{y}(t) = y_r(t)$, $t = 1, 2, \ldots, N$.

Theorem 2: The following ILC rule drives the control error to zero for the desired output at $t = 1, 2, \ldots, N$ after only one learning iteration if the condition of corollary 2 is satisfied:

$$u(t) \Leftarrow u^*(t) + K_1(t+1)\hat{x}(t) \quad (27)$$

where

$$K_2(t) = (C(t)B(t-1))^T[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}$$

$K_1(t) = -K_2(t)C(t)A(t-1)$, and $\hat{x}(t)$ is the state vector of system (26).

Proof: Using the control input (27) to system (11), we have

$$x(t+1) = A(t)x(t) + B(t)\lfloor u^*(t) + K_1(t+1)\hat{x}(t)\rfloor \quad (28a)$$

$$y(t) = C(t)x(t) \quad (28b)$$

It has been shown that the output of the system (26) is identical to the reference output when its input is computed by (25). Hence

$$\begin{aligned} e(t+1) &= \hat{y}(t+1) - y(t+1) \\ &= C(t)\lfloor (A(t) + B(t)K_1(t+1))\hat{x}(t) + B(t)u^*(t)\rfloor \\ &\quad - C(t)\lfloor A(t)x(t) + B(t)(u^*(t) + K_1(t+1)\hat{x}(t))\rfloor \\ &= C(t)A(t)[\hat{x}(t) - x(t)] \end{aligned} \quad (29)$$

From (26) and (28)

$$\hat{x}(t+1) - x(t+1) = A(t)[\hat{x}(t) - x(t)] \quad (30)$$

holds. Since both systems (11) and (26) have the same boundary conditions, $x(0) = \hat{x}(0) = x_0$, we get $x(t) - \hat{x}(t) = 0$ for $t = 0, 1, \ldots, N$. Furthermore, it can be concluded from (29) that $e(t+1) = 0$ for $t = 0, 1, \ldots, N-1$, namely $e(t) = 0$ for $t = 1, 2, \ldots, N$. This completes the proof.

Theorem 2 can be detailed as the following algorithm.

Algorithm 1

(i) At the time-step $t = 0, 1, \ldots, N$ the system matrices $A(t)$, $B(t)$, and $C(t)$, the reference output trajectory $y_r(t)$, any initial input sequences $u(t)$, and the initial state of system $x(0) = x_0$ are given.
(ii) Calculate the learning rule matrices $K_2(t) = (C(t)B(t-1))^T[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}$ and $K_1(t) = -K_2(t)C(t)A(t-1)$ for $t = 1, 2, \ldots, N$.
(iii) Measure $x(t)$ and $y(t)$ of system (11).
(iv) Use (25) to calculate $u^*(t)$, and apply $u^*(t)$ to system (26) and measure $\hat{x}(t)$.
(v) Apply control $u^*(t) + K_1(t+1)\hat{x}(t)$ to system (11).

From corollary 1 and system (21) it is clear that only if there exists a matrix $K_2(t)$ to make $\rho(I - C(t)B(t-1)K_2(t)) \le p < 1$, $t = 1, 2, \ldots, N$, then the ILC rule (23) can drive the error $e(t, k)$ to zero as k increases, and the convergence has nothing to do with the matrix $K_1(t)$. Therefore for simplicity we might as well let $K_1(t) = 0$ and $K(t) = K_2(t)$ in (23). A simpler form of ILC rule with only one parameter is presented in theorem 3.

Theorem 3: For a 2-D ILC model (13), if there exists a matrix $K(t)$ to make $\rho(I - C(t)B(t-1)K(t)) \le p < 1$, $t = 1, 2, \ldots, N$, the ILC rule

$$u(t, k+1) = u(t, k) + K(t+1)e(t+1, k) \quad (31)$$

can ensure $\lim_{k\to\infty} e(t, k) = 0$ for $t = 1, 2, \ldots, N$.

Remark: It is easy to show that a matrix $K(t)$ exists that makes $\rho(I - C(t)B(t-1)K(t)) \le p < 1$ for $t = 1, 2, \ldots, N$ only if matrix $C(t)B(t-1)$ has full-row rank. And in this case one can obtain the whole resulting error matrix $I - C(t)B(t-1)K(t)$ in the required form $\Phi(t)$ with $\rho(\Phi(t)) \le p < 1$, calculating

$$K(t) = (C(t)B(t-1))^T[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}(I - \Phi(t))$$

These ILC strategies can also be used for system identification of time-variant parameter. Consider the following dynamical system with a time-variant parameter $\theta(t)$:

$$x(t+1) = A(u(t), t)x(t) + B(u(t), t)\theta(t) \quad (32a)$$

$$y(t) = C(u(t), t)x(t) \quad (32b)$$

The identification problem for time-variant parameter $\theta(t)$ of system (32) can be described as: Measure output $y_d(t)$ of system (32) for a given input $u_d(t)$ at $t = 0, 1, \ldots, N-1$, then construct an identification device by using the pair of $\{u_d(t), y_d(t), t = 0, 1, \ldots, N\}$ to make its output $\hat{\theta}(t)$ as close to $\theta(t)$ at $t = 0, 1, \ldots, N-1$ as possible.

For a given input $u_d(t)$, $t = 0, 1, \ldots, N-1$, we obtain the following iterative identification model in a 2-D form from the dynamical system (32):

$$x(t+1) = A(t)x(t) + B(t)\theta(t) \quad (33a)$$

$$y(t) = C(t)x(t) \quad (33b)$$

where $A(t) = A(u_d(t), t)$, $B(t) = B(u_d(t), t)$, $C(t) = C(u_d(t), t)$. The desired output of system (33) is $y_d(t)$. Compared with system (11), the identification problem for time-variant parameter $\theta(t)$ of system (32) can be easily transferred to an ILC problem for linear time-variant discrete systems.

4 Simulation examples

4.1 Example 1

Consider the ILC problem for the following linear time-variant discrete system:

$$x(t+1) = \begin{bmatrix} -0.24 & 0.01 \\ 0.2\sin t + 0.04 & -0.35 \end{bmatrix} x(t) + \begin{bmatrix} 0.027t + 1 \\ 0.12 \end{bmatrix} u(t) \quad (34a)$$

$$y(t) = [\,0.45 \quad -0.001t\,]x(t) \quad (34b)$$

where $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and the matrix $C(t)B(t-1)$ has a full-row rank for $t = 1, 2, \ldots, 100$. The desired output is described by $y_r(t) = 1.5\sin 0.06t$, $t = 0, 1, \ldots, 100$. Using the ILC rule (31) we set the initial input sequence of ILC as $u(t, 0) = 0$, $t = 0, 1, \ldots, 99$, and let

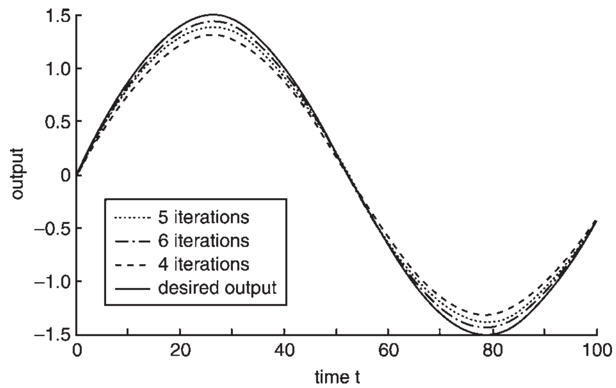

Fig. 1 *Example 1: Tracking performance of ILC system output at different time-steps and iterations using ILC rule (31)*

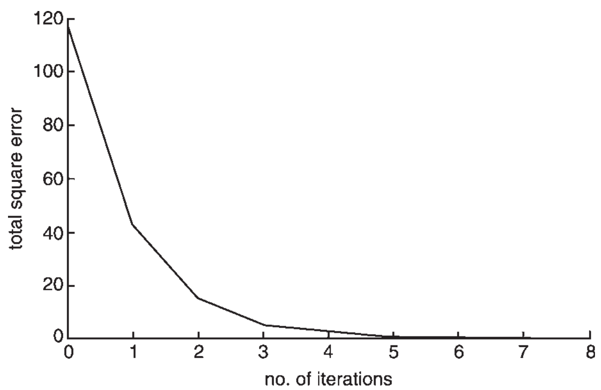

Fig. 2 *Example 1: Total square errors of tracking at different numbers of iteration with proposed ILC rule (31)*

$K(t) = 0.5(C(t)B(t-1))^T\,[C(t)B(t-1)(C(t)B(t-1))^T]^{-1}$. The accuracy of tracking is evaluated by the following total square error of tracking:

$$EE = \sum_{t=0}^{100} [y_r(t) - y(t)]^2$$

Figure 1 shows the tracking performance of the ILC system output at different time-steps and iterations. Also, Fig. 2 shows the total squared error of tracking when ILC rule (31) is iteratively executed at different times. From Figs. 1 and 2 notice that the convergence rate is high and the output is capable of approaching the desired trajectory accurately within few iterations.

4.2 Example 2

To demonstrate the ILC rule (27) (algorithm 1), consider the following linear time-variant discrete system:

$$x(t+1) = \begin{bmatrix} 0.18 & 0 \\ 0.02t & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} 0.1 \\ 0.01t+2 \end{bmatrix} u(t) \quad (35a)$$

$$y(t) = [-0.52 \quad 0]x(t) \quad (35b)$$

where $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and the matrix $C(t)B(t-1)$ has a full-row rank for $t = 1, 2, \ldots, 100$. The desired output is described by $y_r(t) = 0.6e^{0.02t}\sin 0.1t$, $t = 0, 1, \ldots, 100$. Using the ILC rule (27), the initial input sequence $u(t, 0)$, $t = 0, 1, \ldots, 99$ are randomised between 0 and 1. After a single iteration, the total squared error decreased to 0. To verify the robustness of the proposed ILC rule (27), assume that system (35) cannot be exactly known but is estimated as

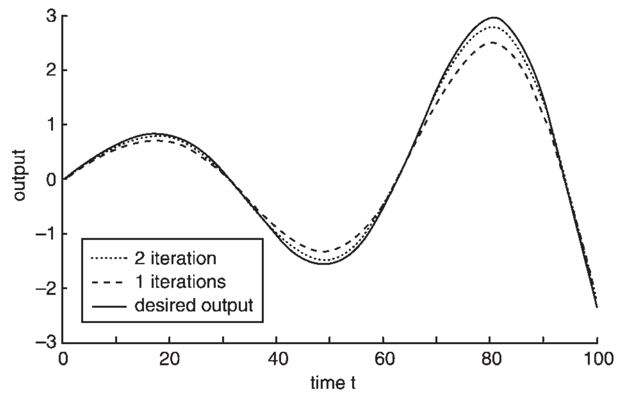

Fig. 3 *Example 2: Tracking performance of ILC system output at different time-steps and iterations using ILC rule (27)*

$$x(t+1) = \begin{bmatrix} 0.2 & 0 \\ 0.02t & -0.46 \end{bmatrix} x(t) + \begin{bmatrix} 0.12 \\ 0.01t+2 \end{bmatrix} u(t) \quad (36a)$$

$$y(t) = [-0.5 \quad 0]x(t) \quad (36b)$$

Applying ILC rule (27) and taking a few more iterations, the tracking error is driven to a very small level. Figure 3 shows the tracking performance of the ILC system output at different time steps as ILC rule (27) is iteratively executed one and two times, respectively. This illustrates that our proposed ILC algorithm is robust with respect to perturbations of system parameters.

5 Conclusions

Based on the study of convergent condition of 2-D linear time-variant discrete systems with only one independent variable, this paper has extended the current ILC techniques for linear time-invariant discrete systems to the cases of linear time-variant discrete systems. By simply reconstructing linear systems with time-variant parameters, the proposed 2-D system theory ILC strategy is also applicable to time-variant parameter identification for discrete dynamical systems.

6 Acknowledgment

The work described in this paper was partially supported by a grant from the Research Council of the Hong Kong SAR Region [RGC Project No. 9040601].

7 References

1 Arimoto, S., Kawamura, S., and Miyazaki, F.: 'Bettering operation of robots by learning', *J. Robot Syst.*, 1984, **1**, pp. 123–140

2 Fang, Y., and Chow, T.W.S.: '2-D analysis for iterative learning controller for discrete-time systems with variable initial conditions', *IEEE Trans. Circuits Syst., I Fundam. Theory Appl.*, 2003, **50**, (5), pp. 722–727

3 Wang, D.: 'A simple iterative learning controller for manipulators with flexible joints', *Automatica*, 1995, **31**, (9), pp. 1341–1344

4 de Luca, A., and Ulivi, G.: 'Iterative learning control of robots with elastic joints'. Proc. IEEE Conf. on Robotics and Automation, Nice, France, 1992, pp. 1920–1926

5 Manayathara, T., Tsao, T., Bentsman, J., and Ross, D: 'Rejection of unknown periodic load distributors in continuous steel casting process using learning repetitive control approach', *IEEE Trans. Control Syst. Technol.*, 1996, **4**, (3), pp. 259–265

6 Dijkstra, B., and Bosgra, O.: 'Noise suppression in buffer-state iterative learning control, applied to a high-precision wafer stage'. Proc. Int Conf. on Control Applications, Vol. 2, 18–20 Sept. 2002, pp. 998–1003
7 Moore, K.L.: 'Iterative learning control for deterministic systems' (Springer, New York, 1993)
8 Chow, T.W.S., and Fang, Y.: 'An iterative learning control method for continuous-time systems based on 2-D system theory', *IEEE Trans. Circuits Syst., I Fundam. Theory Appl.*, 1998, **45**, (4), pp. 683–689
9 Fang, Y., and Chow, T.W.S.: 'Iterative learning control of linear discrete-time multivariable system', *Automatica*, 1998, **34**, (11), pp. 1459–1462
10 Chow, T.W.S., and Xiao-Dong Li: 'A real-time learning control approach for nonlinear continuous-time systems using recurrent neural networks', *IEEE Trans. Ind. Electron.*, 2000, **47**, (2), pp. 478–486
11 Geng, Z., and Jamshidi, M.: 'Learning control system analysis and design based on 2-D system theory', *J. Intell. Robot. Syst.*, 1990, pp. 17–26
12 Geng, Z., Carroll, R., and Xies, J.: 'Two-dimensional model and algorithm analysis for a class of iterative learning control system', *Int. J. Control*, 1990, **52**, pp. 833–862
13 Kurek, J.E., and Zaremba, M.B.: 'Iterative learning control synthesis based on 2-D system theory', *IEEE Trans. Autom. Control*, 1993, **38**, pp. 121–125
14 Choi, J.Y., and Lee, J.S.: 'Adaptive iterative learning control of uncertain robotic systems', *IEE Proc., Control Theory Appl.*, 2000, **147**, (2), pp. 217–223
15 Sugie, T., and Ono, T.: 'An iterative learning control law for dynamic systems', *Automatica*, 1991, **27**, (4), p. 729
16 Hwang, D.H., Bien, Z., and Oh, S.R.: 'Iterative learning control method for discrete-time dynamic systems', *IEE Proc.-D*, 1991, **138**, (2), pp. 139–144
17 Huang, S.N., Tan, K.K., and Lee, T.H.: 'Necessary and sufficient condition for convergence of iterative learning algorithm', *Automatica*, 2002, **38**, pp. 1257–1260

8 Appendix: Proof of (5)

From the definition of state transition matrix $T^{i,j}_{t,k}$ and (4), for $t \geq i \geq 0$ and $k \geq j > 0$, we have

$$\begin{aligned}
T^{i,j}_{t,k} &= A^{1,0}_{t-1,k}T^{i-1,j}_{t-1,k} + A^{0,1}_{t,k-1}T^{i,j-1}_{t,k-1} \\
&= A^{1,0}_{t-1,k}T^{i-1,j}_{t-1,k} + A^{0,1}_{t,k-1}\left(A^{1,0}_{t-1,k-1}T^{i-1,j-1}_{t-1,k-1} + A^{0,1}_{t,k-2}T^{i,j-2}_{t,k-2}\right) \\
&= A^{1,0}_{t-1,k}T^{i-1,j}_{t-1,k} + A^{0,1}_{t,k-1}A^{1,0}_{t-1,k-1}T^{i-1,j-1}_{t-1,k-1} \\
&\quad + T^{0,2}_{t,k}\left(A^{1,0}_{t-1,k-2}T^{i-1,j-2}_{t-1,k-2} + A^{0,1}_{t,k-3}T^{i,j-3}_{t,k-3}\right) \\
&= A^{1,0}_{t-1,k}T^{i-1,j}_{t-1,k} + T^{0,1}_{t,k}A^{1,0}_{t-1,k-1}T^{i-1,j-1}_{t-1,k-1} + T^{0,2}_{t,k}A^{1,0}_{t-1,k-2}T^{i-1,j-2}_{t-1,k-2} \\
&\quad + T^{0,3}_{t,k}\left(A^{1,0}_{t-1,k-3}T^{i-1,j-3}_{t-1,k-3} + A^{0,1}_{t,k-4}T^{i,j-4}_{t,k-4}\right) \\
&= \sum_{h=0}^{j} T^{0,h}_{t,k}A^{1,0}_{t-1,k-h}T^{i-1,j-h}_{t-1,k-h}
\end{aligned}$$

The proof of (5) is complete.