Undersea Cable Path Planning with Curvature Constraints

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ABSTRACT

Undersea optical fiber cables that span vast distances are integral to the Internet's infrastructure. Manual path planning of such cables is an arduous task. The Fast Marching Method (FMM), a precise numerical approach capable of solving the Eikonal equation, offers a viable and efficient alternative by optimizing the path between source and destination, taking account of a summary objective function of costs and risk factors. However, it often neglects the crucial curvature constraints arising from the flexural rigidity of cables and maneuvering capabilities of the laying vessels in real-world scenarios. To address real-world cable path planning, our work incorporates these constraints. We propose modifying our previous FMM-based work to generate paths by systematically expanding forbidden regions and replacing sharp angular changes with circular arcs of a predefined radius to ensure compliance with the curvature constraints. By factoring curvature in cable system design, we ensure robust, real-world applicability of cable path design methods. While this modification may result in a departure from the FMM's optimally calculated path, potentially compromising its cost-effectiveness or risk minimization, the adjusted route adheres to essential curvature constraints and the structural integrity of cables.

Keywords: Undersea optical fiber cables, Fast Marching Method, curvature constraint, path optimization

1. INTRODUCTION

Undersea optical fiber cables, responsible for transmitting over 99% of global voice and data traffic, are indispensable to our global information infrastructure. The intricate process of planning these undersea cable paths entails determining the most efficient path between two or more specified locations, taking into account a variety of hydrogeological and anthropogenic factors such as earthquakes, volcanic eruptions, water depth, seabed slope, sediment hardness, marine protected areas, and activities related to fishing and anchoring [1]–[5].

Currently, the industry standard involves predominantly manual path planning during much of the design process, although tools like MakaiPlan are commonly employed for assistance [6]. However, MakaiPlan does not automate the optimization of cable routes; instead, it calculates the shortest path along the Great Circle as a series of Rhumb lines [6]. This initially calculated path is manually adjusted afterward to mitigate costs associated with various risks. Given the extensive lengths of long-haul cables, which can span thousands of kilometers, the predominance of manual path planning without the advantages of scalable, automated methods might not effectively balance cost against risk.

Scholarly work explores sophisticated models and analyses concerning cable systems and networks. Neumayer *et al.* [7] developed a network model to pinpoint major disruption sites and mitigate disaster impacts. In [8], a risk analysis of network fractures due to seismic activities was conducted, focusing on enhancing network resilience within budget constraints. Zhao *et al.* [9] applied Dijkstra's algorithm (DA) to grid-based path planning to minimize total costs by considering cable length and earthquake resistance. Wang *et al.* [10] addressed a multi-objective optimization problem of cable path planning, simplifying it to an Eikonal equation, which they solved using the Fast Marching Method (FMM) [11]. The FMM, having polynomial complexity similar to DA [11], offers the advantage of not being restricted to grid points at each step and ensures an optimal path on a given triangulated manifold, as demonstrated both theoretically and numerically [3], [11], [12].

Further exploration of FMM-based optimization techniques for undersea cable path planning can be found extensively in [13]–[18]. Wang *et al.* [13] optimized the path planning of undersea cable systems, specifically considering a trunk-and-branch tree topology on the Earth's surface. The research in [14] and [15] focuses on latency constraints across various node pairs within undersea cable networks, while [16] explores cost implications related to branching units and cable landing stations. The study in [17] employs parallel techniques and multi-resolution analysis to enhance computation speed, scalability, and quality of cable path planning. Additionally, Wang *et al.* [18] designed cable systems under bandwidth capacity constraints to optimize network requirements and cost efficiency.

Despite extensive research on point-to-point cable path planning [1]–[5], [9], [10], [12] and cable system optimization [7], [8], [13]–[18], a critical aspect often overlooked is cable bending fitness. When laying cables, curvatures are inevitable due to the need to circumvent obstacles, necessitating that the curvature radius does not surpass the cable's bending radius limitations [19]. The conventional path planning methods typically require manual post-process adjustments to smooth the route, which can lead to suboptimal paths. Addressing this issue, Blaise and Spinewine [20] introduced a method that incorporates angular discretization with a variable resolution, optimizing the least-cost cable path, where DA was used instead of FMM. In this paper, we propose a method that effectively addresses bending challenges in undersea cable path planning. By expanding forbidden regions and smoothing sharp turns into circular arcs of predefined radii, our method complies with curvature constraints, ensuring robust and practical routing solutions.

2. CABLE CURVING IMPLEMENTATION

2.1 Modeling

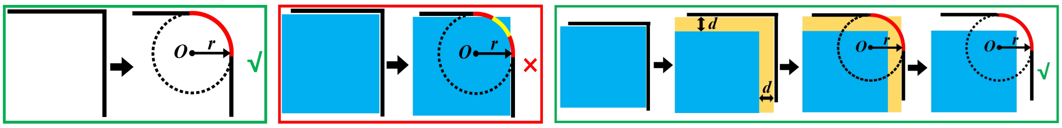
We use a triangulated piecewise-linear 2D manifold $\mathbb{M}$ in a 3D domain $\mathbb{R}$ to represent the Earth's surface. Specifically, every point on $\mathbb{M}$ is assigned to a three-dimensional coordinate (x, y, z) where x and y are the longitude and latitude of this point, and $z = \xi(x, y)$ is the elevation of point (x, y). More details can be found in [3], [4].

We employ a life-cycle cost model that incorporates multiple considerations that impact both the costs and the survivability of undersea cables, providing a thorough representation of their cost-efficiency and longevity. Assuming a set of K design considerations associated with anthropogenic activities or hydrogeological risks, which could impact the undersea cable's durability and economic efficiency, we introduce a non-negative set $\mathcal{W} = \{w_1, w_2, \ldots, w_k\}$. Here, w_k denotes the significance of the k^{th} design factor in the cable path planning process. For a point $X = (x, y, z) \in \mathbb{M}$, we define $C(X) = \sum_{k=1}^{K} w_k c_k(X)$, representing the life-cycle cost per unit length for the cable at location X. More details can be found in [3], [4].

2.2 Curvature constraints

In this subsection, we explore the incorporation of critical curvature constraints stemming from the bending stiffness of the cables and the maneuvering capabilities of the laying vessel in realistic scenarios. Specifically, we consider two distinct cases:

1) For sharp angles not near prohibited areas, as shown in Fig. 1(a), we replace the sharp corner with a circular arc of a predefined radius r. The red arc in Fig. 1(a) indicates the new path segment. This direct substitution is permissible as the modified path does not traverse any prohibited zones.
2) For sharp angles near prohibited areas, as depicted in Fig. 1(b) and Fig. 1(c), direct arc replacement is infeasible since the circular arc may cross prohibited areas (yellow in Fig. 1(b)). To address this, we gradually inflate the prohibited area (blue) until an appropriate inflation distance d is obtained, creating an expanded buffer (yellow). This adjustment is illustrated in Fig. 1(c). We then calculate a new path under the inflated prohibited area constraint and apply the circular arc transformation with radius r for the new path. Finally, we revert the prohibited area to its original size, ensuring that the replacement of sharp angles with predefined bending radius arcs adheres to the curvature constraints and guarantees that the path does not cross prohibited zones.



(a) Replacement of a sharp angle with a circular arc where no prohibited areas are present. (b) Infeasibility of direct sharp angle replacement due to encroachment into a prohibited area. (c) Implementing prohibited area inflation to facilitate safe circular arc substitution for sharp angles.

Figure 1: Circular arc substitution for sharp angles.

2.3 Problem formulation

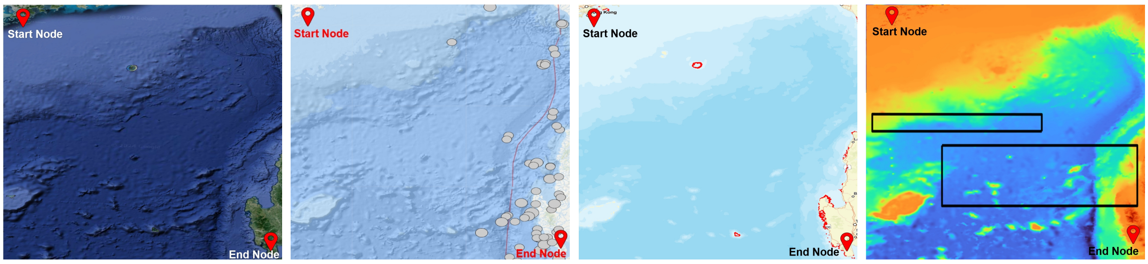
Our objective is to determine the most cost-effective undersea cable path that minimizes the total life-cycle cost between two specified points, A and B. We define our objective function as follows:

$$\phi(S) = \min_{\alpha} \mathbb{C}(\alpha) = \min \int_{l(0)}^{l(\alpha)} C(X(s))ds = \min \int_{l(0)}^{l(\alpha)} \sum_{k=1}^{K} w_k c_k (X(s))\, ds, s.t.\ \alpha(0) = A, \alpha(l) = B, \quad (1)$$

where $l(\alpha)$ represents the total length of the cable path α. Regarding Equation 1, it can be reformulated into a nonlinear partial differential equation, specifically the Eikonal equation, which can be efficiently solved using FMM. Further details are available in [4].

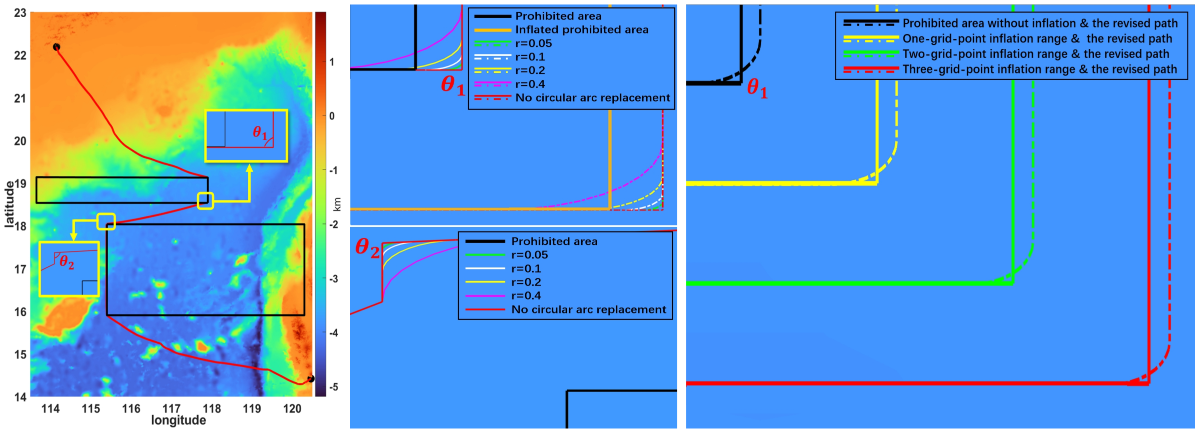
3. APPLICATION

Here we demonstrate our method in region $\mathbb{D}$, covering the South China Sea from $(14°00'00''N, 113°00'00''E)$ to $(23°00'00''N, 120°00'00''E)$, as shown in Fig. 2. Our goal is to design a cable path from Hong Kong SAR at $(22°10'52''N, 114°08'58''E)$ to Bayan ng Mariveles at $(14°25'03''N, 120°27'54''E)$, highlighted by red pins in Fig. 2. We consider six design considerations: basic construction cost, geological hazards, seabed slope, water depth, anthropological hazards, and protected areas. For detailed cost calculation and weights used, see [4].



(a) Geography in Google Earth. (b) Earthquakes (represented by gray circles). Source: United States Geological Survey. (c) Coral reefs (red areas). Source: World Conservation Monitoring Centre. (d) Prohibited areas (represented by two black rectangles).

Figure 2: Maps of region $\mathbb{D}$.



(a) Initial cable path with sharp turns θ_1 and θ_2. (b) Path modifications using circular arcs replacement with different radii and expanded prohibited areas. (c) Path modifications using circular arcs replacement with $r = 0.4$ km and different expanded prohibited areas.

Figure 3: Cable path planning with curvature constraints.

Fig. 3(a) illustrates that under the influence of two prohibited areas, the FMM-generated cable path features two sharp turns, θ_1 and θ_2 (see the magnified section in Fig. 3(a)). In Fig. 3(b), attempts to replace these turns with circular arcs of different bending radii (denoted as r) reveal that revised paths with $r = 0.4$ km and $r = 0.2$ km intersect the prohibited area (marked by black lines). To address this, we inflate the prohibited area by one grid point (indicated by orange lines) before replacing the turns, ensuring that the modified paths for all values of r do not cross the original black-bordered prohibited areas. For sharp turn θ_2 in θ_2, direct replacement of the path segment with circular arcs with all tested radii did not intersect with the prohibited zones, negating the need for expansion. In Fig. 3(c), we investigate how path modifications vary with different expansion ranges, maintaining a bending radius of $r = 0.4$ km for the circular arc replacement. It is observed that as the expansion range increases, although the path deviates further from the initially optimal FMM-generated path, it reduces the likelihood of the modified path with circular arcs intersecting the original prohibited areas, thus allowing for the use of larger bending radii to replace the sharp turn θ_2. Table I presents detailed numerical results for the paths in Fig. 3 where d indicates the expansion range of prohibited areas (e.g., $d = 1$ signifying a one-grid-point inflation).

4. CONCLUSIONS

In this study, we proposed a new method for undersea optical fiber cable path planning that effectively incorporates curvature constraints to meet real-world challenges such as cable bending stiffness and vessel maneuverability. By systematically expanding restricted areas and replacing sharp turns with predefined-radius circular arcs, our method not only ensures the practicality of the cable path planning but also enhances the reliability of the cable structure. Although this method might slightly deviate from the optimal path calculated by FMM, resulting in a minimal increase in cable length and total life-cycle cost, it offers a more practical solution for cable laying and ensuring the survivability of undersea cables.

	$d = 0$, $r = 0$	$d = 1$				$r = 0.4$ km	
		$r = 0.05$ km	$r = 0.1$ km	$r = 0.2$ km	$r = 0.4$ km	$d = 2$	$d = 3$
Length (km)	1707.4550	1710.4009	1710.3627	1710.2863	1710.1335	1713.2204	1715.8033
Life-cycle cost (million $)	47.8090	47.8912	47.8901	47.8880	47.8837	47.9701	48.0425

TABLE I: Numerical results of the paths in Fig. 3.

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REFERENCES

[1] C. Cao, Z. Wang, M. Zukerman, J. Manton, A. Bensoussan, and Y. Wang, “Optimal cable laying across an earthquake fault line considering elliptical failures,” *IEEE Trans. Reliab.*, vol. 65, no. 3, pp. 1536–1550, Sep. 2016.

[2] D. L. Msongaleli, F. Dikbiyik, M. Zukerman, and B. Mukherjee, “Disaster-aware submarine fiber-optic cable deployment for mesh networks,” *J. Light. Technol.*, vol. 34, no. 18, pp. 4293–4303, Sep. 2016.

[3] Q. Wang *et al.*, “Cost-effective path planning for submarine cable network extension,” *IEEE Access*, vol. 7, pp. 61 883–61 895, May 2019.

[4] X. Wang, Z. Wang, E. Tahchi, and M. Zukerman, “Submarine cable path planning based on weight selection of design considerations,” *IEEE Access*, vol. 9, pp. 123 847–123 860, Sep. 2021.

[5] X. Wang, Z. Wang, T. Wang, and M. Zukerman, “Designing cost-effective and reliable submarine communications cable path: Lessons from the Tonga volcano disaster,” *IEEE Commun. Mag.*, vol. 61, no. 7, pp. 179–185, Jul. 2023.

[6] Makai Ocean Engineering Inc., “Makaiplan: Cable route engineering software.” [Online]. Available: https://www.makai.com/brochures/MakaiPlan.pdf

[7] S. Neumayer, G. Zussman, R. Cohen, and E. Modiano, “Assessing the vulnerability of the fiber infrastructure to disasters,” *IEEE/ACM Trans. Netw.*, vol. 19, no. 6, pp. 1610–1623, 2011.

[8] P. N. Tran and H. Saito, “Geographical route design of physical networks using earthquake risk information,” *IEEE Commun. Mag.*, vol. 54, no. 7, pp. 131–137, 2016.

[9] M. Zhao, T. W. Chow, P. Tang, Z. Wang, J. Guo, and M. Zukerman, “Route selection for cabling considering cost minimization and earthquake survivability via a semi-supervised probabilistic model,” *IEEE Trans. Industr. Inform.*, vol. 13, no. 2, pp. 502–511, Apr. 2016.

[10] Z. Wang *et al.*, “Multiobjective path optimization for critical infrastructure links with consideration to seismic resilience,” *Comput.-Aided Civ. Infrastruct. Eng.*, vol. 32, no. 10, pp. 836–855, Oct. 2017.

[11] J. A. Sethian, “Fast marching methods,” *SIAM review*, vol. 41, no. 2, pp. 199–235, Jun. 1999.

[12] Q. Wang *et al.*, “Path planning of submarine cables,” in *Proc. ICTON 2019*, Angers, France, Jul. 2019.

[13] Z. Wang, Q. Wang, B. Moran, and M. Zukerman, “Optimal submarine cable path planning and trunk-and-branch tree network topology design,” *IEEE/ACM Trans. Netw.*, vol. 28, no. 4, pp. 1562–1572, Aug. 2020.

[14] T. Wang, X. Wang, Z. Wang, C. Guo, W. Moran, and M. Zukerman, “Optimal tree topology for a submarine cable network with constrained internodal latency,” *J. Light. Technol.*, vol. 39, no. 9, pp. 2673–2683, Feb. 2021.

[15] T. Wang, Z. Wang, B. Moran, X. Wang, C. Guo, and M. Zukerman, “Latency-aware optimization of submarine communication cable systems with trunk-and-branch topologies,” *J. Light. Technol.*, vol. 40, no. 17, pp. 5825–5841, Sep. 2022.

[16] T. Wang, Z. Wang, B. Moran, and M. Zukerman, “Submarine cable network design for regional connectivity,” *IEEE/ACM Trans. Netw.*, vol. 30, no. 6, pp. 2480–2492, 2022.

[17] X. Wang, G. Cheng, Z. Wang, and M. Zukerman, “A research on submarine cable path planning,” in *Proc. Eighth Symposium on Novel Photoelectronic Detection Technology and Applications*, vol. 12169. SPIE, 2022, pp. 512–519.

[18] T. Wang, B. Moran, and M. Zukerman, “Capacity-aware undersea cable system design,” *J. Light. Technol.*, vol. 42, no. 8, pp. 2648–2656, Apr. 2024.

[19] P. Maioli, “Bending stiffness of submarine cables,” in *Proc. 9th International Conference on Insulated Power Cables (Jicable’15)*, Jun. 2015.

[20] S. Blaise and B. Spinewine, “Efficient curvature-constrained least cost route optimization on parallel architectures,” *Eng. Comput.*, pp. 1–17, Mar. 2021.