Performance Modelling of Diversity Coded Path Protection in OBS/OPS Networks

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Abstract—In bufferless optical burst/packet switched (OBS/OPS) networks, data (bursts or packets) may be lost due to contention or equipment failure. Diversity coding-based path protection schemes can be used to protect data from a single-trunk-failure with potentially more efficient resource utilization compared to dedicated (e.g. 1+1) path protection schemes and help reduce the burst/packet loss ratio. This paper provides a scalable and accurate burst/packet loss ratio approximation based on the Erlang Fixed-Point Approximation for networks that employ protection based on diversity coding and for networks that employ protection based on both diversity coding and 1+1 path protection. We use discrete event simulations to assess the accuracy of the approximation based on a wide range of and scenarios in a 10-node circular lattice and NSFNet networks. Further, we discuss the effect of parameter settings and the effect of the choice of the wavelength selection method on the accuracy of the approximation. We consider scenarios without wavelength conversion and with full wavelength conversion and two types of users: premium and regular. The premium users, typically of mission critical services, receive protection while the regular users do not. We compare diversity coding and 1+1 path protection in a 10-node circular lattice network. The results show that for this example network, for the premium users and under low traffic load, the burst/packet loss ratio in OBS/OPS networks is lower for diversity coding compared to 1+1 path protection. However, under heavy traffic load, we observe a lower loss ratio for the case of 1+1 path protection.

Index Terms—burst loss ratio, optical burst switching, path protection, diversity coding, Erlang Fixed-Point Approximation.

I. INTRODUCTION

With the growth of the Internet, the use of mission-critical services that have high Quality of Service (QoS) requirements has also been growing [1]. Such services must meet stringent resilience requirement. In particular, they still need to meet their QoS requirements even if their network face cascading or simultaneous failures that may be the result of natural or human made causes [2]. Dedicated path protection (e.g. 1+1 path protection) is one of the most popular survivable routing techniques as it provides instantaneous recovery from failures. However, since 1+1 path protection transmits two identical copies of data through two disjoint paths (working and protection paths) simultaneously, the protection overhead is 100%.

We consider a bufferless WDM optical network that can be either burst switched or packet switched. In such networks, data (either bursts or packets) is sent by a source to its destinations without end-to-end capacity reservation of required resources prior to sending the data. Accordingly, bursts/packets may be lost due to contention for a limited capacity. As mentioned above, loss of bursts/packets can also be caused because of equipment failures. Accordingly, in OBS networks, an important performance measure is the burst loss ratio (BLR) which is the ratio of the number of bursts dropped (or dumped) to the total number of bursts generated and sent in the entire OBS network. In Optical Packet Switched (OPS) networks, an equivalent concept to BLR is the packet loss ratio. However, to simplify the exposition, as in [3], we will use the inclusive term burst to mean both a burst in an OBS network, and/or a packet in an OPS network. Similarly, we will also use the inclusive term BLR to include also packet loss ratio.

The BLR is a useful measure for network design and engineering, addressing issues including traffic balancing, congestion control, and network dimensioning [4]. Given the need to meet QoS requirements, the aim is to design OBS networks with sufficient capacity to meet the demand so that the BLR meets QoS requirements. However, even if a network is well dimensioned, link failures may still cause unacceptable BLR for mission-critical services and this is why such premium services require special protection.

This paper provides, for the first time, a comprehensive study on BLR approximation and efficiency of OBS/OPS networks with dynamic traffic demands that use diversity coding for burst protection. In particular, we consider a bufferless OBS network with two types of users called premium and regular. The network employs diversity coding [5–7] for its premium users. Diversity coding is applied to reduce the cost of protection, while still providing instantaneous recovery. The basic idea is splitting a premium burst into N sub-bursts, where each sub-burst is transmitted to the destination node on N edge-disjoint working paths. In addition, at the same time, a coded burst (generated by XORing the N, N ≥ 2 sub-bursts) is transmitted to the destination node, through an additional edge-disjoint path that we call protection path. As shown in Fig. 1 with N = 2, if one of the working paths fails, the destination node can reconstruct the lost sub-burst on the failed path by combining the coded burst and the other received sub-bursts.

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We note that to implement diversity coding in a bufferless OPS networks, we can consider an OBS network where the burst size is equal to \(nN\) for some small integer \(n\). The case, \(n = 1\) is also applicable in which case \(N\) packets are aggregated into bursts and every sub-burst comprises a single packet.

A premium burst may comprise packets of different premium users or may comprise packets from a single premium user (e.g., a data center). The added redundancy in diversity coding helps the premium bursts combatting both link failure and burst contentions. Regular bursts are composed of packets from a single or multiple regular users and are transmitted through the shortest path to their destinations. We consider the full wavelength conversion case as well as the case where no wavelength conversion is available.

Given the requirement of \(N \geq 2\) means that, diversity coding cannot be applied in cases where three edge disjoint paths between origin and destination are not available, for example, if the source or destination have degree of less than three. In such cases, 1+1 protection may be possible. Of course, if a node has degree of one, neither protection scheme is available. Therefore, it is desirable to have a scalable method for BLR approximation for an OBS/OPS network that uses alternative protection schemes for premium bursts between different Origin and Destination (OD) pairs.

![Realization of diversity coding.](image)

The contributions of this paper are as follows.

1) We provide a scalable and accurate BLR approximation for OBS networks that use the RWS scheme for wavelength selection on its first trunk. We provide, for the first time, BLR approximations for scenarios where only diversity coding is used for protection of the premium bursts, as well as where for some OD pairs transmitting premium bursts protection based on diversity coding is used and for others 1+1 protection is used. Our approximations are based on the Erlang Fixed-Point Approximation (EFPA) [9]. We validate the accuracy of our approximations using discrete event simulations of OBS networks with the RWS scheme on a 10-node circular lattice network as well as on a 14-node NSFNet network for a range of traffic load values. Scalable and accurate approximations for performance measures are important because simulations are not scalable to realistic size networks.

2) Recognizing that it is difficult to provide general conclusions on the comparison between diversity coding and 1+1 path protection that apply to all networks, we illustrate such a comparison based on a 10-node circular lattice network. The results indicate that in the case of the 10-node circular lattice network, with low BLR traffic load, diversity coding has lower probability than 1+1 protection; however, with large traffic load, 1+1 protection works better for the premium users.

3) We provide numerical results to illustrate the discrepancy between the BLR obtained by our approximation and that obtained by simulations of networks with different wavelength selection methods. We also provide a detailed discussion on the effect of parameters, such as the number of channels per trunk, on the accuracy of our proposed approximation method.

The remainder of the paper is organized as follows. Information on related work is provided in Section II. Section III provides a description of the network modelling and discusses the assumptions made. In Section IV, we provide details on our methods of BLR approximations. In Section V, numerical results for the 10-node circular lattice network are provided. Finally, the paper is concluded in Section VI.

II. RELATED WORK

There are generally two approaches to provide survivability in optical networks: protection and restoration [10]. The authors of [11], [12] have considered protection techniques that are common for optical circuit switched (OCS) networks, where they reserve resources in advance, and simultaneously transmit data on disjoint working and protection paths. In OBS networks, the protection approaches are different [13]. While in both OCS and OBS networks, disjoint working and protection paths are established in advance. In OBS networks, network resources are not reserved in advance but are reserved hop-by-hop. Research publications on protection technologies can be classified into four categories: path protection, link protection, segment protection and node protection.
Path protection can be further classified into dedicated path protection and shared path protection [14], [15]. Dedicated path protection uses the working and protection paths together to send traffic while shared path protection uses the protection path only if the primary path fails. Since dedicated path protection has less recovery time than shared path protection, we focus on dedicated path protection techniques in this paper. Early dedicated path protection technique, namely, 1+1 automatic protection switching (APS) has been widely used in real networks. In 1+1 APS, a primary path is protected by an edge-disjoint protection path. The protection path carries the same traffic as the primary path. Diversity coding [5–7] was proposed to allocate the bandwidth resources more efficiently for path protection networks.

The idea of using diversity coding for protection against link failure dates back to the early nineties [16], [17]: N working paths are protected by a separate N+1sr protection path which carries the modulo-2 sum, or XOR combination, of subflows in each of the working paths. (The early work on diversity routing did not consider OBS, so the term “subflows” used in the early work is equivalent to the term of sub-bursts used in this paper.) If all of the N+1 paths are edge-disjoint, then a recovery from any single link failure can be achieved by applying the modulo-2 sum over the bits obtained from the unfailed links. Assume that the bits of each of the working paths are b1, b2, b3, . . . , bN and the checksum of the primary bits is [18]

\[ c_1 = b_1 \oplus b_2 \oplus \ldots \oplus b_N = \bigoplus_{j=1}^{N} b_j. \]

In the destination node, if a failure is detected, the destination node applies modulo-2 sum to the remaining N working paths and extracts the failed bit as [18]

\[ c_1 \oplus \bigoplus_{j=1, j \neq i}^{N} b_j = b_i \oplus \bigoplus_{j=1, j \neq i}^{N} (b_j \oplus b_j) = b_i, \]

where i is the failed working path. In diversity coding, synchronization is needed at the destination node between the working and protection paths. Ayanoglu et al [16] explained the need for synchronization and how to implement it.

The capacity utilization of diversity coding is dependent on the topology of the network where it is deployed. As shown in [6] and [19], the capacity utilization of diversity coding correlates positively with the average node degree of the network topology. That is, diversity coding has better capacity utilization in highly connected networks compared to lightly connected networks. This is because an increased average node degree means that there are on average more disjoint paths between node pairs in the network, which in turn means that more data packets can share the same redundancy packet. Moreover, Avci et al. [18] show that the capacity requirement, by using a diversity coding protection scheme, is lower in highly connected networks compared to sparsely connected networks. For networks with average node degree of 2 and above, the capacity utilization of diversity coding is potentially higher than that of 1+1 protection. When the node degree between a node pair is exactly 2, diversity coding is similar to 1+1 protection regarding capacity utilization. This is because there are no extra paths to share the redundancy data when there are only 2 disjoint paths between a node pair. Among the cases studied in [20], the best results for diversity coding with respect to capacity utilization were achieved for the torus topology with node degree 4 (which was the topology with the highest node degree considered in [20]). Even better results in networks with diversity coding are expected for network topologies with a higher node degree than 4.

Avcı et al. [21] developed a diversity coding algorithm for networks with arbitrary topology to minimize spare capacity and demonstrated overall benefit in terms of Quality of Recovery, which considers the two objectives of spare capacity percentages, as well as worst case recovery time. Jose et al. [22] optimized the traffic splitting associated with diversity coding by choosing the N+1 disjoint paths with the aim to minimize cost through efficient utilization of network resources. Pašič et al. [23] proposed a polynomial-time algorithm in sparse topologies to find minimum cost survivable routings in networks with diversity coding. Unlike [23], the present paper focuses on BLR approximation in an OBS network that uses diversity coding. Note that our BLR approximation is versatile and can apply to any routing choices including the one obtained by [23]. Later, Pašič et al. [24] investigated an optimal structure of diversity coding-based survivable routing and the effect of QoS and differential delay bounds on the solution cost. Muktadir and Oki [25] proposed a protection scheme based on diversity coding, which can be used to protect data from multiple link failures.

Note that in our BLR approximation we use shortest path between any SD.

Comparisons of BLR in OBS networks under scenarios with full wavelength conversion and without wavelength conversion, but without consideration to diversity coding or protection have been discussed in [26]. Note also that [8] was probably the first to observe for OCS networks that the benefit in terms of efficiency of wavelength conversion reduces as the number of optical fibers increases. This effect has its root from the scaling property, and related increased efficiency with increased traffic, of the Erlang B model [27], which applies to OBS networks as well as OCS networks. As mentioned in the introduction, the authors of [8] also considered a least loaded method (LL) of wavelength selection. However, as the work of [8] applies to OCS networks, they consider the entire path in their LL approach. As we deal with OBS networks, which are distributed in nature, in our LLWS scheme we only consider the first trunk. Interestingly, even if we consider only the first trunk, we observe significant benefit for the LLWS scheme.

The well-known EFPA method, which was first proposed in 1964 [28], has been extensively studied and used for approximating the loss ratio. The approximations provided by EFPA have been found to be accurate in traditional application areas such as large communication networks [9], [27], [29] and also in optical networks with different topologies, switching strategies, routing algorithms and bandwidth assignment methods [3], [30–33]. EFPA relies on the assumption that the offered load of each source-destination (SD) pair to each trunk follows an independent Poisson process. To calculate
the network BLR, we start with the calculation of the total offered load to each of the trunks in the networks, then use Erlang-B formula to calculate BLR of each trunk (trunk-BLR) individually. With the independent assumption, the probability that a burst successfully passes through all the trunks in a given path and arrives at the destination node is the product of the values of (1 - trunk-BLR) of each trunk along this path; then, the path-BLR is obtained by one minus this product. The network BLR is the weighted average of all the path-BLR values, with the weight for a particular path-BLR value equal to the ratio of the offered load to that path to the total offered load to all the paths. This method of calculating the network BLR only applies to unprotected networks. In the present paper, we extend this approach to protected networks based on diversity coding and hybrid of diversity coding and 1+1 path protection.

The paper closest to the present paper in terms of the method of EFPA application is our earlier publication [3] that applied EFPA to OBS networks with 1+X protection. In [3] we also have two types of users, premium and regular, but in [3] the premium users obtain 1+X protection while in the present paper, the premium users are protected by diversity coding. This requires a further EFPA adaption that differs from [3] in two ways: (1) the way we derive the offered load to each path for each SD pair (the offered load to each path for a SD pair from premium users under diversity coding is reduced to half of that under 1+X protection), and (2) the way we obtain the network BLR based on path BLRs. Notice that while in 1+X protection there are two options for every protected burst transmission: either the entire burst is successful, or the entire burst is lost. By contrast, under diversity coding, if several sub-bursts of a given protected burst are successfully received while others are lost, the data that is successfully received is useful and it is not counted as loss in the BLR calculation. Namely, this protected burst is only partially lost. Such partial loss is considered in the network BLR while the path BLR only accounts for the BLR of individual sub-bursts.

Dao [34] has proposed an Integer Linear Programming formulation for optical network design for OBS networks with static traffic demands under 1+1 protection with XOR network coding. Unlike [34], the present paper focuses on BLR approximation and efficiency studies of OBS/OPS networks with dynamic traffic demands.

III. THE MODEL

The bufferless OBS network that we consider is modeled by a graph \( G = (V, E) \), where \( E \) is a set of trunks and \( V \) is a set of nodes. In the present context, a node may represent an optical cross connect, optical switch, or edge router. Under the most general setting, trunk \( j \in E \) composes \( f_j \) fibers and each of which carries a set of wavelengths \( W_j \). The number of wavelengths in wavelength set \( W_j \) is \( W_j \). The number of channels carried by a wavelength on a given fiber in trunk \( j \) can further increase by considering multiple sub-wavelength channels (e.g.: TDM). However, for ease of exposition, we make the following simplifying assumptions: (1) there are no sub-wavelength channels and a wavelength channel in a given fiber on a given trunk can carry at most one burst at any given point in time, (2) all the fibers carry an equal set \( (W) \) of wavelengths and the number of wavelength channels in set \( (W) \) is equal to \( W \), and (3) all the trunks have the same number \( (f) \) of fibers. Then, under the scenario of full wavelength conversion, the total number of channels on each trunk is equal to \( C = fW \), and under the scenario of no wavelength conversion it is equal to \( C = f \) because only one wavelength per fiber is relevant in this scenario. Note that the analyses in this paper can be extended in the vein of [3] to cases where these simplifying assumptions are relaxed.

For a given network, let \( P \) be the set of all uni-directional SD pairs. Then, for each SD pair \( m \in P \), consider \( N \) working paths and one protection path. We choose the first working path denoted \( U_m^1 \) to be the route that has the least number of hops. If there are more such routes, we randomly choose one of them. Next, we consider a new topology in which all the trunks of the first working path are excluded. Then, we choose the second working path \( U_m^2 \) for this SD pair to be the least-hop route in the new topology. Repeating the above procedure until the working path \( U_m^N \) is found. Finally, we repeat the same procedure to find the protection path \( U_m^P \). All the working paths and the protection path are edge-disjoint [3] to protect the network from single-link failures. To protect the network from node-failure, the working paths and their corresponding protection path need to be both node-disjoint and edge-disjoint. In this paper, we only consider single-link failures.

As mentioned above, two service types are considered: premium and regular that generate premium and regular bursts, respectively. For each SD pair \( m \in P \), we assume that the arrival processes of the premium and the regular bursts follow Poisson processes with arrival rates \( \lambda^p_m \) and \( \lambda^r_m \), respectively. When a premium burst is generated, and diversity coding is used for its protection, the ingress node divides the burst into \( N \) equal-sized premium sub-bursts that are transmitted through the \( N \) working paths. In addition, a coded burst formed by XORing the \( N \) premium sub-bursts is transmitted through the protection path. All these \( N + 1 \) new bursts (the \( N \) sub-bursts and the coded burst) are generated at once and start the transmission process at the same time. This implies that the arrival processes of the sub-bursts in their individual working path and of the coded burst in the protection path also follow Poisson processes all with arrival rate \( \lambda^p_m / N \). If 1+1 protection is used for the premium burst, then two disjoint paths are used (working and protection paths) to transmit two identical copies of the burst. For a new regular burst, the ingress node will transmit the new regular burst through the shortest working path. For all bursts, we ignore the switching configuration time and then the service time of the bursts on a node equals to the transmission delay of the bursts on that node.

Let \( L_m^n \) be the length of the path \( U_m^n \), \( h_m^n \) be the total number of links in the path \( U_m^n \), \( S_{prop} \) be the propagation speed, and \( T_p \) be the processing time of the header in each node, then the end-to-end delay (the time from the source node sending out the header to the destination node receiving the burst) of a burst from SD pair \( m \) on the path \( U_m^n \) is : 

\[
D_m^n(e2e) = L_m^n/S_{prop} + h_m^n \times T_p + D_{trans},
\] 

where \( D_{trans} \) is the transmission delay.
delay. In the destination nodes on the OBS network, each SD pair has an individual buffer. Once a burst is received, it will be put into the corresponding buffer for a maximum duration $T_m(\text{buf})$ that

$$T_m(\text{buf}) = \max_{n=1,\ldots,N,\text{pro}} (D_m^a(e2e)) - \min_{n=1,\ldots,N,\text{pro}} (D_m^b(e2e)).$$

If a burst in the buffer at the destination node on the OBS network is timed out, then the node will reconstruct the original burst by gathering all the related sub-bursts. If one sub-burst is missing, then the decoder will reconstruct the missing sub-burst. Then, in the access network the burst is disaggregated, and the individual packets are sent to their final destinations.

We assume that the service times of all the regular and premium bursts are independent and exponentially distributed with mean 1/µ if they are served at the wavelength capacity. Accordingly, the mean of the premium sub-bursts and the protection burst is equal to 1/(Nµ). However, they are no longer independent and exponentially distributed. On the other hand, in our EFPA based BLR approximation we assume for simplicity that they are independent and exponentially distributed with mean 1/(Nµ), but in our simulations we do not make this simplifying assumption.

Note that the error introduced by the exponential assumption of their service times is not expected to be significant. This is due to the well known insensitivity property of the Erlang-B formula, that implies that for a single bufferless system of a finite number of servers with Poisson arrivals, the BLR, or the blocking probability, depending on the context, is insensitive to the shape of the service time distribution (namely, insensitive to moments of the distribution higher than the first). In [3], simulation results show that even for a network of Erlang-B subsystems, the network BLR is nearly insensitive to the shape of the service time distribution. Then since our network model considered in this paper is also a network of Erlang-B subsystems, we can expect a minimal error to be introduced by this simplifying assumption.

IV. BLR APPROXIMATION

A key assumption of EFPA is that all traffic streams on all the various paths follow independent Poisson processes. This EFPA assumption is adopted also in this paper for our BLR approximation. It is intuitively clear that the traffic streams of the premium sub-bursts on all the working paths and the protection sub-burst(s) on protection path(s) are in fact dependent. However, we still make the simplifying EFPA independence assumption for tractability, and then test by simulations the error in BLR approximation introduced by such assumption. Note that this assumption is applied only to the approximations, but not to the simulations.

We consider an OBS network with the RWS wavelength selection scheme under the scenarios of no wavelength conversion and of full wavelength conversion, and we first describe our BLR approximation method for the case where only diversity coding is used for path protection for premium bursts. Next, we will explain how the BLR approximation is extended to the case of hybrid path protection, namely, the case where some premium bursts are protected via diversity coding and some via 1+1 protection. We remind the reader that some SD pairs may not have three or more disjoint paths required for diversity coding, so if they have only two such paths they can still use 1+1 protection.

A. The case of diversity coding path protection

According to our model, for any given SD pair $m \in P$, the premium offered traffic to all the working paths and the protection path are the same and we denote their value by $ρ_m^\text{pre}$, which is given by $ρ_m^\text{pre} = λ_m^p/(Nµ)$. As for the regular traffic, because a regular burst uses the first working path for transmission, the regular offered traffic to the first working path of SD pair $m$ is given by $ρ_m^r = λ_m^p/µ$.

In the approximation, we assume that when a regular burst is generated and arrives at the first trunk of its path, it randomly and uniformly selects a wavelength in accordance with the RWS scheme, i.e., each wavelength has probability 1/W to be selected. If all the wavelength channels of the selected wavelength on the different fibers are busy, then the new regular burst chooses another wavelength from the remaining wavelengths and repeats the procedure until it finds a wavelength channel that is free. If such wavelength channel cannot be found, i.e., if all the wavelength channels for all the wavelengths on all the different fibers are busy, then the burst is blocked and lost.

For a premium burst that under diversity coding is divided into $N$ equal-sized sub-bursts, and different sub-bursts use different wavelengths. In this case, each premium sub-burst will choose the wavelength as the regular burst does by randomly selecting a wavelength channel with probability 1/W on the first trunk of its path.

Let $U_m^k(1)$ be the first trunk on the path $U_m^k$, $k = 1, 2, \ldots, N, \text{pro}$. The premium traffic of SD pair $m$ offered to the path $U_m^k$, $k = 1, 2, \ldots, N, \text{pro}$, for wavelength $w$ is obtained by

$$a_\text{Pre}(w) = ρ_m^\text{pre} × Q(U_m^k(1), w),$$

where the function $Q(j, w)$ is provided explicitly by (7.31) and it represents the ratio of the traffic offered to all the channels of wavelength $w$ on the various fibers in trunk $j$ to the total traffic offered to trunk $j$. In (7.31), $b_j(w)$ denotes the BLR of wavelength $w$ and $W$ denotes the set of wavelengths on trunk $j$.

The traffic generated by the regular users of SD pair $m$ and offered to the shortest path (the first working path) for wavelength $w$ is obtained by

$$a_\text{Reg}(w) = ρ_m^r × Q(U_m^1(1), w).$$

Let $\bar{a}_j(w)$ be the total traffic offered to wavelength $w$ on trunk $j$. It is given by (8), where $I(i, j, U)$ and $I'(j, U)$ are indicator functions that are defined as follows:

$$I(i, j, U) = \begin{cases} 1, & \text{if } i, j \in E \text{ and trunk } i \text{ strictly precedes trunk } j \\ 0, & \text{otherwise,} \end{cases}$$

and

$$I'(j, U) = \begin{cases} 1, & j \in U \\ 0, & \text{otherwise,} \end{cases}$$

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and $b_j(w)$ is the BLR of wavelength $w$ in link $j$, which is obtainable by the Erlang-B formula as follows

$$b_j(w) = \frac{\bar{a}_j(w)^j}{\sum_{m=0}^{\infty} \bar{a}_j(w)^m / m!}. \tag{3}$$

Note that in our analysis we consider every wavelength separately. In this way, we ensure that under the scenario of no wavelength conversion every regular burst or premium sub-burst uses only one wavelength for each SD path.

As mentioned in Section II, unlike the cases of $1+1$ or $1+X$ protection where a burst is either fully successfully received, or fully lost, under diversity coding a burst can be partially lost because if several of its sub-bursts are successfully received while others are lost, the data that is successfully received is not counted as loss in the BLR calculation.

To calculate the BLR of a premium burst, under diversity coding protection scheme, we consider the following three cases.

1) **The entire premium burst is lost** if either all the premium sub-bursts and the coded burst are lost, or if all the premium sub-bursts are lost and only the coded burst is received.

2) **The transmission of the entire premium burst is successful** if either only one of the premium sub-bursts is lost and the coded burst is successfully received, or if all the premium sub-bursts are successfully received.

3) **The premium burst is partially lost**. In this case, if the number of received premium sub-bursts $N_{receive}$ is smaller than $N-1$, then even if the coded burst is well received, the dropped sub-burst could not be reconstructed.

When one sub-burst is well received, the packets in this sub-burst are useful and, therefore, they are considered as successfully transmitted, then theses packets (or this sub-burst) are not counted as loss. Therefore, in this case, only $(N-N_{receive})/N$ of this premium burst is counted as loss.

Then, the BLR values $B^p_m$ and $B^s_m$ of the premium and regular traffic, respectively, for each SD pair $m \in P$ are obtained using (9) and

$$B^s_m = \sum_{k=1}^{N} B_m(U^s_m), \tag{4}$$

where $B_m(U^k_m)$ indicates the BLR of bursts from SD pair $m$ on path $U^k_m$ as

$$B_m(U^k_m) = \sum_{w=1}^{W} R_{U^k_m}^{m}(1) (1 - \prod_{j \in U^k_m} (1 - b_j(w))). \tag{5}$$

Then the BLR of the entire OBS network is given by

$$B_{network} = \sum_{m \in P} (\rho_m^{pri} \times B^p_m + \rho_m^{r} \times B^s_m). \tag{6}$$

For the case of full wavelength conversion, we can use Eqs. (1) - (9) by replacing $f$ with $fW$ and without separate solutions for different wavelengths. Then, for the full wavelength conversion case, equations (1) - (3) can be solved by the following iterative procedure. We begin by setting initial values of all $b_j(w)$ to be zero. Then, we iteratively update the $b_j(w)$ values using equations (1) - (3). In each iteration, we first use (1), (2) and (3) to obtain the traffic offered to each wavelength on each trunk. Next, we use (5) to obtain updated values for $b_j(w)$. These iterations are then repeated until the difference between the values $b_j(w)$ obtained in two consecutive iterations for all wavelengths and trunks are less than a predefined value. Finally, we use (4), (6) and (9) to obtain the overall BLR of the entire OBS network.

**B. The case of hybrid path protection by diversity coding and $1+1$ path protection**

If in the network, some of the SD pairs use diversity coding and all the others use $1+1$ for protection of premium bursts, then the SD pair set $P$ can be divided into two sets: $P_{1+1}$ that includes all the SD pairs using $1+1$ path protection, and $P_{DC}$ that includes all the SD pairs using diversity coding.

For a SD pair $m \in P_{DC}$, the traffic offered to each wavelength in each related trunk can be calculated by equations (1) and (2). Then equation (8) can be used to calculate $a_{DC}^j(w)$ which is the total offered load from all the SD pairs in $P_{DC}$ to wavelength $w$ in trunk $j$. For a SD pair $m \in P_{1+1}$, the traffic offered to each wavelength in each related trunk can be calculated by equations (1)-(3) in (3). Then equation (4) in (3) can be used to calculate $a_{1+1}^j(w)$ which is the total offered traffic load from all the SD pairs in $P_{1+1}$ to wavelength $w$ in trunk $j$. Therefore, the total traffic offered to each wavelength $w$ in trunk $j$ is

$$a_j(w) = a_{DC}^j(w) + a_{1+1}^j(w),$$

which is a combination of equation (8) and equation (4) in (3).

The BLR of link $w$ in trunk $j$ is calculated using (3). Then, the BLR values $B^p_m(DC)$ and $B^s_m(DC)$ of the premium and regular traffic, respectively, for each SD pair $m \in P_{DC}$ are obtained using equations (9) and (4), and the BLR values $B^p_m(1+1)$ and $B^s_m(1+1)$ of the premium and regular traffic, respectively, for each SD pair $m \in P_{1+1}$ are obtained using equations (9) and (6) in (3). Finally, the network BLR is obtained by equation (10), which is a modification of equation (6) used for the case where $1+1$ protection is excluded.

**V. Numerical Results and Observations**

We begin this section by comparing the BLR results for the three wavelength selection methods (RWS, RCS and LLWS) in OBS networks involving diversity coding. The comparison is made for a range of scenarios with different apportionments of premium versus regular traffic. We define the term $x\%$-premium scenario as the case where $x\%$ of the traffic is generated by premium users. For example, the 100%-premium scenario means that all the traffic in the network is generated by premium users. Then we observe and discuss the effect of the proportion of the traffic of the premium users on the BLR results. Next, a comparison between $1+1$ protection and a protection scheme based on diversity coding is presented. For this comparison, we consider both full and no wavelength conversion conditions, and use the 10-node circular lattice.


\[ Q(j,w) = R_j(w) + \sum_{w_1 \in W \backslash \{w\}} \frac{R_j(w_1)b_j(w_1)R_j(w)}{[1 - R_j(w_1)]} + \sum_{w_1 \in W \backslash \{w\}} \frac{R_j(w_1)b_j(w_1)}{[1 - R_j(w_1)]} \sum_{w_2 \in W \backslash \{w_1,w\}} \frac{R_j(w_2)b_j(w_2)R_j(w)}{[1 - R_j(w_1) - R_j(w_2)]} + \cdots \]

\[ + R_j(w) \sum_{w_1 \in W \backslash \{w\}} \frac{R_j(w_1)b_j(w_1)}{[1 - R_j(w_1)]} \sum_{w_{w-1} \in W \backslash \{w_1,w_{w-1}\}} R_j(w_{w-1})b_j(w_{w-1})R_j(w) \]

\[ \bar{a}_j(w) = \sum_{m \in P} \sum_{k=1}^N l'(j, U^k_m) a_{Pr^R_m}(w) \prod_{i \in E} (1 - I(i, j, U^k_m)b_i(w)) + \sum_{m \in P} l'(j, U^k_m) a_{Pr^R_m}(w) \prod_{i \in E} (1 - I(i, j, U^k_m)b_i(w)) \]

\[ B^m_p = \prod_{k=1}^N B_m(U^k_m) + \frac{N - 1}{N} \left( \sum_{s=1}^N \left( 1 - B_m(U_p^s) \right) \prod_{k \in N \setminus s} B_m(U^k_m) \right) \]

\[ + \frac{N - 2}{N} \sum_{s=1}^N \left( 1 - B_m(U_p^s) \right) \sum_{s_2 \in N \setminus s} \left( 1 - B_m(U_p^{s_2}) \right) \prod_{k \in N \setminus \{s, s_2\}} B_m(U^k_m) + \cdots \]

\[ + \frac{1}{N} \left( B_m(U_p^1) \right) \sum_{s=1}^N B_m(U_p^s) \prod_{k \in N \setminus s} \left( 1 - B_m(U_p^s) \right), \]

\[ (N = \{1, 2, \ldots, N\}), \]

\[ B_{network} = \frac{\sum_{m_1 \in P_{NC}} (p_{m_1}^p \times B^m_{m_1}(DC) + p_{m_1}^p \times B^m_{m_1}(DC)) + \sum_{m_2 \in P_{1:s}} (p_{m_2}^p \times B^m_{m_2}(1 + 1) + p_{m_2}^p \times B^m_{m_2}(1 + 1))}{\sum_{m \in P} (p_{m}^p + p_{m}^f)} \]

Fig. 2. Topology of the 10-node circular lattice network.

network depicted in Fig. 2 for the cases of no failure and single-trunk failure. Again, the effect of the apportionment of premium versus regular traffic on the comparison is studied and discussed. Finally, we validate our proposed approximation under full and no wavelength conversion conditions. The validation is done by comparison with results of discrete event simulations. In fact, all the simulation results presented in this section has been obtained using discrete event simulations with RWS wavelength selection method. We examine the effect of the parameters C and N on the accuracy of the proposed approximation. We also validate the hybrid approximation by comparing its BLR results against results based on discrete event simulation for both full wavelength conversion and no wavelength conversion conditions. We only use traffic loads that result in BLR higher than \(10^{-3}\) to avoid excessive simulation times. We also provide 95% confidence intervals based on the Student’s t-distribution for all the simulation results.

A. Comparison of wavelength selection schemes

We compare here the BLR for the three wavelength selection schemes (RWS, RCS and LLWS) for the first trunk on a path, discussed in Section II for OBS with diversity coding, using discrete event simulations. We first consider the 0%-premium scenario. Each trunk has 5 fibers and each fiber has 4 wavelengths, and there are no wavelength converters in the network. We note that for the case of full wavelength
Traffic offered to each SD pair from regular users

It is observed that LLWS achieves the best performance (lowest BLR) and RWS has performed the worst (highest BLR). This is consistent with the fact that LLWS aims at and achieves the best load balancing of the three methods and RWS the worst. Interestingly, aiming for load balancing on and achieves the best load balancing of the three methods and BLR). This is consistent with the fact that LLWS aims at

3. It is observed that LLWS achieves the best performance wavelength channel in a trunk. The results are shown in Fig. 4. The settings of trunks are the same as for the 0%-premium scenario where each trunk has 5 fibers and each fiber has 4 wavelengths.

Fig. 4. Comparison between wavelength assignment schemes for (a) the non-failure case and (b) single-failure case in the 50%-premium scenario where each trunk has 25 fibers and each fiber has 4 wavelengths.

The results for the 50%-premium scenario are shown in Fig. 4. The settings of trunks are the same as for the 0%-premium scenario (5 fibers, 4 wavelengths per fiber and no wavelength conversion). We set $N = 2$, namely, each SD pair has two working paths and a protection path. From Fig. 4 it is observed that for both premium and regular traffic, LLWS performs the best and RWS the worst in both non-failure case and single-failure case.

Similar conclusion can be observed in Fig. 5 where the number of fibers in each trunk increases to 25.

These comparisons have important implications not only for the choice on wavelength assignment, but also to the way network dimensioning is done using our approximations. Clearly, for realistic size networks, simulations cannot be used, so we must rely on approximations. Recall that our approximations, for tractability, must assume random wavelength selection – RWS. If the network design is based on, e.g. LLWS, then we understand that our network dimensioning is conservative (assuming RWS in the approximation). The results of this section that provide the difference in BLR between RWS and LLWS indicate how much capacity can be saved relative to capacity dimensioning based on the approximation. This enables designers to achieve efficient dimensioning. Consider for example the case of Fig. 4 if we aim for $10^{-3}$ BLR for the premium users, 20% more traffic can be served using LLWS than using RWS. As our approximation is based on RWS, and the network uses LLWS, certain adjustment in capacity dimensioning will be required. We note, however, that there are many other uncertain parameters such as traffic level predictions that are all taken into consideration when designers consider the tradeoff of network costs versus grade of service.

B. Comparison between diversity coding and 1+1 protection approaches

Using discrete event simulations and considering scenarios involving both full and no wavelength conversion, we compare the diversity coding and 1+1 protection approaches with LLWS wavelength selection method. We first consider full wavelength conversion condition, where each trunk has 20 wavelengths channels. To compare the performance of 1+1 protection and protection scheme based on diversity coding, we first consider the case where all the traffic in the network is premium traffic (100%-premium scenario). The results for the cases of non-failure and single-trunk-failure are shown in Fig. 6. We observe that when the traffic offered to each SD pair is low, the protection scheme based on diversity coding performs better than the 1+1 path protection. However, when the offered traffic between each SD pair increases, the performance of the protection scheme based on diversity coding becomes worse than that of the 1+1 path protection. To explain this effect, notice that when the offered traffic is low, the probability that there is more than one sub-burst from the same S-D pair being lost is very small. On the other hand, the protection scheme...
Based on diversity coding is more resource efficient than the 1+1 protection scheme. As a result, the protection scheme based on diversity coding performs better in this situation. The latter can be explained as follows. When the offered traffic is high, in 1+1 path protection, one copy of the two “protection” bursts reaching the destination node means that this burst is transmitted successfully. On the other hand, in diversity coding, even if the protection sub-burst successfully reaches the destination, but the two premium sub-bursts of the original burst are both lost during transmission, the entire burst is lost, which implies that the transmission of such protection burst wastes network resources. Under heavier traffic loads, the probability that the two premium sub-bursts are both lost while the protection sub-burst is successfully transmitted increases, the resources wasted become significant, so that diversity coding performs worse.

The cross-point in Fig. 6(a) is around 0.01 BLR. Since the target BLR range in real networks is usually below 0.01, we could conclude that in this situation, the diversity coding technique may reduce the BLR comparing to the 1+1 path protection technique. In Fig. 6(b), the cross-point is around 0.005 BLR, which is slightly lower than the cross-point in (a).

Next, we examine the 50%-premium scenario. We first consider the case with 5 fibers each trunk and 4 wavelengths on each fiber. The results of the full wavelength conversion case are presented in Fig. 7. In the non-failure case, for the premium users, the cross-point of BLR of the protection scheme based on diversity coding and 1+1 path protection techniques is around $10^{-4}$ (simulation) in the non-failure case, which is lower than the BLR of the cross-point in Fig. 6(a).

This is because in Fig. 6(a), though the BLR of the premium users is around $10^{-4}$, the average network BLR is much higher than that and the resources wasted become significate. This is similar to the situation presented in Fig. 6(a). For the regular users, the protection scheme based on diversity coding outperforms 1+1 path protection when the BLR is bellow 0.1. In the single-failure case, the cross-point is around 0.004 BLR, which is similar to the situation presented in Fig. 6(b).

The results without wavelength conversion are shown in Fig. 8. It is observed that for the premium users, 1+1 path protection performs better, for the cases considered, than the protection scheme based on diversity coding (the crosspoint of the two algorithms can be found using the proposed approximation, but the BLR value of the crosspoint is very low); however, for the regular users, the protection scheme based on diversity coding performs better than 1+1 path protection in both non-failure case and single-failure case. Comparing Figs. 6, 7 and 8, we observe that for the premium users, the diversity coding scheme performs better than 1+1 path protection under low traffic load, but when the offered traffic increases, 1+1 path protection starts to outperform diversity coding in terms of BLR. Furthermore, the cross-point of BLR of the two approaches moves to the left when the proportion of the premium traffic decreases. For the regular traffic, diversity coding outperforms 1+1 path protection when BLR is lower than 0.1 as shown in Figs. 7 and 8.

Then, we increase the number of fibers in each trunk to 25, and the results for the 50%-premium scenario with full wavelength conversion are shown in Fig. 9. For these cases, it is observed that the cross-points of the BLR of the protection
Traffic offered to each SD pair from regular users

BLR

Traffic offered to each SD pair from premium users

BLR

C. Accuracy of the approximation

Here we examine the accuracy of the proposed approximation for diversity coding networks. We also set $N = 2$, $f = 5$ and $W = 4$. The results for the 50%-premium scenario with full wavelength conversion are depicted in Fig. 10. We observe that the approximation underestimates the BLR of both the premium traffic and the regular traffic. This is because for the premium users, the assumption that the traffic offered to all the working paths and the protection path follows independent Poisson process (path independence assumption) implies consideration of less burstiness in the approximation. For sub-bursts, protection-bursts and regular bursts, they have the same priority inside the network. The dependence between sub-bursts and protection-bursts increases on average the burstiness of the traffic inside the network, which affects not only the premium traffic but also the regular bursts. Therefore, the link BLR is actually underestimated, which further causes the underestimation of the BLR for both the premium traffic and the regular traffic.

Fig. 8. Comparison of diversity coding and 1+1 protection approaches with LLWS wavelength selection method for (a) the non-failure case and (b) single-failure case in the 50%-premium scenario under no wavelength conversion condition where $F = 5$ and $W = 4$.

Fig. 9. BLR comparison for (a) the non-failure case and (b) single-trunk-failure case under the 50%-premium scenario with full wavelength conversion where $F = 25$ and $W = 4$.

schemes based on diversity coding and 1+1 path protection are around 0.002 and 0.005 BLR for the non-failure and single-failure cases, respectively. The cross-points move to right comparing with the results in Fig. 7 leading to the observation that increasing the number of wavelength-channels per trunk could improve the performance of the protection scheme based on diversity coding comparing to 1+1 path protection.

We further examine the accuracy of our proposed approximation for the case of no wavelength conversion and the results are shown in Fig. 11. In the simulations, RWS is used for wavelength selection. From the results shown in Fig. 11 we observe that we achieve quite high accuracy of the proposed approximation relative to the simulation results based on RWS. However, we remind the reader that the approximation is limited to RWS scheme for tractability, and although it is accurate with respect to simulation results based on RWS, as we demonstrate above in Section V-A RWS is inferior by comparison to LLWS.

1) The effect of $N$ on the accuracy: To test the effect of $N$ on the accuracy of the approximation, we also examine the
performance of the proposed approximation when \( N = 3 \). The results are shown in Fig. 12. The parameter settings are the same as those used to obtain the results shown in Fig. 10 except for the value of \( N \).

Comparing Figs. 12 and 10, we observe that the accuracy of the approximation slightly decreases when \( N \) increases from 2 to 3. This can be explained by noticing that when \( N \) increases, the underestimation effect of the path independence assumption increases. We also observe that the BLR values increase when we increase \( N \) from 2 to 3. This may be surprising because if all the paths have only one hop, the total offered traffic contribution to the network for the protection of a premium burst reduces to \( 4/3 \) from \( 3/2 \) when \( N \) increases from 2 to 3. However, notice that because in our network topology, in the case \( N = 3 \), the length (number of hops) of the fourth path for a given SD pair is much longer than the length of the first three paths. Then the actual resources used by this burst increase overall because of the length of the fourth path.

Fig. 11. BLR for the cases of (a) no failure and (b) single failure without wavelength conversion under the 50%-premium scenario where \( F = 5 \) and \( W = 4 \).

Fig. 12. BLR for (a) the non-failure case and (b) single-failure case for full wavelength conversion in the 50%-premium scenario where \( F = 5 \), \( W = 4 \) and \( N = 3 \).

Comparing Figs. 12 and 10 we observe that the accuracy of the approximation slightly decreases when \( N \) increases from 2 to 3. This can be explained by noticing that when \( N \) increases, the underestimation effect of the path independence assumption increases. We also observe that the BLR values increase when we increase \( N \) from 2 to 3. This may be surprising because if all the paths have only one hop, the total offered traffic contribution to the network for the protection of a premium burst reduces to \( 4/3 \) from \( 3/2 \) when \( N \) increases from 2 to 3. However, notice that because in our network topology, in the case \( N = 3 \), the length (number of hops) of the fourth path for a given SD pair is much longer than the length of the first three paths. Then the actual resources used by this burst increase overall because of the length of the fourth path.

Fig. 13. BLR for (a) the non-failure case and (b) single-trunk-failure case with full wavelength conversion for the 50%-premium scenario where \( F = 25 \) and \( W = 4 \).

2) The effect of number of channels: To test the effect of number of channels on the accuracy of the approximations, we set \( N = 2 \) and increase the number of channels from 20 to 100 and the results for the 50%-premium scenario with full wavelength conversion are shown in Fig. 13. Proceeding with \( N = 2 \), we also test the case without wavelength conversion and the results are shown in Fig. 14, where each fiber on a trunk has 4 wavelengths and there are 25 fibers per trunk. In this case, each trunk has 25 wavelength channels of the same wavelength in each trunk.

Comparing Fig. 13 with Fig. 10 and Fig. 14 with Fig. 11 we observe that increasing the number of channels has not significantly affected the accuracy of the proposed approximation in the cases considered.

3) Hybrid path protection by diversity coding and 1+1: To test the accuracy of the BLR approximation in the case of the hybrid path protection, we consider a modified topology of Fig. 8 so that some of the nodes only have 2 output trunks and 2 input trunks. The new network topology is shown in Fig. 15. In the new topology, Nodes 1 and 6 have 2 output/input trunks, then any SD pairs that include one of these two nodes
as either source node or destination node, or both the source and destination, can only find 2 trunk-disjoint path. Therefore, these SD pairs can only use 1+1 path protection method. We let the SD pairs that include one of Nodes 1 and 6 as either source node or destination node (or both nodes being the source and destination nodes) use 1+1 and the other SD pairs use diversity coding, then the results for full wavelength conversion and no wavelength conversion case are shown in Figs. 16 and 17, respectively. There are 5 fibers in each trunk and 4 wavelengths in each fiber. We observed from Figs. 16 and 17 that the proposed hybrid approximation algorithm is quite accurate in the cases considered.

We also test the accuracy of the BLR approximation for the case of the hybrid path protection for the 14-node NSFNet shown in Fig. 18. The results of the cases with full wavelength conversion and no wavelength conversion are shown in Figs. 19 and 20, respectively. The results presented in Figs. 19 and 20 again demonstrate the accuracy of our the proposed hybrid approximation algorithm.

VI. Conclusion

We have provided a scalable and accurate BLR approximation for OBS/OPS networks that employ protection based on
Traffic offered to each SD pair from regular users

Fig. 18. Topology of the 14-node NSFNet.

Fig. 19. BLR for the hybrid case with full wavelength conversion in the 50%-premium scenario where \( F = 5 \) and \( W = 4 \) in NSFNet network.

Fig. 20. BLR for the hybrid case without wavelength conversion in the 50%-premium scenario where \( F = 5 \) and \( W = 4 \) in NSFNet network.

1+1 path protection is lower than that of diversity coding when the network traffic is high for premium users. In addition, we have observed that increasing the number of channels per trunk could improve the performance of the protection scheme based on diversity coding as compared with 1+1 path protection. For the regular users in all the test cases, the protection scheme based on diversity coding outperforms 1+1 path protection in all test cases we studied when the BLR is under 0.1.

We have also compared the performance of the three wavelength selection schemes: RWS, RCS and LLWS. In all the test cases, LLWS has the best performance (lowest BLR) since it achieves the best load balancing. By contrast, RWS has the worst performance (highest BLR) in all the test cases. As our approximations are all based on RWS, they provide conservative assessment of the BLR, so designers that use our approximations should consider this fact in resource dimensioning. As simulations are not scalable, approximations are required for realistic size networks. The combination of approximations and thorough assessment of their accuracy, as provided in this paper, provides guidance to network designers in their efforts to develop efficient and cost effective networks that meet required QoS. However, as it is often the case with experimental and simulation based validations, the conclusions are limited to the scenarios studied, and there is always a scope for new insights and conclusions when new scenarios are considered.

REFERENCES


